

Freight consolidation through carrier collaboration

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Freight consolidation through carrier collaboration - A cooperative game

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ABSTRACT

Reducing inefficient truck movements, this research investigates the potential of freight consolidation through carrier collaboration. Considering the financial benefits of consolidation as well as the additional cost arising from collaboration, we introduce a cooperative game in which several carriers can collaborate by pooling transportation capacities. Although the core of this game can be empty, we provide three conditions under which core non-emptiness is preserved. Numerical experiments indicate, moreover, that core non-emptiness is also likely outside these conditions.

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1. Introduction

The rise in e-commerce, generally associated with smaller individual shipment sizes, has led to an increase in less-than-truckload (LTL) shipping, requiring higher levels of coordination between shipments in order to keep costs low [13]. However, within the dynamic e-commerce environment, it is often difficult to achieve such high levels of coordination between orders of a carrier due to short delivery timelines, resulting in inefficient truck movements that do not make good use of the available capacities. Freight consolidation through carrier collaboration offers a promising avenue, in this context, to achieve overall higher shipping volumes without significantly increasing delivery times [12]. Although examples of this type of collaboration already exist (see, e.g., [8,16]), large-scale implementation is still hampered by a lack of trust as well as other practical challenges that prevent data and capacity sharing between companies [3,14]. Emerging technologies, such as Blockchain or the Internet of Things, are helping to alleviate some of these challenges by paving the way for improved data sharing, real-time tracking, and secure and transparent data exchange [4,5,10]. Investing in these new technologies can, how-

ever, be expensive and thus might hinder possible collaborations between carriers.

This paper aims to address this issue by investigating under which circumstances carriers are interested in collaboration, considering both the financial benefits of consolidation and the additional costs arising from collaboration. For this purpose, we study a setting where multiple carriers must deliver goods to the same final location, passing via several intermediate locations (e.g., a hub). In this setting, the carriers can choose to operate independently, solely paying a cost for every unit of good transported, or to collaborate, using spare capacity to consolidate shipments with other players and achieve better resource utilization. Despite its potential benefits from an operational perspective, this consolidation may also trigger additional costs as carriers need to invest in new and often expensive technologies (e.g., secure and transparent data sharing and/or real-time tracking), pay for insurance, and purchase handling equipment to facilitate the transfer of goods between collaborators. We refer to all these costs as *transfer costs*, which include both a variable and a fixed component, where the former depends on the number of units transferred. At the same time, the latter presents a more general investment cost that is independent of the amount of goods shared. Formulating this situation as a cooperative game, we then investigate under which type of cost structures carrier collaboration is stable. We do this by studying the core of our game, which presents the set of all allocations that divides the joint costs amongst the carriers so that no group of carriers has reasons to leave the collaboration. In this context, we demonstrate that collaboration is not always stable, so

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some carriers might prefer to leave the collaboration even if the joint cost reduction is positive (i.e., if the benefits of consolidation outweigh the initial investment costs). At the same time, however, we show that collaboration is stable if fixed transfer costs are very low, very high, or symmetric. In addition, our extensive numerical experiments indicate that collaboration is likely to be sustainable outside of these extreme cases, although exceptions might exist.

1.1. Some related literature

Given its potential for practice, collaboration in transportation systems has received quite some attention within the scientific literature (see, e.g., the literature reviews of [5,21]). This includes the discussion on how parties should compensate each other financially in case of collaboration, which is usually addressed using cooperative game theory [9]. In this context, it is common for studies to investigate the core of the considered cooperative game, corresponding to a specific transport situation in which different parties collaborate. Examples of such situations are the assignment of demands to routes owned by collaborators [2,11], the sharing of vehicle capacity as well as the creation of routes satisfying the joint delivery planning [6,7,17,20]. Based on the properties of the considered situation, some studies focus then on proving core non-emptiness for specific subclasses (e.g., [1,18,23]). In contrast, others present a numerical investigation of the core and its behavior under different properties (e.g., [15,17]). Our work combines these two by proving core non-emptiness for several subclasses while studying its properties numerically outside of these subclasses.

Outside of the transportation context, the setting in our paper has the most overlap with commodity flow games (see, e.g., [1,18]). In these games, there is an underlying network in which players own capacity on the arcs and a set of commodities, for which the transfer of a single unit leads to a player-specific profit. Players can collaborate by sharing arc capacity and transporting only the most profitable commodities through the network. Our work differs in this aspect as we assume that players must transport *all* orders. Moreover, instead of maximizing profit, we minimize the cost associated with transport and transfers by accounting for a fixed cost that needs to be paid whenever orders are transferred between parties. This fixed cost can break the collaboration, whereas collaboration (i.e., core non-emptiness) is guaranteed in commodity flow games, which do not incur such fixed costs.

1.2. Outline of the paper

The remainder of the paper is organized as follows. In § 2, we introduce the transfer and transport situation addressed in our paper before introducing the corresponding cooperative game in § 3. In § 4, we then explore sufficient conditions that lead to core non-emptiness, while § 5 provides the results of a numerical experiment, exploring core non-emptiness based on multiple parameters.

2. The transfer and transport situation

We consider a transportation environment with a set $N \subseteq \mathbb{N}$ of carriers, in which each carrier $i \in N$ needs to transport a specific volume $D_i \in \mathbb{R}_{\geq 0}$ between two points, by passing through a set of points (e.g., a hub location, or a distribution or consolidation center). In this context, we assume that all carriers travel via the same starting, intermediate, and final points. In practice, these points do not need to be precisely the same but may be sufficiently close (e.g., in an urban context, they might be located in the same neighborhood or district of a city). We denote the set of points by $P_f = \{1, 2, \dots, n, n+1\}$ with 1 representing the starting point and

$n+1$ the final point, while the remaining points are intermediate points.

