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SIMULATION RESULTS ON ALBIN & WHITT'S ESTIMATION METHOD FOR AVERAGE WAITING TIMES IN MULTI PRODUCT, SINGLE SERVER QUEUEING SYSTEMS.

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ABSTRACT

In this paper we investigate the quality of Albin and Whitt's estimation method for the average waiting time in a multi-product, single-server queueing system. In the queueing system under study, orders for the different products arrive according to Erlang distributed interarrival times. Upon arrival, the orders are put in queue to be processed in order of arrival. The service times of the orders also follow Erlang-distributions. This system can be modelled as a ΣGI/G/1 queue.

We apply Albin and Whitt's approximation method to estimate the average order waiting times in this system. The approximation method starts by replacing the superposition arrival process by an approximating renewal process that has the same first moment as the original arrival process and an approximated second moment. In this way the ΣGI/G/1 queue is transformed into a GI/G/1 queueing problem. Subsequently, a standard approximation formula for the GI/G/1 queue, based only on the first two moments, is applied to obtain estimates for the average waiting time.

Comparing the resulting waiting time estimates with the outcome of extensive simulation studies shows that the estimation method gives poor approximations for the average waiting times. Since Albin & Whitt's estimation method is an essential building block of many queuing network analysers, our findings seriously compromise the use of these methods for performance evaluation and decision-making in a manufacturing context.

1. Introduction

We investigate the estimation quality of Albin & Whitt's method to estimate the average waiting times in a multi-product, single server queueing system. The queueing system under study is presented in figure 1.1. The system consists of N component arrival processes and a single server. Orders arrive according to Erlang(Q_i,λ_i)-renewal processes. The order arrival process observed by the server is a complex process, consisting of the superposition of N independent Erlang-processes. We assume that orders are served according to Erlang(Q_i,μ_i)-distributed service times and in order of arrival (FCFS).

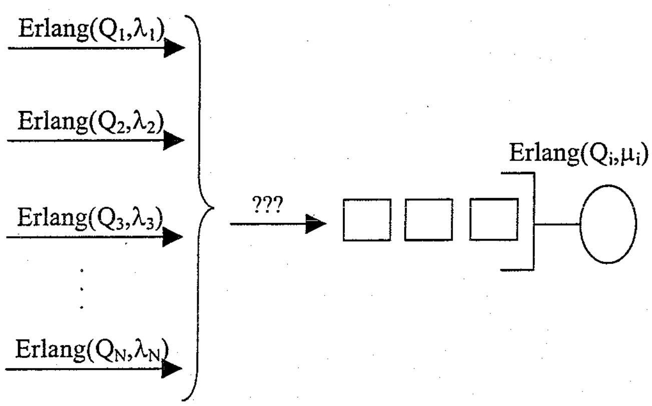

Figure 1.1: Queueing system with superposition arrival process

One particular realisation of this queuing system is a multi-product single server production-inventory system in which the N stock points are controlled by an (s_i,Q_i)-inventory policy, with i = 1, ... , N. This production-inventory system is shown in figure 1.2. Unit sized customer orders for product i arrive at stockpoint i according to exponentially distributed interarrival times with rate λ_i. Each time the inventory position drops below the reorder level s_i, a production order (with lot size Q_i) is placed on the production system. Since customer order interarrival times are exponentially distributed, production orders are generated according to Erlang(Q_i,λ_i) renewal processes. Production orders for different products compete for capacity at the server and are processed in order of arrival (FCFS). The service time of a single product i is assumed to be exponentially distributed with rate μ_i. Consequently, the processing time of a production batch of product i follows an Erlang(Q_i,μ_i) distribution function.

