Clip Correction in Wireless LAN receivers

Clip Correction in Wireless LAN Receivers

Ronald Rietman #1, Jean-Paul Linnartz #*2 Eric Penning de Vries #3

#*Philips Research Laboratories, Eindhoven, The Netherlands*

**Eindhoven University of Technology, Eindhoven, The Netherlands*

1ronald.rietman@philips.com

2j.p.linnartz@philips.com

3eric.penning.de.vries@philips.com

***Abstract*— OFDM signals suffer from a large Peak to Average Power Ratio, which requires large power back-offs in the transmit and receive chains. This paper presents a digital post-processing method that mitigates clipping by the analog-to-digital converter (ADC) in the receiver. Clipped peaks cause spurious signals on empty subcarriers, which can be used to eliminate clipping artifacts and to recover the original signal. Simulations show that a significant reduction of 3 dB in the headroom of the A/D converter (ADC) is possible, when an elaborate MMSE clip correction algorithm is used. A simple algorithm still allows for 1 dB reduction of the headroom. As the ADC is consuming an ever increasing fraction of the total receiver power, the results are believed to be relevant for low–power design of OFDM receivers, for instance to prolong battery life of laptops and other portable WLAN devices.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) modulation has a number of distinct advantages for transmission over multipath channels. However, the nearly Gaussian distribution of the OFDM transmit signal is seen as one of its main disadvantages. Practical systems require a margin of about 8 to 9 dB headroom between the average signal power and the clipping level of the transmitter and the receiver. Many papers have been written about the effect of the Peak to Average Power Ratio (PAPR) of OFDM, and on methods to mitigate this problem. Mostly, the high PAPR is seen as a problem for the linearity and the power consumption of the power amplifier at the transmitter. Several methods have been proposed to reduce the PAPR of the transmitted and many papers have been written on receiver algorithms for mitigating clipping noise, see e.g., [1].

Here we take a receiver-oriented direction. We develop a receiver algorithm that compensates modest clipping *by the ADC*. This approach is relevant for instance for wireless personal and local area network systems which transmit at gigabits per second over a very short distance, the power consumption of the ADC in the receiver may exceed the power consumed by the transmit power amplifier. If a moderate amount of clipping could be compensated digitally, the ADC's saturation level could be lowered, and hence its power consumption reduced. This has been a motivation for us to develop a signal processing algorithm that attempts to reconstruct clipped peaks of the OFDM signal. In order to effectively shape the emitter spectrum, all wireless OFDM standards use empty, i.e. unmodulated, subcarriers at the outskirts of their spectrum. We pursue the idea that clippig causes spurious components which occur as non-zero subcarrier signals. Hence these can assist the receiver to reconstruct the original (not clipped) signal components.

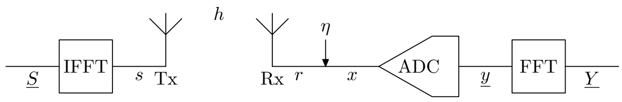

Fig. 1. System diagram at baseband level

In Section II we give a baseband model of an OFDM communication system and a statistical model of the OFDM signal at the receiver, including the receiver's ADC. Then, in Section III, we describe our clipping correction algorithms and evaluate their performance for an idealized DVB-H system. In Section IV we present the results of an 802.11a WLAN system simulation in which we included our simple clip correction algorithm.

II. Model of the communication system

We give a baseband model for the transmitter, the channel and the receiver of an OFDM system, including the receiver's ADC. The system diagram is given in fig. 1.

The number of subcarriers is denoted by N_{s}, which is assumed to be even. The transmitted OFDM symbol has duration $T_{\mathrm{symbol}} = 1/f_{\mathrm{s}}$, where f_{s} is the subcarrier spacing. The subcarriers are indexed from $-N_{\mathrm{s}}/2$ to $N_{\mathrm{s}}/2-1$, and subcarrier m has subcarrier frequency mf_{s}. In an OFDM symbol, N_{s} complex numbers $\underline{S} = (S_{-N_{\mathrm{s}}/2}, \ldots, S_{N_{\mathrm{s}}/2-1})$ are transmitted, by transmitting S_m on subcarrier m. Actually only a subset of the subcarriers is used: most standards require that the leftmost M_L and the rightmost M_R subcarriers be unmodulated, so that the corresponding $S_m = 0$. For IEEE 802.11a, $N_{\mathrm{s}} = 64$, $M_L = 6$ and $M_R = 5$. The set of empty subcarriers is denoted by $\mathcal{M}_0$, its size $|\mathcal{M}_0| = M_L + M_R$ is denoted by M.