In addition, we assume that each carrier $i \in N$ has capacity $Q_i^p \in \mathbb{R}_{\geq 0}$ available to transport demand from point $p \in P_f \setminus \{n+1\}$ to $p+1$, and the cost of transporting one unit of volume equals $c_i^p \in \mathbb{R}_{\geq 0}$. For notational convenience, we let $P = P_f \setminus \{n+1\}$ be the set of points where transfers can occur since there is no need to transfer volume once the destination has been reached. Moreover, we assume that carriers always have sufficient capacity to serve their own volume (i.e., $D_i \leq Q_i^p$ for all $i \in N$ and all $p \in P$). Yet, instead of transporting their own volume, carriers may *transfer* all or part of their volume at any point $p \in P$ to another carrier, depending on available capacity. Transferring between parties, however, incurs costs which need to be taken into account. We distinguish between two types of transfer costs, namely a variable cost s_{ij}^p and a fixed cost t_{ij}^p . In this case, the variable cost refers to the handling costs when transferring the goods from one truck to another, while the fixed cost presents an investment cost, e.g., for secure and transparent data sharing and/or real-time tracking. The amount of volume transferred from carrier $i \in N$ to carrier $j \in N$ at point $p \in P$ is denoted by $x_{ij}^p \in \mathbb{R}_{\geq 0}$, whereas the amount of volume that carrier $i \in N$ decides to keep and transport to the next point by itself is denoted by $x_{ii}^p \in \mathbb{R}_{\geq 0}$. The costs associated with this internal transfer also consist of a variable component $s_{ii}^p \in \mathbb{R}_{\geq 0}$ and a fixed component $t_{ii}^p \in \mathbb{R}_{\geq 0}$. Fig. 1 presents a stylized representation of the situation.

It should be noted that our model is also able to deal with situations where some carriers enter their volume later in the network (e.g., at point 2), which can be integrated by setting transportation costs and internal variable and fixed transfer costs of these carriers equal to zero up until their point of entry in the system, while external variable and fixed transfer costs should be sufficiently high in order to not be considered.

The carriers aim to jointly determine how to *transport* and *transfer* the entire volume such that the associated cost is minimized. This optimization problem, which we will refer to as Problem (1), can be formulated as the following mixed integer linear programming problem:

$$\min \sum_{p \in P} \sum_{i \in N} \left(c_i^p \sum_{j \in N} x_{ji}^p + \sum_{j \in N} (s_{ji}^p x_{ji}^p + t_{ji}^p z_{ji}^p) \right) \quad (1a)$$

$$\text{s.t. } \sum_{j \in N} x_{ji}^p \leq Q_i^p \quad \forall i \in N \quad \forall p \in P \quad (1b)$$

$$\sum_{j \in N} x_{ji}^p = \sum_{j \in N} x_{ij}^{p+1} \quad \forall i \in N \quad \forall p \in P \setminus \{n\} \quad (1c)$$

$$\sum_{j \in N} x_{ij}^1 = D_i \quad \forall i \in N \quad (1d)$$

$$x_{ij}^p \leq M z_{ij}^p \quad \forall i \in N \quad \forall j \in N \quad \forall p \in P \quad (1e)$$

$$x_{ij}^p \in \mathbb{R}_{\geq 0} \quad \forall i \in N \quad \forall j \in N \quad \forall p \in P \quad (1f)$$

$$z_{ij}^p \in \{0, 1\} \quad \forall i \in N \quad \forall j \in N \quad \forall p \in P \quad (1g)$$

In this formulation, the objective function (1a) aims to minimize the total cost, while constraints (1b) represent the available capacity per carrier to transport demand between points. Constraints (1c) ensure that the entire volume transported by a carrier is transferred to the next intermediate point, and constraints (1d) enforce that the entire volume of each carrier is assigned at the starting point. The big-M constraints (1e) ensure that a fixed transfer cost is paid whenever some positive volume is transferred between carriers, with $M \geq \sum_{i \in N} D_i$ for all $i, j \in N$ and $p \in P$. Finally, all variables are non-negative, and z_{ij}^p is binary.

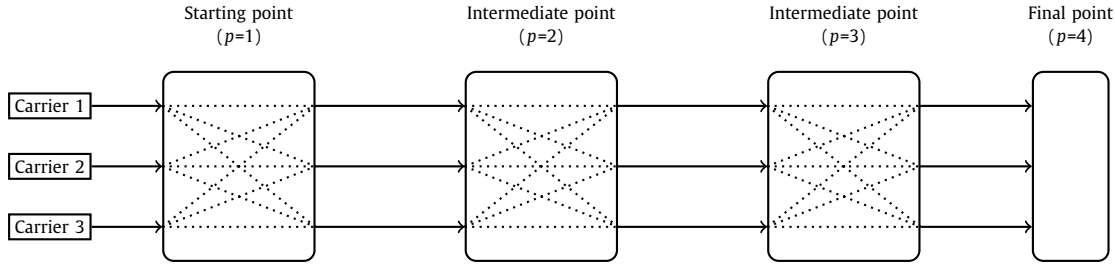


Fig. 1. Representation with three carriers, three transfer points, and the final point, with $n = 3$. Note that transfers are not considered at the final point.

Table 1
Sets, parameters, and variables.

