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Experimental and Numerical Investigation of the Acoustic Response of Multi-Slit Bunsen Burners

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Abstract

Experimental and numerical techniques to characterize the response of premixed methane-air flames to acoustic waves are discussed and applied to a multi-slit Bunsen burner. The steady flame shape, flame front kinematics and flow field of acoustically exited flames, as well as the flame transfer function and matrix are computed. The numerical results are compared with experiments. The influence of changes in the mean flow velocity, mixture equivalence ratio, slit width and distance between the slits on the transfer function is studied, both numerically and experimentally. Good agreement is found which indicates the suitability of both the experimental and numerical approach and shows the importance of predicting the influence of the flow on the flame and vice versa. On the basis of the results obtained, the role and physical nature of convective flow structures, heat transfer between the flame and burner plate and interaction between adjacent flames are discussed. Suggestions for analytical models of premixed flame-acoustics interaction are formulated.

Key words: premixed Bunsen flames, thermo-acoustic transfer function, experimental and numerical research.

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1. Introduction

Acoustic instability of combustion processes is a phenomenon which can occur in practically any type of combustion device where the flame (or, more generally, a source or sink of energy) is enclosed in a vessel. In some applications the pulsating combustion is specially arranged to achieve high performance of the combustion and heat-transfer processes and to minimize harmful emissions. However, in most practical combustors, thermo-acoustic instability is an undesirable effect, which is often difficult to predict and eliminate.

A general explanation of thermo-acoustic instability was first given by Rayleigh at the end of 19th century. The physical nature of the problem is related to the positive closed loop of interactions between acoustic waves in the complete system (vessel) and the flame. Acoustic waves can lead to an increasing heat release rate of the flame, which can amplify the acoustic wave, leading to an increase of the acoustic energy in the system, etc., provided the well-known Rayleigh criterion is satisfied.

In spite of significant progress in the theory of thermo-acoustic instability, the problem of combustion-generated noise is still not well understood, both on a fundamental level and in practical applications. A fundamental description of flame-acoustic wave interaction is still lacking, and consequently also proper designing guidelines for combustion devices, because there are many possible causes for the feedback mechanism between flame and acoustic waves. There exist many ways to arrange this flame-acoustic wave coupling, depending on the specific flame and/or burner. Therefore, each type of flame requires a separate investigation.

In the present paper we will consider the response of fully premixed laminar Bunsen-type flames on a multi-slit burner to a fluctuating velocity field, resembling an acoustic wave. For these particular flames two different phenomena of flame-acoustics interaction can be expected a priori. First, small flames on the multi-slit burner can display an oscillating heat release rate due to flame surface undulations, as is
typical for Bunsen flames. Second, the heat loss rate to the burner deck near the flame foot can oscillate, resembling the case of a flat flame stabilized on a surface burner. These two phenomena are expected to be responsible for a fluctuating heat release.

The choice of flames on a multi-slit burner as an object of investigation is motivated by the following factors. First, from a practical point of view, similar flames are widely used in the burners of small and moderate scale combustors, like domestic and district-heating boilers, dryers, etc. Second, with regard to fundamental research purposes, the slit configuration provides a close to two-dimensional geometry which is convenient for numerical simulation in Cartesian coordinates. Furthermore, the multitude of flames allows applying symmetry boundary conditions, thus facilitating the numerical simulations significantly.

The approach in this paper is to investigate the response of the flames to a fluctuating velocity field, both experimentally and numerically. In the numerical simulations, a model is used which includes the Navier-Stokes and transport equations for a two-dimensional flame on a burner, in which the full interaction between flame, flow and burner is taken into account. The experimental research includes both global characterization of the flame thermo-acoustic behavior and detailed spatially and temporally resolved measurements of the flame front. Real and numerical experiments are conducted, some of which have identical burner geometry, flow and excitation parameters. In this way we can directly compare experimental and numerical results of the flame front motion and flow field. Furthermore, the measured and numerically computed thermo-acoustic transfer function (TF) and transfer matrix (TM), which characterize the flame as a lumped element, are compared and analyzed.

In the present contribution a numerical model, which provides the possibility to reproduce experimental results, will be developed and validated. Next, using the synergy of both numerical and experimental results, some insight in the physics of thermo-acoustic behavior will be given.
The paper is organized as follows. In Section 2, a genesis and the current state of knowledge of thermo-acoustic behavior of laminar Bunsen-type flames is presented. The experimental configuration is reported in the next section, including an outline of the experiments performed. In Section 4, the model used in our numerical simulations together with the numerical data post-processing procedures are described. The core of the paper consists of a comparison between experimental and numerical results of the flame thermo-acoustics, and will be presented in the Section 5. This contribution ends with a discussion of the results obtained followed by conclusions.

2. Thermo-acoustics of premixed flames: survey of existing models

Recent reviews of different aspects of flame-acoustics interaction, including Bunsen-type flames, were presented by Candel [1] and Lieuwen [2]. Below we will attempt to trace how progress in the understanding of the physical nature of this interaction was incorporated into several models. On the basis of this analysis together with results of the present study, we will suggest possible ways to improve both models and experiments.

2.1. Approaches to characterize flame thermo-acoustics

The three most widely used methods to characterize the interaction between flames and acoustic waves in the linear limit are based on, (i) the thermo-acoustic transfer function, (ii) the acoustic transfer matrix and (iii) the pyro-acoustic amplification factor.

The main part of our discussion will be based on the transfer function concept. It means that the response of a flame to acoustic waves is characterized in the frequency domain by the so-called flame thermo-acoustic transfer function, which is defined as the ratio of the relative heat release rate perturbation (response) to the relative flow velocity perturbation (stimulus), i.e.,

\[ TF(f) := \frac{q'/\dot{q}}{u'/\dot{u}}, \]  

(1)
where \( q \) is the heat release, \( u \) the velocity and \( f \) the frequency of the perturbation. A prime (') denotes the perturbation and a bar (\( \bar{\cdot} \)) the average value of a variable. If the response \( q' \) at a forcing frequency \( f \) is linearly proportional to the perturbation \( u' \), then the TF is independent of the amplitude of the perturbation. This linear regime will be considered in the present paper. TF(\( f \)) is a complex number and can be characterized either by its magnitude (gain) and phase (Fig. 1(a,b)), or by its real and imaginary parts (Fig. 1(d)).

The concept to characterize the thermo-acoustics of a flame via an acoustic transfer matrix is a purely acoustical approach where parts of an acoustic system (vessel) are considered as lumped acoustic elements, so-called two-ports. These elements relate the input vector of acoustic variables, the acoustic pressure and velocity \( (p'_{up}, u'_{up}) \) to the output vector \( (p'_{down}, u'_{down}) \). The TM approach is most suitable to describe longitudinal acoustic waves in the linear regime of oscillations. The acoustic elements are represented by the so-called transfer matrix, which provides the relation between input and output, i.e.,

\[
\begin{bmatrix}
  p'_{down} \\
  u'_{down}
\end{bmatrix}
= M(f)
\begin{bmatrix}
  p'_{up} \\
  u'_{up}
\end{bmatrix},
\quad
M(f) = \begin{bmatrix}
  M_{pp}(f) & M_{pu}(f) \\
  M_{up}(f) & M_{uu}(f)
\end{bmatrix}.
\]

(2)

It is possible to show (see e.g. [3, 4, 5, 6]), that if the flame/burner is an acoustically compact element, then, in the limit of low Mach numbers, only the element \( M_{uu}(f) \) is sensitive to the presence of a flame (heat release). This element relates the oscillating (acoustic) components of the velocity in the downstream (burnt) and upstream (unburnt) parts of the flame/burner, where the acoustic fields are assumed to be planar waves. The element \( M_{uu}(f) \) is a complex number with magnitude equal to \( |u'_{down}/u'_{up}| \) and phase equal to the phase difference between \( u'_{down} \) and \( u'_{up} \).

