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Citation for published version (APA):
Hou, Q., Zhang, L. X., Tijsseling, A. S., & Kruisbrink, A. C. H. (2013). *SPH simulation of free overfall in open channels with even and uneven bottom*. (CASA-report; Vol. 1320). Technische Universiteit Eindhoven.

Document status and date:
Published: 01/01/2013

Document Version:
Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

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EINDHOVEN UNIVERSITY OF TECHNOLOGY
Department of Mathematics and Computer Science

CASA-Report 13-20
August 2013

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ISSN: 0926-4507

SPH simulation of free overfall in open channels with even and uneven bottom

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Keywords: SPH, free overfall, even and uneven bottom, subcritical flow, critical flow, supercritical flow.

Abstract. The free overfall can be used as a simple and accurate device for flow measurement in open channels. In the past, the solution to this problem was found mainly through simplified theoretical expressions or on the basis of experimental data. In this paper, using the meshless smoothed particle hydrodynamics (SPH) method, the free overfall in open channels with even and uneven bottom is investigated. For the even bottom case, subcritical, critical and supercritical flows are simulated. For the uneven bottom case, supercritical flows with different Froude numbers are considered. The free surface profiles are predicted and compared with theoretical and experimental solutions in literature and good agreements are obtained.

Introduction

The definition sketch of a typical overfall in a Cartesian coordinate system is shown in Fig. 1, where the hydraulic aspects and the variation of streamlines curvature are included too.

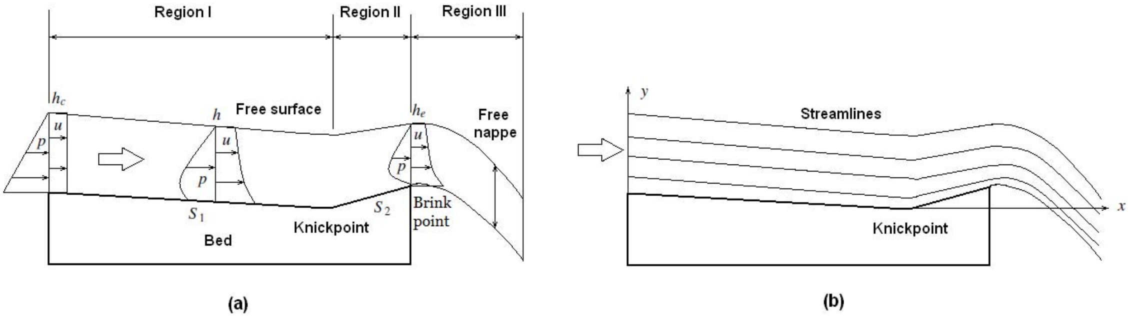

Fig. 1. Definition sketch of a typical free overfall with (a) hydraulic aspects and (b) streamline pattern.

Three distinct regions of flow are associated with this problem. The upstream region (Region I) is characterized by a free top surface and a channel bed with slope S_1. The middle region (Region II) is characterized by a free top surface and a bottom with slope S_2. The downstream region (Region III) is characterized by two free surfaces and is referred to as free nappe. The streamline curvature is finite at the free surfaces and zero at the channel bed. There are two singular points that need special attention. One is the knickpoint between the two bottoms, due to which a transition zone of the streamlines is expected. The other one is the end point of Region II and is generally referred to as the brink point. The flow in the vicinity of the brink is strongly affected by gravity.

When the two bottoms have the same slope, i.e. even bottom ($S_1 = S_2$), the problem reduces to a traditional free overfall problem, for which an elegant review was given by Dey [1]. Although several different approaches can be utilized to formulate the free overfall with uniform slope as summarized in [1], they are mainly theoretical methods based on various assumptions or empirical formulations calibrated by experimental data. In addition, only a final steady-state solution can be obtained with these approaches. Webster and Zhang [2] studied a waterfall with an uneven bottom ($S_1 \neq S_2$) using the Green-Naghdi method developed by Naghdi and Rubin [3]. The flow is assumed to be inviscid and hence governed by the Euler equations. A straightforward numerical simulation by solving the Euler equations is not possible for traditional methods, because the free-surface profiles are not known in advance, and hence boundary conditions are difficult to impose. Mohapatra et al. [4] simulated the flow passing a free overfall using the MAC method together with the VOF surface-tracking technique. With a turbulence model in VOF, Ramamurthy et al. [5] and Guo [6] predicted the hydraulic characteristics of a free overfall in a rectangular open channel. The MAC method with VOF is a powerful and robust numerical tool for free-surface flows but it suffers from complexities in surface tracking and difficulties in the treatment of irregular boundaries.

