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EXACT SOLUTION OF THE REFLEXION PROBLEM IN NON-LINEAR OPTICS

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In non-linear optics one considers media in which the polarisation is not proportional to the electric field. Since the advent of lasers this no longer seems to be a purely academic problem.

Non-linear wave equations have been treated in other disciplines (gas-dynamics, hydraulics) by means of techniques based on the properties of characteristics. It might be advantageous to adapt these methods to non-linear optics.

As an example we treat the reflexion of a plane polarised wave at normal incidence. The dielectric occupies the half space $x \geqslant 0$ and the Maxwell equations for the transmitted wave reduce to

$$\frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} = 0 \, , \qquad \frac{\partial B}{\partial x} - \mu \frac{\partial D}{\partial t} = 0 \, . \tag{1}$$

We now consider D to be a given function of E. Defining a function $c(E)$ by the relation

$$c^{-2} = \mu \frac{\mathrm{d}D}{\mathrm{d}E} \, , \tag{2}$$

we write eqs. (1) in the form

$$\frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} = 0 \, , \qquad \frac{\partial B}{\partial x} - \frac{1}{c^2} \frac{\partial E}{\partial t} = 0 \, . \tag{3}$$

Using the method of characteristics (see, e.g., ref. [1]) it is easily shown that the solutions of (3) must satisfy the relations

$$\mathrm{d}E = c \, \mathrm{d}B \quad \text{for} \quad \frac{\mathrm{d}x}{\mathrm{d}t} = c \, , \qquad \mathrm{d}E = -c \, \mathrm{d}B \quad \text{for} \quad \frac{\mathrm{d}x}{\mathrm{d}t} = -c \, . \tag{4}$$

There exist solutions for which $\mathrm{d}E + c \, \mathrm{d}B = 0$ throughout. These so-called simple waves travel to the right with local speed c. They represent a pure transmitted wave. If the boundary condition is $E = F(t)$ for $x = 0$, this solution is, in implicit form

$$E = F\left(t - \frac{x}{c(E)}\right) .$$

It remains to join this solution to the vacuum fields. These can be written in the form

$$E_{\mathrm{vac}} = f_{\mathrm{i}}\left(t - \frac{x}{c_0}\right) + f_{\mathrm{r}}\left(t + \frac{x}{c_0}\right) , \qquad B_{\mathrm{vac}} = -\frac{1}{c_0} f_{\mathrm{i}}\left(t - \frac{x}{c_0}\right) + \frac{1}{c_0}\left(t + \frac{x}{c_0}\right) , \tag{5}$$

where $f_{\mathrm{i}}(t)$ and $f_{\mathrm{r}}(t)$ denote the time dependence of incident and reflected waves. The former function is arbitrary, and the problem is to express f_{r} in terms of f_{i}.

We take μ to be equal in vacuum and dielectric. The joining conditions then simply are

$$E_{\mathrm{vac}} = E \, , \quad B_{\mathrm{vac}} = B \quad \text{for} \quad x = 0 \, .$$

Using (2) and (4) we find

$$B_{\mathrm{vac}} = B = -\int_0^{E_{\mathrm{vac}}} \frac{\mathrm{d}E}{c} = -\mu^{\frac{1}{2}} \int_0^{E_{\mathrm{vac}}} \left(\frac{\mathrm{d}D}{\mathrm{d}E}\right)^{\frac{1}{2}} \mathrm{d}E \, .$$

From (5) it is then seen that the solution of our problem is

$$f_{\mathrm{i}} - f_{\mathrm{r}} = c_0 \, \mu^{\frac{1}{2}} \int_0^{f_{\mathrm{i}}+f_{\mathrm{r}}} \left(\frac{\mathrm{d}D}{\mathrm{d}E}\right)^{\frac{1}{2}} \mathrm{d}E \, .$$

References

1) R. Courant and K.O. Friedrichs, Supersonic flow and shock waves (New York, 1948).

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