Combating media noise for high-density optical recording

Citation for published version (APA):

DOI:
10.1109/ICC.2007.1036

Document status and date:
Published: 01/01/2007

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.
Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.

Download date: 05. Aug. 2019
Combating Media Noise for High-Density Optical Recording

Kui Cai
Data Storage Institute,
Singapore 117608

Yong Ning Foo
Singapore-MIT Alliance,
National Univ. of Singapore,
Singapore 117576

J.W.M. Bergmans
Elect. Engg. Faculty
Technical Univ. of Eindhoven,
The Netherlands

Abstract—Media noise is the dominant noise that severely degrades the channel detector’s performance for high-density optical recording. In this paper, we first propose a simple but general channel model for rewritable optical recording systems with media noise. We further develop various detection approaches to combat media noise. In particular, two novel modifications to the channel detector are proposed: i) monic constrained minimum mean square error (MMSE) based equalization with media noise considerations, and ii) a modified Viterbi detector (VD) accounting for data-dependent noise variance. The proposed modifications provide significant performance gain over the conventional detection approaches that are designed for additive, white and Gaussian noise (AWGN), without increasing the size of the VD trellis. Furthermore, a new constrained parity-check (PC) code and a data-dependent post-processing scheme are proposed based on the modified detection approaches. Simulation results show that compared to the system designed without considering media noise and without PC codes, the proposed scheme achieves an overall performance gain of more than 11 dB at high media noise levels.

I. INTRODUCTION

At high recording densities, data-dependent media noise becomes dominant in optical recording systems [1]. Being correlated, data-dependent, and non-stationary in nature, the media noise seriously degrades the performance of channel detectors and post-processors that are designed for stationary additive, white and Gaussian noise (AWGN). The media noise is a physical property of the storage media, and hence it exhibits different characteristics for different systems. For example, in magnetic recording systems, the media noise is mainly caused by transition jitter, which causes the edges of magnetic transitions to be randomly displaced [2]. There have been several channel models that incorporate media noise for magnetic recording, such as the position-jitter and width-variation model [2]. However, for optical recording, except for the transition noise model proposed in [3] for magnetooptic (MO) systems, no other channel models have been developed.

Several approaches have been proposed in the literature to deal with the degradation in the detector’s performance due to media noise. In [4], the effect of various constraints on the partial response (PR) target is studied for channels with jitter media noise, and the superiority of the monic constraint over other constraints is reported. The maximum-likelihood sequence detector (MLSD) for media noise channels is derived in [5], under the assumption that the noise can be modeled as a data-dependent finite-order Markov process. In [6], the same detector structure has been derived from the data-dependent noise prediction viewpoint. The data-dependent branch metric calculation has also been used by the soft-output post-processor proposed in [7], to correct dominant error events at the output of a conventional Viterbi detector (VD). These methods have been shown to significantly improve the performance, however, at the expense of a significant increase of computational complexity. This is because the conditional statistics of media noise depend on a large span of the input data, which leads a considerable increase in the size of VD trellis. Furthermore, most of the reported detection approaches are proposed for magnetic recording systems, and very little work has been done for the design of detectors for optical recording channels with media noise, whose channel characteristics and code constraints are significantly different from those of magnetic recording systems. There is also no reported work on parity-check (PC) coding and post-processing for optical recording channels with media noise.

In this paper, a simple but general channel model is first presented to incorporate media noise for rewritable optical recording systems. An analytical approach is then proposed for jointly designing the PR target and equalizer, according to the minimum mean square error (MMSE) criterion with monic constraint [4], taking into account the data-dependence property of the media noise. A modified VD is further proposed which changes the noise variance in its branch metrics depending on the input data. Moreover, to further improve the system’s performance with media noise, a new constrained PC code is designed for error detection, and a data-dependent post-processing scheme is developed for error correction.

