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Abstract

Sales in European retail environments typically follow a weekly pattern having high sales at the end of the week and low sales at the start of the week. In this paper we compare two different ways of setting safety stock norms in a retail environment with weekly sales patterns. The first option is to set a single safety stock norm, which is constant throughout the week. The second option is a safety stock norm which is dynamic since it depends on the weekday. The inventory is controlled periodically and a lost sales environment is assumed. We study the impact of the dynamic safety stock on the inventory holding and shortage costs as well as on drivers of handling costs like the number of orderlines and the workload balance, since earlier research has shown that handling costs are relatively large for retailers. We use a full factorial experiment and simulation to evaluate both inventory replenishment strategies.

Keywords: Retail, Inventory, Replenishment, Dynamic, Week pattern, Safety stock

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1. Introduction

In this paper we analyse two different ways of setting safety stock norms in a retail environment with weekly sales patterns. Basically we compare static and dynamic safety stock norms in inventory models with periodic data.

In this section we will first describe the research environment, the research questions and a review of the literature. In section 2 we will describe the two safety stock policies in more detail. The full factorial design and the simulation, which are used to determine the performance of the static and dynamic safety stock policies are described in section 3. In section 4 the results are presented and finally conclusions are given in section 5.

The research environment

Sales at European retailers typically follow a weekly pattern having high sales in the second half of the week and low sales in the first half of the week (see Broekmeulen (2007), Raman and Zotteri (2000) and East et al. (1994)). According to Kahn and Schmittlein (1989) and Raman and Zotteri (2000), sales in the United States hardly depend on the weekday. Therefore in this paper we focus on European retailers. More specifically, we focus on inventory control at European retailers who are dealing with a weekly sales pattern, multiple delivery moments per week (based on a delivery schedule per store), fixed case pack sizes and limited shelf space per SKU. We assume that if an item is out-of-stock at a retailer, this demand is lost for this SKU (e.g. because the customer bought another SKU instead).

For many European retailers capacity costs are at least as important as inventory holding costs. For example, Van Zelst et al. (2007) report transportation costs plus handling costs in the retailers' DC and the stores which are seven times as high as the inventory holding costs. As a result a substantial part of this paper is devoted to analysing the impact of dynamic safety stocks on capacity requirements. Capacity requirements for handling in retail stores typically consists of two parts: labor needed for the first replenishment (i.e. the time needed to stack the items which fit on the shelf on the first attempt after the goods arrived at the store) plus for the second replenishment (i.e. the time needed to stack the items which did not fit on the shelves at the first attempt and which were brought to the backroom). As shown in Van Zelst et al. (2007), capacity requirements for stacking items on the shelves in the first replenishment depend on the number of consumer units to be handled, the number of orderlines, the casepacksize, the employee and the way the consumer units are stacked on the shelves. The second replenishment process has not been modelled yet in the literature. The capacity requirements for handling in DC's depend on the number of orderlines, the number of units to be handled, the picking strategy and the worker productivty (see Tompkins (2003)). The capacity costs for handling in the stores, for handling in the DC and for transportation do not only depend on the capacity requirements (i.e. total time needed), but also on the workload pattern during the week. E.g. Gaur and Fisher (2004) showed that due to workload differences, consolidation should be applied for the transportation from DC to stores in non-busy periods, leading to higher tansportation costs. Van Donselaar et al (2007) showed that store managers take into account the workload pattern when they make their inventory replenishment decisions: they systematicaly changed advices from the Automated Store Ordering system to smoothen the workload, even though this resulted in higher inventory holding costs.

Due to the fact that not all capacity requirements in the retail supply chain have been modelled to the same level of detail and due to the fact that the capacity costs are a function of the workload pattern during the week while the European labor market does not have a simple uniform cost structure, total capacity requirements cannot be translated easily in a cost function. For readers, who are less familiar with the European labor market, we give a brief description below.

The European labor market has a limited flexibility (see Wallace (2003)). This makes it difficult to fully match the hired number of employees and their hours worked with the dynamic workload resulting from the weekly sales pattern and the replenishment strategy used by the retailer. In general, contingent work is not readily available to European retailers. Hiring and firing and the type of labor contracts that can be offered to employees are limited by the Employment Protection Legislation. Temporal flexibility, such as overtime and setting of work schedules is in most European countries limited by collective agreements between employers and trade unions. Finally, the collective agreements also limit the wage flexibility, such as adjusting the hourly wage to the workload and/or the productivity of the worker.