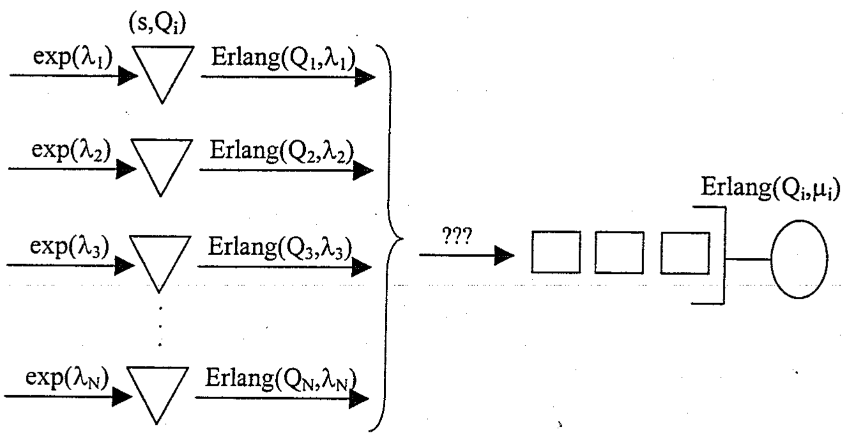

Figure 1.2: Production-inventory system with superposition arrival process

The described queueing systems can be denoted as $\Sigma E_i/E_i/1$-queues. To the best of our knowledge, no exact results exist for this queueing problem. Therefore, we assume N general independent arrival processes and general service times: $\Sigma GI/G/1$. Albin (1982, 1984) and Whitt (1982, 1983) jointly developed an approximation procedure for the expected waiting times in this queueing model. Whitt and Albin claim that their approximation procedure yields an average absolute error of 4 percent.

In this paper we evaluate the estimation quality of this approximation procedure by comparing the estimates with the outcome of an extensive simulation study. This allows us to verify if the method yields the claimed 4-percent average absolute error in a broader context. Our simulation study, consisting of 50 different parameter settings for the factors (i) utilization of the server, (ii) number of component processes and (iii) squared coefficient of variation of the component arrival processes, reveals that the approximation formulae proposed by Whitt and Albin perform poorly. Our simulation study shows substantial discrepancies - about 22 percent on average - between the estimates for the average waiting time obtained by approximation and those obtained by simulation.

The remainder of this paper is organized as follows: section 2 discusses Albin & Whitt's approximation procedure; section 3 presents the design of the simulation study; section 4 shows the outcome of the simulation experiments and compares them with the estimates of the approximation method; in section 5 we try to improve the performance of the estimation procedure by applying alternative expressions for the weighting function; section 6 discusses the impact of our findings on queueing network analysers; finally, section 7 summarizes the major findings of this paper.

2. Outline of Albin & Whitt's approximation procedure for ΣGI/G/1 queues

In this section we investigate the approximation method for ΣGI/G/1 queues proposed by Kuehn (1979) and refined by Whitt (1982, 1983) and Albin (1982, 1984).

The approximation method starts by replacing the superposition arrival process by an approximating renewal process that has the same first moment as the original arrival process and an approximated second moment. In this way, the ΣGI/G/1 queue is transformed in a GI/G/1 queueing problem. Subsequently, a standard approximation formula for the GI/G/1 queue, based only on the first two moments, can be applied to obtain estimates for the average waiting time in the system.

A potential weakness of this method is that the superposition of N renewal processes is not a renewal process, except for the special case when all component processes are Poisson. As Albin remarks: "If at least one component process is not Poisson, then the superposition process is not renewal and the intervals between (arrival) points are identically distributed, but not independent". Ignoring this interdependence between interarrival intervals may seriously deteriorate the waiting time estimates. Livny et al. (1993) pointed out that the injection of autocorrelation into interarrival times can have a dramatic impact on performance measures.

In his paper of 1982, Whitt proposes two different methods to specify the first two moments of the approximating renewal arrival process. His stationary-interval method equates the moments of the renewal interval with the moments of the stationary interval in the point process to be approximated. The asymptotic method determines the moments of the renewal interval by matching the asymptotic behaviour of the moments of the sums of successive intervals. In this way, it attempts to account for the dependence among successive intervals. The stationary and asymptotic method coincide for the first moment but yield different expressions for the second moment. The proposed methods have the analytical properties that the stationary-interval approximation is asymptotically correct as the number of component arrival processes N approaches infinity and that the asymptotic approximation is asymptotically correct as the utilization of the server ρ approaches the critical value 1. Moreover, simulation experiments suggest that the simulation estimates for the average number of customers in queue fall between the approximations determined by the basic methods.

Building on these observations, Albin (1984) developed a hybrid method that combines the squared coefficients of variation (scv) of the basic methods by a weighting function w to obtain the hybrid approximation of the scv c_H^2 by:

$$c_H^2 = wc_A^2 + (1-w)c_S^2.$$

Notation:

- c_A^2: scv of the approximating renewal arrival process determined by Whitt's asymptotic method;
- c_S^2: scv of the approximating renewal arrival process determined by Whitt's stationary interval method;
- $w \equiv w(\rho, n^*)$: waiting function, a simple function of the server utilization ρ and the "effective" number of component process n^*, given by $n^* = \left[\sum_{i=1}^{n} (\lambda_i / \lambda)^2\right]^{-1}$.