II. SYSTEM MODELING

A. Media Noise in Rewritable Optical Recording Systems

In rewritable optical recording systems, phase changes due to local differences in material structure are used to represent information [8]. The active layer of the disc is first initialized to the crystalline phase. During the writing process, a high power laser beam heats the material locally and makes the material melt, and the amorphous marks are formed. Previously written amorphous marks are erased by heating the material using a medium power laser. This converts the material back to the crystalline state, and enables direct overwrite. The noise in
this type of medium is mainly caused by scattering of light at
grain boundaries and variation of material refractive index due
to the random orientation of crystallites. Experimental study
shows that these noises cause fluctuations in the reflectivity of
the disc [1], which further lead to random fluctuations in the
amplitude of the replay signal (or photodetector voltage).

Moreover, the media noise in the crystalline state has been
found to be significantly higher than that in the amorphous
state [1]. This is due to the fact that the polycrystalline grain
noise is absent in the amorphous marks, and the influence of
crystal orientation variations on the disc reflectivity is more
significant in the crystalline state than that in the amorphous
state [1]. Therefore, the media noise for rewritable systems
is mainly caused by random fluctuations in the amplitude of
the replay signal from the crystalline state. The fluctuations
of the replay signal from the amorphous state are relatively
weak, and hence can be ignored. In this paper, we focus on
combating this type of media noise. In the study, without loss
of generality, we associate the crystalline state with a channel
bit of \( a_k = +1 \) and the amorphous state with \( a_k = -1 \).

### B. Channel Modeling for Media Noise

For simplicity we first consider only the erased track, which
corresponds to the all ‘+1’ data sequence. We will then extend
the study to the recorded track (i.e. arbitrary data sequence).
Fig. 1 depicts the continuous-time channel model with media
noise, for an erased track. The user bits, at the rate \( 1/T_u \)
bits/second, are coded by a modulation encoder with code
rate \( R_c \), resulting in the channel bits \( a_k \in \{-1, +1\} \) at the
rate \( 1/T \). Thus, \( T = RT_u \). The quantities \( c(t) \) and \( f(t) \)
model the linear pulse modulator and impulse response of the
channel, respectively. The linear pulse modulator transforms
the channel coded data sequence \( a_k \) into a binary write signal
\( s(t) \). The signal component \( x(t) \) of the channel readback signal
\( z(t) \) is generated according to \( x(t) = (s \otimes f)(t) \), with ‘\( \otimes \)’
denoting convolution.

The channel symbol response \( h(t) \) is obtained by convolv-
ing \( f(t) \) with \( c(t) \). In this study, it is assumed that the optical
read-out is linear and a generalized Braat-Hopkins model [9]
is used to describe the channel. The Fourier transform of the
channel symbol response is given by

\[
H(\Omega) = \left\{ \begin{array}{ll}
\frac{2RT_u \sin(\omega t)}{\pi} & |\Omega| < \frac{1}{RT_u} \\
\arccos(\frac{\Omega}{RT_u}) - \sqrt{1 - \left(\frac{\Omega}{RT_u}\right)^2} & \text{for } |\Omega/RT_u| < 1, \\
0 & \text{for } |\Omega/RT_u| \geq 1,
\end{array} \right.
\]

where \( \Omega \) is the frequency normalized by the channel bit
rate. The quantity \( \Omega_u = f_c T_u \), which is the optical cut-off
frequency \( f_c \), normalized by user bit rate \( 1/T_u \), is a measure
of the recording density. For an optical recording system using
a laser diode with wavelength \( \lambda \) and a lens with numerical
aperture \( NA \), the normalized cut-off frequency is given by
\( \Omega_u = \frac{2N\lambda}{L_u} \), where \( L_u \) is the spatial length of one user bit.
For the blue-ray disc (BD) systems using the rate 2/3 17PP code
[8] with the minimum runlength constraint of \( d = 1 \), and with
\( \lambda=405 \text{ nm}, \text{NA}=0.85 \) and \( L_u=112.5 \text{ nm}, \) we get \( \Omega_u \approx 0.5 \). In

![Fig. 1. Continuous-time model of the optical recording channel with media
noise, for an erased track.](image)

In this work, a cut-off frequency of \( \Omega_u = 0.375 \) is considered,
which corresponds to recording systems with a high density,
according to current standards [8].