The research questions

Since sales and demand are dynamic during the week, we will consider two ways of setting safety stocks. The first option is to set a safety stock norm, which is constant throughout the week. The second option is a safety stock norm which is dynamic since it depends on the weekday. Although at first sight it seems natural to use a dynamic safety stock if demand is dynamic, the retail environment has some important characteristics which may limit the potential benefits. For example, the periodic cycle is short and case pack sizes are relatively large (Van Donselaar et al. (2007) report a median value of more than one week for case pack size dived by average demand). As a result it is not clear beforehand how large the potential savings in inventory holding and shortage costs can be. Furthermore, the dynamic safety stock policy may also affect the number of orderlines and the workload pattern and we have argued that these consequences cannot be translated easily into a cost function. Therefore in this paper we will address the following two research questions:

1. How much can inventory holding and shortage costs be reduced in a retail environment by using a dynamic safety stock and how does this reduction depend on the system parameters?
2. How does the dynamic safety stock policy affect other critical performance indicators, like the weekly workload pattern and the number of order lines.

We use a full factorial experiment and simulation to evaluate both policies for setting safety stocks.

Review of the literature

The literature on inventory control in environments with periodic data is extremely sparse. Karlin (1960a, b) and Zipkin (1989) show that a periodic critical-number policy is optimal for inventory models with periodic data, no fixed ordering costs (i.e. no lot-sizing) and backordering. For inventory models with non-stationary but independent demand and fixed ordering costs, a state-dependent (s,S)-policy is optimal, see eg. Zipkin (2000). Given this optimal policy, in the last decade papers on inventory models with non-stationary demand aim to find good heuristics and approximations for setting the parameters. see eg Bollapragada and Morton (1999) and Levi et al. (2007). These papers in general do not assume fixed casepack sizes nor lost sales and they do not address periodic demand patterns specifically (including the impact of replenishment strategies on capacities). If the case pack size is fixed, excess demand is backordered, and there are no fixed costs, it is optimal to order the minimal number of casepacks which is needed to raise the inventory position at or above a reorder level. This result holds under fairly general assumptions on demand (Veinott (1965)). Under lost sales, the optimal policy with fixed case pack sizes is not known, although according to Hill and Johansen (2006), the type of policy which is optimal for the model with backordering also performs very well in a lost sales environment, provided there is not more than one replenishment order outstanding at any time.

2. The two safety stock policies

Before describing the two safety stock policies in more detail, we define the variables used in this paper in the following table:

SKU	Stock Keeping Unit	*IP*	Inventory Position [CU]
V	Shelf capacity [CU]	s	Reorder level [CU]
Q	Case pack size [CU]	ss	safety stock [CU]
q	Order quantity [CU]	P_2	Fill rate
h	Inventory holding cost [€/CU/yr]	σ	Std. deviation of demand per week [CU/week]
p	Penalty cost per unit lost sales [€/CU]	R	Review period [days]
		L	Leadtime [days]

Table 1: Definitions of the variables used

We consider a periodic review inventory system with stochastic demand, which is stationary at the week level. Within the week demand follows a pattern with high sales on Friday and Saturday and low sales on Monday and Tuesday. Demand which cannot be satisfied from the shelf is lost. We will evaluate the performance of the safety stock policies for three different replenishment strategies. Each replenishment strategy is based on a reorder level s_t, which is equal to the average demand during the next $L+R$ periods plus ss_d, the safety stock at day t. If a static safety stock is used, then the safety stock at day t is simply equal to a constant:

$$ss_t = c \qquad (1)$$

If a dynamic safety stock is used, the safety stock at day t is equal to**:**

$$ss_t = k * \sigma_{t,L+R} \qquad (2)$$

with k a constant safety factor and $\sigma_{t,L+R}$ the standard deviation of the demand during periods (t+1,t+L+R).

The three different replenishment strategies for which we compare the static and dynamic safety stock policies are the (R,s,nQ)-strategy, the Full Service strategy and the Efficient Full Service strategy. In all strategies we assume that the ordering decision is made at the end of the day. Below we explain each of these three strategies.

The $\underline{(R,s,nQ)}$ -replenishment strategy works as follows: if at a review moment on day t the inventory position drops below the reorder level s_t, then an integer multiple (n) of the fixed case pack size Q is ordered, such that the inventory position after ordering is again at or above the reorder level. So with a (R,s,nQ)-strategy and an inventory position equal to IP_t, the order quantity q_t is equal to:

$$q_t = \max\left\{\left\lceil \frac{(s_t - IP_t)}{Q} \right\rceil \cdot Q, 0\right\} \qquad (3)$$

Next to the (R,s,nQ)-strategy, we consider two alternative replenishment strategies: the Full Service (FS) and the Efficient Full Service (EFS) strategy, as defined in Van Donselaar and Broekmeulen (2008). Roughly speaking, the Full Service strategy keeps the shelves as full as possible, so it orders as soon as one or more case packs fit on the shelf or even earlier if the inventory position drops below the reorder level. The Efficient Full Service strategy on the other hand waits till the last possible moment to replenish the shelves in order to get relatively large order quantities and thereby reducing the handling time in the store. The latter is based on the result derived in Van Zelst et al. (2007) that the handling time needed per year decreases if the number of order lines decreases, while the total amount of case packs remain the same.