The transmitter adds a cyclic prefix of duration T_{guard}, so the transmitted signal can be written in complex baseband representation as

$$s(t) = \sum_{m=-N_{\mathrm{s}}/2}^{N_{\mathrm{s}}/2-1} S_m e^{2\pi i m f_{\mathrm{s}} t}, \quad t \in [-T_{\mathrm{guard}}, T_{\mathrm{symbol}}). \qquad (1)$$

The signal passes through a channel with impulse response $h(\tau)$, where it is assumed that $h(\tau)$ is zero for τ outside of

$[0,T_{\text{guard}})$. The analog transmit and receive filters are included in $h(\tau)$. Noise from the receiver's low-noise amplifier and interference from adjacent channel users is denoted by η. The receiver ignores the signal received in the interval $[-T_{\text{guard}},0)$, since it suffers from inter-symbol interference, and samples and discretizes the in-phase and quadrature components of the complex signal in the interval $[0,T_{\text{symbol}})$ at the ADC. The complex baseband signal entering the ADC is $x(t) = \eta(t)+r(t)$ with

$$r(t) = \sum_{m=-N_s/2}^{N_s/2-1} R_m e^{2\pi i m f_s t}, \quad t \in [0,T_{\text{symbol}}) \tag{2}$$

where

$$R_m = S_m \int_0^{T_{\text{guard}}} h(\tau)e^{-2\pi i m f_s \tau}\, d\tau =: S_m H_m. \tag{3}$$

Assuming sampling at the Nyquist frequency, $T_{\text{sample}} = (N_s f_s)^{-1}$ and the n-th sample is $x_n = x(nT_{\text{sample}}) = r_n + \eta_n$, $n = 0,\ldots,N_s-1$. The set of all N_s samples is denoted by $\underline{x} = (x_0,\ldots,x_{N_s-1})$. The in-phase and quadrature components of each sample x_n are discretized. Discretization of the signal value w gives the best approximation in a discrete set D with L different quantisation levels, $D = \{d_1,\ldots,d_L\} \subset \mathbb{R}$ with $d_1 < d_2 < \ldots < d_L$, according to

$$q(w) = \arg\min_{d\in D} |d-w|.$$

After the ADC, the complex baseband representation of the sample is

$$y_n = q(\mathrm{Re}(x_n)) + i q(\mathrm{Im}(x_n)) =: Q(x_n), \tag{4}$$

or

$$\underline{y} = Q(\underline{x}), \tag{5}$$

where $\underline{y} = (y_0,\ldots,y_{N_s-1})$.

The discretized samples are Fourier transformed, which yields the received signal on the subcarriers:

$$Y_m = \frac{1}{N_s}\sum_{n=0}^{N_s-1} y_n e^{-2\pi i m n/N_s}, \quad m = -N_s/2,\ldots,N_s/2-1. \tag{6}$$

We now return to the quantization of the signal. We take $L > 2$ and a uniform level spacing: $d_\ell = (2\ell-L+1)C/(L-2)$, so that $(d_1+d_2)/2 = -C$, $q(w) = d_1$ if $w < -C$, and $q(w) = d_L$ if $w > C$. If the signal value w is less than C in absolute value, the discretization error $|q(w)-w|$ is at most equal to $C/(L-2)$, but if the input sample $w > C$, or $w < -C$ the error is unbounded and the ADC is said to *clip* the sample. For an unclipped signal, the conditional expected squared error $W_u = \mathbb{E}[|w-q(w)|^2 \,|\, |w| < C]$ is well approximated by $C^2/3(L-2)^2$, for a clipped signal the conditional expected squared error depends on the tail of the distribution of w.

If we ignore, for the moment, the correlation of different samples due to the empty subcarriers and, motivated by the central limit theorem which holds in the limit $N_s \to \infty$, assume that the $n = 2N_s$ samples are independently and identically distributed as a zero-mean Gaussian with variance $P/2$, then the probability that a given sample is clipped is given by

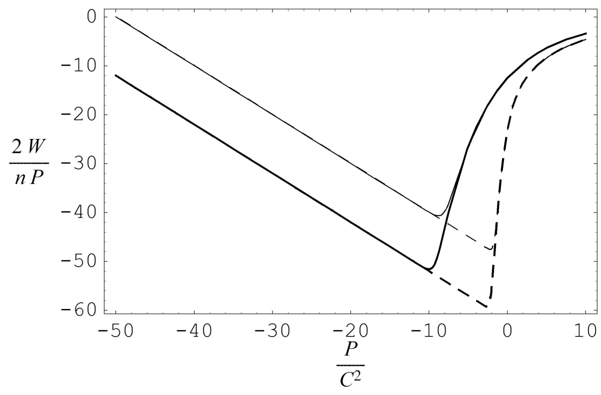

Fig. 2. The relative error due to clipping and quantization according to (10) (in dB) as a function of P/C^2 (in dB) for $n = 2N_s = 128$ and $(L = 256, m = 0)$ (thin line), $(L = 1024, m = 0)$ (thick line), $(L = 256, m = 20)$ (thin dashed line) and $(L = 1024, m = 20)$ (thick dashed line).