In this paper the two-dimensional free overfall problem is simulated with SPH and the solutions are compared with theoretical and experimental results. The semi-discrete SPH equations are presented in Section 2, the results in Section 3 and the conclusion in Section 4.

Semi-discrete SPH Equations

SPH Equations. The modeled two-dimensional flow is assumed to be weakly compressible. The governing equations in discrete SPH form for mass density ρ, velocity vector $\mathbf{v}$, particle position $\mathbf{r}$

and pressure p read [7, 8]:

$$\frac{D\rho_i}{Dt} = \sum_j m_j \mathbf{v}_{ij} \cdot \nabla_i W_{ij}, \tag{1}$$

$$\frac{D\mathbf{v}_i}{Dt} = -\sum_j m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij}, \tag{2}$$

$$\frac{D\mathbf{r}_i}{Dt} = \mathbf{v}_i, \tag{3}$$

$$p_i = c_0^2 (\rho_i - \rho_0), \tag{4}$$

where subscripts i and j denote a reference particle i and its neighbors j, $\mathbf{v}_{ij} := \mathbf{v}_i - \mathbf{v}_j$, $W_{ij} := W(\mathbf{r}_i - \mathbf{r}_j, h)$ is the kernel and $\nabla_i W_{ij}$ is the gradient of the kernel taken with respect to the position of particle i. The smoothing length h is a size scale of the kernel support and determines the degree to which a particle interacts with its neighbors; Π_{ij} is an artificial viscous term of the general form

$$\Pi_{ij} = \frac{-\alpha c_0 h}{\bar{\rho}_{ij} \left(r_{ij}^2 + 0.01h^2 \right)} \min\left(\mathbf{v}_{ij} \cdot \mathbf{r}_{ij} \right), \tag{5}$$

in which α is a problem-dependent constant [8], c_0 is the speed of sound, ρ_0 is a reference density, $\mathrm{r}_{ij} := \mathrm{r}_i - \mathrm{r}_j$, $r_{ij} := |\mathrm{r}_{ij}|$ and $\bar{\rho}_{ij} := (\rho_i + \rho_j)/2$. Here we take $\alpha = 0.1$ as in [9, 10].

The Euler forward method is used in Eqns. (1)-(4) for time marching. The temporal discretization is dynamically linked to the spatial particle approximation by satisfying stability constraints related to the speed of sound and convective velocity of the flow [7]. Starting from an initial particle distribution ($\mathbf{r}_i$) with given masses m_i (remaining constant), densities ρ_i, pressure p_i and velocities $\mathbf{v}_i$, the system (1)-(4) is solved at each time step for each particle.

Boundary Conditions. There are four different kinds of boundary conditions for the problem considered herein. The free surface boundary condition is enforced implicitly in SPH. The image particle method is applied to treat the inflow and outflow boundaries. The mirror particle method is used to impose the free-slip wall boundary condition (see Section 3.4.8 in [7] for details).

Numerical Results

The following variables scaled by the critical depth h_c (see Fig.1a), i.e. $X := x/h_c$ and $Y := y/h_c$, are used.

Subcritical Flow. For the case of subcritical flow, the SPH method is applied to the experimental setup of Rajaratnam [11], in which the bed slopes S_1 and S_2 are both equal to zero. The inflow depth and discharge are equal to 0.132 m and 0.143 $\mathrm{m}^3\mathrm{s}^{-1}\mathrm{m}^{-1}$, respectively. The corresponding Froude number is $F = V/(gh_c)^{1/2} = 0.953$. The computed free-surface profiles including the lower and upper nappes are in Fig. 2 compared to numerical [4], experimental [11] and theoretical [12] results. The present SPH results agree better with the experimental [11] and theoretical [12] results than with the numerical solution of Mohapatra et al. [4]. As an important parameter in the discharge estimation [13], the brink depth is predicted very well with an error of only 2 percent relative to the theoretical solution.