In the Full Service (FS) strategy, the order quantity at a review period is equal to:

$$q_t = \max\left\{\left\lfloor \frac{(V - IP_t)}{Q} \right\rfloor \cdot Q, \left\lceil \frac{(s_t - IP_t)}{Q} \right\rceil \cdot Q, 0\right\} \qquad (4)$$

with $\lfloor x \rfloor$ resp. $\lceil x \rceil$ representing the nearest integer less or equal to x resp. greater or equal to x. The first term on the right-hand side reflects the basic idea behind the FS strategy: at a review period we order as many case packs that fit on the shelf, given the current inventory position IP_t. The second term on the right-hand side is needed in order to satisfy the requirement that the fill rate P_2 is at least equal to the target fill rate P_2^* for a *SKU*. So if the inventory position IP_t is less than the reorder level s_t, which is determined as described above for the (R,s,nQ)–strategy, we need to order the minimum number of case packs which is needed to raise IP_t back to or above the reorder level. Finally the third term reflects the notion that the order quantity should always be non-negative (due to the dynamic s_t the IP_t may sometimes be larger than $s_t + Q$).

The replenishment strategy described above is a generalization of the Full Service strategy as described by Cachon (2001) by incorporating the possibilities that the reorder level may be larger than the shelf space minus one case pack size, s_t may be dynamic and/or case pack sizes may be larger than one consumer unit.

The Efficient Full Service strategy (EFS) is similar to the FS strategy, but aims to minimize the number of order lines per year, while still guaranteeing the target service level. If at a review period the inventory position is strictly below the reorder level s_t, we order the maximum number of case packs such that the inventory position (IP_t) after ordering is less than or equal to the shelf capacity V. Unless this IP_t is still below s_t, i.e., the shelf is not large enough to accommodate all units, then we order as many case packs as needed to bring the inventory position after reordering to (or just above) s_t. In summary: if at a review period IP_t is strictly less than s_t, the order quantity becomes:

$$q_t = \max\left\{\left\lfloor \frac{(V - IP_t)}{Q} \right\rfloor \cdot Q, \left\lceil \frac{(s_t - IP_t)}{Q} \right\rceil \cdot Q, 0\right\} \quad \text{if} \quad IP_t < s_t \qquad (5)$$

Note that this is the same formula as in Full Service, but now the order is only triggered when $IP_t < s_t$. This EFS strategy extends and generalizes the (s, S, Q)-strategy proposed by Hill (2006) by including situations in which the shelf space is smaller than the reorder level, demand is not stationary or in which the shelf space and reorder level are not a strict multiple of Q.

3. Datasets and Simulation

In order to compare the performance of the replenishment policies, we measured the long-term average inventory holding and shortage costs C. The costs incurred during year T are given by:

$$C_T = h \cdot I_T + p \cdot B_T \qquad (6)$$

with I_T the average inventory and B_T the lost sales in units in year T. In the simulation experiment we assumed a year consists of 50 weeks.

We did a factorial experiment in which we tested several levels for each of the nine input parameters. The experimental setup is given in Table 1. The parameters for the average demand, variance to mean ratio, case pack size and shelf capacity are based on parameters reported in Van Donselaar et al. (2007), Van Donselaar and Broekmeulen (2008) and Ehrhardt (1979). Demand is probabilistic with a time-varying demand pattern during the week. The weekly demand has mean μ and variance σ^2, with f_d the fraction of the weekly demand for weekday d. We modeled the demand for each weekday d with a Gamma distribution (cf. Burgin (1975)), with mean $\mu_d = f_d \cdot \mu$ and variance $\sigma_d^2 = f_d \cdot \sigma^2$. The week length is equal to six days with $\{f_d\} = \{0.08, 0.08, 0.11, 0.19, 0.30, 0.24\}$ the week pattern taken from an European grocery retailer, described by Broekmeulen et al. (2007). To test the sensitivity of the results for the week pattern we also simulated each scenario with demand having a smoothed week pattern, i.e. $\tilde{f}_d = (f_d + 1/6)/2$. For the delivery schedule to the store we considered daily delivery and every other day delivery. This last delivery schedule with three deliveries per week was tested with an early pattern, i.e., Monday-Wednesday-Friday, and a late pattern, i.e., Tuesday-Thursday-Saturday. The lead-time we encountered at grocery retailers is often equal to one day. To test the sensitivity of the results we also consider a lead-time equal to two days.

Table 1: Input parameters for the simulation experiment.