For an all-polycrystalline track, the continuous-time noise
caused by fluctuations of the reflectivity of the disc can be
represented as a stationary random signal \( u(t) \). We assume
that \( u(t) \) is white Gaussian with power spectrum density \( U_0 \).
The value of \( U_0 \) depends on various properties of the laser
and system, such as the laser wavelength \( \lambda \), the numerical
aperture \( NA \), the effective incident laser power, as well as the
optical efficiency of the system [1]. The noise component \( y(t) \)
of the channel readback signal \( z(t) \) is obtained by filtering
\( u(t) \) with the impulse response \( f(t) \) of the channel. This is
because media noise is generated at the disc and read circuit
side, rather than the write circuit side. Note that for the all ‘+1’
data sequence, \( y(t) \) is stationary. Furthermore, for given media
and system, \( y(t) \) is independent of the channel bit interval \( T \).

Fig. 2 shows the discrete-time counterpart of the channel
model of Fig. 1. It is obtained as follows. Since \( x(t) \) has no
excess bandwidth [8], we can apply it to an ideal low-pass filter
of bandwidth \( 1/(2T) \) and a symbol rate sampler to obtain an
equivalent discrete-time sequence \( x_k \). Hence, we get the lower
path of Fig. 2, with \( h_k \) being the \( T \)-spaced sampled version
of \( h(t) \). Furthermore, the discrete-time Fourier transform of \( h_k \)
is given by

\[
H_d(e^{j2\pi \Omega}) = \frac{1}{T} \sum_{n=-\infty}^{n=-\infty} C(\Omega + n)F(\Omega + n),
\]

where \( C(\Omega) \) and \( F(\Omega) \) are the Fourier transforms of \( c(t) \) and
\( f(t) \), respectively. In the absence of excess bandwidth, \( c(t) \)
(2) simplifies to \( H_d(e^{j2\pi \Omega}) = \frac{1}{T}C(\Omega)F(\Omega) \), for \( |\Omega| \leq 0.5 \).

For recording densities of practical interest, \( F(\Omega) \) has no excess
bandwidth, and \( C(\Omega) \approx T \) for \( \Omega \) such that \( F(\Omega) \) is significant.

Hence we have the approximation

\[
H_d(e^{j2\pi \Omega}) \approx F(\Omega).
\]

Similarly, by sampling \( y(t) \) at rate \( 1/T \), we obtain the
sampled noise sequence \( y_k \), whose power spectrum density is
given by

\[
P_d(e^{j2\pi \Omega}) = \frac{1}{T} \sum_{n=-\infty}^{n=-\infty} U_0|F(\Omega + n)|^2.
\]
In the case of no excess bandwidth, (4) simplifies to

\[ P_d(e^{j2\pi\Omega}) = \frac{U_0}{T} |F(\Omega)|^2, \quad \text{for } |\Omega| \leq 0.5. \quad (5) \]

In the discrete-time channel model, it is desirable to model \( y_k \) by passing a white Gaussian discrete-time random sequence \( \varepsilon_k \) through a linear filter with impulse response \( q_k \). In that case, \( y_k \) has power spectrum density

\[ P_d(e^{j2\pi\Omega}) = \sigma^2_{\varepsilon}\left|Q_d(e^{j2\pi\Omega})\right|^2, \quad (6) \]

where \( \sigma^2_{\varepsilon} \) is the variance of \( \varepsilon_k \), and \( Q_d(e^{j2\pi\Omega}) \) is the Fourier transform of \( q_k \). A logical way to equate (5) and (6) is to choose

\[ \sigma^2_{\varepsilon} = \frac{U_0}{T}, \quad \text{and} \quad Q_d(e^{j2\pi\Omega}) = F(\Omega). \quad (7) \]

In view of (3), we obtain the approximate equivalent discrete-time model for the media noise part, depicted by the upper path of Fig. 2.

