Isogeometric analysis of drop deformation

Technische Universiteit Eindhoven University of Technology

Isogeometric analysis of drop deformation

A. A. Joneidi
C. V. Verhoosel
P. D. Anderson

Introduction

Understanding the dynamics of cells is important in many different areas. We use Isogeometric Analysis (IGA) to mimic the motion of a drop in shear flow. This motion is characterized by Taylor deformation $D = \frac{L-B}{L+B}$ where L and B are length and breadth respectively.

Boundary integral method

As schematically shown in Figure 1, we consider a drop (Ω_2) suspended in matrix fluid (Ω_1).

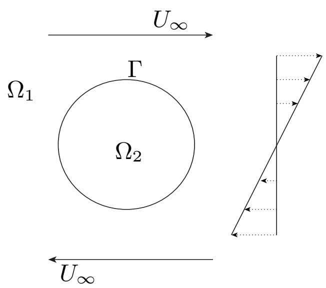

Figure 1 *Schematic representation of the fluidic domains.*

The drop and matrix fluids are Newtonian with the same viscosity while inertia effects are neglected. We employ the boundary integral method [1] in order to solve the governing Stokes equations. For any point $\boldsymbol{x_0} \in \Gamma$, the velocity, $\boldsymbol{u}(\boldsymbol{x}_0)$, is given by

$$\boldsymbol{u}(\boldsymbol{x}_0) = \boldsymbol{u}_\infty(\boldsymbol{x}_0) - \frac{1}{8\pi}\int_\Gamma \boldsymbol{G}(\boldsymbol{x},\boldsymbol{x}_0)\boldsymbol{f}(\boldsymbol{x})d\Gamma(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{u}_\infty(\boldsymbol{x}_0)$ is the imposed shear velocity and $\boldsymbol{G}(\boldsymbol{x},\boldsymbol{x}_0)$ is the Green's function given by

$$\boldsymbol{G}(\boldsymbol{x},\boldsymbol{x_0}) = \boldsymbol{G}(\hat{\boldsymbol{x}}) = \frac{\boldsymbol{I}}{|\hat{\boldsymbol{x}}|} + \frac{\hat{\boldsymbol{x}}\otimes\hat{\boldsymbol{x}}}{|\hat{\boldsymbol{x}}|^3}, \tag{2}$$

with $\hat{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x_0}$. $\boldsymbol{f}(\boldsymbol{x})$ is the stress discontinuity across the interface which is $\boldsymbol{f}(\boldsymbol{x}) = \frac{2}{\mathrm{Ca}}\kappa(\boldsymbol{x})\boldsymbol{n}(\boldsymbol{x})$ where $\kappa(\boldsymbol{x})$ is the local mean curvature, $\boldsymbol{n}(\boldsymbol{x})$ is the unit normal vector and Ca is the capillary number defined as a measure of the ratio between viscous and surface tension forces.

Isogeometric analysis

The key ingredient of this isogeometric analysis is that we employ a B-spline representation of the droplet surface. The B-spline basis functions are defined recursively beginning with piecewise constant:

$$N_{i,0}(\zeta) = \begin{cases} 1 & \zeta_i \le \zeta < \zeta_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where ζ_i is the i^{th} knot, i is the knot index and p is the polynomial order. The higher degree B-spline shape functions then follow as [2]:

$$N_{i,p}(\zeta) = \frac{\zeta-\zeta_i}{\zeta_{i+p}-\zeta_i}N_{i,p-1}(\zeta) + \frac{\zeta_{i+p+1}-\zeta}{\zeta_{i+p+1}-\zeta_{i+1}}N_{i+1,p-1}(\zeta) \tag{4}$$

Results

In Figure 2 we show the Taylor deformation as a function of time with 200 elements when Ca $= 0.2$. It is observed that the flow gradually elongates and re-orients the drop while the drop keeps its centroid in the same position. The stable Taylor deformation, $D = 0.2235$, closely resembles results reported in [3].

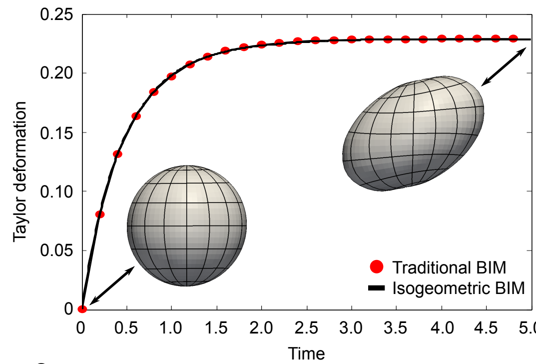

Figure 2 *Taylor deformation for a drop in shear flow with Ca $= 0.2$.*

As a preliminary result we consider the computation of the first- and second-order gradients of the curvature over an element (see Figure 3). As a result, the error of $\boldsymbol{\Delta}_s\kappa = 0.0231$ with 720 elements and fourth-order splines where $\boldsymbol{\Delta}_s$ denotes surface laplace-Beltrami operator. The error is defined as

$$\mathcal{E} = \frac{\|\boldsymbol{\Delta}_s\kappa - \boldsymbol{\Delta}_s\kappa_{ana}\|_{\hat{\Gamma}}}{\|\boldsymbol{\Delta}_s\kappa_{ana}\|_{\hat{\Gamma}}}, \tag{5}$$

with

$$\|\square\|_{\hat{\Gamma}} = \sqrt{\int_{\hat{\Gamma}} (\square)^2\, d\hat{\Gamma}} \tag{6}$$

the L^2-norm over the parameter domain $\hat{\Gamma}$.

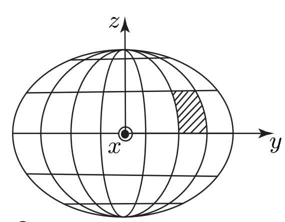

Figure 3 *Initial configuration of the ellipsoid.*

Conclusion

An advantage of IGA is that it allows the unambiguous evaluation of normal vectors and curvatures. This advantage will be more pronounced when higher-order gradients appear in the surface force term, which can be found in red blood cell or vesicle membrane modeling.

References:

[1] J. M. Rallison, A. Acrivos, 1987 *J. Fluid Mech.* **89**, 191-200

[2] M. G. Cox 1972 *J. Inst. Math. Appl.* **10**, 134-147

[3] P. J. A. Janssen, P. D. Anderson 2007 *Phys. fluids* **19**, 043602