

A priori error estimate and control in the eigencurrent expansion method applied to Linear Embedding via Green's Operators (LEGO)

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**A priori error estimate and control
in the eigencurrent expansion method applied to
linear embedding via Green's operators (LEGO)**

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Introduction

Linear embedding via Green's operators (LEGO) [1,2] is a domain decomposition method in which the electromagnetic scattering by an aggregate of N_D bodies (immersed in a homogeneous background medium) is tackled by enclosing each object within an arbitrarily-shaped bounded domain \mathcal{D}_k (brick), $k = 1, \dots, N_D$ (e.g., see Fig. 1). The bricks are characterized electromagnetically by means of scattering operators \mathbf{S}_{kk} , which are subsequently combined to form the total inverse scattering operator \mathbf{S}^{-1} of the structure [1]. Finally, we use the eigencurrent expansion method (EEM) [1,3] to solve the relevant equation involving \mathbf{S}^{-1} , viz.,

$$\mathbf{S}^{-1}q^s = q^i, \quad (q^{s,i})_k = q_k^{s,i}, \quad q_k^{s,i} = \begin{bmatrix} \sqrt{\eta} \mathbf{J}_k^{s,i} \\ -\mathbf{M}_k^{s,i} / \sqrt{\eta} \end{bmatrix}, \quad (1)$$

where $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the background medium. In the EEM we rely on the Method of Moments (MoM) to obtain a set of basis and test functions which are approximations to the eigencurrents (i.e., eigenfunctions) of \mathbf{S}^{-1} . The eigencurrents serve as basis functions to expand $q^{s,i}$ [1].

In [1, 2, 4] we numerically demonstrated that the eigencurrents can be separated into two sub-sets, namely, coupled and uncoupled eigencurrents. In practice, only the coupled eigencurrents contribute to the multiple scattering occurring among the bricks which model the structure. By contrast, the uncoupled eigencurrents do not interact with one another and give rise to a diagonal system of equations [1] — which renders the LEGO/EEM approach computationally efficient. In fact, ordinarily, the number of coupled eigencurrents, $N_C N_D$, required to attain convergence of the solution (q^s) is far smaller than the number of Rao-Wilton-Glisson (RWG) [5] functions, $2N_F N_D$, used for the underlying MoM. As a result, we are able to solve relatively large 3-D scattering problems.

The accuracy of the computed currents q^s is expected to improve when an increasingly larger number of coupled eigencurrents $N_C N_D$ is employed in the numerical solution of (1). Hence, it is important to have a criterion for selecting N_C *a priori*. To this purpose, we devised several case studies, which we solved applying LEGO/EEM. Two meaningful examples are reported in [4], whereas in this communication we discuss a case study which involves penetrable bodies enclosed in bricks with varying sizes. By gathering results from all of the scattering problems we analyzed, we have found out that a simple relationship exists between the 2-norm error on q^s and the eigenvalue of $[S_{kk}]$ (i.e., the algebraic counterpart of \mathbf{S}_{kk}) associated

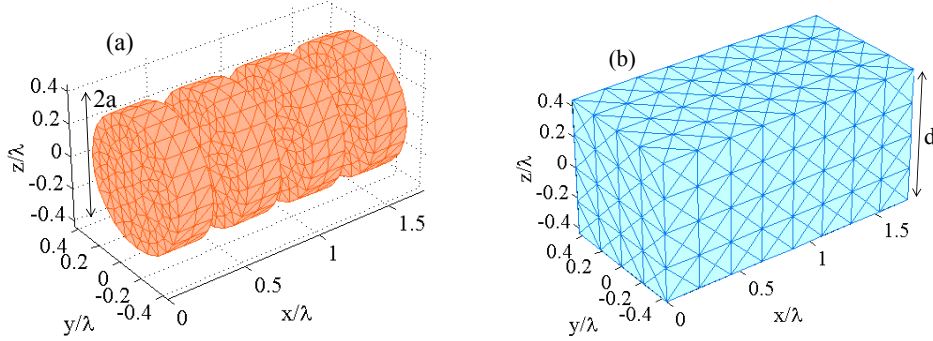


Figure 1: Case study: (a) four dielectric cylinders and (b) LEGO model comprised of as many bricks with increasing edge length (the instance $d/a = 2.22$ is shown).

with the last coupled eigencurrent retained in the calculations. In what follows we derive such a relationship and we briefly elaborate on its usage for estimating and controlling the error.