Let us now consider the recorded track, which corresponds to an arbitrary data sequence. Since media noise is mainly caused by fluctuations in the reflectivity of the crystalline marks (i.e. '+'1's), and no such fluctuations arise from the amorphous marks (i.e. '-'1's), we can extend the above derived model to arbitrary data. The resulting discrete-time channel model with media noise and AWGN is shown in Fig. 3. In the figure, \( n_k \) is a discrete-time AWGN with power spectrum density (two sided) \( N_0/2 \). As illustrated by Fig. 3, we model the fluctuations in the reflectivity of the disc as an additive white Gaussian random process \( \{\varepsilon_k\} \) with variance \( \sigma^2_{\varepsilon} \). We model the data-dependent nature of the media noise by multiplying \( \varepsilon_k \) with \( \frac{1+\alpha_k}{2} \). That is, the input data \( a_k \) is additively corrupted by \( \varepsilon_k \) only when \( a_k = +1 \), which corresponds to the crystalline state. The readback signal can be written as

\[ r_k = \sum_{i=-\infty}^{\infty} (a_i + \frac{1+a_i}{2}\varepsilon_i)h_{k-i} + n_k \quad (8) \]

with \( m_k = \frac{1+\alpha_k}{2}\varepsilon_k \). Therefore, the media noise results in an additive signal-dependent noise component \( \sum_{i=-\infty}^{\infty} m_i h_{k-i} \).

The variance \( \sigma^2_{\varepsilon} \) of \( \varepsilon_k \), according to (7), is given by \( \sigma^2_{\varepsilon} = \frac{U_0}{T} \). At the user side, the corresponding noise power in the user bandwidth is given by \( \sigma^2_{\varepsilon} = \frac{U_0}{T} \square \). With \( T = R T_u \), we get \( \sigma^2_{\varepsilon} = \frac{\sigma^2_{\varepsilon}}{R} \). Moreover, we wish to further normalize \( \varepsilon_k \) into a dimensionless quantity. For simplicity, we choose to specify \( \sigma_{\varepsilon} \) with respect to the amplitude of the channel input data, i.e. \( |a_k| = 1 \). We thus get

\[ \tilde{\sigma}_{\varepsilon} = \frac{\sigma_{\varepsilon}}{|a_k|} = \frac{\sigma}{\varepsilon}. \quad (9) \]

At the channel side, we have \( \Delta_{\varepsilon} = \frac{\sigma_{\varepsilon}}{|a_k|} = \tilde{\Delta}_{\varepsilon}/\sqrt{R} \).

For the channel with both media noise and AWGN, we use the signal-to-noise-ratio (SNR) definition that includes the electronics noise only. The influence of media noise is indicated separately by the quantity \( \tilde{\Delta}_\varepsilon \). Similar definitions can be found in [10]. Thus, the variance \( \sigma^2 \) of \( n_k \), is determined by the user SNR defined as \( \text{SNR}_u(dB) = 10 \log_{10} \left( \frac{\sum h_k^2}{\sigma^2} \right) \) and

\[ \sigma^2 = \frac{1}{R} \sigma^2_u, \quad \text{with} \quad \sigma^2_u \text{ being the power of electronics noise in the user bandwidth,} \quad h_{ku} \text{ is the channel symbol response for} \quad R = 1 \quad [9]. \]

When studying the performance over different user densities, the reference signal power in the user SNR needs to be independent of density. For this, \( h_{ku} \) is evaluated for a particular user density, e.g. \( \Omega_u = 0.33 \), which is independent of the densities at which the channel and receiver are tested.

In this work, we take two different normalized variances of disc reflectivity fluctuations, i.e. \( \tilde{\Delta}_\varepsilon = 14\% \) and \( \Delta_{\varepsilon} = 17\% \), as examples to study the detector’s performance. It can be verified that in both cases, the value of \( \sigma^2_{\varepsilon} \) is larger than that of \( \sigma^2_u \) considered in this work. In particular, with \( \Delta_{\varepsilon} = 17\% \), the media noise is significantly prominent. This reflects the characteristics of high-density optical recording systems [1].

**III. DETECTION FOR MEDIA NOISE CHANNELS**

**A. Monic Constrained MMSE Equalization**

In this section, we present an analytical approach for jointly designing the channel PR target and equalizer, based on the media noise model presented in Section II-B. We choose to use the monic constrained MMSE design criterion, since the monic constraint has good noise whitening ability.