The approach of flame pyro-acoustic amplification is presented in [7] and will be not used here. A detailed comparison between the different methods is the subject of a separate investigation; however, some of the aspects will be addressed below. A relation between the TF and TM approaches is discussed...
2. Experimental evidence of thermo-acoustics of laminar premixed flames

The acoustic response of Bunsen-type flames has been studied intensively over the years. Putnam’s experiments [8] with many different systems in which a self-sustained acoustic instability can be observed, forced him to conclude that there is a combustion process time lag and that this time lag is equal to a traveling time of gas particles from the burner outlet to the mean position in the flame zone. By nature this time lag is a ‘system time delay’, in other words, the flame effectively responds to a flow perturbation some time after it is applied at the burner outlet. This observation has been verified in many experimental studies afterwards. Among these early measurements the experiments of Matsui and Sugimoto [7, 9] and Goldschmidt [10] are representative. In the last decade the TF of several Bunsen-type premixed flames was intensively studied, such as conical, inverted conical (V-form) and M-form flames, as well as flames anchored at multiple perforation burner decks; see [11, 12, 13, 14, 15].

Summarizing the cumulative experimental material of different Bunsen-type flames the following qualitative conclusions can be drawn. Besides the time delay property of the Bunsen-type flame TF, which was observed by practically all researchers, there are several typical features of the flame TF which also require physical interpretation; see Fig. 1. First of all, the TF gain shows a global decay when the perturbation frequency increases. This ‘low-pass behavior’ could be anticipated _a priori_; however, the decay behavior is non-trivial. The gain of the TF has distinct minima and maxima. Furthermore, the presence of a weakly frequency dependent component of the TF can also be recognized. It is evident from the high frequency saturation followed by a slow drift of the TF phase and gain. Another feature, typical for V-form, M-form and multiple flame configurations but absent for a free conical flame, is the overshoot of the TF gain above 1 for some excitation frequency.

Regarding the thermo-acoustic TF of flat burner surface stabilized flames the following main features
can be observed; see Fig. 1. The flame TF exhibits a low-pass filter behavior, shows a limited phase change $\phi(f)$, asymptotically approaching $\pi$ at high frequencies, and displays a large gain $G(f)$ at low frequencies (resonance, [6]).

2.3. The genesis of thermo-acoustic models of premixed Bunsen-type flames

A first attempt to build a theory of the response of conical flames to an acoustic flow excitation was developed by Merk [16] on a semi-phenomenological basis. The proposed model leads to a first-order equation for the heat release rate and accordingly to low-pass filter features of the flame response to excitation. For the characteristic time of the flame response Merk proposed to use $1/3$ of the ratio of the flame height $H$ to the flow velocity $V$ at the flame cone base, the so-called ‘convective time of the flame cone’. Not only the physical background of this estimation is obscure but, furthermore, the physical nature of this time is a system relaxation time and not a system response time lag. A manifest discrepancy between experiment and theory is that for the high frequency limit Merk’s theory predicts a phase for the transfer function which is asymptotically approaching $\pi/2$, while Putnam’s experiments gave evidence to a much larger phase.

The next stage in modeling was based on the observations of perturbed flame form motion. It was reported by Markstein [17] and Blackshear [18] that flame front undulations periodically raise the flame cone anchoring zone and travel towards the flame tip with a velocity close to the convective flow velocity. Accordingly, theories developed in the seventies prescribe the flame front perturbation as convective waves on the surface of the flame cone. The flame heat release rate perturbation can be calculated assuming that the flame surface area variation is the main cause of the heat release oscillation.

Different variants of these models were reviewed, further developed and presented in a clear way by Matsui [7, 9]. The results of these models suffer from the same shortcoming as Merk’s model. The models predict a low-pass filter behavior of the flame TF. At the same time, direct measurements of the
TF by Matsui and Sugimoto [9] indicate that the gain of the TF has a complicated jagged form with multiple minima and maxima while the TF phase shows an almost linear increase with the perturbation frequency, typical for systems with a time delay. To eliminate this grave discrepancy between theory and experiment Matsui and Sugimoto proposed a modification of the model by including an exponential growth of the flame surface perturbation amplitude as it propagates along the flame cone. They have affirmed their idea by measurements; however, the conclusion of an increasing perturbation amplitude was an artifact of their data processing procedure [14, 19] and was not confirmed by later research.

A new stage in modeling was initiated by the idea to describe the kinematics of the excited flame front by the so-called G-equation [20]. The fundamentally new idea in this approach is that the flame front perturbations are not prescribed \textit{a priori}, but naturally arise as the flame front responds to an applied flow perturbation. Therefore, in the framework of kinematic models the input is the oscillating flow field and the output is the flame surface area oscillation, which is directly proportional to the heat release rate, assuming that the normal flame speed is constant over the flame surface. In early versions of the kinematic models, the flow perturbations were prescribed as bulk streamwise oscillations [21]. The complete theory is presented in [22]. The model predicts the formation and further convection of flame front undulations which was prescribed in previous models and observed experimentally. However, the resulting TF once again displays a low-pass filter behavior and a phase saturation at $\pi/2$.

The reason why convection of the flame cone undulations does not lead to a convective time delay behavior of the TF is hidden in the nature of the surface area calculation; for more details see [14]. The conclusion which is important is that convection of flame front perturbations is not sufficient to explain the time delay property of the flame cone area TF.

Several ideas were proposed to overcome the discrepancy between the results of modeling and experiments, see the review in [14]. Currently, the most plausible explanation is that the flow field at the burner
outlet creates periodical perturbations of a convective nature superimposed on the acoustic wave. These perturbations resemble the peristaltic motion of a jet and can be described as convective traveling waves. The corresponding velocity field inside the flame cone was observed and characterized experimentally [23, 24, 25]. It was shown that the measured oscillating velocity field as input to the kinematic model results in a TF which exhibits the proper time delay behavior close to experimental results [26]. Simplifying the velocity field even allows to derive an analytical expression for the flame TF [22], having the same time delay features. Furthermore, it was demonstrated experimentally, that an excited flame front without convective waves yields a TF which does not display the observed time delay property [25].

It is interesting to note that the parameter values and the qualitative behavior of the convective waves in the flow upstream of the flame cone differ from the waves of the well-known convective structures at the outlet of an excited cold jet, referred to as vortex shedding [24, 27]. Currently, it is not clear what causes the difference. It can be either the mean effect of the pressure change due to gas expansion (acceleration) at the flame front, or the effect of the time-dependent (oscillating) flame retroaction to the upstream flow. Regarding the last possibility, two hypotheses are formulated. The first one concerns the effect of the curved flame front on the flow upstream of the flame [24]. The second hypothesis considers the radial motion (in and out) of the anchoring point as the main driver of the peristaltic jet motion [25].

It was shown [14] that the nonmonotone form of the TF gain is probably a direct consequence of the fact that the TF of Bunsen-type flames contains in addition to a purely kinematic part an additive component, which is relatively small and has a weaker frequency dependence; see the convergence point of the TF spiral in Fig. 1(c). This TF ‘offset’ also shows up at saturation followed by slow drift of the TF gain and phase when the excitation frequency increases. There is evidence that the TF offset originates from the flame foot region and is probably associated with the motion of the flame anchoring point [14]. In this zone the role of heat transfer between the flame and burner surface plays a dominant role and
therefore we conjecture that the TF offset resembles the TF describing the interaction of a flat surface burner stabilized flame with acoustic waves. There is only one, first attempt to build a model which combines both effects of flame cone kinematics and oscillating heat loss to the burner surface [28].

The TF gain overshoot above 1 for a certain frequency range was theoretically described in the framework of kinematic models for V-flames [22], which is in accordance with experimental observations. The cause of the amplification of the response over its quasi-steady value lies in the interference between perturbations which originate from the anchoring point and travel along the flame, and a convective perturbation of the flow pattern [29]. A TF gain above 1 was also measured for multiple flame burner heads [15, 30]. At the same time, both the theoretical and the experimental TF of a single conical flame shows a TF gain less than 1 for the whole frequency range.