Sets	
N	Set of carriers
P	Set of points
Variables	
x_{ij}^p	Volume transferred from carrier i to j at point p
z_{ij}^p	1 if there is a positive volume transferred from carrier i to j at point p , 0 otherwise

Parameters	
D_i	Demand that carrier i has to deliver
c_i^p	Cost of transporting a unit of demand for carrier i from point p to point $p + 1$
Q_i^p	Transport capacity of carrier i from point p to point $p + 1$
s_{ij}^p	Variable costs for transferring from carrier i to carrier j at point p
t_{ij}^p	Fixed costs for transferring from carrier i to j at point p
M	Big-M (i.e., a sufficiently large number)

We refer to our setting as a *transfer and transport situation* $\theta = (N, P, D, Q, c, s, t)$ with $D = (D_i)_{i \in N}$, $Q = (Q_i^p)_{i \in N, p \in P}$, $c = (c_i^p)_{i \in N, p \in P}$, $s = (s_{ij}^p)_{i \in N, j \in N, p \in P}$ and $t = (t_{ij}^p)_{i \in N, j \in N, p \in P}$, and denote the set of all transfer and transport situations by Θ .

Table 1 presents an overview of all the parameters of θ and variables used.

Relatively to Problem (1), since there is no cost incentive to transport or transfer more than needed through the network, it is possible to relax equality constraints (1c) and (1d) of Problem (1) without changing its optimal value. We describe this formally in Observation 1.

Observation 1. In Problem (1), “=” constraints (1c) can be relaxed into “ \leq ” constraints and “=” constraints (1d) can be relaxed into “ \geq ” constraints, without changing its optimal value.

From now onwards, when referring to Problem (1), we refer to the optimization problem in which constraints (1c) and (1d) are replaced by inequality constraints (as in Observation 1).

In the following, we present an illustrative example of our setting, for which the parameters do not necessarily represent reality but are selected to keep calculations simple and easy to follow.

Example 1. Consider a transfer and transport situation $\theta \in \Theta$ with $N = \{1, 2, 3\}$, $P = \{1, 2\}$, and the remaining parameters being equivalent to the values shown in Table 2.

Table 2
Parameters of the problem of Example 1.

i	1	2	3	Q_i^p	$i=1$	2	3	c_i^p	$i=1$	2	3
$p=1$	6	4	3	2	7	8	7	6	1	9	9
D_i	6	4	3	2	7	6	7	2	9	2	4

(a) Carriers' demands (b) Carriers' capacities (c) Carriers' costs

t_{ij}^1, s_{ij}^1	$j=1$	2	3	t_{ij}^2, s_{ij}^2	$j=1$	2	3
$i=1$	0,0	4,2	6,1	$i=1$	0,0	6,4	9,4
2	9,4	0,0	7,1	2	5,4	0,0	5,3
3	5,4	5,3	0,0	3	9,3	5,2	0,0

(d) Fixed and variable costs on the first point (e) Fixed and variable costs on the second point.

Using a standard solver package for solving integer linear programming problems, we can find an optimal solution for this problem with an objective value of 120. We present this solution visually in Fig. 2.

If the carriers decide to transport their volume individually, carrier 1 ends up with a cost of 90 (i.e., $6 \cdot (6 + 9)$), carrier 2 with a cost of 12 (i.e., $4 \cdot (1 + 2)$) and carrier 3 with a cost of 39 (i.e., $3 \cdot (9 + 4)$). Carriers can thus benefit from collaborating (i.e., transferring their volume to others), as $90 + 12 + 39 > 120$. Δ

This example demonstrates that carriers can benefit from coordinating transport. However, to sustain such a collaboration, it is important that the associated costs (e.g., the 120 of Example 1) are allocated in a stable way (i.e., in a way that sustains the collabora-

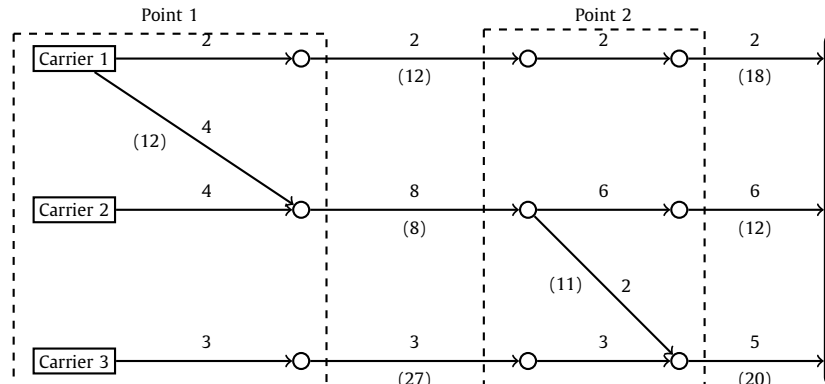


Fig. 2. Optimal solution graph of Example 1. The numbers without brackets represent the volume flow, and the numbers within brackets represent the costs related to that flow, which are omitted if there is no cost.

tion). In the upcoming section, we use cooperative game theory to study this cost allocation problem.