Fig. 4 shows the block diagram of an optical recording channel with media noise \( \beta_k \) and electronic noise \( n_k \), and the partial response maximum-likelihood (PRML) receiver. The media noise \( \beta_k \) can be given as

\[ \beta_k = \sum_{i=0}^{L_u-1} h_i m_{k-i} = h^T m_k, \quad (10) \]
where $h = [h_0, h_1, \ldots, h_{L_h-1}]^T$ is the channel symbol response of length $L_h$, and $m_k = [m_k, m_{k-1}, \ldots, m_{k-L_m+1}]^T$. Since $a_k$ and $\varepsilon_k$ are mutually uncorrelated, the autocorrelation of $m_k$ can be given by

$$E[m_k m_l^T] = E\left[ \frac{1 + a_k}{2} \varepsilon_k \right] \left[ \frac{1 + a_l}{2} \varepsilon_l \right]\begin{cases} \frac{1}{2} \sigma^2_e, & k = l \\ 0, & k \neq l. \end{cases}$$

The equalizer output $q_k$, PR target output $c_k$, and error signal $e_k$ are given by $q_k = \sum_{i=0}^{L_q-1} w_i r_{k-i} = w^T r_k$, $c_k = \sum_{i=0}^{L_q-1} g_i a_{k-i} = g^T a_k$, and $e_k = q_k + m_0 - c_k = w^T r_{k+L_m-1} - c_k$, where $r_k$ is the equalizer input samples given by (8), $g = [g_0, g_1, \ldots, g_{L_q-1}]^T$ is the PR target of length $L_q$, $w = [w_0, w_1, \ldots, w_{L_w-1}]^T$ is the equalizer of length $L_w$, $a_k = [a_k, a_{k-1}, \ldots, a_{k-L_m+1}]^T$, and $m_0$ is the delay introduced by the channel and equalizer. The mean square error (MSE) is thus given by

$$E[e_k^2] = w^T R w + g^T A g - 2 w^T P g,$$ \hspace{1cm} (11)

where $R = E[r_{k+m_0} r_{k+m_0}^T]$ is a $L_w \times L_w$ autocorrelation matrix of $r_k$, $A = E[a_k a_k^T]$ is a $L_g \times L_g$ autocorrelation matrix of $a_k$, and $P = E[r_{k+m_0} a_k^T]$ is a $L_w \times L_g$ cross-correlation matrix between $r_k$ and $a_k$. Since $a_k$, $\beta_k$, and $\eta_k$ are mutually uncorrelated, the $(i,j)^{th}$ elements of these matrices can be calculated as

$$R_{i,j} = E[r_{k+m_0-i} r_{k+m_0-j}] = E[r_{k-i} r_{k-j}] = \begin{cases} h^T E[a_{k-i} a_{k-j}^T] h + h^T E[m_{k-i} m_{k-j}^T] & \text{if } i \neq j, \\ E[n_{k-i} n_{k-j}], & \text{if } i = j, \end{cases}$$

$$A_{i,j} = E[a_{k-i} a_{k-j}],$$

$$P_{i,j} = E[r_{k+m_0-i} a_{k-j}], = E[h^T a_{k-m_0-i} + \beta_k + n_{k+m_0-i}] a_{k-j} = h^T E[a_{k+m_0-i} a_{k-j}].$$

By imposing the monic constraint on $g$, and minimizing the $MSE$ in (11), we obtain the optimum PR target and equalizer as $g_{opt} = (A - P^T R^{-1} P^T)^{-1}$, $w_{opt} = R^{-1} P g_{opt}$, and $\xi_{min} = \frac{1}{t^T (A - P^T R^{-1} P^T)^{-1} t},$ where $i = [1, 0, \ldots, 0]^T$ is a $L_g \times 1$ vector, and $\xi_{min}$ is the resulting minimum $MSE$.