Until very recently the thermo-acoustic behavior of a configuration of multiple identical flames has been interpreted on the basis of single (individual) flame behavior. The qualitative features of a single flame and identical multiple flames are similar. In both cases, the gain of the TF as a function of frequency has a complicated jagged form and the TF phase has a time delay behavior. In spite of the qualitative resemblance the two flame configurations are not identical. At least, one more burner parameter, viz. the inter-flame spacing (pitch) becomes important for the TF of a multiple flame burner [15]. The overshoot above 1 of the TF gain of multiple flames is an extra evidence of the qualitative difference between the single and multiple flame configurations. To our knowledge there are no models of flame thermoacoustics that include flame-to-flame interaction.

Summarizing the progress in understanding, we can conclude that thermo-acoustic models for premixed Bunsen-type flames evolved from the prescription of the flame surface area dynamics in the early models, towards the prescription of the flame form perturbation in the theories in the period 1970-1980 to the prescription of the oscillating flow field in modern theories. A next logical step would be to pre-
3 EXPERIMENTAL METHOD

dict the flow dynamics starting from the prescribed plane acoustic field at some position upstream of the burner. This step will probably require the incorporation of the flame retroaction on the upstream flow. Other necessary extensions of the models should include (i) intrinsic mechanisms of flame anchoring to the burner (or flame-holder) which accounts for heat transfer between flame and burner surface and (ii) mutual interaction between adjacent flames, probably via the downstream part of the flow. The aim of the present research is to provide results which can serve as basis for future analytical investigation/models.

3. Experimental method

3.1. Experimental setup

The burner which was used in the present study consists of a vessel with a flat perforated disk inserted on top of it; see Fig. 2(a). The disk contains a series of 12 mm long rectangular slits, each of width $d$, whereas $l$ is the distance between adjacent slits. The pitch then equals $l+d$ and the porosity $\xi = d/(d+l)$. The slits are perforated in a steel disk of 0.5 mm thickness. A mixture of methane and air at ambient conditions ($p = 1.0 \text{ atm}$ and $T = 293 \text{ K}$), with an equivalence ratio $\Phi$ and a velocity $\bar{u}_{in}$ (approaching the bulk velocity below the plate, leading to an average velocity $V$ in the slit) is used to stabilize the steady flames. The burning velocity of the mixture is $s_L$. The burner plate temperature was measured by a K-type thermocouple embedded in the center of the burner plate. The temperature reaches $100^\circ \text{ C}$ to $150^\circ \text{ C}$ due to steady combustion. The gas flows are controlled with mass flow controllers (MFC) installed far enough from the burner to allow a perfect mixing and to avoid a possible acoustic influence on $\Phi$ and/or $\bar{u}_{in}$. To impose a flow velocity perturbation $u'$, a loudspeaker operated by a pure tone generator was installed upstream in the mixture supply tube. Because the mixture composition is unperturbed and ambient air entrainment cannot be significant it is known that to measure the flame heat release rate, the chemiluminescence intensity of $\text{OH}^*$ can be used as an appropriate indicator (see e.g. [31] and further references). To monitor the velocity oscillation a hot-wire anemometer was installed 10 mm upstream,
3 EXPERIMENTAL METHOD

just beneath a slit in the burner vessel (see Fig. 2(a)).

Flame front kinematics and flow field visualizations were performed via the direct phase-locked video-recording of the flame natural emission (a CCD camera, Kodak Megaplus ES1.0, was used) and flow velocity field reconstruction was done by the conventional Particle Image Velocimetry (PIV) technique. The flow was seeded with Al$_2$O$_3$ particles with average size of approximately 1µm. Both the upstream and flame region of the flow were illuminated by a light sheet of an Nd:YAG pulsed laser. For this purpose the upper part of the burner vessel is made optically transparent. Care was taken to minimize the light reflections on the entrance and exit windows of the laser sheet which were made in the upper upstream burner section. The processing of raw PIV image pairs was conducted by the PIVview2C software.

3.2. Procedure of TF data processing

The experimental TF is found by measuring the flame response at a large number of different frequencies. According to definition (1) of the thermo-acoustic TF, its amplitude (gain) and argument (phase) can be reconstructed from the measured time series of the relative flow velocity perturbation $u'/\bar{u}$ and the relative heat release rate perturbation $q'/\bar{q}$. Raw experimental data consist of 0.5 s samples of $u'(t)$ and $I'_{OH^*}(t)$ time histories digitized with a sampling rate of 20 kHz. A perturbation amplitude $u'/\bar{u}$ of approximately 5-10% was chosen, for which the flame response can be considered linear [14]. The gain of the TF was calculated as the ratio of the amplitudes of the Fourier transform of $I'_{OH^*}(t)$ and the Fourier transform of $u'(t)$. The phase difference between both signals was determined by a cross-correlation analysis. The measured values can be presented either in the form of a frequency dependent gain $G(f)$ and phase delay $\phi(f)$ (see Fig. 1(a,b)) or in a polar plot where $G(f)$ represents the radial length and $\phi(f)$ the angle (see Fig. 1(c)).
4. Numerical modeling

The code LAMFLA2D [6, 32] is used to simulate the response of lean methane-air flames to velocity perturbations. The code solves the primitive variable formulation of the conservation laws for two-dimensional, low-Mach number reacting flow. It is based on a one-step chemical reaction model for the species CH$_4$, O$_2$, CO$_2$, H$_2$O and N$_2$. LAMFLA2D uses the following numerical methods: a second order finite volume/complete flux scheme for space discretisation, the implicit Euler method for time integration, a pressure-correction method to decouple the pressure computation and a nonlinear multigrid method and GMRES to solve the discretized system. For more details see [6, 32, 33].

The most important physical and chemical models used are presented in the following. The diffusion fluxes are modeled using a Fick-like expression with the mixture-averaged diffusion coefficients $D_{i,m}$ given by

$$D_{i,m} = \frac{(1 - Y_i)}{\sum_{j \neq i} X_j / D_{ij}},$$

(3)

where $X_i$ and $Y_i$ are the mole and mass fractions, respectively, and $D_{ij}$ are the binary diffusion coefficients [34]. The transport equations for the species CH$_4$, O$_2$, CO$_2$ and H$_2$O are solved. The mass fraction of the $N$th, abundant, species N$_2$ is computed from the constraint $\sum_{i=1}^{N} Y_i = 1$ to assure that the sum of all diffusion fluxes equals 0. A semi-empirical formulation is applied for the conductivity, i.e.,

$$\lambda = \frac{1}{2} \left( \sum_{i=1}^{N} X_i \lambda_i + \left( \sum_{i=1}^{N} X_i / \lambda_i \right)^{-1} \right),$$

(4)

where $\lambda_i$ is the thermal conductivity of the $i$th species. The transport coefficients $D_{ij}$ and $\lambda_i$ are tabulated in terms of polynomial coefficients, similar as in the CHEMKIN package [35]. The thermodynamic properties are also tabulated in polynomial form [36].

We apply the following one-step overall irreversible reaction mechanism in the numerical study:

$$\text{CH}_4 + 2\text{O}_2 \rightarrow 2\text{H}_2\text{O} + \text{CO}_2,$$

(5)
with the reaction rate of methane given by [37]:

\[
\dot{\rho}_{\text{CH}_4} = -A \rho^{m+n} \rho_{\text{CH}_4}^m \rho_{\text{O}_2}^n \exp\left(-\frac{E_a}{RT}\right).
\]

The reaction parameters were fit to experiments to predict the correct relation between the burning velocity and flame temperature, for flat adiabatic and burner-stabilized flames, in the range \(0.8 \leq \Phi \leq 1.2\) and optimized for \(\Phi = 0.8\) [38], resulting in \(m = 2.8\), \(n = 1.2\), \(E_a = 138\ \text{kJ/mol}\) and \(A = 2.87 \times 10^{15} (\text{kg/m}^3)^{1-m-n} \text{s}^{-1}\). The effect of heat losses is incorporated in the fitting procedure to make sure that the flame stabilizes due to heat transfer to the burner. Accordingly, we can surmise that the phenomenon of flame anchoring, i.e., the stand-off distance, is accurately modeled. It was shown in earlier studies [38] that this model accurately describes the global behavior of steady burner-stabilized flames. In [39] it was shown that this mechanism is also well-suited to model the response of one-dimensional lean methane-air flames to low-frequency acoustic perturbations. In the current paper we restrict ourselves to premixed methane-air flames with \(\Phi = 0.8\).

