ClassONN

ClassONN: Classification with Oscillatory Neural Networks using the Kuramoto Model

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***Abstract*—Over the recent years, networks of coupled oscillators or oscillatory neural networks (ONNs) emerged as an alternative computing paradigm with information encoded in phase. Such networks are intrinsically attractive for associative memory applications such as pattern retrieval. Thus far, there are few works focusing on image classification using ONNs, as there is no straightforward way to do it. This paper investigates the performance of a neuromorphic phase-based classification model using a fully connected single layer ONNs. For benchmarking, we deploy the ONN on the full set of 28x28 binary MNIST handwritten digits and achieve around 70% accuracy on both training and test set. To the best of our knowledge, this is the first effort classifying such large images utilizing ONNs.**

***Index Terms*—Image classification, oscillatory Hopfield neural networks, Kuramoto model**

I. Introduction

Image classification is considered as one of the most classical edge artificial intelligence problems. Typically, one turns to convolutional neural networks (CNNs) to solve such tasks [1]. This paper investigates a new model for classification via coupled oscillators called oscillatory neural network (ONN).

This single layered network of fully connected oscillators is based on Hopfield neural networks [2, 3]. Despite particularly exceling at auto-associative memory tasks, further research needs to be done on these energy-based models regarding classification. Authors in [4, 5] were the first to investigate how to use HNNs and ONNs for classification.

Belyaev et al. [4] performed pattern classification with HNNs on binary pre-processed 14x14 MNIST digits achieving an accuracy of 61%. Similarly, Abernot et al. [5] used HNNs and ONNs to classify binary pre-processed 10x10 MNIST digits to reach precisions of 67% and 62.6%, respectively.

This paper expands on the two aforementioned papers by introducing ClassONN, a classification model with ONNs utilizing the Kuramoto neuron model to classify the full 28x28 MNIST binary data set. There are countless attempts at benchmarking on this dataset with CNNs or other various models [6, 7, 8], though to the best of our knowledge, this is the first attempt at classifying the whole MNIST dataset with ONNs.

II. Background

A. Oscillatory neural networks

Oscillatory neural networks comprise fully coupled neurons in a single layer. Being based on the Ising model [9, 10], the aim lies in minimizing the following underlying Hamiltonian:

The authors acknowledge the support by the European Union's Horizon EU research and innovation programme Project No. 101092096 PHASTRAC.

$$H = -\sum_{(i,j)} J_{ij}\sigma_i\sigma_j - \mu\sum_i h_i\sigma_i \tag{1}$$

Given some couplings J_{ij} and a magnetic field $\vec{h}$, the aim is to find an arrangement of binary spins σ_i which minimizes H. This corresponds to finding the state the system naturally strives to evolve into. The evolution of oscillators can be expressed by neuron models, Kuramoto being one of them [11, 12].

B. Kuramoto model

The dynamics of a phase-locked loop neural networks in the phase model can be described by the following ODE [13]:

$$\dot{\theta}_i = \omega + V(\theta_i)\sum_{j=1}^{n} W_{ij}V(\theta_j - \frac{\pi}{2}) \tag{2}$$

The phases θ_i of oscillators are coupled through W with ω standing for the natural frequency of the system. Being derived based on the phase response curves (PRC) of the system, $V(\theta)$ is an odd 2π-periodic waveform function.

In the Kuramoto model, $V(\theta)$ is a sinusoidal function. By using phase deviations ϕ_i (derived with $\theta_i(t) = \omega t + \phi_i$) and introducing the subharmonic injection locking (SHIL) with K_S being its scaling factor, we rewrite Eq. 2 in the following way:

$$\dot{\phi}_i = \frac{1}{2}\sum_{j=1}^{n} W_{ij}\sin(\phi_j - \phi_i) - K_S\sin(2\phi_i) \tag{3}$$

III. Image classification with Kuramoto

Generally, machines work with binary values $[+1, -1]$. Regarding oscillators, the phases $[0°, 180°]$ of the waves encode these binary values. Figure 1 illustrates the idea behind assigning binary values to oscillators based on their phases.