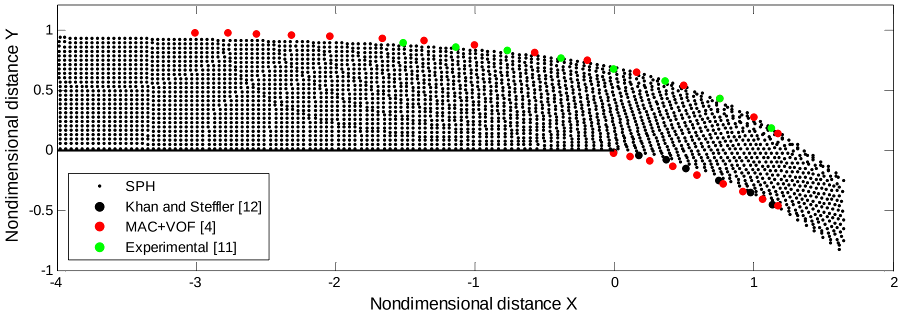

Fig. 2. Free surface profile in an even free overfall: subcritical flow.

Critical Flow. For the case of critical flow, we consider the problem presented by Smith and Abd-el-Malek [14], which was solved by, among others, Goh and Tuck [15]. The inflow depth and discharge are equal to 0.1 m and 0.99 $m^3s^{-1}m^{-1}$, respectively.

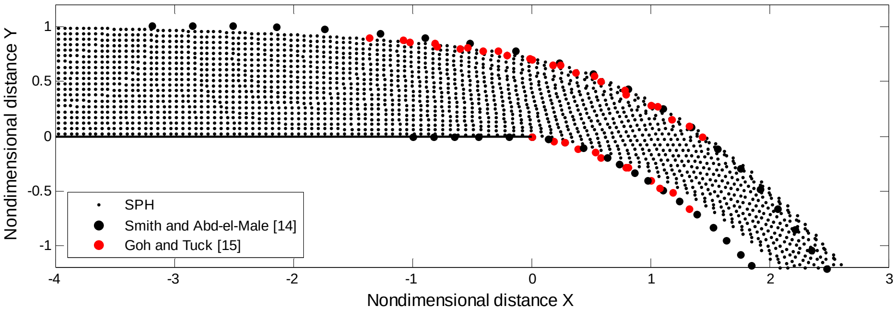

Fig. 3. Free surface profile in an even free overfall: critical flow.

The computed free-surface profiles are compared to theory [14, 15] in Fig. 3. The computed free-surface profiles match well with the theoretical results. The best agreement is obtained with the theoretical solution of Goh and Tuck. Potential flow theory was used in both [14] and [15], but different numerical methods were used to solve the governing equations. The solution in [14] is very close to the experimental results, which are also available for this case. For the brink depth, the difference between the present result and Smith and Abd-el-Malek's prediction is less than 2 percent. For the profiles far downstream of the brink point, the agreement is less. The outflow section possibly is not taken far enough away and affecting the results. This is only approximately enforced in SPH (also in VOF) as the outflow section cannot be exactly perpendicular to the flow velocity as required.

Supercritical Flow. The theoretical solution for the free overfall with a supercritical upstream flow is taken from [2]. Figure 4 shows the computed and theoretical water surface profiles. The agreement between the numerical and theoretical results is excellent with the maximum error less than 1 percent.

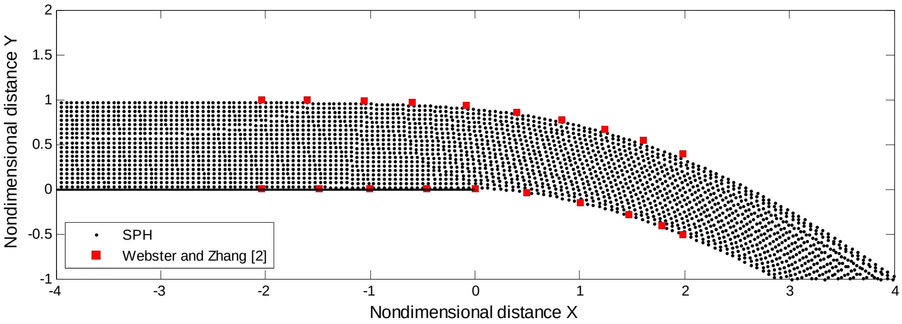

Fig. 4. Free surface profile in an even free overfall: supercritical flow.