Only a small part of the repetitive flame structure is computed on a domain of width half the pitch \((l + d)/2\), using symmetry boundary conditions at both sides; see Fig. 2(b). The inflow below the burner plate is also taken into account in the simulations using a flat velocity profile with perturbation, i.e., \(u = \bar{u} + u'\) as inflow condition. This condition accurately models the approaching acoustic wave at the inflow, since acoustic waves have infinite wave length and pressure fluctuations are not needed in the limit of the Mach-number \(Ma \to 0\). The outflow is simply modeled by setting to 0 the normal derivatives of the velocity and the combustion variables, which is accurate if the outflow boundary is sufficiently far away from the flame front. The temperature of different burner decks was not computed but was set at 100°C. The influence of the burner deck temperature and thickness was not studied here. However, as follows from the results in [13], the influence of the thickness of the burner plate on the burner thermo-acoustics is weak and becomes noticeable when it is significantly larger than the perforation size.
4.1. **Post-processing of numerical data**

Most flame TF and TM are computed during a single computation, i.e., by modeling the flame response to a small but instantaneous velocity change at the inlet. This perturbation contains the response information of all frequencies which can be found by Fourier analysis. With respect to calculation time, applying broad band excitation (like a jump of the inlet velocity) is much more effective than the calculation of the flame response for each pure tone separately. However, the requirements on the response linearity are much more restrictive in the case of broad band excitation than in the case of single frequency response.

The perturbations should be small enough to avoid nonlinear flame response which can induce spurious components of the response at higher harmonics. Application of low magnitude perturbations causes the problem of signal-to-numerical noise ratio. The detailed analysis of this question can be found in [40]. The correctness of the flame TF reconstruction from the response to a stepwise perturbation is checked by varying the velocity perturbation magnitude. The time series of the flame heat release rate was calculated via the integration over the entire calculation domain.

The time history of inlet and outlet velocities required for the burner TM calculation were determined in the corresponding zones of the calculation domain, when a stepwise excitation was applied. We checked that almost plane distributions of mean and oscillating velocities were retrieved in the far downstream part of the calculation domain.

Flame front kinematics and flow pattern features under the action of pure tone excitation were visualized during both numerical and real time (phase) and spatially resolved experiments. In numerical experiments, five periods of oscillation were calculated which was sufficient to reach the steady-state periodic response of the flame and flow. As initial condition the corresponding steady flame was used. To minimize the transition time to the steady-state oscillation a smoothly growing amplitude of the tone perturbation was imposed.
Besides the heat release transfer function $TF_A$, we introduce the transfer function $TF_A$, which relates the relative response of the flame surface area $A$ (defined in several different ways, see below) to the velocity perturbation, i.e.,:

$$TF_A(f) := \frac{A'/\bar{A}}{u'/\bar{u}}.$$  \hspace{1cm} (7)

Flame front kinematics and the calculation of $TF_A$ supposes the flame front to be a line (in two dimensions) or a surface (in three dimensions) which separates burnt and unburnt gases. Accordingly, a procedure to determine this line in the reacting flow field should be specified. For the case of Bunsen-type flames it is natural to take for the position of the flame front the line where the value of the heat release rate distribution has its maximum. However, the flame front is not attached to the burner surface and a location for the flame end (foot) should be specified as well. When the flame front and flame end point are reconstructed, the flame surface area can be determined as the length of the flame line from its tip to the end point.

The following criterion was used for the flame anchoring point: it is the point where the heat release rate, as a function of the coordinate along the flame front, has the maximal rate of change. The physical meaning of such a definition is the following: the end point is the location where the reaction rate along the flame front has its maximal gradient of decay. A similar procedure of flame front tracking was used before [14, 19] for the analysis of experimental data in the case of much larger Bunsen flames. However, for the slit burner configuration considered here the noise-to-signal ratio of experimental images of perturbed flames is not sufficient to obtain the accuracy of the flame front kinematics reconstruction which is necessary to compute the transfer function. Therefore, only the results of numerical simulations will be post-processed. Furthermore, the accuracy of numerical simulations was not sufficient to calculate the flame surface area $TF_A$ from the numerical experiment with a stepwise excitation of the inlet velocity. Because of this, the calculation of the flame response to a pure tone excitation in a limited range of
frequencies was used to calculate the flame area oscillation.

Besides the physically reasonable definition described above, some other, easier, ways to define the flame anchoring point were also examined. Two other methods are tested: (i) the flame is truncated by the vertical lines emanating from the slit edge, which is equivalent to the artificial ‘attachment’ to the slit edges and (ii) the flame front was reconstructed from the 1200 K isotherm (see the line in Fig. 10 (c)). This temperature was arbitrarily selected as one where typically the reaction layer of the methane-air flame starts.

5. Results

Two classes of experimental and/or numerical results will be presented. The first one concerns measurements of the flame TF for a set of burner, flow and excitation parameters. This is a global flame analysis which allows us to assess the experimental and numerical methods to determine the flame TF. Some hypotheses about the physical mechanisms of the flame response will be proposed and examined on the basis of the TF. Moreover, the numerically computed TF and TM will be compared and the relation between them will be discussed. The second class deals with a detailed comparison between steady and acoustically exited flames, i.e., the flame motion and the flow are resolved temporally and spatially. The resulting data will lead to a better understanding of the thermo-acoustic behavior of flames.

5.1. Comparison of experimental and simulated flame TF and TM

First, the measured flame TF is analyzed and compared with numerical results for a flame with $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$ and $l = 3.0 \text{ mm}$; the so-called representative case. Next, a parameter study is presented for the TF with varying $\Phi$, $V$, $d$ and $l$. Moreover, the measured and computed shape of the steady flame is presented, in order to facilitate further analysis of the TF. The TM was only determined via numerical simulations.

*Flame TF for the representative case*
Figure 3 compares the steady flame shapes (c) and TF (gain $G(f)$ (a) and phase $\phi(f)$ (b)) for the representative case. The experimental part shows a chemiluminescence photograph while the chemical source term is displayed in the modeling results of Fig. 3(c). The correspondence is reasonable despite that different quantities are visualized. The experimental flame has a height $H$ of approximately 4.5 to 4.7 mm while the numerically computed flame is slightly smaller with a height of 4.5 mm. Note also that individual flames stabilize on the slits and do not merge near the foot in both the experiment and the numerical simulations.

The experimental and numerical flame TFs are compared in Fig. 3 (a,b). The correspondence of gain and phase is reasonable. All qualitative features of the experimental TF are captured by numerical simulation. The time delay property (almost linear increase of the TF phase with frequency) is accurately predicted. The effect of the TF gain overshoot of the quasi-steady response is also accurately computed.

The TF phase saturation property is somewhat overpredicted, which is reflected by the fact that the numerical TF phase approaches a constant phase of approximately $11\,\text{rad}$ for $f > 450\,\text{Hz}$, while the experimental TF phase is still increasing. Another consequence of the TF offset overprediction is the wiggly behavior of the computed TF gain compared to the experimental one. The difficulty to predict the TF offset accurately is due to its high sensitivity to small inaccuracies of the flame (mixture) parameters, both experimentally and numerically.

