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Analytical Passive Mixer Power Gain Models

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***Abstract*—According to the well-known Friis equation the available power gain should be maximized to reduce the overall noise figure. Therefore, in receivers where an LNA is not present or its gain is low, the available power gain of the passive mixer is of interest. However, only the voltage gain is presented in many papers. In this paper an analytical model is presented for the power gain, voltage gain and input and output impedance of a passive mixer. The model is obtained using time-domain analysis since the mixer is periodically time variant. The validity of the models is checked using CMOS 90nm transistor simulations.**

I. Introduction

The most important design goal of a wireless sensor is low power consumption. To increase the battery life time, the receiver power consumption should be minimized. An often used solution is to duty-cycle the radio. Another solution is to add a wake-up receiver which consumes very little power and wakes up the main radio when a wake-up call is received. The wake-up receiver requirements can be different from the requirements for the main radio. The bit rate and signal bandwidth of the wake-up receiver is usually small and thus the noise figure requirement is more relaxed.

Since the noise figure requirements are relaxed in these applications, it becomes possible to remove the Low Noise Amplifier (LNA) and sacrifice noise performance for a lower power consumption. To further decrease the power consumption, a passive mixer can be employed. As proposed in [1] and [2]. Without the LNA, the mixer is directly connected to the antenna with only a matching network in between. Using the well-known Friis equation it can be concluded that the available power gain should be maximized in order to decrease the overall noise figure. Additionally, the mixer should be matched to the antenna. At the same time the noise figure of the mixer should be minimized, which means the transducer power gain should be maximized because the mixer is a passive circuit.

In [3], [4] and [5] the voltage conversion gain is obtained for different local oscillator (LO) wave forms. However, the papers assume that the source impedance is zero and the load impedance is infinite and therefore can be neglected. In this paper models for the power gain, the input and output impedances are presented. The presented models take the LO waveform and finite source and load impedances into account. Additionally, solutions are obtained for block and sine wave LO signals. Using the obtained models, optimal choices for the load and source resistance are presented. In order to validate the analytical models simulations are carried out using CMOS 90nm transistor models.

II. Mixer model

A double balanced passive mixer circuit, without matching and biasing circuits, is shown in Fig. 1. The load conductance G_L and capacitance C_L include the input admittance of the following baseband amplifier as well as parasitic components. The RC load and the MOSTs act as a low pass baseband filter.

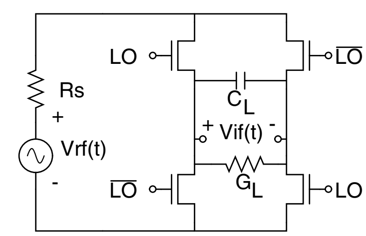


Fig. 1. Double balanced mixer circuit.

The passive mixer is considered to be a linear time-variant circuit. The approximation that the path from the RF input to the IF input is linear holds when the input power is small, e.g. smaller than the 1dB compression point. Since the mixer is time-variant the conversion gain is obtained using time domain analysis.

Additionally the model assumes that the RF frequency ω_{RF} and LO frequency ω_{LO} are approximately equal, which holds when the signal bandwidth and IF frequency is small compared to the RF frequency. This is the case for zero and low IF receivers.

A. Transistor model

The transistors are biased in deep triode region and are modeled as voltage dependent conductances with conductance $g(t)$, which is given by (1) assuming $V_{ds} \approx 0$V and $V_b = V_s + V_{th}$.

$$g(t) \approx \begin{cases} \frac{kW(V_{LO}(t)-V_b)}{L}, & V_{LO}(t) > V_b \\ 0, & V_{LO}(t) \leq V_b \end{cases} \quad (1)$$

Only the on-conductance is taken into account, not the parasitic capacitances. This is no longer valid when the load and source impedances both become very large and the RC constants of the mixer approaches the time constant of the RF signal. However, this region is not reached in practical circuits.