Input parameter	Levels
Mean week demand μ	$\{1, 8, 64\}$
Variance-to-mean ratio σ^2/μ	$\{1, 2, 4\}$
Case pack sizes Q	$\{1, 6, 12, 24\}$
Shelf capacity V	$\{9, 18, 36\}$
Delivery schedule	$\{Daily, Mo-We-Fr, Tu-Th-Sa\}$
Lead-time L	$\{1,2\}$
Week pattern	$\{European, Smoothed\}$
Holding cost h	$\{0.1, 0.2\}$
Lost sales cost p	$\{0.01, 0.05, 0.25\}$

All cost parameters in the model are normalized on the purchasing costs, which is set equal to one cost unit. For example, the inventory holding costs mainly consist of interest costs and therefore the annual holding costs are varied between 10% and 20% of the purchasing costs. The penalty for lost sales was varied between 1% and 25% of the purchasing costs. In general, a service level between 85% and 99% is reasonable for many regular grocery products. High service levels are typically reserved for new items with a high profit margin to stimulate sales.
The timing of events in the simulation is: during opening hours inventory decreases due to customers' demand and after closing the store the service level is calculated, inventory is counted, goods arrive in the backroom and are stacked on the shelves and finally orders are placed before opening the store. SKU's for which the daily demand is larger than the available shelf space are also replenished from the backroom during the day to prevent out-of-stocks.

Following Law and Kelton (2000), the reported values for the simulation are the averages from at least 10 replications. In each replication, the first 50 weeks were the warming-up periods and statistics are recorded for the last 1000 weeks. We replicated until we reached an absolute precision for the customer service level $P_2 \pm 0.002$ with 95% confidence. P_2 is the fraction of demand delivered from stock, also known as the fill rate.
We ran each of the 7776 simulation experiments for the following six scenarios:

I. Base (R,s,nQ) policy with static safety stocks;
II. Base (R,s,nQ) policy with dynamic safety stocks;
III. Full Service policy with static safety stocks;
IV. Full Service policy with dynamic safety stocks;
V. Efficient Full Service policy with static safety stocks;
VI. Efficient Full Service policy with dynamic safety stocks

In all six scenario's we determined for each parameter setting the optimal safety stock levels ss_t, which minimized the average simulated costs. In the case of static safety stocks, we used (1) and for dynamic safety stocks we used (2).

4. Results

In this section, we show the results obtained in the analysis of the static and the dynamic safety stock policies. First we compare the cost difference between the safety stock policies for each of the three replenishment strategies. When comparing the inventory holding plus shortage costs of the two safety stock policies, we will report the percentual difference in costs $\Delta(C_{stat}, C_{dyn})$, with

$$\Delta(C_{stat}, C_{dyn}) = \frac{(C_{stat} - C_{dyn})}{C_{stat}} \cdot 100 \tag{6}$$

Table 2: Cost difference between the two safety stock policies for three inventory replenishment strategies.

Percentile	(R,s,nQ)	FS	EFS
0.00	-0.95	-0.95	-1.53
0.05	0	0	0
0.10	0	0	0
0.15	0	0	0
0,20	0	0	0
0,25	0	0	0
0,30	0	0	0
0,35	0,09	0	0,04
0,40	0,18	0	0,14
0,45	0,30	0	0,23
0,50	0,40	0	0,33
0,55	0,52	0	0,43
0,60	0.64	0.16	0.56
0.65	0.77	0.34	0.69
0.70	0.94	0.52	0.85
0.75	1.14	0.72	1.05
0.80	1.42	0.99	1.29
0.85	1.75	1.36	1.60
0.90	2.23	1.82	2.09
0.95	3.15	2.79	3.05
1.00	8.06	8.06	8.06
Average	0.80	0.55	0.74

Our first conclusion is that the (R, s, nQ) and EFS strategies have almost the same average cost difference: 0.80% resp. 0.74%. For these strategies the dynamic safety stock policy has a small advantage over the static safety stock policy. For the majority of experiments with the FS strategy, there is no cost difference at all. This is due to the fact that in many of these experiments the inventory level is determined by the shelf capacity V instead of s_t, as shown in (4) and (5), and therefore the safety stock plays no role in these cases. Due to the fact that the cost performance of the (R, s, nQ) and EFS strategy are similar, while the FS strategy shows very limited cost differences, we restrict our more detailed analysis of the cost differences (see Table 3) to the (R, s, nQ) strategy. In Table 3, we show how the cost difference between static and dynamic safety stocks depends on the different input parameters.

Table 3: The average cost reduction of the dynamic safety stock policy compared to the static policy for the (R, s, nQ) strategy.

Parameter	Level	Average	Median	Maximum
Weekly demand	1	0.23	0.00	3.93
	8	0.77	0.56	4.77
	64	1.40	0.94	8.06
Variance-to-mean ratio	1	1.03	0.54	8.06
	2	0.84	0.46	7.26
	4	0.54	0.32	3.62
Case pack size	1	1.05	0.64	8.06
	6	0.92	0.55	7.71
	12	0.72	0.32	7.59
	24	0.52	0.04	4.95
Shelf space	9	0.80	0.40	8.06
	18	0.80	0.40	8.06
	36	0.80	0.40	8.06
Delivery schedule	Daily	1.06	0.59	8.06
	Mo-We-Fr	0.63	0.30	4.58
	Tu-Th-Sa	0.71	0.36	6.18
Lead-time	1	0.98	0.55	8.06
	2	0.63	0.29	5.83
Week pattern	European	1.17	0.77	8.06
	Smoothed	0.43	0.17	4.56
Holding cost	0.1	0.91	0.51	8.06
	0.2	0.70	0.30	7.82
Penalty cost	0.01	0.38	0.00	4.77
	0.05	0.80	0.48	7.36
	0.25	1.22	0.76	8.06