*Effects of $\Phi$, $V$, $d$ and $l$ on the TF*

Figure 4 shows measurements and simulation results of the flame TF for varying $V$ while $\Phi = 0.8$, $d = 2.0\,\text{mm}$ and $l = 3.0\,\text{mm}$. The figure indicates that the numerical simulation is very well capable to predict the behavior of the experimental results for all cases. The TFs for the lowest velocities look very similar to those of a flat burner-stabilized flame (low phase for all frequencies and high gain at small frequencies; see lines 2 in Fig. 1), since the individual flames are very small and the major part of
the combustible mixture is consumed as in a flat surface burner. For higher velocities the flame height increases and much more of the mixture is consumed by the longer Bunsen-type flames, and as a result, the contribution of the Bunsen flame TF to the multi-slit TF increases. Therefore, the phase becomes very similar to the Bunsen-type flame behavior, with a constant slope at low frequencies and saturation at a constant phase at higher frequencies (see lines 1 in Fig.1). The slope of the phase, proportional to the convective time $\tau := H/V$, does not change when $V$ is varied, because the laminar flame height $H$ also increases for increasing velocity. The saturation level increases with $V$ like in the case of a single Bunsen flame due to the shift of the cutoff frequency of the low-pass filter to higher frequencies. A similar effect is seen in the gain: at low velocities, $G(f)$ looks like the surface burner gain, while the influence of the Bunsen-type flame gain becomes more important for increasing $V$. This leads to the observation that the change in gain $G(f)$ is more pronounced than in the case of a single Bunsen flame [14]. The frequency range where the TF gain exceeds 1 enlarges with increasing flow velocity.

Figure 5 shows measurement data and numerical results of the flame TF for varying slit width $d$ and distance $l$ with $l/d = 1.5$, implying that the porosity is constant, while $\Phi = 0.8$ and $V = 100 \text{ cm/s}$. In this experiment the effect of purely geometrical scaling is revealed. The flame height $H$ increases with increasing $d$ while $V$ remains constant so that the slope of $\phi(f)$, proportional to $\tau$, increases. The frequency range where the TF gain exceeds 1 is larger for smaller flames. The correspondence between experimental data and numerical results is again excellent.

Figure 6 shows measurements and numerical results of the flame TF for varying distance between the slits $l$ while $\Phi = 0.8$, $V = 100 \text{ cm/s}$ and $d = 2.0 \text{ mm}$. Increasing the distance between the slits leads to a separation between the attachment points of individual flames, a higher flame foot position and a larger flame height. This, once more explains the increasing slope in the phase plots, whereas the gain is hardly influenced. The frequency range where the TF gain overshoots 1 is almost the same for all
values of $l$.

Finally, Fig. 7 shows measurements of the flame TF for varying $\Phi$ while $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$ and $l = 3.0 \text{ mm}$. No comparison with numerical results are given here, since we only did numerical simulations for $\Phi = 0.8$. The slope of the phase $\phi(f)$ is proportional to $\tau$, which decreases for increasing $\Phi$ because the flame length $H$ becomes smaller. A smaller flame is a result of a higher burning velocity corresponding to a richer mixture. The gain is hardly influenced by changes in $\Phi$ as for the case of a single Bunsen flame [14]. The region where the TF exceeds 1 decreases with increasing $\Phi$ for the burner and flow parameters used in Fig. 7.

**Numerical simulation of TM versus TF**

The calculation of the element $M_{uu}$ of the TM was included in the numerical simulation of the TFs presented in Fig. 4(c,d) for various velocities. The magnitude and phase of $M_{uu}$ are presented in Fig. 8(c,d) by solid lines together with the corresponding TFs, presented in Fig. 8(a,b). Note that the scale of the TF gain is now linear. The quasi-stationary limit $f \to 0$ corresponds to the ratio of the densities of unburnt and burnt gases which is the same as the reciprocal ratio of the corresponding temperatures. In the high frequency range the magnitude of $M_{uu}$ approaches 1 and the phase tends to 0 modulo $2\pi$.

This means that the flame does not alter acoustic waves of sufficiently high frequency. For all lines in Fig. 8 (c), except the line for $V = 150 \text{ cm/s}$, the frequency range where the magnitudes of $M_{uu}$ exceeds the quasi-stationary response level is slightly narrower than the corresponding range of the TF gain (see Fig. 8(a)). For high inlet velocity ($V = 150 \text{ cm/s}$) a secondary maximum of the magnitude of $M_{uu}$ in the frequency range 200 − 300 Hz can be recognized.

It is interesting to compare the phases of the TF and TM, see Fig. 8(b,d). For the smallest flame ($V = 50 \text{ cm/s}$) the phase of the TF saturates to a level slightly smaller than $\pi$ whereas the phase of the TM decreases to 0. For other inlet velocities the absolute value of the TF phase typically saturates to a
level between $\pi/2$ and $\pi$, modulo $2\pi$, whereas the corresponding phase of $M_{uu}$ tends to 0 modulo $2\pi$.

However, the slopes of the TF and TM phases in the linear regime are the same, which implies the same time lag.

Based on the Rankine-Hugoniot jump conditions, a relation between the flame heat release TF and the element $M_{uu}$ of the acoustic TM can be established; see equation (8). The dotted lines depicted in Fig. 8 (c,d) are obtained using (8). The temperature ratios $T_{burnt}/T_{unburnt}$ which are required to link $M_{uu}$ with the corresponding TF are calculated within the same numerical simulations. The temperature ratio slightly increases as function of the imposed flow velocity from 6.39 for the case of $V = 50 \text{ cm/s}$ till 6.56 for $V = 150 \text{ cm/s}$.

5.2. Oscillating flow field and flame front kinematics for the representative case

Only the representative case is examined for a few frequencies and amplitudes of perturbation. As the qualitative conclusions for the different flames were the same, we will restrict ourselves in this section to a steady flame and a flame exited at 200 Hz. All other parameters are the same as for the representative case.

Steady-state flame

Figure 9 presents the raw PIV image (a) and processed results in the form of a vector field overlaid by a grey-scales plot of the streamwise (b) and transversal (c) components of the measured velocity field, for the steady flame. Parts of the flow inside the burner plate, as well as approximately 0.5 mm above and 1 mm beneath the deck cannot be visualized because of difficulties due to laser light and image reflections on the surface of the plate. Figure 9(d) represents the computed and measured vertical (streamwise) component of the velocity along the slit centerline, marked by ‘y’ in Fig. 9(b,d). The measurement domain in streamwise direction is smaller than the computational domain. Therefore, line 1 in Fig. 9(d) is shorter than line 2. Because the experimental configuration is not entirely two-dimensional and some
flow contraction and expansion occurs in the direction perpendicular to the measured section plane, the measured inflow (upstream) and outflow (downstream) velocities are smaller than the computed velocities. However, the flow velocity in the vicinity of the slit plate and inside the flame cone is accurately predicted.

As can be expected, contraction of the flow beneath the slit leads to a gradual increase of the vertical velocity and a horizontal component in the deck upstream part of the flow. Inside the flame cone the velocity is almost constant, weakly decreasing (circa 20%) towards the flame front, followed by a rapid acceleration due to gas expansion. The velocity in the flame tip increases roughly by a factor 2, which is much less than the temperature ratio: $T_{\text{burnt}}/T_{\text{unburnt}} \approx 6$. At the same time the total flow acceleration (ratio of far downstream to far upstream velocities) corresponds well to the gas expansion factor due to the temperature rise.

After the flame zone the flow quickly recovers an almost flat velocity profile, see upper line in Fig. 9(c), where the three lines represent the distributions of the streamwise velocity along three horizontal planes (parallel to the burner plate). The corresponding computed velocity field approximates the measured velocity profiles quite well (not shown in the figure).

Figure 10(a,b) presents a comparison of the chemiluminescence image of the stationary flame (Fig. 10(a)) and the measured flow field overlaid with the contour plot of the flow dilatation rate (Fig. 10(b)). The difference between the position of maximal dilatation and heat release rate is of the order of the flame thickness, which is of the order of the measurement accuracy. Accordingly, both quantities are equally suitable as indicator for the flame front position.

In the flame foot region a gas recirculation zone can be observed. Figure 10(g) shows a close view of the PIV velocity field near the flame anchoring point. Numerical simulation allows resolving this flow phenomenon close to the burner deck in great detail; see Fig. 10(h). The layer of rapid temperature
increase, see Fig. 10(d,e), is much thicker here, which is due to a lower heat release rate (see Fig. 10(c)) and hot gas recirculation. The pressure field (Fig. 10(f)) is also depicted for the sake of completeness.

From the analysis above we conclude that the spatial structure of the steady flame matches the experiment well. Many flow and flame parameters are predicted qualitatively and quantitatively correct. This gives confidence that the time-dependent flame and flow can be also accurately computed.