$$p_c = \frac{2}{\sqrt{\pi P}}\int_C^\infty \exp[-w^2/P]\,\mathrm{d}w = \mathrm{erfc}(C/\sqrt{P}). \tag{7}$$

The expected squared error, given that a sample is clipped, equals

$$W_c = \frac{\int_C^\infty (w-d_L)^2 \exp[-w^2/P]\,\mathrm{d}w}{\int_C^\infty \exp[-w^2/P]\mathrm{d}w}. \tag{8}$$

The expected total squared error is thus given by

$$\begin{aligned} W &= \sum_{k=0}^{n}\binom{n}{k}(1-p_c)^{n-k}p_c^k[kW_c+(n-k)W_u] \\ &= n[p_c W_c + (1-p_c)W_u]. \end{aligned} \tag{9}$$

Note that the relative errors W_u/P and W_c/P depend on the power P and clip level C only through the ratio P/C^2.

Now suppose that we have an algorthm that is able to fully mitigate the effect of up to m clippings, e.g., as in (14) in the next section. Then that algorithm reduces the expected error W to

$$W = nW_u + \sum_{k=m+1}^{n}\binom{n}{k}(1-p_c)^{n-k}p_c^k(k-m)(W_c-W_u). \tag{10}$$

As an example, we have plotted in fig. 2 the relative squared error per complex sample $2W/nP$ (in dB) as function of P/C^2 (in dB) for $n = 128$, $L \in \{2^8, 2^{10}\}$ and $m \in \{0, 20\}$. We want to operate the ADC in a regime where this relative error is well below the relative error due to thermal noise, for instance $2W/nP \ll -30\,\mathrm{dB}$. The graphs show that the lower limit of this regime depends on the number of quantization levels, L, whereas the upper limit appears to be independent of L. It also appears that a clip mitigation algorithm can increase the upper limit of the regime by an amount that does not depend on L, whereas the lower limit is unaffected. Futhermore the results suggest that a good clip correction algorithm may give a gain (at the upper limit) that is similar to the gain (at the lower limit) that would be obtained if the number of bits of the

ADC were increased by two. In other words: a good clipping algorithm may allow for a reduction in the power consumption of the ADC by 6 dB (by reducing the number of bits) without significantly lowering its dynamic range.

We assume from now on that L is large enough to consider only the error introduced by clipping, i.e.:

$$q(w) = \begin{cases} -C & \text{if } w \le -C \\ w & \text{if } -C < w < C \\ C & \text{if } w \ge C. \end{cases} \tag{11}$$

III. Clipping compensation algorithms

The most promising approach to clip correction is to find an estimate of the received signal, given that it is observed after being disturbed by additive thermal noise and the clipping ADC. This MMSE algorithm is worked out in [2]. In that letter we show that the MMSE algorithm has close to optimal performance, but is hard to implement in practice. Here we focus on a simple clip correction algorithm that can be implemented easily in the digital receiver. It estimates the degree of clipping independently for all subcarriers involved and weighs the measured out-of-band artifacts equally.

We assume that there is no noise, i.e., $\eta = 0$ so that any sample that is not clipped by the ADC is not distorted at all. Then we can write $r_n = y_n + c_n$ with

$$\begin{aligned} \mathrm{Re}(c_n) &= 0 \text{ if } |\mathrm{Re}(y_n)| < C, \\ \mathrm{Im}(c_n) &= 0 \text{ if } |\mathrm{Im}(y_n)| < C. \end{aligned} \tag{12}$$

Denoting the number of clippings by N_{c}, there are N_{c} non-zero parameters, one for each clipped real or imaginary part of a sample.