As mentioned, the average cost difference for the (R,s,nQ) strategy is very low: 0.80%. For 29% (2166/7776) of the cases there is no cost difference at all. This is mainly due to the fact that the safety stock norms for all weekdays are equal to zero for both policies in 72% (1556/2166) of these cases. From Table 3, we see that the median is (almost) zero for low weekly demand, a large case pack size, and low penalty costs. When the case pack size is large compared to the average demand, the cycle stock (partly) functions as a safety stock, thereby reducing the relevance of the safety stock, especially with low penalty costs.

On the other hand, if demand is relatively high, larger cost differences may occur (all experiments with cost difference>5% have the highest average weekly demand), but these are still below 10%. The reason for this is the fact that if average demand is high, the coefficient of variation for demand is also low. This can be seen in the table below, which shows the coefficient of variation for weekly demand as a fucntion of the average demand (μ) and the variance-to-mean-ratio (σ^2/μ). Due to the low coefficient of variation for fastmovers, relatively few safety stock is needed to prevent lost sales due to out-of-stocks. Therefore even for fastmovers the gains from using dynamic safety stocks rather than static safety stocks are limited.

Table 4: The coefficient of variation of weekly demand as a function of the average demand and the variance-to-mean ratio.

$\mu\ \rightarrow$		1	8	64
$\sigma^2/\mu\ \downarrow$	1	1	0,35	0,13
	2	1,41	0,5	0,18
	4	2	0,7	0,25

Next, we address the second research question. It turns out that the dynamic safety stock policy also has an impact on three other performance indicators: 1. the average number of orderlines, 2. the range of the handling workload during the week and 3. the range in the inventory during the week. We will use the number of consumer units (CU's) ordered as the primary indicator for the amount of handling workload on a particular day and calculate the range of the workload during the week based on these numbers. First we will explain how we calculated the above three performance indicators using an example. Then we discuss the results for each of the above 3 performance indicators in detail for the (R, s, nQ) strategy. Finally, at the end of this section we will discuss the results for the other two strategies: the EFS and FS strategy.

To report on the above 3 performance indicators we first selected the experiments with a positive cost difference. Then we calculated per experiment per weekday the average number of orderlines, the average order size (i.e. the average number of CU's ordered) and the average inventory (in CU's).
Since we report results for subset of experiments (this is necesary to make fair comparisons as we will see later on), we take the average performance of all experiments in the subset. Table 5 shows an example of the results from these calculations.

The first 8 rows in Table 5 show the results for the subset which consists of all experiments with positive cost difference and with (R, s, nQ)-strategy, European weekpattern, daily delivery schedule and Leadtime=1. Rows 3 upto 8 show the results for weekday 1 to 6. In row 9 we determine the week-average of the inventory (INV), ordersize (OS) and number of orderlines (OL) and in row 10 we determine the range for the inventory and the ordersize (with range= (maximum value-minimum value)/average value). To show the impact of the delivery schedule, we also give the results for a similar subset in which we replaced the daily schedule by a bi-daily delivery schedule. It clearly shows that a reduction in delivery frequency has a very strong impact on the range in inventory, the range in order size as well as the average number of orderlines, irrespective whether a static or dynamic safety stock policy is applied. This underscores the need to report results per delivery schedule.

Table 5: Effect on inventory, order sizes and order lines during the week for the dynamic safety stock policy and the static policy for the (R, s, nQ) strategy for experiments with cost difference > 0%.

Strategy	Week Pattern	Delivery Schedule	Lead Time	Week day	Inv Stat	Inv Dyn	OS Stat	OS Dyn	OL Stat	OL Dyn
RsnQ	European	Daily	1	1	14,39	14,37	2,96	2,51	0,32	0,26
RsnQ	European	Daily	1	2	14,57	13,17	5,58	6,39	0,48	0,47
RsnQ	European	Daily	1	3	14,23	12,39	8,74	10,73	0,55	0,62
RsnQ	European	Daily	1	4	14,11	13,08	7,35	7,65	0,56	0,56
RsnQ	European	Daily	1	5	13,85	14,79	2,79	1,54	0,29	0,17
RsnQ	European	Daily	1	6	14,01	15,23	2,59	1,20	0,29	0,13
				Average	14,20	13,84	5,00	5,00	**0,42**	**0,37**
				Range [%]	**5,09**	**20,50**	**122,96**	**190,43**		