Based on the properties of the basic methods, the following properties are imposed on the waiting function $w \equiv w(\rho, n^*)$:

(i) $0 \leq w \equiv w(\rho, n^*) \leq 1$.

(ii) As $\rho \to 1$, $w \equiv w(\rho, n^*) \to 1$.

(iii) As $\rho \to 0$, $w \equiv w(\rho, n^*) \to 0$.

(iv) As $n^* \to \infty$, $w \equiv w(\rho, n^*) \to 0$.

Albin generated a list of candidate waiting functions, all satisfying the listed properties. Subsequently, simulation is used to determine which candidate weighting function yields the smallest average absolute approximation error. So, simulation is not just applied to check the approximation quality of the method, it is an essential building block in the development of the approximation method. Albin conducted a simulation study consisting of 180 $\Sigma GI_i/M/1$ queues with various utilizations, numbers of component arrival processes and interarrival time distributions. Albin varied the utilization ρ over 5 levels: 0.3, 0.5, 0.7, 0.8 and 0.9. The four levels of the number of component processes N are: 2, 4, 8 and 16. Albin uses c_A^2, the scv of the approximating renewal arrival process determined by Whitt's asymptotic method, as a measure for the variability of the arrival process. The three levels of the c_A^2 are 2, 4 and 6. Finally, there are three levels of distributions in Albin's experiment: (i) all component processes are identical and have hyperexponentially distributed intervals; (ii) all component processes have hyperexponentially distributed intervals, but half of the component processes have c_1^2 and the other have c_2^2; (iii) half of the component processes are hyperexponentially distributed and half are Erlang distributed with $c^2 =$ 0.5.

Albin tested her approximation on 216 ΣGI/G/1-queues. She varied the utilization level over 70 – 90 percent and the number of component processes over the values 2, 4 and 16. In the experiments, six possible component interarrival distributions and six possible service-time distributions are tested. Each of the distributions have c^2 equal to 5, 2 or 0.5. Albin reports that the average absolute error made by the hybrid approximation of the expected number of customers in queue is 3% , compared to the errors of the basic methods that are 20-30%.

In the Queueing Network Analyser, developed by Whitt (1983), the hybrid approximation procedure of Albin is used in a somewhat simplified form by combining the asymptotic approximation with a Poisson approximation to the superposition process. This modification is based on the results of Albin (1982). He reports on a slight degradation of the estimation quality of the approximation procedure when compared with the procedure proposed by Albin (1984). The average absolute error rises from 3 to 4 percent but the approximation is considerably easier to compute.

The hybrid approximation for the arrival rate is given by:

$$\lambda = \sum_{i=1}^{N} \lambda_i ;$$

and the approximation for the scv of the interarrival times is given by:

$$c_H^2 = w \cdot c_A^2 + (1 - w)$$

where

$$c_A^2 = \sum_{i=1}^{N} \left(\frac{\lambda_i}{\sum_{k=1}^{N} \lambda_k} \right) \cdot c_i^2 \text{ and } w = \left[1 + 4(1-\rho)^2 (v-1)\right]^{-1},$$

where

$$v^{-1} = \sum_{k=1}^{N} \left(\frac{\lambda_k^2}{\left(\sum_{i=1}^{N} \lambda_i \right)^2} \right).$$

Notation:

- λ_i = arrival rate of component arrival process i,
- c_i^2 = scv of interarrival times of component arrival process i,
- N = number of component arrival processes.

Once the arrival process to the server has been approximated by a simpler renewal process characterised by the first two moments, the approximation method continues by applying two-moment approximations for the GI/G/1 queue, e.g. Buzacott and Shanthikumar (1993), De Kok (1989), Kraemer and Langenbach-Belz (1976), to find results for the average waiting times, sojourn times, etc. As proposed by Albin and Whitt, we will make use of an adaptation of the Kraemer and Langenbach-Belz formula to approximate the average waiting times in queue, that is:

$$EW = \frac{\rho(c_H^2 + c_S^2)}{2\mu(1-\rho)} g$$

where $g \equiv g(\rho, c_H^2, c_S^2)$ is defined as

$$g(\rho, c_H^2, c_S^2) = \begin{cases} \exp\left[-\frac{2(1-\rho)}{3\rho} \frac{(1-c_H^2)^2}{(c_H^2 + c_S^2)} \right], & c_H^2 < 1. \\ 1, & c_H^2 \geq 1. \end{cases}$$

Notation:

- ρ = utilization of the server,
- μ = mean service rate of a customer (so that $1/\mu$ is the mean service time),
- c_H^2 = scv of interarrival intervals determined by Albin & Whitt's procedure,
- c_S^2 = scv of variation of service times.

