Isogeometric analysis for vesicle deformation

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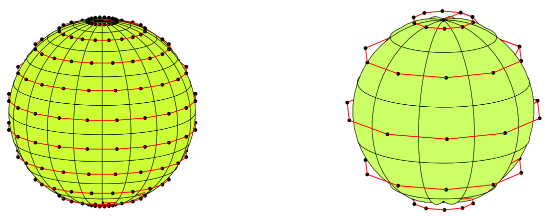
Introduction

Isogeometric analysis has been proposed by Hughes et al. [1]. This is a computational approach which offers us to work with the higher order definition of the interface. In this method, a control net is needed to make a desired interface (see Fig. 1).



(a) (b)

Fig. 1 *Spheres which are created by Bézier curves. The red nets depict the control net and the black dots are control points*
(a) 240 control points (b) 70 control points

Objective

To model vesicle deformation in flow using isogeometric analysis

B-spline curve

The B-spline basis functions are defined recursively beginning with piecewise constant (p=0):

$$N_{i,0}(\zeta) = \begin{cases} 1 & \zeta_i \le \zeta < \zeta_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Where ζ_i is the i^{th} knot, i is the knot index, $i = 1, 2, ..., n+p+1$, p is the polynomial order and n is the number of basis functions needed to make the B-spline curve. For $p = 1, 2, 3, ...$, they are defined by

$$N_{i,p}(\zeta) = \frac{\zeta - \zeta_i}{\zeta_{i+p} - \zeta_i} N_{i,p-1}(\zeta) + \frac{\zeta_{i+p+1} - \zeta}{\zeta_{i+p+1} - \zeta_{i+1}} N_{i+1,p-1}(\zeta) \tag{2}$$

Method

To describe the dynamic deformation of the vesicle in Stokes flows, the boundary integral method is implemented. In this method the computations will apply directly on the membrane. The membrane velocity is [2]:

$$\boldsymbol{u}(x_0) = \boldsymbol{u}_\infty - \frac{1}{8\pi} \int \boldsymbol{f}(x).G(x, x_0) dS(x) \tag{3}$$

where $\boldsymbol{u}_\infty$ is the velocity of imposed flow, $\boldsymbol{f}$ is the force term across the membrane and G is the Greens function [3].

Results

Fig. 2 shows the increment of the number of control points highly influences on the accuracy of the obtained curve.

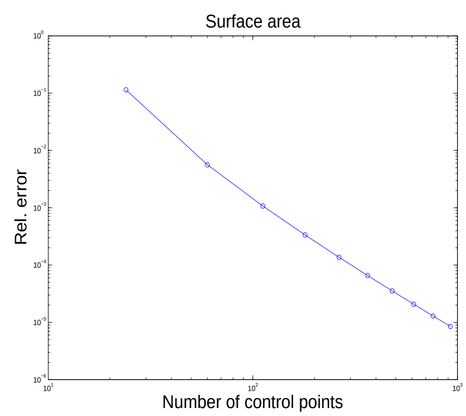


Fig. 2 *Accuracy of surface area by Isogeometric analysis for a unit circle.*

Conclusion

The study demonstrates the accuracy of the curve by increasing the number of control points.

References:

[1] T. J. R. Hughes, J. A. Cottrell, Y. Bazilevs 2005 Isogeometric analysis: CAD, nite elements, NURBS, exact geometry, and mesh renement. *Computer Methods in Applied Mechanics and Engineering* **194**, 41354195

[2] C. Pozrikidis 1992 Boundary-Integral and Singularity methods for Linearized Viscous Flow. *Cambridge U.P., Cambridge*

[3] P. J. A. Janssen, P. D. Andersona 2007 Boundary-integral method for drop deformation between parallel plates. *Phys. fluids* **19**, 043602