3. The cooperative game

We now associate to each transfer and transport situation $\theta \in \Theta$ a transferable utility cooperative game (N, C^θ) . We call this game the Cooperative Transporting and Transferring (CTT) game. In this game, N denotes the set of carriers, which we will now refer to as players, and $C^\theta : 2^N \rightarrow \mathbb{R}$ denotes the characteristic cost function associating a real value to any coalition $S \subseteq N$, representing a subset of players. This cost function $C^\theta(S)$ reflects the optimal cost value that can be achieved when the players in S decide to cooperate, i.e., the optimal value of Problem (1) restricted to the players participating in coalition S . Defining $C^\theta(\emptyset) = 0$, and coalition $S \subseteq N, S \neq \emptyset$, we then obtain the following formulation based on the logic of Problem (1) and the result of Observation 1:

$$C^\theta(S) = \min \sum_{p \in P} \sum_{i \in S} \left(c_i^p \sum_{j \in S} x_{ji}^p + \sum_{j \in S} (s_{ji}^p x_{ji}^p + t_{ji}^p z_{ji}^p) \right) \quad (2a)$$

$$\text{s.t. } \sum_{j \in S} x_{ji}^p \leq Q_i^p \quad \forall i \in S \quad \forall p \in P \quad (2b)$$

$$\sum_{j \in S} x_{ji}^p \leq \sum_{j \in S} x_{ij}^{p+1} \quad \forall i \in S \quad \forall p \in P \setminus \{n\} \quad (2c)$$

$$\sum_{j \in S} x_{ij}^1 \geq D_i \quad \forall i \in S \quad (2d)$$

$$x_{ij}^p \leq M z_{ij}^p \quad \forall i \in S \quad \forall j \in S \quad \forall p \in P \quad (2e)$$

$$x_{ij}^p \in \mathbb{R}_+ \quad \forall i \in S \quad \forall j \in S \quad \forall p \in P \quad (2f)$$

$$z_{ij}^p \in \{0, 1\} \quad \forall i \in S \quad \forall j \in S \quad \forall p \in P \quad (2g)$$

The central question in this paper is whether $C^\theta(N)$ can be allocated in a stable way amongst the players. In the literature, it is common to address this question by investigating the core of the associated game, as the core represents the set of allocations for which no individual player, nor a group of players (i.e., coalition) has incentives to break from the grand coalition [22]. For our CTT game, the core is defined as:

$$\mathcal{C}(N, C^\theta) = \left\{ u \in \mathbb{R}^N \mid \sum_{i \in N} u_i = C^\theta(N), \sum_{i \in S} u_i \leq C^\theta(S) \quad \forall S \subseteq N, S \neq \emptyset \right\}. \quad (3)$$

Example 2 shows a CTT game with an allocation in the core.

Example 2. Reconsidering the setting of Example 1, we know the value of $C^\theta(\{1\})$, $C^\theta(\{2\})$, $C^\theta(\{3\})$, and $C^\theta(N)$ from the example. To determine the values of $C^\theta(\{1, 2\})$, $C^\theta(\{1, 3\})$, and $C^\theta(\{2, 3\})$, we solve the associated optimization problem (see Problem (2)). The values of the characteristic cost function are presented in Table 3, with the corresponding solutions reported in Appendix A. We see that $u = (80, 5, 35) \in \mathcal{C}(N, C^\theta)$ and so the core is non-empty. \triangle

Table 3
Coalitional costs for the problem of Example 1.

S	1	2	3	1,2	1,3	2,3	N
$C^\theta(S)$	90	12	39	86	129	42	120

However, not all CTT games have a non-empty core, which we demonstrate in Example 3.

Example 3. Consider a transfer and transport situation $\theta \in \Theta$ with $N = \{1, 2, 3\}$, $P = \{1, 2\}$, and the other parameter settings as shown in Table 4. The coalitional values for this example are reported in Table 5, while the solutions of the associated optimization problem per coalition are reported in Appendix B.

Table 4
Parameters of the problem of Example 3.

Q_i^p				c_i^p			
i	1	2	3	i=1	1	2	3
D_i	3	3	6	2	1	2	3
				7	7	9	
				6	5	9	

(a) Carriers' demands				(b) Carriers' capacities				(c) Carriers' costs					
t_{ij}^1, s_{ij}^1		j=1	2	3	t_{ij}^2, s_{ij}^2		j=1	2	3	c_i^p			
i=1	2	0,0	9,4	5,1	i=1	2	0,0	7,2	8,4	1	2	3	
2	3	7,2	0,0	7,2	2	3	5,1	0,0	8,4	1	9	7	2
3		8,1	8,2	0,0	3		4,1	7,2	0,0	2	2	8	6

Table 5
Coalitional values for the empty core example.

S	1	2	3	1,2	1,3	2,3	N
$C^\theta(S)$	33	45	48	68	66	85	110

Suppose now that $\mathcal{C}(N, C^\theta) \neq \emptyset$, and $u \in \mathcal{C}(N, C^\theta)$. Then by stability and efficiency, we know that $u_i \geq C^\theta(N) - C^\theta(N \setminus \{i\})$ for all $i \in N$. Hence, $u_1 \geq C^\theta(N) - C^\theta(\{2, 3\}) = 110 - 85 = 25$, $u_2 \geq C^\theta(N) - C^\theta(\{1, 3\}) = 110 - 66 = 44$ and $u_3 \geq C^\theta(N) - C^\theta(\{1, 2\}) = 110 - 68 = 42$. Thus $u_1 + u_2 + u_3 \geq 25 + 44 + 42 = 111 > 110$, which is a contradiction. Therefore, the core is empty. \triangle

Example 3 thus illustrates that a stable allocation of the joint costs is not always possible, which may hinder potential collaboration. Note that this result is in sharp contrast to commodity flow games, for which core non-emptiness is guaranteed.

Observation 2. Based on Example 2, we can conclude that CTT games are generally not monotonic, while Example 3 allows us to conclude that CTT games are generally not concave (as concavity implies core non-emptiness). However, CTT games are subadditive, as an optimal flow of two disjoint coalitions represents a feasible flow within the joint coalition.