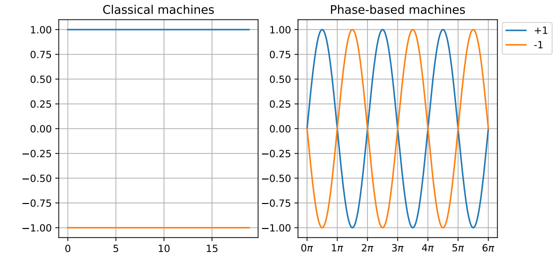

Fig. 1. Encoding of information, classically and with oscillators[5]

Since ONNs are intrinsically not meant for pattern classification, we need to rethink the structure of our model. Belyaev

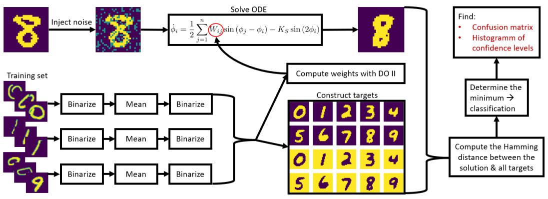

Fig. 2. Classification model of this paper

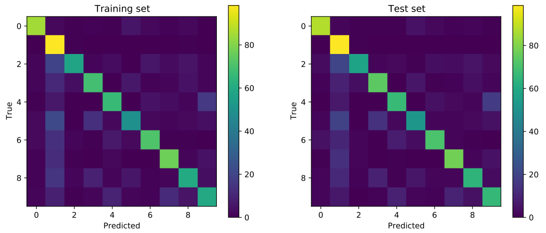

Fig. 3. Confusion matrices of the training and test set

et al. [4] proposed a method how to classify samples using oscillatory neural networks. This approach boils down to each class having its own pattern ("target"), instead of the model outputting a label. Given some query, the model tries to retrieve one of the class targets instead of the corresponding pattern.

Our algorithm goes as follows: first, we determine the average of all the binarized MNIST training images based on their given class. These averages are subsequently binarized yielding the targets. Given these targets, we compute the weight matrix $W_{i,j}$ with the Diederich-Opper II learning rule [14]. This learning rule is superior to Hebbian [15] and Storkey [16] due to its larger capacity and ability to deal with correlations.

Next, we solve Eq. 3 with a corrupted binary MNIST image. Due to the nature of the model, it may well happen that the solution of the ODE has a few spurious pixels and/or converges to an inverse of one of the targets. That is the reason why we include the inverse of the targets in the classification process.

Hence, a pattern is classified in the following manner: first, we compute the Hamming distance between the solution of Eq. 3 and all the targets. Given the Hamming distances, we determine the minimum, which corresponds to the predicted class. While classifying the samples, we keep track of the confidence level of each classification and construct a confusion matrix. Fig. 2 summarizes the classification algorithm.

IV. Results

Fig. 4 presents the final relative confusion matrices for the classification of the MNIST data sets. The values along the rows sum up to 100%. Generally, one could say that the model yields identical confusion matrices. The accuracies are the following:

$$\text{Training set: } 69.81\%$$
$$\text{Test set: } 71.88\%$$

Now we know how successful the model is in the classification process. However, let us investigate the confidence of each classification, i.e. to which extent (in $\%$) does the solution agree with one of the targets, to verify the model indeed converges to the set out targets. These can be seen in Fig. 4 for both sets and separately for correct and false predictions therein. Generally, it can be stated that only a few pixels are spurious at the end of the computation. However, the plot also proves that the model tends to converge for all pixels to an incorrect target, as the highest bar for false predictions appears at 100%.

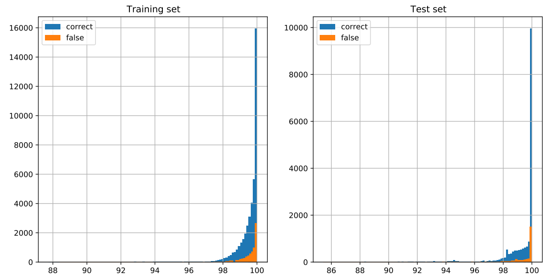

Fig. 4. Histogram of the confidence levels for the training and test set

V. Conclusions

For the first time, we have presented ClassONN in this paper - a single layer model of fully connected coupled Kuramoto oscillators to perform image classification on the full 28x28 MNIST dataset. Using the Diederich-Opper II learning rule and subharmonic injection locking, we have achieved an accuracy of around 70% on the training and 72% on the test set.

There are various neuron models such as Kuramoto, Izhikevich [17] or van der Pol [18]. Thanks to their rich dynamics, these various models have the potential to make the phase-based paradigms more expressive. Despite the simplicity of Kuramoto's model, this paradigm shows promising results. Many questions remain unanswered ranging from an effective learning algorithm, multi-layered structure and neuron model.

Despite not having achieved the high accuracies typical for CNNs, this is the first attempt at classifying images with such a large resolution utilizing ONNs. Furthermore, it motivates to further investigate this topic, as the phase-based paradigm provides an attractive alternative to classical computing.

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