Strategy	Week Pattern	Delivery Schedule	Lead Time	Week day	Inv Stat	Inv Dyn	OS Stat	OS Dyn	OL Stat	OL Dyn
RsnQ	European	Mo-We-Fr	1	1	14,03	14,65	0,00	0,00	0,00	0,00
RsnQ	European	Mo-We-Fr	1	2	17,82	16,00	13,98	16,38	0,61	0,67
RsnQ	European	Mo-We-Fr	1	3	14,59	12,80	0,00	0,00	0,00	0,00
RsnQ	European	Mo-We-Fr	1	4	22,92	23,53	9,31	9,37	0,56	0,57
RsnQ	European	Mo-We-Fr	1	5	14,14	14,72	0,00	0,00	0,00	0,00
RsnQ	European	Mo-We-Fr	1	6	16,34	16,98	6,17	3,72	0,49	0,32
				Average	16,64	16,45	4,91	4,91	**0,28**	**0,26**
				Range [%]	**53,45**	**65,25**	**284,78**	**333,50**		

Using the calculation method as explained just now, we get the following results for the (R, s, nQ)-strategy for our three performance indicators:

Number of orderlines

Table 6 reports the effects of the dynamic and static safety stock policy on the number of order lines for the (R, s, nQ) strategy for experiments with cost difference > 0%. The results are reported for subsets which differ in the weekpattern, the delivery schedule and the leadtime.

Columns five and six in Table 6 report the number of orderlines in an environment with a static resp. a dynamic safety stock, while column 6 reports the ratio between them. The overall average reduction in number of orderlines is 5.7% if we apply a dynamic instead of a static safety stock policy for those experiments which show a positive cost difference.

This reduction in numer of order lines strongly depends on the weekpattern: for the European weekpattern the reduction is 8.4% versus 2.9% for the smoothed weekpattern. Also the delivery frquency has a large impact. For daily delivery and European weekpattern, the reduction is even as large as 11.2% or 12.1%. Daily delivery typically occurs at large European supermarkets which are being delivered from a central DC. Finally we note that a reduction in orderlines not only leads to a reduction in handling time in the stores, but also in the DC.

Table 6: Effect of the dynamic and static safety stock policy on the number of order lines for the (R, s, nQ) strategy for experiments with cost difference > 0%.

Strategy	Week Pattern	Delivery Schedule	Lead Time	StatOL	DynOL	DynOL/Stat
RsnQ	European	Daily	1	2,49	2,22	1,121
RsnQ	European	Daily	2	2,54	2,29	1,112
RsnQ	European	Mo-We-Fr	1	1,66	1,55	1,073
RsnQ	European	Mo-We-Fr	2	1,69	1,61	1,048
RsnQ	European	Tu-Th-Sa	1	1,63	1,50	1,092
RsnQ	European	Tu-Th-Sa	2	1,68	1,59	1,060
						1,084
RsnQ	Smoothed	Daily	1	2,82	2,71	1,040
RsnQ	Smoothed	Daily	2	2,78	2,68	1,037
RsnQ	Smoothed	Mo-We-Fr	1	1,86	1,81	1,029
RsnQ	Smoothed	Mo-We-Fr	2	1,68	1,65	1,014
RsnQ	Smoothed	Tu-Th-Sa	1	1,87	1,80	1,039
RsnQ	Smoothed	Tu-Th-Sa	2	1,95	1,91	1,017
						1,029
				Overall average		**1,057**

So far, all results point in the direction of using a dynamic safety sock policy.

Range in handling workload

In Table 7, the range in number of consumer units ordered is reported in the columns starting with RangeOS and is used as an indictor for the range in handling workload.. The last column shows the change in this range if we move from a static to a dynamic safety stock policy. The table clearly shows that a dynamic safety stock policy leads to an increase in the range in handling workload and therefore a static policy is preferred over a dynamic policy if the workforce is highly inflexible.

Table 7: Effect of the dynamic and static safety stock policy on the range in Inventory and Workload for the (R, s, nQ) strategy for experiments with cost difference > 0%.

Strategy	Week Pattern	Delivery Schedule	Lead Time	RangeInv Stat	RangeInv Dyn	RangeInv Dyn/Stat	RangeOS Stat	RangeOS Dyn	RangeOS Dyn/Stat
RsnQ	European	Daily	1	5,09	20,50	4,03	122,96	190,43	1,55
RsnQ	European	Daily	2	4,44	16,69	3,76	121,70	181,10	1,49
RsnQ	Smoothed	Daily	1	2,15	14,05	6,52	67,56	108,29	1,60
RsnQ	Smoothed	Daily	2	1,71	12,33	7,23	66,28	100,61	1,52
				3,35		**5,38**			**1,54**
RsnQ	European	Mo-We-Fr	1	53,45	65,25	1,22	284,78	333,50	1,17
RsnQ	European	Mo-We-Fr	2	50,64	54,64	1,08	289,51	310,16	1,07
RsnQ	Smoothed	Mo-We-Fr	1	44,07	52,16	1,18	247,77	278,26	1,12
RsnQ	Smoothed	Mo-We-Fr	2	37,09	40,41	1,09	253,79	266,47	1,05
				46,31		**1,14**			**1,10**
RsnQ	European	Tu-Th-Sa	1	41,65	56,10	1,35	315,00	370,14	1,18
RsnQ	European	Tu-Th-Sa	2	41,13	55,39	1,35	319,99	371,47	1,16
RsnQ	Smoothed	Tu-Th-Sa	1	36,50	45,46	1,25	257,53	291,58	1,13
RsnQ	Smoothed	Tu-Th-Sa	2	36,90	44,18	1,20	262,27	291,35	1,11
				39,04		**1,28**			**1,14**

The range in handling workload can increase substantially if we change from a static to a dynamic safety stock policy. Particularly in case of daily delivery, the range increases by 54%. With a lower delivery frequency, this effect decreases rapidly: in case of bi-daily delivery the range is only 10% or 14%.