The models presented in this paper are valid for every kind of LO signal, but as examples, a block and sine wave LO are used. The corresponding transistor conductances are given below.

$$g(t)|_{sine} = \begin{cases} g_{max}\sin\left(\frac{2\pi}{T_{LO}}t\right), & t < \frac{T_{LO}}{2} \\ 0, & t \geq \frac{T_{LO}}{2} \end{cases} \tag{2}$$

$$g(t)|_{block} = \begin{cases} g_{max}, & t < \frac{T_{LO}}{2} \\ 0, & t \geq \frac{T_{LO}}{2} \end{cases} \tag{3}$$

B. Equivalent mixer model

The Thévenin equivalent mixer circuit is depicted in Fig. 2, where $g_T(t)$ is the Thévenin source impedance and $v_T(t)$ the equivalent source voltage, which are given by (4) and (5) respectively. In these equations, it is assumed that the on-period of two parallel transistors do not overlap. With this assumption

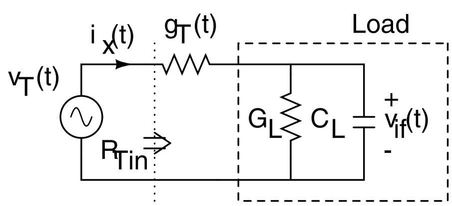


Fig. 2. Thevenin equivalent mixer circuit with load impedance.

it can be seen that the Thévenin equivalent conductance $g_T(t)$ is a series connection of R_S and two switch conductances.

$$g_T(t) = \frac{g(t) + g\left(t - \frac{T_{LO}}{2}\right)}{2 + \left[g(t) + g\left(t - \frac{T_{LO}}{2}\right)\right]R_S} \tag{4}$$

$$v_T(t) = m(t)v_{rf}(t) \tag{5}$$

$$m(t) = \begin{cases} 1, & g(t) > 0, g\left(t - \frac{T_{LO}}{2}\right) = 0 \\ -1, & g(t) = 0, g\left(t - \frac{T_{LO}}{2}\right) > 0 \\ 0, & g(t) = 0, g\left(t - \frac{T_{LO}}{2}\right) = 0 \end{cases} \tag{6}$$

In the models presented in this paper, the average Thévenin conductance $\overline{g_T}$ is of interest and is given by

$$\overline{g_T} = \frac{\omega_{LO}}{2\pi}\int_0^{\frac{2\pi}{\omega_{LO}}} g_T(t)dt \tag{7}$$

Therefore, the average conductances $\overline{g_T}$ for a sine and block LO signals can be found as

$$\overline{g_T}|_{\text{block}} = \frac{g_{max}}{2 + g_{max}R_S} \tag{8}$$

$$\overline{g_T}|_{\text{sine}} = \frac{1}{R_S}\left(1 - \frac{2}{\pi}\frac{\arccos\left(\frac{g_{max}R_S}{2}\right)}{\sqrt{1 - \left(\frac{g_{max}R_S}{2}\right)^2}}\right) \tag{9}$$

III. Conversion Gain

A. Voltage conversion gain

Since the mixer will be used as a frequency down-converter only the down converted frequency component $\omega_{RF} - \omega_{LO}$ is of interest; the component at $\omega_{RF} + \omega_{LO}$ is filtered out by the low pass filter. The definition of the voltage gain is given by (10), where $X(\omega)$ is the Fourier transform of $x(t)$ at frequency ω.

$$G_c = \left|\frac{V_{IF}(\omega_{RF} - \omega_{LO})}{V_{RF}(\omega_{RF})}\right| \tag{10}$$

Because the output is filtered and $\omega_{RF} - \omega_{LO} \ll \omega_{RF}$, voltage $v_{if}(t)$ is approximately constant during a period of the RF input signal. In this section the gain and input impedance is obtained for $\omega_{RF} = \omega_{LO}$, but the results will still be valid when the frequency difference is smaller than the mixer IF bandwidth. This is the case for the desired RF signal.