4. Sufficient conditions for core non-emptiness

Given that the core of our CTT game can be empty, we now focus on identifying sufficient conditions for which the core of CTT games is non-empty.

4.1. No fixed transfer costs

We start with a special class of transfer and transport situations $\Theta_{NF} \subset \Theta$, namely the ones without fixed transfer costs, i.e., $t_{ij}^p = 0$ for all $i, j \in N$ and $p \in P$. Note that for any $\theta \in \Theta_{NF}$, constraints (2e) and variables $(z_{ij}^p)_{i,j \in N, p \in P}$ are redundant. It turns out that any CTT game (N, C^θ) with $\theta \in \Theta_{NF}$ can be recognized as a so-called linear production game. In a linear production game, there is a group of players $N \subseteq \mathbb{N}$ with each player $i \in N$ owning a vector $b^i = (b_j^i)_{j=1}^{|R|} \in \mathbb{R}^{|R|}$ of resources, with R the set of resources. The resources are used to produce a set K of products, with $p_k \in \mathbb{R}$ being the price of product $k \in K$, and matrix $A \in \mathbb{R}^{|R| \times |K|}$ representing the number and type of resources needed to produce specific products. Instead of working independently, players can

bundle their resources and make products together. If a coalition $S \subseteq N$ decides to bundle resources, the profit is given by

$$v^{LP}(S) = \max \sum_{k \in K} p_k y_k \quad \text{s.t.} \quad \sum_{k \in K} a_{jk} y_k \leq \sum_{i \in S} b_j^i \quad \forall j \in R, y \in \mathbb{R}_{\geq 0}^{|K|}$$

Proposition 1. Every CTT game (N, C^θ) with $\theta \in \Theta_{NF}$ is a linear production game.

Proof. Proof of Proposition 1 is presented in Appendix C. \square

We want to stress that this result is not a direct consequence of the fact that the optimization problem of a coalition is a linear programming problem, like the linear programming problem in a linear production game. For instance, in a linear production game, the size of the objective function does not depend on the size of the coalition, while it does in CTT games.

Given that all CTT games (N, C^θ) with $\theta \in \Theta_{NF}$ are linear production games, we can now apply properties of linear production games within our context. More precisely, from [19], we know that the Owen set is a subset of the core of linear production games corresponding to the set of optimal solutions of the dual of the grand-coalition problem. The dual of the grand-coalition problem for our CTT game reads,

$$\max \sum_{i \in N} (D_i \eta_i) - \sum_{i \in N} \sum_{p \in P} (Q_i^p \gamma_i^p) \quad (4a)$$

$$\text{s.t.} \quad \varphi_j^{p-1} - \varphi_i^p - \gamma_i^p \leq c_i^p + s_{ji}^p \quad \forall i \in N \forall j \in N \forall p \in P \setminus \{1, n\} \quad (4b)$$

$$\varphi_j^{n-1} - \gamma_i^n \leq c_i^n + s_{ji}^n \quad \forall i \in N \forall j \in N \quad (4c)$$

$$\eta_j - \varphi_i^1 - \gamma_i^1 \leq c_i^1 + s_{ji}^1 \quad \forall i \in N \forall j \in N \quad (4d)$$

$$\gamma_i^p, \varphi_i^p, \eta_i \in \mathbb{R}_+ \quad \forall i \in N \forall p \in P \quad (4e)$$

where any optimal solution $(D_i \eta_i - \sum_{p \in P} (Q_i^p \gamma_i^p))_{i \in N}$ with η_i, γ_i for $i \in N$ derived from solving the dual formulation in Problem (4), presents a core element of (N, C^θ) with $\theta \in \Theta_{NF}$. We can interpret η_i as the price a carrier i needs to pay for each unit of demand shipped, γ_i as a discount received for the capacity provided by carrier i to support the coalition, while φ_i^p represents how price and discounts are transferred among carriers given their costs and capacities.

Observation 3. It is well-known that linear production games are totally balanced (see [19]), implying that CTT games (N, C^θ) with $\theta \in \Theta_{NF}$ are totally balanced as well.

4.2. Unlimited fixed transfer cost

In addition to games without fixed transfer costs, we also explore a class of transfer and transport situations $\Theta_{UTC} \subset \Theta$ for which $t_{ij}^p > D_i \cdot \sum_{p \in P} (c_i^p + s_{ii}^p) + \sum_{p \in P} t_{ii}^p$ for all $i, j \in N$ with $i \neq j$ and all $p \in P$, i.e., where the fixed transfer costs are extremely high compared to the transport costs. Because these transfer costs are so high, there is no reason for carriers to transfer units amongst each other. As a consequence, every carrier is transporting its own demand. This also leads to a CTT game for which the core is non-empty.

Proposition 2. Let $\theta \in \Theta_{UTC}$. Then, the corresponding CTT game has a non-empty core.

Proof. Proof of Proposition 2 is presented in Appendix C. \square

We want to stress that a similar argument can be used to argue that a CTT game with extremely high variable transfer costs has a non-empty core.

4.3. Unlimited capacity with equal fixed and variable transfer costs

Finally, we discuss a class of transfer and transport situations $\Theta_{UCETC} \subset \Theta$ for which $Q_i^p > \sum_{i \in N} D_i$ for all $p \in P$ and $t_{ij}^p = t_{i'j'}^p$ and $s_{ij}^p = s_{i'j'}^p$ for all $i, i', j, j' \in N$ and all $p \in P$. This describes a situation where due to the lack of capacity restrictions, in combination with the equal (fixed and variable) transfer costs, the entire volume will follow the same transfer policy after consolidation at point 1.