3. The simulation study

In our simulation study, we consider a large variety of multi-product single-server production-inventory systems. Each simulation experiment is characterised by the following factors: (i) utilization of the server ρ; (ii) number of component arrival processes N; (iii) scv of the component arrival processes c_i^2. We assume symmetrical component arrival processes, i.e. identical rates, scv's, etc. Service times are Erlang(Q_i,μ_i)-distributed where μ_i is kept constant at one time unit and the scv of the service times equals the scv from the component arrival processes. The reorder level s_i is fixed at 10,000 for all experiments and all products.

In the simulation experiments, the scv of the component arrival processes and the number of component processes are varied over 5 different levels: c_i^2 = 0.500, 0.333, 0.100, 0.010, 0.001 and N = 5, 10, 25, 50, 100. The utilization takes on two values; ρ = 0.7 and 0.9. Since the interarrival times of the component processes are Erlang(Q_i, λ_i)-distributed, the Erlang distribution function parameters Q_i are given by $c_i^2 = 1/Q_i$. Hence, in the simulation study, the Erlang distribution function parameters Q_i are varied over the values 2, 3, 10, 100, and 1000.

Each experiment is replicated 10 times, which leads to a general factorial experiment with a total number of 5 x 5 x 2 x 10 = 500 observations. The simulation period lengths are chosen such that in each replication at least 350,000 orders are simulated.

Before making our observations, we validated the simulation program in several ways, by: (i) assuming exponential interarrival and service times ($c_i^2 = 1$) for which exact results are available; (ii) setting the number of component processes N equal to 1 so that the queueing reduces from an intractable $\Sigma E_Q/E_Q/1$-queue to an $E_Q/E_Q/1$-queue for which numerical results are available in e.g. Buzacott and Shanthikumar (1993); (iii) comparing the output of the Simula-model with the output of a similar simulation model built in Delphi.

4. Comparison of Albin & Whitt's estimates with simulation results

The average waiting times in queue (AW_{sim}) observed through simulation are presented in table 4.1. Next, Albin & Whitt's formulae are applied to obtain approximations for the 50 ΣGI/G/1-queues in our experiment. The resulting estimates for the average waiting times, $AW_{a\&w}$, are displayed in table 4.1.

The table also gives the relative difference between Albin & Whitt's approximations and the outcome of the simulation, e_{total}:

$$e_{total} = 100 \cdot \frac{(AW_{a\&w} - AW_{sim})}{AW_{sim}}.$$

At the bottom of the table, some summary statistics on the relative errors are provided. The indices ABS indicate that the absolute value of the relative difference has been used to compute the summary statistics.

From the summary statistics, we learn that the approximating method proposed by Whitt and Albin yields average absolute approximation errors of 24.3 and 19.7 percent for the case of 70 and 90 percent utilization respectively. The statistics indicate that the errors in the low utilization case are considerably higher than in the high utilization case. Errors can become exceedingly high, up to 57.5 percent. In either way, our simulation shows that the average absolute errors of Albin & Whitt's approximation method are dramatically worse than the reported 4 percent.

In figures 4.1 and 4.2 the average of the absolute value of the relative difference is plotted in function of respectively the number of products N and the scv of interarrival times of the component processes. From figure 4.1 it arises that the relative error is high when the number of component processes is small and the server utilization is low. For the case of low utilization, the relative error declines as the number of component processes increases. For the case of high utilization no clear pattern arises: the relative error is small when the number of component processes is low (5 – 10). The error sharply increases when the number of component arrival processes is increased to 25 and 50. A further increase of the number of component processes decreases the relative error. Figure 4.2 shows the average relative error in function of the scv of the component arrival processes. The relative errors are clearly rising if the scv of the component processes is declining, but it also appears that the relative error is more or less converging if the scv is made arbitrarily small.