Since the load consists of a capacitor and a resistor differential equations are needed to find the conversion gain. The differential equation for the mixer is given by equation (11); the impulse response of $v_{if}(t)$ as function of $v_T(t)$ is derived by solving this differential equation.

$$C_L\frac{dv_{if}(t)}{dt} = g_T(t)v_T(t) - (g_T(t) + G_L)\, v_{if}(t) \tag{11}$$

The equation is solved similarly as described in [3]. The impulse response is given by (12), where h_{lpf} is the impulse response of a low pass filter with a 3dB bandwidth given by (13).

$$h(t - \tau) = h_{lpf}(t - \tau)\frac{g_T(\tau)}{\overline{g_T} + G_L} \tag{12}$$

$$\omega_{3dB} = \frac{\overline{g_T} + G_L}{C_L} \tag{13}$$

As the mixer is assumed to be linear the output voltage can be obtained using the convolution integral given by

$$v_{if}(t) = \int_{-\infty}^{t} h(t - \tau)v_T(\tau)d\tau \tag{14}$$

Using the substitutions for $m'(t)$ and H_{load} given below, the conversion gain is calculated. It can be seen that H_{load} is a resistive divider.

$$m'(t) = \frac{g_T(t)}{\overline{g_T}}m(t)\sin(\omega_{RF}t) \tag{15}$$

$$H_{load} = \frac{\overline{g_T}}{\overline{g_T} + G_L} \tag{16}$$

$$v_{if}(t) = H_{load}V_{rf}\int_{-\infty}^{t} h_{lpf}(t - \tau)m'(\tau)d\tau \tag{17}$$

The convolution integral given by 17, for the conversion gain can easily be transformed to the frequency domain using Fourier analysis. Due to the fact that only the low frequency component passes the low pass filter, the conversion gain is given below.

$$G_c|_{\omega_{RF} \approx \omega_{LO}} = H_{load}\left|M'(0)\right| \tag{18}$$

The voltage conversion gain for different types of LO signals are given by (19) and (20). The parameter γ, defined by(21), is used for clarity and is plotted in Fig. 3. In case the switch resistance is much smaller than the source and load resistances,

γ is close to one and the conversion gain of the sine wave driven and block wave driven mixers are approximately equal.

$$G_c|_{\text{block}} = \frac{2R_L}{\pi} \frac{1}{\frac{2}{g_{max}} + R_S + R_L} \tag{19}$$

$$G_c|_{\text{sine}} = \frac{2R_L}{\pi} \frac{1 - \frac{\pi\gamma}{g_{max}R_S}}{R_S + R_L\gamma} \tag{20}$$

$$\gamma = 1 - j\frac{4}{\pi} \frac{\ln\left(\frac{j g_{max}R_S + \sqrt{4-(g_{max}R_S)^2}}{j g_{max}R_S - \sqrt{4-(g_{max}R_S)^2}}\right)}{\sqrt{4-(g_{max}R_S)^2}} \tag{21}$$

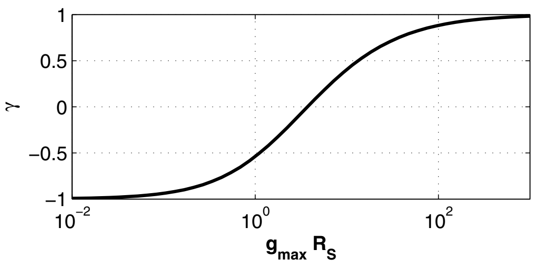


Fig. 3. Plot of γ as function of $g_{max}R_S$, note that the X axis has a logarithmic scale.

B. Power conversion gain

The transducer power gain as function of the voltage gain, load and source impedance is given by (22), whereas the available power gain is given by (23).

$$G_T = 4G_c^2 \frac{R_S}{R_L} \tag{22}$$

$$G_A = G_c^2 \frac{R_S}{R_L} \frac{(R_{out} + R_L)^2}{R_{out}R_L} \tag{23}$$

The available and transducer power gains are equal when the output of the mixer is matched. Therefore, maximizing the gain while keeping the input and output matched leads to an optimal configuration.