Kinematics of excited flames

Both real and numerical experiments on pure tone excited flames show the periodic formation of undulations. These waves travel along the flame front with a velocity of the order of magnitude of the mean flow velocity. The shape and temporal evolution of these perturbations is extensively studied and reported, and we will not repeat this. Instead, we will concentrate on new aspects, which could be useful for future flame models.

The important question for the development of kinematic-type models is: suppose that a model correctly predicts the flame front motion, then does the flame surface area oscillation adequately determine the heat release rate oscillation? Physically this is equivalent to the assumption of constant $s_L$ over the entire flame front.

To check the validity of the kinematic approach, the deviation of the flame area response from the heat release rate response should be studied. The presence of such a difference could indicate that effects of nonconstant $s_L$ and/or heat transfer from flame to burner deck play a significant role.

Figure 11 presents the time history of the simulated response of the representative flame to a velocity oscillation. The flame surface area is calculated using the flame end point definition as the heat release rate inflection point. The relative value of a variable is defined as its current deviation from the mean value scaled with this mean value. For a frequency of 200 Hz two periods of oscillation were sufficient to reach a steady oscillation. The relative heat release oscillation $q'(t)$ and downstream velocity fluctuation
$u'_d(t)$ are in agreement with the corresponding points of the TF and TM element $M_{u_d}$ presented in Fig. 8.

The relative oscillation of the flame surface area $A'(t)$ has amplitude and phase which are close to the corresponding values of the relative heat release fluctuation for this particular excitation frequency. All the responses are delayed relative to the imposed velocity oscillation as expected. Conducting similar calculations for a range of excitation frequencies, we can compare the thermal TF and flame area TF$_A$. Figure 12 present the results of this calculation. Points marked by open circles correspond to the thermal TF obtained via pure tone high amplitude (10%) excitation of the flame. The points agree well with the TF calculated via the low amplitude excitation of the flame by a stepwise velocity jump. This observation cross-validates the correctness of both TF calculation procedures. Points marked by squares represent the flame area TF. Both gain and phase are in good agreement with the thermal TF, which emphasizes the dominant role of the surface area oscillation in flame thermo-acoustic behavior.

For this example it is useful to demonstrate the effect of the method to determine the flame end point. As mentioned above, the surface area time history, indicated by $A'(t)$ in Fig. 11 and marked by squares in Fig. 12, is calculated via the length of the flame line from the tip to the end point, for which we take the inflection point of the heat release. Despite the large amplitude of the velocity excitation and response, the noise level of the flame area signal is significant. This fact hampers using a stepwise flame perturbation to calculate the flame area TF.

One of the sources of (numerical) noise of the flame area time evolution is the difficulty to accurately determine the (motion of) the flame end point. Therefore, two other methods were tested. The dash-dotted lines in Fig. 12 indicated by $A'_{\text{truncated}}$ and the dotted line indicated by $A'_{\text{isotherm}}$ are computed using procedure (i) and (ii), respectively, explained in the Section 4.1.

All flame area TFs give similar values for the time lag but different saturation frequencies. Quali-
tatively, the general trend of all flame area TF is similar. However, quantitatively the gain of the TFs reconstructed from $A_{\text{truncated}}'$ is significantly overpredicted.

**Acoustically excited flow**

Both experimental and numerical results of periodically excited flow show the presence of structures which can be described in terms of convective traveling waves. This kind of flow behavior has been observed earlier and is well-documented [20, 24, 25]. However, in previous research only the downstream part, relative to the burner outlet, of the jet was accessible for measurements, and therefore the question remains: what is the ‘starting point’ of the convective wave? In this study, measurements and simulations of oscillating flow, both upstream and downstream of the burner deck and flame, allow us to investigate the details of the traveling wave formation and may support the development of a new theory for thermo-acoustic behavior of flames.

Both real and numerical experiments show the same flow structure. However, numerical results are more detailed and we will only consider these. Due to flow contraction/expansion the mean and oscillating components of the axial velocity vary significantly along the streamlines. Therefore, a more convenient indicator to trace the oscillating flow structure is the ratio of the oscillating component and the mean value of the axial velocity. The mean velocity is calculated as the time average (over one period) of the velocity, at each point in space. The oscillating component is the difference between the (local) mean and instant velocity.

Figure 13 presents the distribution of the relative flow oscillation along the centerline of the burner slit. Different lines correspond to progressing phases of the imposed oscillation. Far upstream of the burner plate the perturbation is uniform and in phase. This plane acoustic wave feature of the flow persists to the vicinity close upstream of the burner plate. Waves of convective nature are formed in the immediate proximity of the cross-section of the burner deck. The waves convectively travel to the flame
front where they disappear and once again a plane wave, with a new amplitude and phase, is recovered.

The large jump in the relative oscillations at the flame tip is an artefact of the data processing procedure. In Fig. 13 this part is covered with the grey rectangle. The reason for this is that the measurement points in this zone are alternatingly in the burnt and unburnt gas mixture, which leads to an artificially large relative amplitude of the velocity oscillation.

As indicated above, the flame-flow interaction in the flame foot zone could hypothetically determine the TF offset characteristics and the flame retroaction to the upstream flow. Because of this it is interesting to have a look at the oscillating flow dynamics in the burner plate vicinity. Figure 14 presents the time evolution of the oscillating part of the flow velocities computed at three points near the slit. All velocities are scaled with the mean flow velocity at the inflow. The line marked as $V_s^C$ represents the axial oscillating velocity in the slit center point (C in Fig. 14). The lines indicated as $V_t^U$ and $V_t^D$ are the transversal component of the oscillating flow velocity at 0.5 mm upstream (point U) and downstream (point D) of the burner deck, respectively; see the right part of Fig. 14. The oscillating vertical velocity component on the centerline is in antiphase with the transversal upstream velocity, which is intuitively clear - the moment when the vertical velocity in the slit reaches the largest upward value coincides with the maximum of the inward directed value of the upstream transversal velocity. At the same time, the flow dynamics in the near downstream part of the flow is more complicated. The phase of oscillation of the horizontal velocity in point D is some value ahead of the oscillation in the slit center point.

6. Discussion

The good agreement between experimental and numerical results for the TF, flow fields, stationary and excited flames obtained for a wide range of parameters, convinces us that both the experimental and numerical approaches are adequate to describe the essential phenomena in thermo-acoustic flame behavior of multi-slit Bunsen flames.
Regarding the measurement procedure the following questions have been clarified. First, is chemiluminescence of the flame a direct and appropriate indicator of the heat release rate for the practically relevant case of small flames? This question should be answered affirmative because in our model there are no assumptions concerning the relation between chemiluminescence and heat release rate. The second controversial aspect of the TF measurement approach is: were is the ‘input’ of the burner-flame system, or more precisely, where should the flow velocity perturbation be measured? Results above show that for the tested flame-burner configuration, the appropriate location to measure the velocity perturbation is not too close to the burner deck. This is valid, if the aim of the flame characterization is to relate the heat release rate to the plane wave acoustic perturbation. For our case this location should be more than a few millimeters upstream of the burner plate. In the close vicinity of the flame, a convective flow perturbation is generated and the plane acoustic wave behavior is affected. This result is in contradiction with [3] where the recommended position to measure the TF is as close as possible to the flame cone. As follows from the spatially resolved study of the excited flow (see Fig. 13) the flow inside the slit and close up/down-stream is dominated by convective waves. If the flame TF is measured inside the burner deck or close above the slit outlet then the relation between the local velocity and acoustic velocity in a plane wave in the burner upstream part should be incorporated. In [13, 30] such a strategy was successfully applied, however the question remains whether it is justified to use cold flow theory to estimate the acoustic features of a flame.