Level of factors			Average waiting time				Average waiting time		
c_i^2	N	ρ	AW_{sim}	$AW_{a\&w}$	e_{total}	ρ	AW_{sim}	$AW_{a\&w}$	e_{total}
0.500	5	70.0	2.4	3.0	23.8	90.0	9.1	9.5	3.7
0.500	10	70.0	2.6	3.2	23.2	90.0	9.3	10.1	8.0
0.500	25	70.0	2.9	3.4	17.0	90.0	9.8	11.2	13.4
0.500	50	70.0	3.1	3.4	11.5	90.0	10.2	12.0	16.8
0.500	100	70.0	3.2	3.5	6.6	90.0	10.9	12.6	15.2
0.333	5	70.0	2.6	3.6	38.0	90.0	9.4	9.9	5.6
0.333	10	70.0	3.0	4.1	34.1	90.0	9.9	11.1	12.4
0.333	25	70.0	3.6	4.4	22.5	90.0	11.0	13.3	21.3
0.333	50	70.0	4.0	4.5	14.1	90.0	11.9	14.9	25.0
0.333	100	70.0	4.3	4.6	7.7	90.0	13.2	16.1	22.3
0.100	5	70.0	5.1	8.1	57.5	90.0	12.5	12.7	1.7
0.100	10	70.0	7.1	10.2	44.1	90.0	15.7	18.3	16.4
0.100	25	70.0	9.4	11.7	24.6	90.0	21.3	28.1	32.3
0.100	50	70.0	10.8	12.3	14.0	90.0	25.8	35.5	37.5
0.100	100	70.0	11.7	12.5	7.3	90.0	31.8	41.2	29.3
0.010	5	70.0	43.8	65.2	48.9	90.0	67.3	49.9	-25.9
0.010	10	70.0	63.8	88.8	39.1	90.0	107.1	110.5	3.1
0.010	25	70.0	85.8	105.5	22.9	90.0	172.3	218.8	27.0
0.010	50	70.0	98.4	111.5	13.4	90.0	228.8	300.2	31.2
0.010	100	70.0	107.3	114.6	6.8	90.0	286.7	363.2	26.7
0.001	5	70.0	431.5	636.5	47.5	90.0	630.8	424.6	-32.7
0.001	10	70.0	641.6	874.6	36.3	90.0	1046.6	1031.9	-1.4
0.001	25	70.0	846.3	1043.5	23.3	90.0	1687.4	2126.0	26.0
0.001	50	70.0	966.4	1104.5	14.3	90.0	2256.0	2948.1	30.7
0.001	100	70.0	1050.4	1135.8	8.1	90.0	2831.5	3583.7	26.6
		AVG			24.3	AVG			14.9
		MIN_{abs}			6.6	MIN_{abs}			1.4
		MAX_{abs}			57.5	MAX_{abs}			37.5
		AVG_{abs}			24.3	AVG_{abs}			19.7

Table 4.1: Comparison of simulation and approximation estimates for 50 ΣGI/G/1 queues

