

Exact solution of the reflexion problem in non-linear optics

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EXACT SOLUTION OF THE REFLEXION PROBLEM
IN NON-LINEAR OPTICS

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In non-linear optics one considers media in which the polarisation is not proportional to the electric field. Since the advent of lasers this no longer seems to be a purely academic problem.

Non-linear wave equations have been treated in other disciplines (gas-dynamics, hydraulics) by means of techniques based on the properties of characteristics. It might be advantageous to adapt these methods to non-linear optics.

As an example we treat the reflexion of a plane polarised wave at normal incidence. The dielectric occupies the half space $x \geq 0$ and the Maxwell equations for the transmitted wave reduce to

$$\begin{aligned} \frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} &= 0, \\ \frac{\partial B}{\partial x} - \mu \frac{\partial D}{\partial t} &= 0. \end{aligned} \tag{1}$$

We now consider D to be a given function of E . Defining a function $c(E)$ by the relation

$$c^{-2} = \mu \frac{dD}{dE}, \tag{2}$$

we write eqs. (1) in the form

$$\begin{aligned} \frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} &= 0, \\ \frac{\partial B}{\partial x} - \frac{1}{c^2} \frac{\partial E}{\partial t} &= 0. \end{aligned} \tag{3}$$

Using the method of characteristics (see, e.g., ref. 1)) it is easily shown that the solutions of (3) must satisfy the relations

$$\begin{aligned} dE = c dB \quad \text{for} \quad \frac{dx}{dt} = c, \\ dE = -c dB \quad \text{for} \quad \frac{dx}{dt} = -c. \end{aligned} \tag{4}$$

There exist solutions for which $dE + c dB = 0$ throughout. These so-called simple waves travel to the right with local speed c . They represent a pure transmitted wave. If the boundary condition is $E = F(t)$ for $x = 0$, this solution is, in implicit form

$$E = F \left(t - \frac{x}{c(E)} \right).$$

It remains to join this solution to the vacuum fields. These can be written in the form

$$\begin{aligned} E_{\text{vac}} &= f_i \left(t - \frac{x}{c_0} \right) + f_r \left(t + \frac{x}{c_0} \right), \\ B_{\text{vac}} &= -\frac{1}{c_0} f_i \left(t - \frac{x}{c_0} \right) + \frac{1}{c_0} f_r \left(t + \frac{x}{c_0} \right), \end{aligned} \tag{5}$$

where $f_i(t)$ and $f_r(t)$ denote the time dependence of incident and reflected waves. The former function is arbitrary, and the problem is to express f_r in terms of f_i .

We take μ to be equal in vacuum and dielectric. The joining conditions then simply are

$$E_{\text{vac}} = E, \quad B_{\text{vac}} = B \quad \text{for} \quad x = 0.$$

Using (2) and (4) we find

$$B_{\text{vac}} = B = - \int_0^{E_{\text{vac}}} \frac{dE}{c} = - \mu^{\frac{1}{2}} \int_0^{E_{\text{vac}}} \left(\frac{dD}{dE} \right)^{\frac{1}{2}} dE.$$

From (5) it is then seen that the solution of our problem is

$$f_i - f_r = c_0 \mu^{\frac{1}{2}} \int_0^{f_i + f_r} \left(\frac{dD}{dE} \right)^{\frac{1}{2}} dE.$$

References

- 1) R. Courant and K.O. Friedrichs, Supersonic flow and shock waves (New York, 1948).