IV. Impedance

A. Input impedance

The input impedance is approximately real, assuming that the switch parasitic capacitances can be neglected

Using the approximation that the circuit is linear the mixer input impedance can be obtained from the impedance R_{Tin} given in Fig. 2 by removing the serially connected R_S,

$$R_{in} = R_{Tin} - R_S \tag{24}$$

To calculate the input impedance it is more convenient to convert the output IF signal to the RF frequency domain by multiplying it with the mixing function $m(t)$. Using this trick the input impedance is given by (26), where $\hat{I_X}$ and $\hat{V_{RF}}$ are the complex fundamental Fourier coefficients of the time domain signals for the input current i_x and voltage v_{rf}, respectively.

$$i_x(t) = \left(v_{rf}(t) - m(t)v_{if}(t)\right) g_T(t) \tag{25}$$

$$R_{in} = \frac{\hat{V_{RF}}}{\hat{I_X}} - R_S \tag{26}$$

Solving the input impedance equation given above gives,

$$R_{in} = \frac{1}{\overline{g_T}} \frac{1}{1 - \frac{4}{\pi}G_c} - R_S \tag{27}$$

B. Output impedance

The impedance seen at the output of the mixer is periodic time variant. Since the mixer is symmetric, it can be shown that the period of the impedance is half the LO period which is much shorter than the IF period. Furthermore, the IF signal can be assumed constant during half a LO period, since $\omega_{IF} \ll \omega_{LO}$. Using this we only have to average over half an LO period and the output impedance including the parallel capacitive load is then

$$\overline{Z_{out}} \approx \frac{1}{\overline{g_T} + j\omega_{IF}C_L} \tag{28}$$

Often, it can be assumed that the IF frequency is very low and the the load capacitance can be neglected leading to a purely resistive output impedance.

V. Optimization

A. Maximal transducer power gain

Given a load resistance R_L and switch conductance g_{max}, the optimal source resistance which leads to the maximal transducer power gain can be obtained. To derive the optimal source resistance, equation (29) has to be solved.

$$\frac{\partial G_T(R_L, g_{max})}{\partial R_S} = 0 \tag{29}$$

The solutions for the different LO wave-forms are given below. For the sine wave driven mixer it is assumed that the switch resistance is much lower than source resistance.

$$R_S|_{\text{block}} = R_L + \tfrac{2}{g_{max}} \tag{30}$$

$$R_S|_{\text{sine}} \approx R_L \tag{31}$$

B. Matching

In order to maximize the available power gain of the stage following the mixer, the output should be matched. Furthermore, matching is important to minimize the power reflections. To achieve a simultaneous input and output match the load given by (28) is substituted in the input resistance given by (26) and $R_{in} = R_S$ is solved for R_S. It is interesting to note that there is only one R_S, R_L combination where a good input and output match is obtained. The combinations for the two different LO signals are given in table I, where it is assumed that $R_S \gg \frac{1}{g_{max}}$ for the sine case.

The circuit impedances for the sine wave driven mixer are larger than for the block wave driven mixer which can be understood by noticing that the average on-conductance over one LO period is lower for the sine wave case.

Additionally, it is interesting to note that the mixer can not be matched at both the input and output and have a maximal transducer power gain at the same time. This is a different

LO Signal	R_L	R_S
Block	$R_S + \frac{2}{g_{max}}$	$\frac{1}{g_{max}} \frac{2\pi^2}{\pi^2 - 8}$
Sine	R_S	$\approx \frac{55}{g_{max}}$