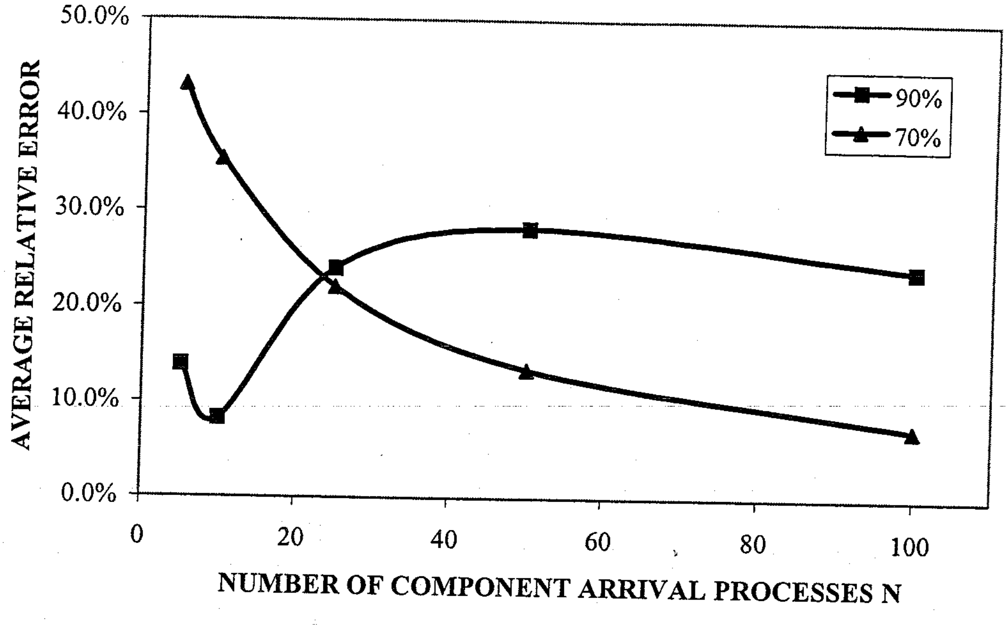

Figure 4.1: Average error in function of number of component arrival processes

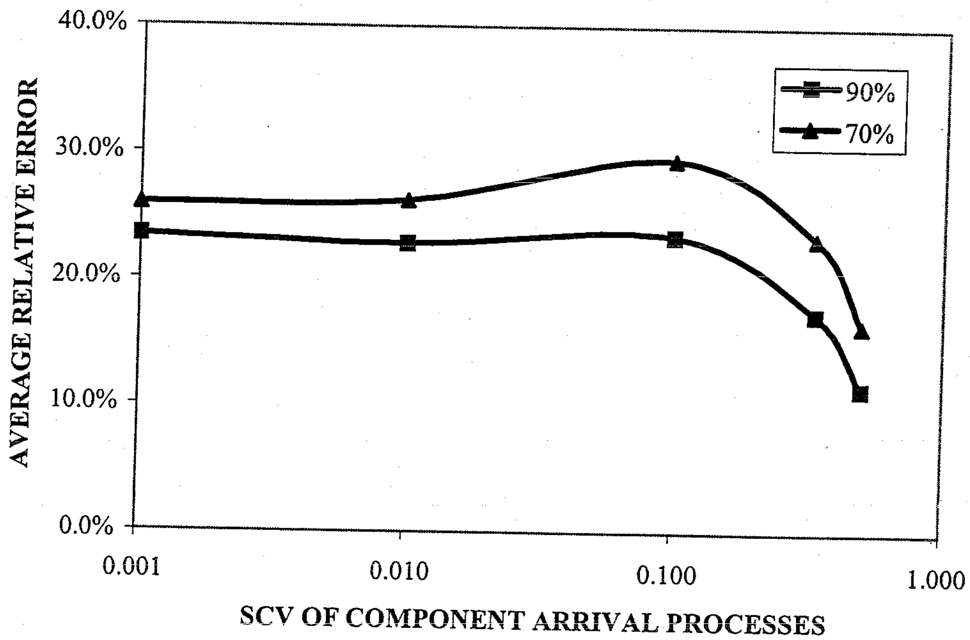

Figure 4.2: Average error in function of scv of component arrival processes

5. Alternative weighting functions

In this section we investigate whether it is possible to reduce the average absolute approximation errors by applying alternative weighting functions. We start by modifying the parameters a and b of the weighting function proposed by Albin and Whitt, i.e. $w = \left[1 + a(1-\rho)^b(\nu - 1)\right]^{-1}$ (5.1). Next we try to find an alternative expression for the weighting function, so that it minimizes the average absolute approximation error of the approximation method on the set 50 ΣGI/G/1-queues in our simulation experiment.

Table 5.1 lists the average approximation error for 5 different settings of the parameters a and b. Remember that the case (a = 4, b = 2) is the parameter setting proposed by Albin and Whitt. In the table below we increased and decreased both parameters by 0.5. It appears that the average approximation error is very sensitive to the specific parameterisation of the weighting function. Also, it seems that the parameter setting proposed by Albin & Whitt yields reasonably good results when compared to alternative parameter settings.

Parameter setting		**Low utilization (70 %)**		**High utilization (90 %)**	
a	**b**	**Avg. error (%)**	**Max. error (%)**	**Avg. error (%)**	**Max. error (%)**
3.5	**1.5**	32.0	83.1	56.9	108.3
4.5	**1.5**	35.3	97.7	67.5	139.4
4	**2**	24.3	57.5	19.7	37.5
3.5	**2.5**	10.5	20.8	27.2	93.1
4.5	**2.5**	14.4	27.6	22.1	87.8

Table 5.1: Sensitivity of approximation for parameterisation of weighting function

However, some experimenting teaches us that considerable improvements can be realised by a further modification of the parameters. Table 5.2 summarizes the results. Setting parameters a = 1.4 and b = 1.8 ultimately yields the lowest average absolute approximation error. Parameter setting a = 1.3 and b = 1.6 combines low average absolute approximation errors, small maximum errors and error margins that are relatively stable in function of the utilization of the server.

Parameter setting		Low utilization (70 %)		High utilization (90 %)	
a	b	Avg. error (%)	Max. error (%)	Avg. error (%)	Max. error (%)
1.4	1.8	9.7	19.2	16.1	71.8
1.3	1.6	14.1	31.7	16.3	30.9
4	2	24.3	57.5	19.7	37.5

Table 5.2: Alternative parameterisation of weighting function 5.1

Finally, the basic form of the weighting function is modified. We changed the basic form into the following expression: $w = c\left[1 + a(1-\rho)^b \nu\right]^{-1}$ (5.2). After some experimentation, we found that setting the parameters a = 0.72, b = 1.7 and c = 0.88 strongly reduces the average and maximum absolute errors. Table 5.3 summarizes the resulting approximation errors. The alternative weighting function yields approximation errors that are stable in the utilization of the server. Figures 5.1 and 5.2 indicate that the approximation error is decreasing in the number of products and slightly increasing in the scv of the component arrival processes.