Description and discussion of a case study

We consider an aggregate [Fig. 1(a)] of $N_D = 4$ x -aligned penetrable cylinders ($\varepsilon = 4.8\varepsilon_0$, radius a , height h , $a/h = 1.8$) illuminated by the plane wave $\mathbf{E}^i = 1\hat{\mathbf{z}} \exp(-j2\pi x/\lambda)$ [V/m], with λ the wavelength in the medium comprising the cylinders ($a/\lambda = 0.394$). In accordance with LEGO [1], we begin solving this scattering problem upon embedding each cylinder within a rectangularly-shaped brick (edge d , height $H = 2h$) [Fig. 1(b)]. We allow for four different lengths of d , namely, $d/a \in \{2.22, 2.77, 3.33, 3.88\}$: This will enable us to assess the effect of d/a on the spectrum of a brick's scattering operator $[S_{kk}]$ [1, Eq. (21)].

To compute $[S_{kk}]$, we model the surface of a cylinder and its enclosing brick with a 3-D triangular-facet mesh [1] on which we define N_O and $2N_F$ RWG functions [5], respectively, to expand both electric and magnetic surface current densities. In particular, $N_O = 2 \times 573 = 1146$, whereas $2N_F \in \{768, 1080, 1440, 1848\}$, so as to ensure a constant mesh density over a brick's surface as d is increased. Notice that Fig. 1(b) shows the bricks along with their meshes in the instance $d/a = 2.22$ and $2N_F = 768$. The next step consists of applying the EEM to the equation [1, Eqs. (23)-(25)]

$$[S]^{-1} [q^s] = [q^i], \quad (2)$$

which constitutes the weak form of (1). The EEM entails, among others, computing the eigenvalues λ_p , $p = 1, \dots, 2N_F$, of $[S_{kk}]$.

Now, to investigate the convergence of $[q^s]$, we first obtain a reference solution by solving (2) with as many coupled eigencurrents as possible [4], i.e., $N_{C,\max}N_D = \min\{N_ON_D, 2N_FN_D\}$. Secondly, we repeatedly invert (2) employing an increasing number of coupled eigencurrents N_CN_D : We choose ten values of $N_C < N_{C,\max}$ by retaining all the eigencurrents whose corresponding eigenvalues satisfy $|\lambda_p| \geq |\lambda_{N_C}| = 10^{-t}|\lambda_1|$, $t = 1, \dots, 10$. Lastly, we compute the relative error on the calcu-