Regarding the numerical simulations, the agreement between experimental and simulation results confirms that the underlying model, assuming incompressible (low-Mach number) reacting flow with one-step chemistry, is sufficient to reproduce the global flame thermo-acoustic properties as well as the spatially and temporally resolved flame structure and flow. The fact that the measured shape and height of the flames are correctly reproduced by our model is direct evidence of the adequacy of the
chemical kinetics used. The model captures the thermo-acoustic TF of the multiple flame configuration. Furthermore, parameters for stationary and excited flames are also well reproduced. Accordingly, we conclude that this model includes all important physical phenomena, and therefore provides a convenient tool to investigate flame-flow interaction. Currently there are no arguments to include detailed chemistry and/or sophisticated diffusion models. Furthermore, the computation of the TF does not require the inclusion of compressibility effects. However, this might not be true for the TM. This question was also discussed in [3] and requires further investigation.

An analytical model of the interaction between premixed Bunsen-type flames and flow oscillations requires the identification of those phenomena which are responsible for the flame response. The key question is: what is the level of complexity required to accurately predict the thermo-acoustic response of the flame?

Nowadays, kinematic models to describe the premixed flame-acoustics interaction are widely studied. Are these models adequate to predict the thermal TF of practically interesting configurations where the perforations, and accordingly also the flame heights, are of the order of \(1\,\text{mm}\)? This question can be split in two parts. The first one is: how to model properly the flame sheet kinematics? The second question is: how do we relate the heat release rate fluctuations to the movement of the flame sheet?

With regard to the first question we note the following. All experimental and numerical results of the TF show that the time delay is proportional to the convective time \(\tau = \frac{H}{V}\). This time delay might vary due to variations in flow velocity (Fig. 4), slit width (Fig. 5), pattern pitch (Fig. 6) or mixture equivalence ratio (Fig. 7). Nowadays, it is hypothesized that the time lag should be explained in terms of convective structures of the upstream flow. The observation that convective waves also occur in our flame configuration and that they originate close to the burner deck (see Fig. 13) does not contradict this hypothesis. Accordingly, in order to predict the correct time delay, analytical models should include
The occurrence of maxima and minima in the TF gain as well as the saturation behavior of the TF gain and phase can be attributed to a weakly frequency dependent offset component of the TF. Once again, the smaller the flame the more pronounced the TF offset is. The saturation frequency is lower for the flames with smaller velocity (Fig. 4), smaller slit width (Fig. 5), smaller pitch (Fig. 6) or richer mixture composition (Fig. 7). This observation is in agreement with the earlier mentioned hypothesis that the TF offset is related to the flame foot dynamics and probably resembles the mechanism describing the interaction between a surface burner stabilized flame and acoustics waves. Accordingly, to predict the TF offset feature, analytical models should incorporate flame foot behavior.

The available data are not conclusive of the cause of the TF gain overshoot. Two hypotheses, which require further investigation, can be proposed. First, the overshoot is due to the low-frequency features of the offset component in the TF. The other possible reason is the interaction of adjacent flames. The flame-to-flame influence should be included anyhow in an analytical model in order to predict the influence of the pitch on the TF (see Fig. 6).

The second aspect is how to relate the flame surface area fluctuations to the heat release rate oscillations. As is apparent from Fig. 12 the TFs for flame area and heat release rate correlate well. It means that the problem of variation of the normal flame propagation speed over the flame surface due to heat transfer to the burner deck or flame stretch (because of flow straining and flame curvature) is not very pronounced, even for small flames. Instead, another problem arises: how to specify the flame end point? For example, the physically plausible way to end the flame surface in the point of maximal decay of the heat release (inflection point) leads to a good prediction of the flame TF. However, this choice requires an accurate numerical simulation of the flame foot zone. The artificial truncation of the flame front by the slit edges can result in a significant error in the computed TF (see Fig. 12). Taking an isotherm as
DISCUSSION

flame front, e.g. the 1200 K isotherm, hardly can be accepted, since it would imply that the flame front is defined even in points where there is no heat release at all; see Fig. 10(c). Our results reveal these problems, show possible causes of discrepancies but give no answers how to eliminate these.

The difficulty to identify the flame TF in a consistent way and subsequently use it to calculate the TM was recognized earlier for compressible flow simulation of a flame excited by the pure tone with a single frequency [3]. It was surmised that, because of reflections of the acoustic wave in the downstream section of the burner, the point to measure the excitation velocity should be as close as possible to the flame, in order to obtain a Helmholtz number $He$ less than $10^{-2}$. It is defined as $He := L/\lambda$, where $L$ is the distance between a mean flame position and the measurement point and $\lambda$ the wavelength of the acoustic perturbation. In the incompressible flow approximation used here the speed of sound, and accordingly the acoustic wavelength, are assumed to be infinitely large leading to $He = 0$.

Another aspect of the problem to link the flame TF and TM for small flames is related to the influence of the heat transfer between the flame and burner deck, which should be considered as well, just as for flat burner surface stabilized flames [6, 39]. However, for larger flames, when the role of the flame foot zone is less important the relation between TF and TM can be derived from the Rankine-Hugoniot jump conditions. Discussion of the link between the TF and TM can be found in [3], details of the derivation and further references are presented in [5]. The resulting formula shows the linear proportionality of the TF and $M_{uu}$, i.e.,

$$M_{uu} = \left(\frac{T_{burnt}}{T_{unburnt}} - 1\right) TF + 1. \tag{8}$$

For the flames studied here the comparison of the directly measured (simulated) element $M_{uu}$ of the TM (see solid lines in fig. 8 c,d) with the one calculated from the corresponding TF according to (8) (dotted lines in the same figures) shows perfect correlations even for the smallest tested flame size/flow velocity. This fact, on the one hand, once again confirms the consistency of the results obtained, and,
on the other hand, supports the conclusion that the flame to burner deck thermal interaction effects are not too substantial for the flames studied here. Certainly, this conclusion should not be extended to any small flames. Admittedly, the link between TF and TM for the case of small Bunsen flames is some 'superposition' of the Rankine-Hugoniot and the relation in [6].

If the hypothesis about the creation of traveling waves via the jet contraction-expansion modulation is valid, then at least a periodic in-out motion of gas along the burner plate should be observed. Figure 14 shows this type of motion near the flame foot and just beneath the slit. On the basis of the available data it is difficult to assess whether this effect is due to the acoustic jet or simply the presence of the flame. Clarification of this question requires a special numerical experiment with a carefully tuned model.

7. Conclusions

The objective of our experimental and numerical investigation of the thermo-acoustic behaviour of multiple-slit Bunsen flames is to assess the applicability of the thermal transfer function concept as well as to validate the experimental and simulation tools used.

The investigation shows that our (numerical) model is capable to reproduce all aspects observed in the experiments. A parametric study is carried out, where the mean flow velocity $V$, the equivalence ratio $\Phi$, the slit width $d$ and the distance between slits $l$ are varied. It is shown that the measured and computed flame transfer function are qualitatively the same. Furthermore, good qualitative agreement between measured and computed parameters is achieved.

For the flames studied here, relation (8), which is based on the Rankine-Hugoniot jump conditions, proved to accurately reproduce the relation between the thermo-acoustic TF and the element $M_{uu}$ of the acoustic TM.

The spatially resolved steady flame and the temporally resolved acoustically excited flame indicate that all essential effects of flame-acoustics interaction are captured by our model of incompressible flow,
with one-step chemical kinetics and a simple diffusion model. The comparison also gives additional confidence in the measurement techniques (i.e. chemiluminescence and heated wire techniques) used during the experiments.

Synergy of numerical simulation and experiments allows us to obtain insight into the physics of flame-acoustics interaction and, subsequently to formulate recommendations for analytical models. In particular, we recommend the following. First, to predict the time lag it is necessary to combine the kinematic model with a model for the creation of convective waves at the burner outlet. Second, to predict the TF saturation, the jagged behavior and, probably the effect of the TF gain overshoot, it is necessary to develop a method to describe the dynamics of the flame anchoring zone. Finally, inclusion of a model for the flame retroaction to the upstream flow and flame-to-flame interaction in the multiple flame configurations could improve the accuracy and capability of the kinematic model to predict the flame TF of practically interesting burners.

An additional result of our investigation is the clarification of the difficulties encountered in the description of flame thermo-acoustics interaction. One of the difficulties is how to relate the flame surface area oscillation to the heat release rate oscillation. This is partially due to the ambiguity of the flame anchoring point definition. Explanation of these issues requires an additional investigation.