Parameter setting			Low utilization (70 %)		High utilization (90 %)	
a	b	c	Avg. error (%)	Max. error (%)	Avg. error (%)	Max. error (%)
0.72	1.7	0.88	7.4	18.0	8.1	17.3
4	2	-	24.3	57.5	19.7	37.5

Table 5.3: Performance of alternative weighting function 5.2

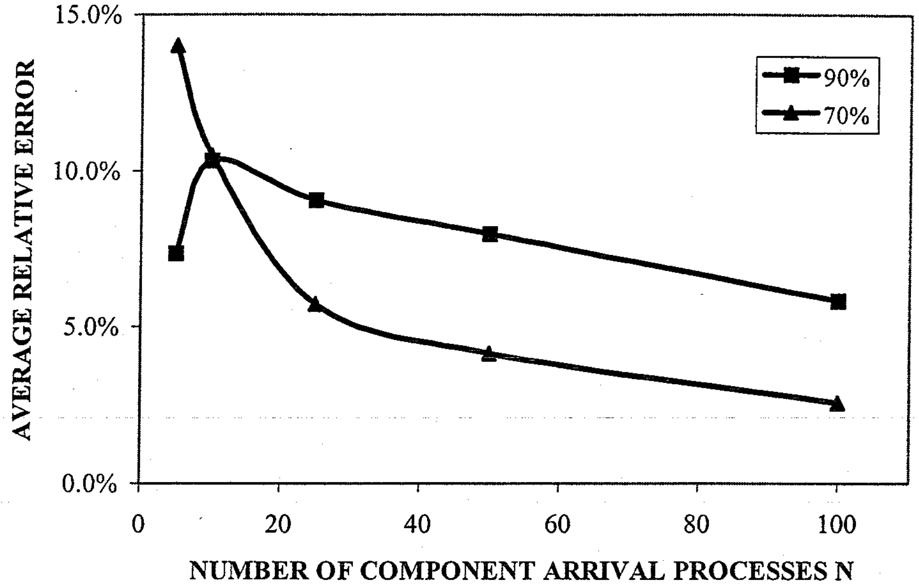

Figure 5.1: Average error in function of number of component arrival processes for weighting function 5.2

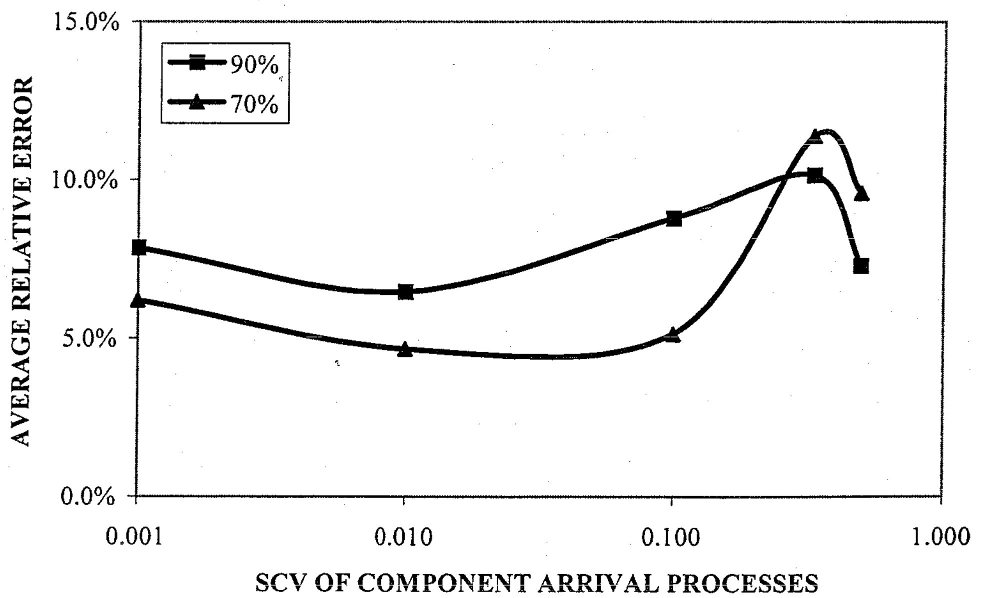

Figure 5.2: Average error in function of scv of component arrival processes for weighting function 5.2