**B. Viterbi Detection with Data-Dependent Noise Variance**

To further account for the data-dependent nature of the media noise, while keeping the increase in complexity minimal, in this section, we propose a modified VD with data-dependent noise variance. The principle of the modified VD is as follows. We assume that the total noise $e_k$ at the input of VD, which consists of the equalized electronics noise, equalized media noise and residual intersymbol interference (ISI), to be a sequence of independent Gaussian random variables. Furthermore, the variance $\sigma^2_e(d_k)$ of each $e_k$ depends on the input data pattern $d_k = [a_{k+L_1}, a_{k+L_2}, \ldots, a_{k+L_1}]^T$, with $L_1$ and $L_2$ being nonnegative integers. Therefore, the joint probability density function (pdf) of the VD input samples $q = [q_1, q_2, \ldots, q_{N+L_1}]^T$, conditioned on the input bit sequence $a = [a_1, a_2, \ldots, a_N]^T$, is given by

$$p_q(q|a) = \prod_{k=1}^{N+L_1} \frac{1}{\sqrt{2\pi \sigma_e(d_k)}} \exp \left( -\frac{(q_k - \bar{q}_k)^2}{2\sigma^2_e(d_k)} \right),$$ \hspace{1cm} (12)

where $\bar{q}_k = \sum_{i=0}^{L_q-1} g_i a_{k-i}$ is the reconstruction of the signal part of $q_k$ based on the assumed data sequence $a$. According to the MLSD [11], we need to find a particular $a$ such that $p_q(q|a)$ is maximized over all possible data sequences. Maximizing (12) is equivalent to minimizing

$$\sum_{k=1}^{N+L_1} \left( \ln \sigma_e(d_k) + \frac{(q_k - \bar{q}_k)^2}{2\sigma^2_e(d_k)} \right).$$

Thus, the branch metric of the modified VD is given by $\ln \sigma_e(d_k) + \frac{(q_k - \bar{q}_k)^2}{2\sigma^2_e(d_k)}$, instead of $(q_k - \bar{q}_k)^2$ in the conventional VD.

To implement the modified VD, the data-dependent noise variance $\sigma^2_e(d_k)$ needs to be computed. For the system shown in Fig. 4, we have

$$e_k = q_k + m_0 - c_k = w^T r_{k+L_m-1} + g^T a_k - c_k = w^T r_{k+L_m-1} + g^T a_k - \sum_{i=0}^{L_q-1} g_i a_{k-i},$$

where $p_k = \sum_{i=0}^{L_q-1} h_i w_{k-i}$ is the equalized channel response with length $L_p = L_h + L_w - 1$, and $v_k = p_k - g_k$ is the residual ISI channel with

$$g_k = \begin{cases} 0, & 0 \leq k < m_0 - 1 \\ g_k, & m_0 \leq k \leq m_0 + L_q - 1 \\ 0, & m_0 + L_q \leq k \leq L_p - 1. \end{cases}$$

Therefore, the noise variance is given by

$$\sigma^2_e(d_k) = E[e_k^2|d_k] - E[e_k^2],$$ \hspace{1cm} (13)

where

$$E[e_k^2|d_k] = E\left[ \left( p^T m_{k+m_0} + w^T n_{k+m_0} + v^T a_{k+m_0} \right)^2 \right] d_k = p^T E[m_{k+m_0} m_{k+m_0}^T] p + w^T E[n_{k+m_0} n_{k+m_0}^T] w + v^T E[a_{k+m_0} a_{k+m_0}^T] v,$$

$$E[e_k|d_k] = E\left[ \left( p^T m_{k+m_0} + w^T n_{k+m_0} + v^T a_{k+m_0} \right)^2 \right] d_k = v^T E[a_{k+m_0} d_k].$$
All the above conditional expectations can be calculated from the knowledge of the data pattern \(d_k\), the autocorrelation function of the input data \(a_k\), and the autocorrelation and conditional autocorrelation functions of the media noise \(m_k\). In particular, the conditional autocorrelation of \(E[m_i m_j | a_i, a_j]\), with \(k - I_1 \leq i, j \leq k + I_2\), is given by

\[
E[m_i m_j | a_i, a_j] = E \left[ \left( \frac{1 + a_i}{2} \right) \left( \frac{1 + a_j}{2} \right) \varepsilon_j \right] | a_i, a_j
\]

\[= \begin{cases} 
\frac{1}{2} (1 + a_i)^2 \sigma^2_{\varepsilon}, & i = j, \\
0, & i \neq j.
\end{cases}
\]

The choice of \(I_1\) and \(I_2\) is a trade-off between the performance gain and computation complexity. In this work, we set \(I_1 = L_q - 1\) and \(I_2 = 0\). This ensures that the modified branch-metric can be easily incorporated into conventional VD without increasing the trellis size.