Supercritical Flow with Uneven Bottom. We also examine supercritical flows with uneven bottom. Although there is no limitation for the bed slopes, we restrict our simulations to $S_1 = 0$ and $S_2 > 0$ without losing generality. The SPH solver is applied to the test cases of Webster and Zhang [2] for a bottom slope $S_2 = 0.1$. This is a problem that has not received much attention either through theoretical investigation or experimentation. To our best knowledge, there is no numerical simulation available in literature. Such a flow is related in a way to the flow in a plunging breaking wave and important in ocean engineering [2]. With a large bottom slope S_2 (positive or negative), it is also related to a weir flow problem [16]. The computed results for different Froude numbers are shown in Fig. 5 and compared to the available theory. For all the simulated cases, good agreement is obtained with a maximum error in surface shape less than 2.5 percent.

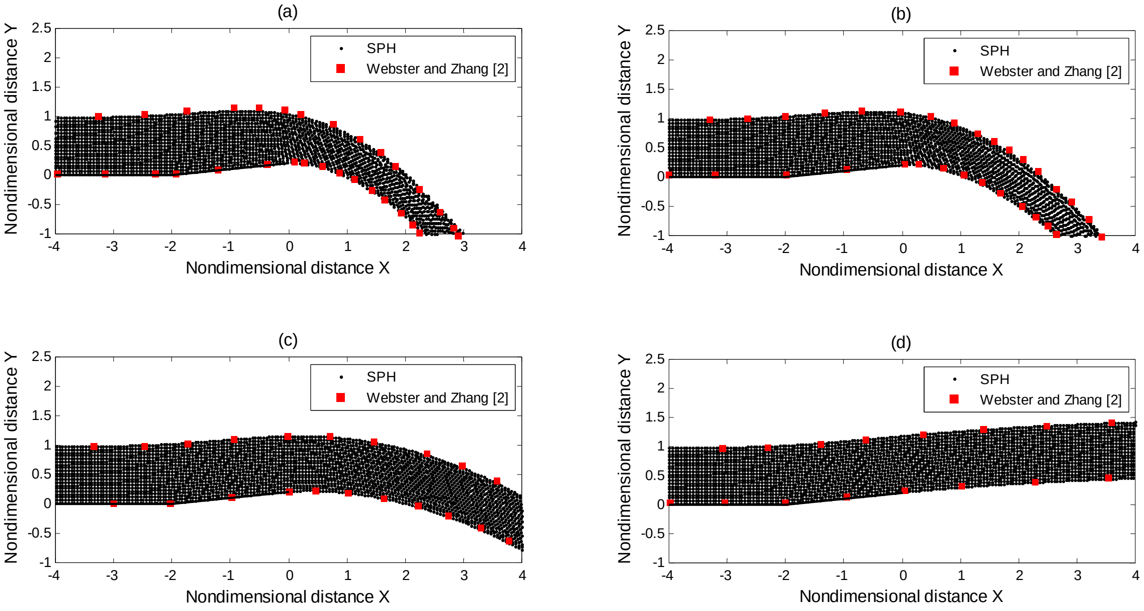

Fig. 5. Free surface profiles in free overfalls with uneven bottoms:
(a) $F = 1.25$, (b) $F = 1.5$, (c) $F = 2.5$, (d) $F = 8.0$.

Conclusion

The successful simulation of a variety of overfall problems confirms that the current SPH solver is a powerful tool for computing inertia driven flows with free surfaces.

Acknowledgments

This work is supported in part by the National Natural Science Foundation of China (No. 51279071) and National Basic Research Program of China (No. 2013CB329301). The first author is grateful to the China Scholarship Council (CSC) for financially supporting his PhD studies at Eindhoven University of Technology, The Netherlands.

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