Range in inventory
The range in inventory, also reported in Table 7, is clearly driven by the delivery frequency: if a static policy is used, the range in inventory is 3.35% on average for daily delivery and 46.31% or 39.04% for bi-daily delivery. Due to the fact that the range in inventory is already very high for bi-daily delivery, the relative increase in the range in inventory if we change from a static to a dynamic policy is much higher for daily delivery (+438%) compared to bi-daily delivery (only 14% or 28%). An increase in the range in inventory implies that more items do not fit on the shelves during the first replenishment and have to be stored in the backroom before being stacked on the shelves in the second replenishment.

Results for alternatve replenishment strategies
Appendix 1 shows how the results derived above for the (R, s, nQ)-strategy, change if the EFS or the FS replenishment strategy is being applied. It turns out that the results hardly change. The main difference is that with the FS strategy there are fewer cases with a positive cost difference, as we have seen before in Table 2. If there is a positive cost difference however, the change in number of orderlines, range in handling workload and range in inventory is similar to the change reported for the (R, s, nQ)-strategy.

6. Conclusions and future research

Using dynamic safety stocks rather than static safety stocks in a European retail environment has both advantages and disadvantages. The advantages are small reductions in the inventory holding plus shortage costs (less than 1% on average) and a substantial reduction in the number of orderlines which reduce the handling capacity requirements in the retailers' stores and DC. The negative effects from dynamic safety stocks are an increase in the range in the handling workload and the range in the inventory during the week. The latter effects are strongest for retailers with a daily delivery frequency. Given the complex interaction between capacity requirements and capacity costs and given the fact that labor market conditions differ per country in Europe, each retailer should make its own trade-off between the advantages and disadvantages reported in this paper. Future research in this area is needed to model the second replenishment process in the stores and to study the complex interaction between capacity requirements and capacity costs.

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Appendix 1

Table for OL

Strategy	Week Pattern	Delivery Schedule	Lead Time	#Cases	StatOL	DynOL	DynOL/StatOL
RsnQ	European	Daily	1	507	2,49	2,22	1,121
RsnQ	European	Daily	2	499	2,54	2,29	1,112
RsnQ	European	Mo-We-Fr	1	504	1,66	1,55	1,073
RsnQ	European	Mo-We-Fr	2	487	1,69	1,61	1,048
RsnQ	European	Tu-Th-Sa	1	497	1,63	1,50	1,092
RsnQ	European	Tu-Th-Sa	2	462	1,68	1,59	1,060
FS	European	Daily	1	302	3,25	2,92	1,111
FS	European	Daily	2	322	3,20	2,89	1,108
FS	European	Mo-We-Fr	1	289	2,03	1,88	1,077
FS	European	Mo-We-Fr	2	310	2,10	2,01	1,048
FS	European	Tu-Th-Sa	1	310	2,03	1,86	1,089
FS	European	Tu-Th-Sa	2	310	2,08	1,97	1,057
EFS	European	Daily	1	492	2,17	1,93	1,121
EFS	European	Daily	2	493	2,23	2,01	1,111
EFS	European	Mo-We-Fr	1	486	1,47	1,37	1,070
EFS	European	Mo-We-Fr	2	463	1,53	1,46	1,045
EFS	European	Tu-Th-Sa	1	486	1,43	1,32	1,085
EFS	European	Tu-Th-Sa	2	463	1,50	1,42	1,057
							1,082
RsnQ	Smoothed	Daily	1	452	2,82	2,71	1,040
RsnQ	Smoothed	Daily	2	462	2,78	2,68	1,037
RsnQ	Smoothed	Mo-We-Fr	1	432	1,86	1,81	1,029
RsnQ	Smoothed	Mo-We-Fr	2	314	1,68	1,65	1,014
RsnQ	Smoothed	Tu-Th-Sa	1	416	1,87	1,80	1,039
RsnQ	Smoothed	Tu-Th-Sa	2	334	1,95	1,91	1,017
FS	Smoothed	Daily	1	299	3,55	3,41	1,040
FS	Smoothed	Daily	2	303	3,47	3,35	1,037
FS	Smoothed	Mo-We-Fr	1	275	2,24	2,18	1,024
FS	Smoothed	Mo-We-Fr	2	192	2,15	2,12	1,015
FS	Smoothed	Tu-Th-Sa	1	274	2,23	2,16	1,031
FS	Smoothed	Tu-Th-Sa	2	251	2,32	2,28	1,016
EFS	Smoothed	Daily	1	459	2,44	2,34	1,044
EFS	Smoothed	Daily	2	441	2,52	2,43	1,036
EFS	Smoothed	Mo-We-Fr	1	393	1,72	1,68	1,023
EFS	Smoothed	Mo-We-Fr	2	296	1,52	1,50	1,013
EFS	Smoothed	Tu-Th-Sa	1	402	1,65	1,60	1,031
EFS	Smoothed	Tu-Th-Sa	2	356	1,74	1,71	1,017
							1,028
					Overall average		**1,055**