IV. PARITY-CHECK CODES AND POST-PROCESSING FOR MEDIA NOISE CHANNELS

In this section, we develop PC codes and post-processors for media noise channels, based on the equalization and modified VD approach proposed in Section III.

A. A New Constrained Parity-Check Code

Prior to designing PC codes and post-processors, the dominant error events at the output of VD need to be determined. We perform the simulations for \(\Delta_e = 14\%\) and \(\Delta_v = 17\%\), with \(\Omega_u = 0.375\). For the modified VD approach, the dominant error events turn out to be \(\pm\{2\}, \pm\{2, 0, -2\}\) and \(\pm\{2, 0, -2, 0, 2, 0, -2\}\), which can all be detected by a cyclic redundancy check (CRC) code with a generator polynomial \(g(x) = 1 + x + x^2\). By using the code design method proposed in [12], we design a new rate 135/198 code with generator polynomial \(g(x) = 1 + x + x^2\). The new code includes two component codes. A rate 9/13 \(d = 1\) code with 5 states proposed in [13] is used as the normal constrained (NC) code, since its code rate is 3.85% higher than that of the rate 2/3 17PP code used in BD. A new rate 9/16 \(d = 1\) code with 5 states is designed as the parity-related constrained (PRC) code, which requires only 1.5 channel bit per parity bit with respect to the rate 2/3 \(d = 1\) code. The overall efficiency of the new constrained PC code is 99.65%, with respect to the capacity proposed in [12].

B. Data-Dependent Post-Processing

For channels with AWGN only, the PC-based post-processing can be implemented by using a simple matched-filtering type post-processor [14]. However, this post-processor is no longer optimum in the presence of media noise. In this section, we propose a maximum-likelihood (ML) based post-processing scheme to account for the data-dependence of the noise variance.

The ML based decision rule for post-processing can be expressed as [11]

\[
\hat{e} = \arg \max_{e_i^j} p(q | e_i^j, \hat{a}),
\]

where \(p(q | e_i^j, \hat{a})\) is the pdf of the VD input samples \(q\) conditioned on the detected bits \(\hat{a} = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_N]^T\) and an assumed error event \(e_i^j\) of length \(L_q\) and type \(j\), starting at position \(i\), with \(i = 1, 2, \ldots, N - L_q + 1\). Using the same assumptions as in Section III-B for the total noise \(e_k\) at the VD input, we can rewrite (14) as

\[
\hat{e} = \arg \max_{e_i^j} \frac{N + L_q + L_q^2 - 2}{2} \frac{1}{2 \pi \sigma_e} \exp \left( -\frac{(q_k - \hat{q}_i^j)^2}{2 \sigma^2_e(\hat{d}_{k,i}^j)} \right),
\]

\[= \arg \min_{e_i^j} \sum_{k=1}^{N + L_q + L_q^2 - 2} \left( \ln \sigma_e(\hat{d}_{k,i}^j) + \frac{(q_k - \hat{q}_i^j)^2}{2 \sigma^2_e(\hat{d}_{k,i}^j)} \right),
\]

(15)

where \(\hat{q}_i^j = \sum_{l=0}^{L_q-1} g_l(\hat{a}_{k-l} + e_i^j)\) are the re-constructed VD input samples based on \(e_i^j\), and \(\hat{d}_{k,i}^j = \hat{d}_k + e_i^j\), with \(\hat{d}_k = [\hat{a}_k + f_2, \hat{a}_k + f_2 - 1, \ldots, \hat{a}_k - f_1]^T\). In (15), unlike the case with AWGN, where the variance of \(e_k\) is a constant, \(\sigma^2_e(\hat{d}_{k,i}^j)\) varies with \(\hat{d}_{k,i}^j\) for media noise channels, and therefore cannot be dropped from the cost function.