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References


REFERENCES


REFERENCES


8. List of captions

**Figure 1**: Experimental flame TF gain (a) and phase (b); polar plot (c); real and imaginary parts (d) of the TF for a single Bunsen flame on a tube (lines 1 for $\Phi = 0.9$, $V = 150$ cm/s and $d = 1.0$ cm) and a flat flame stabilized on a burner with perforated brass deck (lines 2 for $\Phi = 0.9$, $V = 10$ cm/s and $T = 260^\circ$C).

**Figure 2**: Burner setup (a) and calculation domain (b).

**Figure 3**: Comparison of experimental and numerical flame TF gain (a) and phase (b) for the representative case, including a comparison of the steady numerical and experimental flame structure (c). Parameter values are: $\Phi = 0.8$, $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm.

**Figure 4**: Comparison of experimental flame TFs (a,b) with computed TFs (c,d) for varying velocity, i.e., $V = 50, 62.5, 75, 100, 125, 150$ cm/s. Other parameter values are: $\Phi = 0.8$, $d = 2.0$ mm and $l = 3.0$ mm; TF gain (top, a,c), TF phase (bottom b,d).

**Figure 5**: Comparison of measured flame TF (thick lines) with computed TF (thin lines) for varying $d$ and $l$ with $l/d = 1.5$, $\Phi = 0.8$ and $V = 100$ cm/s; TF gain (a), TF phase (b), experimental and numerical steady flame structure (c).

**Figure 6**: Comparison of measured flame TF (thick lines) with computed TF (thin lines) for varying $l$ with $\Phi = 0.8$, $V = 100$ cm/s and $d = 2.0$ mm; TF gain (a), TF phase (b).

**Figure 7**: Measured flame TF for varying equivalence ratio $\Phi$ with $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm; TF gain (a), TF phase (b).

**Figure 8**: Comparison of simulated flame TF (a,b) and TM (c,d) for the cases presented in Fig. 4.

**Figure 9**: Measured flow field of the stationary flame; raw seeded flow image (a), PIV restored streamwise (b) and transversal (c) component of the velocity; comparison of measured (line 1) and computed (line 2) distribution of streamwise velocity along the slit centerline (d). Parameter values are: $\Phi = 0.8$, 
$V = 100 \text{ cm/s}, d = 2.0 \text{ mm} \text{ and } l = 3.0 \text{ mm}$.

**Figure 10**: Comparison of measured (a,b,g) and computed (c,d,e,f,h) results for the steady flame. (a) chemiluminescence image; (b) velocity vector field and dilatation rate; (c) heat release rate and isotherm of 1200 K; (d) velocity vectors overlaid with temperature field; (e) temperature field; (f) pressure field; (g) and (h) close view of flame anchoring zone. Parameter values are: $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$ and $l = 3.0 \text{ mm}$ (representative case, steady flame).

**Figure 11**: Temporal evolution of the relative upstream $u'_\text{up}$ and downstream $u'_\text{d}$ velocity, the flame heat release rate $q'$ and surface area $A'$ subject to a perturbation with a frequency of 200 Hz. Parameter values are: $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$ and $l = 3.0 \text{ mm}$; the amplitude of excitation is 10% of the mean velocity.

**Figure 12**: Comparison of the heat release rate $TF$ with the flame surface area $TF$ computed using several definitions of the flame end point. Parameter values are: $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$ and $l = 3.0 \text{ mm}$.

**Figure 13**: Distribution of relative streamwise flow velocity along the slit centerline. Different lines correspond to four different phases of the applied perturbation, with frequency 200 Hz and relative amplitude 0.1. Parameter values are: $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$ and $l = 3.0 \text{ mm}$.

**Figure 14**: Temporal evolution of the relative streamwise velocity component velocity at the slit center point line, point C; transversal velocity component in the near upstream flow line, point U; transversal component in the near downstream location line, point D. Parameter values are: $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$, $l = 3.0 \text{ mm}$ and $f = 200 \text{ Hz}$.
9. Figures

Figure 1: Experimental flame TF gain (a) and phase (b); polar plot (c); real and imaginary parts (d) of the TF for a single Bunsen flame on a tube (lines 1 for $\Phi = 0.9$, $V = 150$ cm/s and $d = 1.0$ cm) and a flat flame stabilized on a burner with perforated brass deck (lines 2 for $\Phi = 0.9$, $V = 10$ cm/s and $T = 260^\circ$C).

Figure 2: Burner setup (a) and calculation domain (b).
Figure 3: Comparison of experimental and numerical flame TF gain (a) and phase (b) for the representative case, including a comparison of the steady numerical and experimental flame structure (c). Parameter values are: $\Phi = 0.8$, $V = 100\, \text{cm/s}$, $d = 2.0\, \text{mm}$ and $l = 3.0\, \text{mm}$.

Figure 4: Comparison of experimental flame TFs (a,b) with computed TFs (c,d) for varying velocity, i.e., $V = 50, 62.5, 75, 100, 125, 150\, \text{cm/s}$. Other parameter values are: $\Phi = 0.8$, $d = 2.0\, \text{mm}$ and $l = 3.0\, \text{mm}$; TF gain (top, a,c), TF phase (bottom b,d).
Figure 5: Comparison of measured flame TF (thick lines) with computed TF (thin lines) for varying $d$ and $l$ with $l/d = 1.5$, $\Phi = 0.8$ and $V = 100 \text{ cm/s}$; TF gain (a), TF phase (b), experimental and numerical steady flame structure (c).

Figure 6: Comparison of measured flame TF (thick lines) with computed TF (thin lines) for varying $l$ with $\Phi = 0.8$, $V = 100 \text{ cm/s}$ and $d = 2.0 \text{ mm}$; TF gain (a), TF phase (b).
Figure 7: Measured flame TF for varying equivalence ratio $\Phi$ with $V = 100\, \text{cm/s}$, $d = 2.0\, \text{mm}$ and $l = 3.0\, \text{mm}$; TF gain (a), TF phase (b).

Figure 8: Comparison of simulated flame TF (a,b) and TM (c,d) for the cases presented in Fig. 4.
Figure 9: Measured flow field of the stationary flame; raw seeded flow image (a), PIV restored streamwise (b) and transversal (c) component of the velocity; comparison of measured (line 1) and computed (line 2) distribution of streamwise velocity along the slit centerline (d). Parameter values are: $\Phi = 0.8$, $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm.
Figure 10: Comparison of measured (a,b,g) and computed (c,d,e,f,h) results for the steady flame. (a) chemiluminescence image; (b) velocity vector field and dilatation rate; (c) heat release rate and isotherm of 1200 K; (d) velocity vectors overlaid with temperature field; (e) temperature field; (f) pressure field; (g) and (h) close view of flame anchoring zone. Parameter values are: $\Phi = 0.8$, $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm (representative case, steady flame).

Figure 11: Temporal evolution of the relative upstream $u'_u$ and downstream $u'_d$ velocity, the flame heat release rate $q'$ and surface area $A'$ subject to a perturbation with a frequency of 200 Hz. Parameter values are: $\Phi = 0.8$, $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm; the amplitude of excitation is 10% of the mean velocity.
Figure 12: Comparison of the heat release rate TF with the flame surface area TF computed using several definitions of the flame end point. Parameter values are: $\Phi = 0.8$, $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm.

Figure 13: Distribution of relative streamwise flow velocity along the slit centerline. Different lines correspond to four different phases of the applied perturbation, with frequency 200 Hz and relative amplitude 0.1. Parameter values are: $\Phi = 0.8$, $V = 100$ cm/s, $d = 2.0$ mm and $l = 3.0$ mm.
Figure 14: Temporal evolution of the relative streamwise velocity component velocity at the slit center point line, point C; transversal velocity component in the near upstream flow line, point U; transversal component in the near downstream location line; point D. Parameter values are: $\Phi = 0.8$, $V = 100 \text{ cm/s}$, $d = 2.0 \text{ mm}$, $l = 3.0 \text{ mm}$ and $f = 200 \text{ Hz}$. 
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