After the Fourier transform, we have that $R_m = Y_m + C_m$. Since the real and imaginary parts of R_m are zero if m corresponds to an empty subcarrier, we have that $C_m = -Y_m$ for $m \in \mathcal{M}_0$. These equations are equivalent to $2M$ equations for N_{c} unknown parameters, which can be written in matrix form as

$$\underline{v} = A\underline{u}, \tag{13}$$

where $\underline{v}$ is a known $2M$-vector with the real and imaginary parts of $-Y_m$ for the subcarriers where $R_m = 0$, $\underline{u}$ is an N_{c}-vector with the unknown values of $\mathrm{Re}(c_n)$ or $\mathrm{Im}(c_n)$ at a clipped sample and A is an $2M \times N_{\mathrm{c}}$-matrix of which the elements are the appropriate real and imaginary parts of the Fourier-transform matrix. If $N_{\mathrm{c}} \le 2M$, $\underline{u}$ can be uniquely determined by solving any subset of N_{c} equations, or by

$$\underline{u} = (A^T A)^{-1} A^T \underline{v}. \tag{14}$$

If $N_{\mathrm{c}} > 2M$, $A^T A$ has $N_{\mathrm{c}} - 2M$ eigenvalues equal to zero, so $\underline{u}$ cannot be determined uniquely and this equation-solving clip correction algorithm doesn't work. The above results suggest that each empty subcarrier allows for the correction of two clippings, so that at most $2M$ clippings can be corrected for.

We now make an approximation that simplifies the clip correction significantly. Each of the diagonal elements of $A^T A$ is equal to M/N_{s}^2. The off-diagonal elements are *partial* correlations of an integer number of rotations of complex exponentials, so these are usually non-zero. However, off-diagonal components are smaller than the diagonal elements. Approximating $A^T A$ by its diagonal leads to the simpler approximation

$$\tilde{u} = \frac{(N_{\mathrm{s}})^2}{M} A^T v. \tag{15}$$

The corresponding clip-corrected complex-valued signal in the frequency domain is given by

$$\tilde{R} = Y - \frac{N_{\mathrm{s}}}{M} F P_{\mathrm{clip},\underline{y}} F^{-1} P_{\mathrm{empty}} Y, \tag{16}$$

where F is the Fourier transform matrix, with elements $F_{mn} = \exp(-2\pi i mn/N_{\mathrm{s}})/N_{\mathrm{s}}$, F^{-1} its inverse, P_{empty} projects onto the M empty subcarriers, i.e., $(P_{\mathrm{empty}}Y)_m = Y_m$ if $m \in \mathcal{M}$ and zero otherwise, and $P_{\mathrm{clip},\underline{y}}$ projects the real and imaginary parts of its argument onto the positions where the corresponding real or imaginary parts of $\underline{y}$ are clipped, i.e.,

$$\begin{aligned} \mathrm{Re}(P_{\mathrm{clip},\underline{y}}x)_n &= \begin{cases} \mathrm{Re}(x_n) & \text{if } |\mathrm{Re}(y_n)| = C \\ 0 & \text{otherwise} \end{cases} \\ \mathrm{Im}(P_{\mathrm{clip},\underline{y}}x)_n &= \begin{cases} \mathrm{Im}(x_n) & \text{if } |\mathrm{Im}(y_n)| = C \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{17}$$

Intuitively, this algorithm is equivalent to making the approximation that the out-of-band clip artifacts u for every clip location are orthogonal, so that clipping can be corrected by inverting the clip artifacts one by one. Note that this algorithm only uses simple digital processing and is of order $O(N_{\mathrm{s}} \log(N_{\mathrm{s}}))$,

IV. Clip correction in an IEEE 802.11a simulation

In order to evaluate the effectiveness of the simple clip correction algorithm in a somewhat more realistic scenario, we introduced the correction (16) our WLAN simulation code. This code simulates a complete 802.11a WLAN system, with optional MIMO extensions. The digital processing chain of the receiver is shown in fig. 3.

We have simulated a scenatio where clipping is the dominant source of errors, therefore we assume perfect synchronisation of transmitter and receiver, perfect channel estimation at the receiver and an ADC with enough bits so that only clipping needs to be considered. The transmitter sends 8000 bit packets at the highest data rate of 54 Mb/s, the channel is modeled as having an exponential delay profile with RMS delay of 50 ns. The automatic gain controller in the receiver scales the received signal power to $10^{-\mathrm{BackOffdB}/10}$, where BackOffdB is the back-off in decibel. Fig. 4 show the resulting packet error rate (PER) versus the SNR for various values of the back-off, both with and without clip correction. The PER was obtained by simulating random channel and noise realizations until 100 packet errors were observed, hence the estimates of PER are accurate to about 10% at low PER, which explains the non-smoothness of the graphs al high SNR. Nonetheless, these graphs suggest that the simple clip correction algorithm allows for a reduction of back-off by about 1 dB at a SNR of 30 dB while delivering the same PER.