The proposed data-dependent post-processing scheme is summarized as follows.

(1) For each of the dominant error events, determine all valid starting positions of the event so that the error event produces the same non-zero syndrome as the parity-check result. Furthermore, the corrected version of the detected bits should not violate the \(d = 1\) constraint. A set of candidate error events is thus obtained.

(2) Compute \(\sum_{k=1}^{N + L_q + L_q^2 - 2} \left( \ln \sigma_e(\hat{d}_{k,i}^j) + \frac{(q_k - \hat{q}_i^j)^2}{2 \sigma^2_e(\hat{d}_{k,i}^j)} \right)\) for each error event in the set of candidate error events. The error event with the minimum distance is determined as the most likely event.

V. BER SIMULATION RESULTS

In this part, we illustrate the improvement in bit error rate (BER) performance achieved due to the detector modifications as well as PC coding and post-processing. The BERs for different media noise powers are illustrated in Fig. 5. Observe that with \(\Delta_e = 14\%\), the ‘prop. monic MMSE’ (i.e. MMSE equalization with a 5-tap target, accounting for media noise [Section III-A], conventional VD which assumes the total noise \(e_k\) at its input as white Gaussian and data-independent, Curve 2) outperforms the ‘conv. monic MMSE’ (i.e. MMSE equalization with a 5-tap target, without accounting for media noise, conventional VD, Curve 1) by more than 1 dB at BER = 10\(^{-4}\). The modified VD (i.e. monic MMSE equalization of Section III-A combined with the modified branch-metric VD of Section III-B, Curve 3) provides an additional 0.5 dB gain. The performance gains become very significant in the case of \(\Delta_e = 17\%\). In this case, the ‘prop. monic MMSE’ achieves a gain of more than 5 dB compared to ‘conv. monic MMSE’. The additional gain provided by modified VD is more than 5 dB compared to ‘conv. monic MMSE’.
For the case with the new constrained 2-bit PC code, both post-processors achieve significant performance gains over the systems without PC codes. Furthermore, the data-dependent post-processor (legend: DD-PP, Curve 5) provides much better performance than the conventional matched-filtering type postprocessor (legend: MF-PP, Curve 4). With $\Delta_e = 14\%$, the PC code with the ‘MF-PP’ achieves more than 1 dB improvement over the modified VD without PC codes, at BER = $2 \times 10^{-5}$. The corresponding gain of ‘DD-PP’ is more than 2 dB. With $\Delta_e = 17\%$, the corresponding improvements of the two post-processors are more than 2 dB and 4 dB, respectively. If we compare with the conventional detection approach without PC codes (i.e. Curve 1), the overall performance gain is larger than 11 dB, at BER = $10^{-4}$.

VI. CONCLUSION

In this paper, we have studied various signal processing approaches to combat media noise for high-density optical recording. As the first step, we proposed a new channel model to describe rewritable optical recording systems with media noise, which provides a fair basis for the performance comparison of detection schemes over different coding schemes and recording densities. As the second step, we proposed two novel modifications to the channel detector, namely, the monic constrained MMSE equalization taking into account the media noise, and the modified VD with data-dependent noise variance. Simulation results show that these approaches provide significant performance gain over the conventional detection approaches that are designed without considering the media noise. Furthermore, they are simple and can be easily incorporated into existing PRML systems without increasing the size of VD trellis. As the third step, the dominant error events of the proposed detection approach have been found, and a new constrained PC code has been designed. A data-dependent post-processing scheme has been further developed which outperforms the conventional matched-filtering type post-processors by accounting for the data-dependence of the media noise variance. Compared to the system designed without considering media noise and without PC codes, the overall performance gain of the developed scheme is more than 11 dB at BER = $10^{-4}$, at high media noise levels.

ACKNOWLEDGMENT

The authors would like to thank Dr. G. Mathew and S. van Beneden, for their valuable advices and help to this work.

REFERENCES