Proposition 3. Let $\theta \in \Theta_{UCETC}$. Then, the corresponding CTT game has a non-empty core.

Proof. Proof of Proposition 3 is presented in Appendix C. \square

In this case, we have not only seen that the core is non-empty, but we have also found a core allocation based on the optimal path of each carrier and a division of the fixed costs.

5. Numerical experiments

In the previous section, we investigated sufficient conditions for core non-emptiness. In particular, we showed that the core is non-empty when the fixed transfer costs are very low, very high, or symmetric. In this section we will investigate how much we can generalize core non-emptiness when these conditions are not met. In the following, we first explain how we construct the instances used for our experiments, specifying the parameters and the related transfer and transport situations before presenting the results of these experiments in § 5.2.

5.1. Instance design

For our experiments, we randomly generate instances with three, four, or five carriers (i.e., $|N| \in \{3, 4, 5\}$) that need to transport their volume via three, four, or five points (i.e., $|P| \in \{3, 4, 5\}$). In this context, we assume that carriers can operate in two types of markets:

- **A symmetric market**, where all carriers need to transport demand volume drawn from the same probability distribution; in our case, we set $D_i \sim D = U[80, 120]$ for all $i \in N$. In this case, all carriers have, on average, the same presence in the market.
- **A market with a dominant carrier**, where one carrier $i^* \in N$, referred to as the *dominant carrier* transports on average around 50% of the total demand volume, so that $D_{i^*} \sim (|N| - 1) \cdot D$, while the demand volume of all other carriers follows the probability distribution D as proposed in the symmetric market. In this way, we can study the impact of an asymmetric market on core non-emptiness.

Moreover, to observe the impact of capacity on core non-emptiness, we consider instances that range from settings where each carrier has sufficient capacity to serve all the demand to settings where each carrier has little or no space to serve extra demand.

Table 6

Parameter configurations used for our experiments.

Experimental Parameter	Values
Number of carriers	3, 4, 5
Number of points	3, 4, 5
Demand distribution	Symmetric, Big carrier
Fixed cost ratio (fix-r)	0.1, 1, 10, 100, 1000, 10000, 100000
Transportation cost ratio (trans-r)	1, 10, 100
Internal fixed costs scenario	Standard, No internal, Group
Capacity ratio (cap-r)	1.1, 1.25, 1.5, 2, 2.5, 3, 6

As such, to obtain the capacity Q_i^p of carrier $i \in N$ between points $p \in P$ and $p + 1$, we first draw a number from the demand distribution D and subsequently multiply this with a capacity ratio, which is $\text{cap-r} \in \{1.1, 1.25, 1.5, 2, 2.5, 3, 6\}$. A low capacity ratio of $\text{cap-r} = 1.1$ would reflect a setting where each carrier has little or no space to transport additional volumes, while a capacity ratio of $\text{cap-r} = 6$ would reflect the case where each carrier can serve the entire volume in the system. To avoid infeasibility, we set the generated demand of a carrier as a lower bound, so that $Q_i^p \sim \max\{D_i, D \cdot \text{cap-r}\}$ for all $i \in N$ and $p \in P$.

To investigate the impact of different cost structures on core non-emptiness, we consider variable transfer costs as a reference and propose two ratio parameters that relate transportation costs with variable transfer costs and fixed transfer costs with both demand and variable transfer costs. To generate the variable costs of transferring one unit of volume from i to j at point $p \in P$, we draw a number $s_{ij}^p \sim S = U[80, 120]$ for all $i, j \in N$ and $p \in P$. Next, to generate c_i^p , which represents the cost of transporting one unit of volume of carrier $i \in N$ from $p \in P$ to $p + 1$, we draw a number from S and multiply this value with a ratio $\text{trans-r} \in \{1, 10, 100\}$, so that $c_i^p \sim \text{trans-r} \cdot S$ for all $i \in N$ and all $p \in P$. Differently from the transportation costs, we want to scale the fixed transfer costs to both the variable transfer costs and the demands. Thus, to generate t_{ij}^p , i.e., the fixed transfer costs of transferring volume from i to j at point $p \in P$, we multiply a number in the interval of the variable transfer cost (i.e., $s_{ij}^p \sim S$) with a number in the interval of the demand within a symmetric market (i.e., $D_i \sim D$) and subsequently multiply this with a ratio $\text{fix-r} \in \{0.1, 1, 10, 100, 1000, 10000, 100000\}$, so that $t_{ij}^p \sim \text{fix-r} \cdot S \cdot D$ for all $i, j \in N$ and all $p \in P$. Furthermore, we consider two special cases for the fixed transfer costs while referring to the one we have described so far as the “Standard” scenario:

- **No internal fixed transfer cost**, i.e., the setting with $t_{ii}^p = 0$ for all $i \in N$ and $p \in P$, while all other fixed transfer costs are generated in the same way as described earlier. This describes the case where no extra investment is needed for internal transferring. We call this scenario “No internal”.
- **A group discount**, which reflects a setting where carriers already collaborate and, thus, the fixed transfer costs associated with them might be lower. To account for this, we randomly divide the group of carriers N into two subgroups N_1, N_2 , with $N_1 \cup N_2 = N$ and $|N_1| = \lceil |N|/2 \rceil$. Within the two groups, N_1 and N_2 , we reduce the fixed transfer cost between each pair of carriers, t_{ij}^p with 50%. Formally, for all $i, j \in N_1$, all $i, j \in N_2$ and all $p \in P$, we set $t_{ij}^p \sim 1/2 \cdot \text{fix-r} \cdot S \cdot D$. We assume that all other fixed transfer costs are generated in the same way as before. We call this scenario “Group discount”.