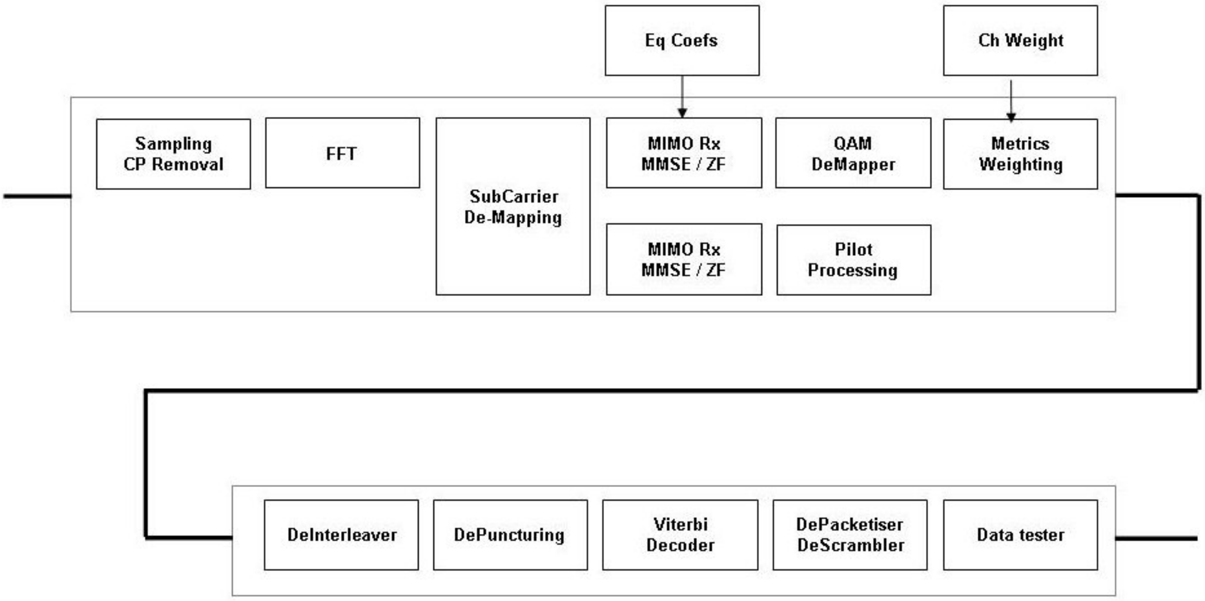

Fig. 3. Block diagram of the digital part of the WLAN receiver simulation chain.

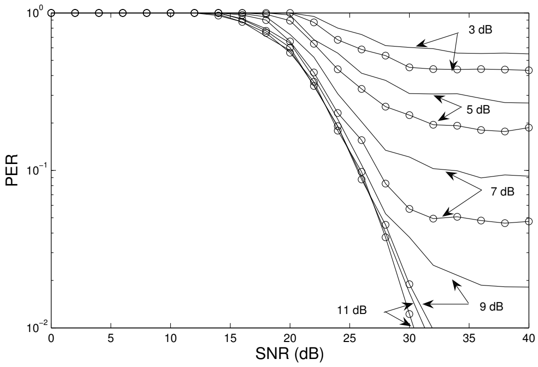

Fig. 4. Packet error rate (PER) versus SNR for an 802.11a WLAN system simulation for varioous values of the AGC back-off from 3 to 11 dB, without using any clip correction (lines) and using the simple clip correction algorithm (lines with circles).

V. Concluding Remarks

The advantage of a clip compensation algorithm is that the design specifications of the ADC can be relaxed.

We found that for systems operating at high SNR the clipping threshold can be reduced by up to 3 dB, when an advanced clip correction algorithm is used. So for a specified signal-to-quantization noise, one may use only 70% of the number of quantization steps needed hitherto. As the power consumption of an ADC is proportional to the number of quantization steps, this corresponds to a reduction of power consumption by the ADC by 30%. Nonetheless our solution demands more digital signal processing operations. We have developed a much simpler algorithm that gives less gain, about 1 dB, but does so for very little signal processing cost. Our proposed algorithms do not require any modification to existing OFDM standards. Its effectiveness has been tested in our simulation chain for 802.11a WLAN.

References

[1] D. Kim and G. L. Stüber, "Clipping noise mitigation for OFDM by decision-aided reconstruction," *IEEE Communications Letters*, vol. 3, no. 1, pp. 4–6, Jan. 1999.
[2] R. Rietman and J. P. Linnartz, "Peak restoration in OFDM receiver with clipping A/D converter," to appear in IEEE Trans. Wireless Comm.