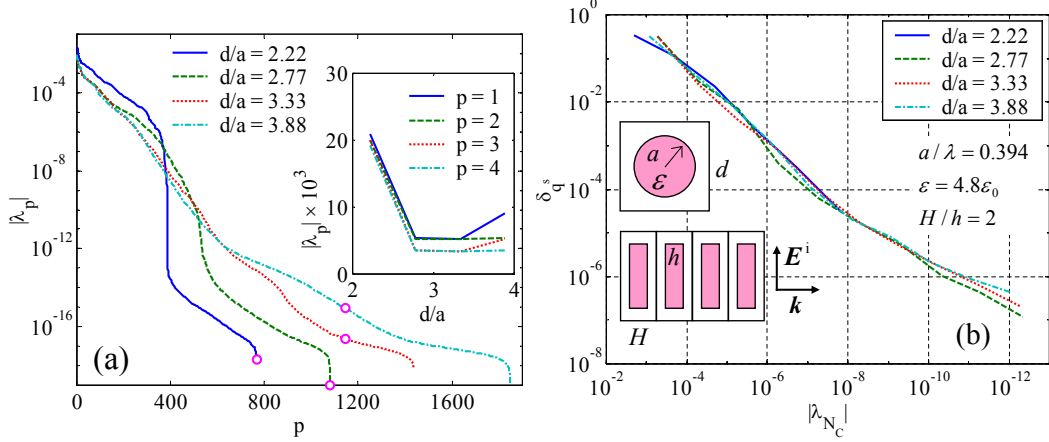


Figure 2: LEGO/EEM convergence: (a) Eigenvalues of $[S_{kk}]$ vs their indices for different values of d/a . Inset: first four eigenvalues vs d/a . (b) 2-norm error δ_{q^s} vs $|\lambda_{N_C}|$ for different values of d/a . Inset: cartoon of the cylinders and their bricks and incident plane wave. (See text for discussion.)

lated $[q^s]$ as [4]

$$\delta_{q^s} = (\| [q^s] - [q_{\text{MoM}}^s] \|_2) / \| [q_{\text{MoM}}^s] \|_2, \quad (3)$$

where $[q_{\text{MoM}}^s]$ is the reference solution mentioned above and $\| \cdot \|_2$ denotes the vector 2-norm in the space spanned by the rows of $[S]^{-1}$.

In Fig. 2(a) we have plotted the eigenvalues ($|\lambda_p|$) of $[S_{kk}]$ versus their index p ; the parameter of the lines is the ratio d/a . The round mark (\circ) on each line denotes the index $p = N_{C,\text{max}} = \min\{N_O, 2N_F\}$. Besides, the inset of Fig. 2(a) shows the first four eigenvalues versus d/a . Fig. 2(b) displays the error δ_{q^s} as a function of $|\lambda_{N_C}|$. The latter is the magnitude of the eigenvalue associated with the last coupled eigencurrent (contributed by a brick) retained in the calculation of $[q^s]$.

From Figs. 2(a), 2(b) we now observe:

1. Unlike the numerical experiments discussed in [4] in the present case the spectrum of $[S_{kk}]$ depends strongly on the relative shape and dimensions of the bricks and the objects inside, and especially so when $\partial\mathcal{D}_k$ “closely” wraps the object (here, for $d/a = 2.22$).
2. Remarkably enough, a simple (linear) relationship still exists between δ_{q^s} and $|\lambda_{N_C}|$, despite the conspicuous variations of the spectrum of $[S_{kk}]$ with d/a . Moreover, since the error curves plotted in Fig. 2(b) are not different from those drawn in [4, Figs. 5, 10] — which we obtained for *other* case studies — once again we are led to conclude that the linear behavior of δ_{q^s} must be an inherent property of the EEM applied to LEGO.

Criterion for estimating and controlling the error a priori

As argued in [4], we can exploit the error diagram of Fig. 2(b) in two manners:

Error estimate: Regarding the mapping with $|\lambda_{N_C}|$ fixed, we can assess the accuracy of $[q^s]$ computed through the EEM applied with $N_C N_D$ coupled eigen-currents.

Error control: Regarding the mapping with the desired value of δ_{q^s} fixed, we can read off $|\lambda_{N_C}|$, whence we determine N_C , i.e., the number of coupled eigen-currents (for each brick) necessary to attain a given level of accuracy.

Finally, Fig. 2(b) points to a mathematical dependence of δ_{q^s} on $|\lambda_{N_C}|$ in the form

$$\delta_{q^s} \approx |\lambda_{N_C}| \times 10^\alpha, \quad (4)$$

where α (in light of the previous discussion and the results in [4]) is a parameter weakly dependent on geometrical and physical quantities. We have numerically found that we can conservatively set α to ~ 3 , when the relevant parameters take on values in the range of interest and applicability of LEGO/EEM. Thus, (4) represents a criterion for choosing N_C (i.e., for truncating the sub-set of coupled eigen-currents) *a priori*. For the sake of argument, if one wants to obtain $[q^s]$ with an accuracy of about 10^{-3} , from (4) one derives $|\lambda_{N_C}| \approx 10^{-6}$. This means that all of the eigen-currents whose eigenvalues are in magnitude larger than 10^{-6} are to be employed in the EEM.

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