Table for Inv and OS

Strategy	Week Pattern	Delivery Schedule	Lead Time	RangeInv Stat	RangeInv Dyn	RangeInv Dyn/Stat	RangeOS Stat	RangeOS Dyn	RangeOS Dyn/Stat
RsnQ	European	Daily	1	5,09	20,50	4,03	122,96	190,43	1,55
RsnQ	European	Daily	2	4,44	16,69	3,76	121,70	181,10	1,49
RsnQ	Smoothed	Daily	1	2,15	14,05	6,52	67,56	108,29	1,60
RsnQ	Smoothed	Daily	2	1,71	12,33	7,23	66,28	100,61	1,52
FS	European	Daily	1	6,92	18,66	2,70	121,47	182,09	1,50
FS	European	Daily	2	4,37	18,26	4,18	125,48	181,34	1,45
FS	Smoothed	Daily	1	2,45	12,79	5,22	63,16	102,57	1,62
FS	Smoothed	Daily	2	1,95	14,25	7,30	67,90	99,39	1,46
EFS	European	Daily	1	6,36	14,50	2,28	115,91	181,62	1,57
EFS	European	Daily	2	4,76	13,70	2,88	121,13	178,09	1,47
EFS	Smoothed	Daily	1	2,38	10,48	4,40	63,44	103,76	1,64
EFS	Smoothed	Daily	2	1,74	10,91	6,28	66,12	99,51	1,50
				3,69		**4,73**			**1,53**
RsnQ	European	Mo-We-Fr	1	53,45	65,25	1,22	284,78	333,50	1,17
RsnQ	European	Mo-We-Fr	2	50,64	54,64	1,08	289,51	310,16	1,07
RsnQ	Smoothed	Mo-We-Fr	1	44,07	52,16	1,18	247,77	278,26	1,12
RsnQ	Smoothed	Mo-We-Fr	2	37,09	40,41	1,09	253,79	266,47	1,05
FS	European	Mo-We-Fr	1	58,55	71,68	1,22	288,18	334,04	1,16
FS	European	Mo-We-Fr	2	57,26	62,04	1,08	291,96	310,63	1,06
FS	Smoothed	Mo-We-Fr	1	49,07	57,94	1,18	249,67	277,95	1,11
FS	Smoothed	Mo-We-Fr	2	42,49	46,09	1,08	253,31	265,27	1,05
EFS	European	Mo-We-Fr	1	48,47	56,99	1,18	280,70	326,18	1,16
EFS	European	Mo-We-Fr	2	47,39	50,59	1,07	289,74	308,98	1,07
EFS	Smoothed	Mo-We-Fr	1	41,58	48,79	1,17	247,39	277,26	1,12
EFS	Smoothed	Mo-We-Fr	2	34,27	37,13	1,08	253,28	265,66	1,05
				47,03		**1,14**			**1,10**
RsnQ	European	Tu-Th-Sa	1	41,65	56,10	1,35	315,00	370,14	1,18
RsnQ	European	Tu-Th-Sa	2	41,13	55,39	1,35	319,99	371,47	1,16
RsnQ	Smoothed	Tu-Th-Sa	1	36,50	45,46	1,25	257,53	291,58	1,13
RsnQ	Smoothed	Tu-Th-Sa	2	36,90	44,18	1,20	262,27	291,35	1,11
FS	European	Tu-Th-Sa	1	47,32	63,34	1,34	318,65	369,08	1,16
FS	European	Tu-Th-Sa	2	45,61	60,74	1,33	321,04	369,30	1,15
FS	Smoothed	Tu-Th-Sa	1	41,16	50,03	1,22	259,17	290,04	1,12
FS	Smoothed	Tu-Th-Sa	2	40,22	47,68	1,19	262,07	289,63	1,11
EFS	European	Tu-Th-Sa	1	37,61	49,64	1,32	313,49	366,86	1,17
EFS	European	Tu-Th-Sa	2	37,19	49,49	1,33	318,49	369,44	1,16
EFS	Smoothed	Tu-Th-Sa	1	33,29	41,20	1,24	257,04	290,50	1,13
EFS	Smoothed	Tu-Th-Sa	2	34,00	40,64	1,20	261,85	291,16	1,11
				39,38		**1,27**			**1,14**