It appears from the results in this section that the estimation quality of Albin & Whitt's approximation procedure can be considerably improved by applying alternative expressions for the weighting function. However, we do not propose using weighting function (5.2) in a general context, since it has not been tested thoroughly. Moreover, weighting function (5.2) does not possess all analytical properties a weighting function should exhibit, c.f. page 7. Our research rather indicates that the approximation procedure proposed by Albin & Whitt is very sensitive to the specific form and parameterisation of the weighting function. Moreover, the results in this paper show that Albin & Whitt's weighting function is not applicable in a general context. Recall that Albin developed her weighting function based on a limited set of simulation experiments. Therefore, we think that alternative expressions for the weighting function should be developed for problem instances in which the parameters lie beyond the range of the parameters in the experiments of Albin's simulation study. This implies that further research, analytical as well as simulation research, will be necessary to improve the approximation procedure and to make it applicable in a more general context.

6. Impact of findings on the performance of queueing network analysers

Our findings may have considerable impact on the use of queueing networks for performance evaluation and decision-making in a manufacturing context. After all, the approximation procedure of Whitt and Albin is one of the building blocks of queueing network analysers based on the parametric decomposition method. In the late seventies and early eighties, extensive research has been executed on the parametric decomposition approach, amongst others by Chandy and Sauer (1978), Kuehn (1979), Shanthikumar and Buzacott (1981) and Whitt (1983). In the late eighties, Bitran and Tirupati (1988) refined the earlier methods by including the phenomenon of interference among products and its effect on the departure process. They also provide an excellent review of the literature on network analysers. In the mid-nineties, Whitt (1995) introduced variability functions that address long-range variability effects in a network of queues. Finally, Lambrecht et al. (1998) included lot-sizing issues in a queueing network analyser. They derive expressions for the expected lead-time and its variance as a function of the lot size, which allows them to find optimal lot sizes. A complete description of the parameteric decomposition approach is given in Suri (1993).

The parametric decomposition methods suggest approximating all the flows in the queuing network by renewal processes characterized by two parameters, representing the rate and variability. The methods start by analysing the interaction between different stations in the queueing network. In this step, the arrival process at each station is approximated by a renewal process using the approximation method by Whitt and Albin. Next, the network is decomposed into subsystems of individual stations. Two-parameter approximations, e.g. Kraemer and Langenbach-Belz, are applied to obtain performance measures for these subsystems. Finally, the results for the individual stations are combined to obtain estimates for the performance of the whole network. The parametric decomposition approach relies on the conditions that (i) all the nodes in the queueing can be considered to be stochastically independent and that (ii) two-parameter approximations yield relatively good results. However, our research indicates that condition (ii) is not always satisfied, which may seriously compromise the estimation quality of the queueing network analysers.

7. Conclusions

We investigated the performance of Albin and Whitt's estimation method for the average waiting times in a multi-product, single-server queueing system and verified the claim that their method yields an average absolute error of 4 percent. For this purpose, an extensive simulation study is executed. Each simulation experiment is characterised by the following factors: (i) utilization of the server, (ii) number of component processes and (iii) scv of the component arrival processes. We varied the levels of the factors over a wide range of values, which lead to 50 different problem instances. The simulation estimates were compared with the approximations by Albin & Whitt's procedure for all 50 $\Sigma GI/G/1$ queues in the simulation study.

The comparison indicates that the average absolute approximation error made by Albin & Whitt's procedure is about 22 percent on the set of experiments. Hence we state that Albin & Whitt's performance claim (4 percent average absolute error) is not valid on our set of experiments. Our findings prompt us to conclude that Albin & Whitt's approximation procedure is not applicable in a general context. This conclusion seriously compromises the use of queueing networks for performance evaluation and decision-making in a manufacturing context, since Albin & Whitt's procedure is an essential part of many queueing network analysers.

Some experimentation with the form and parameterisation of the weighting function of Albin & Whitt's procedure illustrates that the procedure can benefit substantially from the application of alternative expressions for the weighting function. Our preliminary research indicates that it might be sensible to develop alternative expressions for the weighting function in function of the specific ranges of the input parameters. This would ensure that the approximation procedure can be applied successfully to more general situations than those in which Albin and Whitt developed their procedure. However, the developement of the alternative expressions necessitates further research, analytical as well as simulation research.

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