Table 6 presents an overview of the configurations used to generate the instances for our experiments. We create one instance for each possible combination of parameters in Table 6.

5.2. Results

In Fig. 3, we present in white bars the frequency of instances for which $C(N) < \sum_{i \in N} C(\{i\})$, which we refer to as “real collaborations”, and in the grey bars we present the percentage of these real-collaboration instances for which the core is non-empty. Based on this, we observe that the majority of games with real collaboration have a non-empty core, with the highest percentage (99.9%) observed for instances with a fix-r ratio equal to 0.1 and the lowest percentage (73.55%) observed for a capacity ratio of 2.5. These numbers indicate that stable collaborations are still very likely, even without very low, very high, or symmetric fixed transfer costs. At the same time, the impact of the number of carriers, the number of points, the transportation cost ratio, and the market type seems to be minor in comparison to the impact of the capacity, the fixed transfer costs as well as the considered fixed transfer cost scenarios on both real collaborations as well as the frequency of non-empty cores.

Investigating the impact of capacity in more detail, we first observe a decrease in core non-emptiness (from 99.75% to 73.55%) as capacity increases. This is followed by an increase, which peaks at 94.5% for the case where each carrier has sufficient capacity to serve all demand ($\text{cap-r} = 6$). These numbers indicate that the more extreme cases provide more stable conditions for real collaboration, while the range in between creates conditions in which a stable collaboration is less likely. The underlying reasoning for this drop might be that some coalitions can transport their volume over the network without suffering from capacity constraints, while others do not. That is, some coalitions find an optimal transport solution for which none of the capacity constraints are binding, while for other coalitions some of these capacity constraints are binding. Consequently, the coalitions without binding capacity constraints benefit more from the collaboration than the others, which can break stability. This effect disappears when capacity becomes sufficiently large ($\text{cap-r} = 6$) or very tight ($\text{cap-r} = 1.1$).

Focusing on different transfer cost settings, we observe that the absence of internal fixed transfer costs considerably reduces the frequency of real collaborations (22.41% of the cases instead of 99.36% and 98.94%), in comparison to the “Standard” and “Group discount” scenarios. This relatively low frequency of real collaborations follows from the fact that transporting volume individually (which is not a real collaboration) is often cheaper. Despite real collaborations being sparse whenever no internal fixed transfer costs are considered, the core is often non-empty in case of real collaborations with higher probabilities than in the “Standard” and “Group discount” scenarios (99.66% instead of 87.68% and 86.25%, respectively). Looking in more detail at the fixed costs ratio, we observe a decrease in the frequency of core non-emptiness as well as real collaborations as fix-r increases, i.e., the core is non-empty in 99.9% of the cases for fix-r = 0.1 and drops to 80.93% for fix-r = 100,000, and real collaborations drop from 88.71% for fix-r = 0.1 to 65.7% for fix-r = 1000. This is, however, not true if we only focus on the fixed cost scenario “No internal”. This is illustrated in Fig. 4, where we present the results separated by the considered fixed cost ratio scenarios, i.e., “Standard”, “Group” and “No internal”, while using a more detailed scale of fix-r (i.e., we reran our experiments with a more detailed scale of fix-r ranging between 0.1 and 100). From Fig. 4, we learn that the decrease in core non-emptiness is only caused by the fixed cost scenario “Standard” and “Group”, while the frequency of core non-emptiness for the fixed cost scenario “No Internal” is always above 95% and relatively stable.

In summary, the numerical experiments indicate that collaboration is very likely, even without very low, very high, or symmetric fixed transfer costs. However, exceptions exist, most often due to partial capacity restrictions.