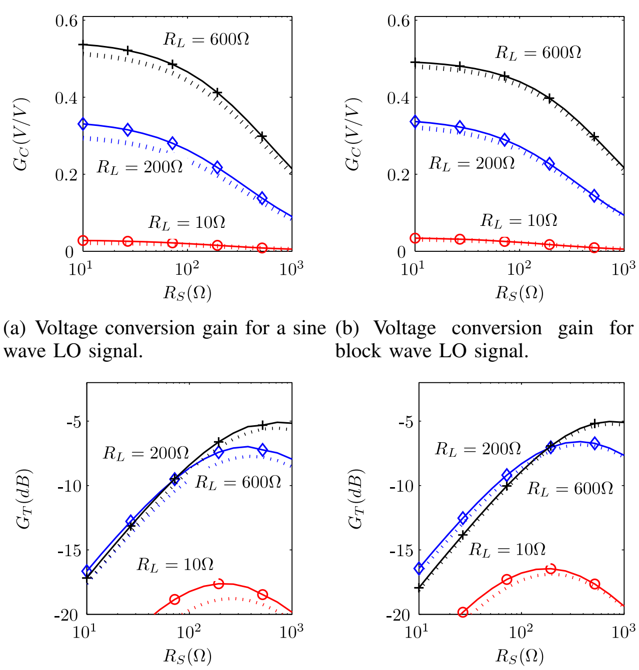
TABLE I
SOURCE AND LOAD LEADING TO IN- AND OUTPUT MATCH.

situation than for linear time-invariant devices such as LNA's. For the mixer this is not the case, because the input impedance depends on the source impedance. Only when $\frac{1}{g_{max}} \ll R_S$ and $\frac{1}{g_{max}} \ll R_L$ the in- and output can be matched while the transducer gain is maximal and equal to the available power gain.

VI. SIMULATION RESULTS

The presented models are compared to simulation results obtained with cadence using TSMC 90nm technology. In the simulations real transistor models were used. The used RF frequency is 1GHz and the transistor widths are $20\mu m$. Furthermore, the LO amplitude was only 300mV and the minimal on resistance of the switches was 80Ω, whereas the source resistance was swept from 10Ω to 1kΩ.

The transducer power gain and voltage conversion gain for both a sine and block wave driven mixer are shown in figure 4. The solid line with markers and dotted line represent the model and simulation results, respectively. The lines are on top of each other, which confirms the accuracy of the proposed models.



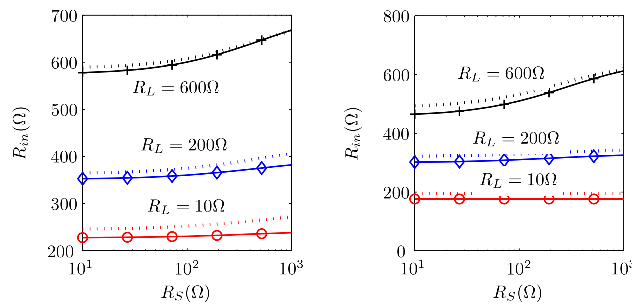
(a) Voltage conversion gain for a sine wave LO signal.
(b) Voltage conversion gain for a block wave LO signal.
(c) Transducer power gain for a sine wave LO signal.
(d) Transducer power gain for a block wave LO signal.

Fig. 4. Simulation and modeled voltage conversion gain and transducer power gain.

The results obtained using the presented model fit well with the simulation results. The error increases when both the source and load impedances increase. In those cases the gate-drain and gate-source capacitances of the switches can no longer be neglected. The error increases because the circuit RC constants approach the LO period and the mixer can not switch instantaneous anymore. In a practical design this situation should be avoided.

The transducer power for the sine wave and block wave driven mixers are very similar. Thus from a gain point of view, the type of LO signal does not matter much.

The modeled and simulated input impedances are shown in figure 5. The simulation results are depicted by the dotted line and the modeled results by the marked solid line. The modeled result is quite close to the simulated results as can be seen in the figures. The error for the sine wave driven mixer is slightly larger.



(a) Sine wave LO signal.
(b) Block wave LO signal.

Fig. 5. Simulation and modeled input resistances.

VII. CONCLUSION

In this paper analytical models were presented for the input and output impedances and for the transducer power gain of the passive mixer for different LO types. The models were validated using CMOS 90nm transistor models. It was proved that the R_S, R_L combination for maximal power gain is different than the combination that leads to the maximal voltage gain. Therefore, when no LNA is present, the power gain should be maximized not the voltage conversion gain. In order to obtain both matching and maximal available power gain, the transistor on-resistance has to be close to zero.

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