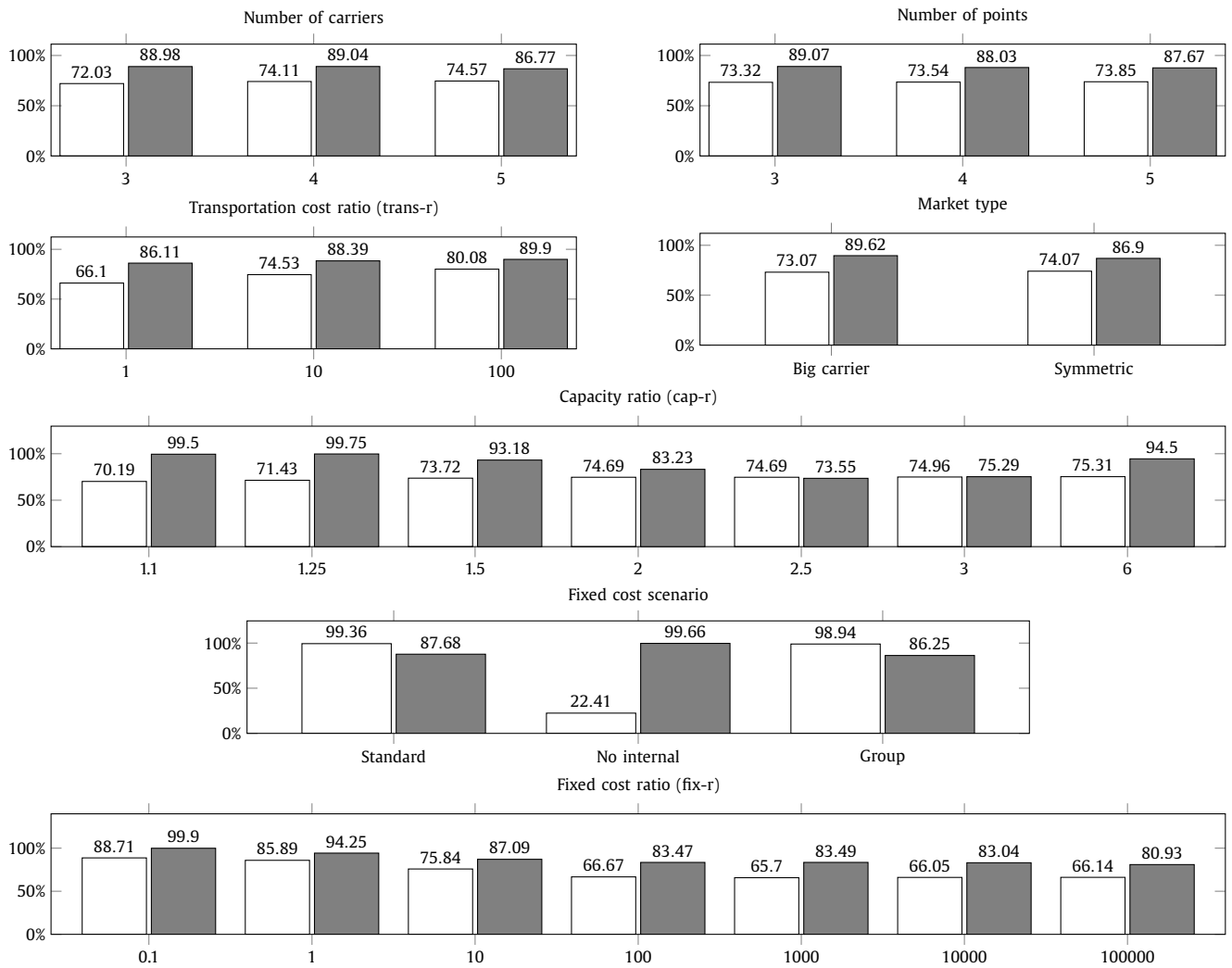


Fig. 3. Non-additive games (white bar) and non-empty core (grey bar) frequencies in the first experiment.

Fig. 4a: Fixed cost ratio (fix-r) with internal fixed cost scenario "Standard"

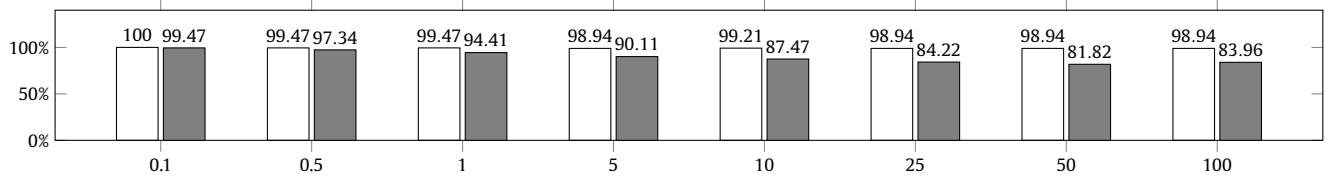


Fig. 4b: Fixed cost ratio (fix-r) with internal fixed cost scenario "No internal"

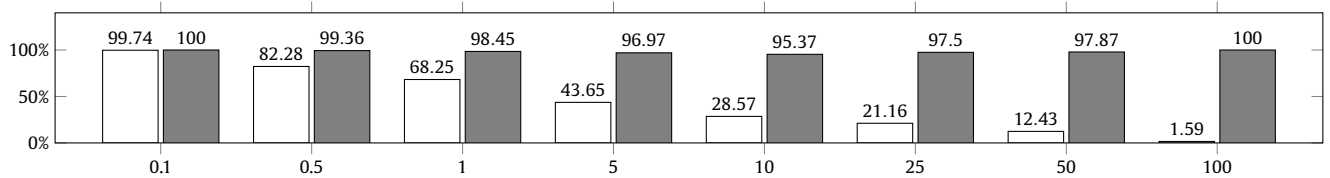


Fig. 4c: Fixed cost ratio (fix-r) with internal fixed cost scenario "Group"

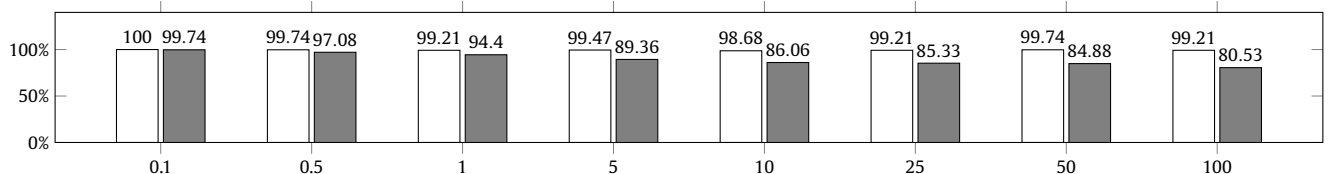


Fig. 4. Comparison of the impact of fix-r between the three discount scenarios.

CRediT authorship contribution statement

Fabio Mercurio: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Loe Schlicher:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Sonja U.K. Rohmer:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Tom Van Woensel:** Writing – review & editing, Writing – original draft, Validation, Supervision, Conceptualization.

Data availability

Data will be made available on request.

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Appendix. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.orl.2024.107142>.

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