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Estimating stock levels in periodic review inventory systems

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Abstract

Calculating the mean physical stock in a simple periodic review inventory system seems to be straightforward. However, the standard approximate expressions appear to yield inaccurate results, especially for low service levels. Low service levels are frequently encountered at intermediate nodes in cost-optimal solutions for multi-echelon systems. In this note, we present an improved approximate method that is both simple and accurate.

Keywords: inventory, approximations, operating characteristics

1. Introduction

In this note we consider a periodic review, order-up-to (R, S) inventory system. The analysis of this system is a traditional problem that has been addressed in many standard textbooks, such as Hadley and Whitin [1] and Silver and Peterson [5]. The emphasis is usually on the determination of the control parameter S such that total costs are minimised or some target service level, such as the fill rate or the stockout probability, is attained. The mean physical stock is generally calculated by a simple approximation, such as the average of the inventory at the start and at the end of a replenishment cycle (= the period between the arrival of two consecutive

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replenishment orders). However, such a simple method can be very inaccurate, as has also been recognised by Nahmias and Smith [4]. This is in particular true for low service levels, whereas it has been shown that cost optimal control policies in multi-echelon systems often imply low service levels at intermediate nodes. Still these simple methods are frequently used to analyse various single stockpoint and multi-echelon inventory (R, S) systems.

Next to the simple approximations, exact methods to calculate the mean physical stock have been derived as well. However, these methods are available for specific cases only, such as:

- Poisson demand and constant lead times (Hadley and Whitin [1])
- Negative binomial demand and zero lead time (Nahmias and Smith [4])
- Compound Poisson demand with Gamma distributed demand per customer and random lead times (Van der Heijden and De Kok [2])

The latter method seems widely applicable at first sight, but it requires a numerical procedure to calculate an infinite sum with incomplete Gamma functions. Hence this method is computationally not very efficient and therefore less useful in practice. However, the method is perfectly fit for benchmarking purposes.

In this note we shall focus on improved simple approximations for the mean physical stock in an (R, S) inventory system. First we shall introduce some basic notation and assumptions (section 2). In section 3 we describe various (approximate) methods to calculate the mean physical stock. Numerical results are presented in section 4. Finally we give our conclusions (section 5).

2. Basic notation and assumptions

We shall use the follow basic notation in the sequel:

R : = review period

S : = order-up-to level

D(t) : = demand during a time interval with length t, a random variable with mean $\mu_D(t)$ and standard deviation $\sigma_D(t)$

L : = lead time of a replenishment order, a random variable with probability distribution function G(t), mean μ_L and standard deviation σ_L.

β : = fill rate, i.e. the fraction of demand delivered directly from stock on hand

Ψ : = mean physical stock

ψ_t : = expected physical stock at time t within a replenishment cycle ($L \le t \le L+R$)

X^+ : = max {X, 0} for any expression X

With respect to the random lead times, we make the common assumption, satisfied in almost all practical situations, that consecutive replenishment orders do not pass in time. Further we assume that the order-up-to level S has already been determined using one of the standard methods (cf. Silver and Peterson [5]). We focus on the determination of the mean physical stock, given S.

3. Calculation methods

Exact

The following exact expression for the mean physical stock can be derived, given the assumptions from the previous section (cf. Van der Heijden and De Kok [2]):

$$\Psi = \frac{1}{R}\int_0^\infty\int_0^S [G(t) - G(t-R)] * \Pr\{D(t) \le S - x\}\, dx\, dt \tag{1}$$

This expression can be numerically evaluated for specific cases only, such as Compound Poisson demand with Gamma distributed demand per customer. Then we need the values $Z_i(\lambda)$, defined by

$$Z_i(\lambda) = \int_0^\infty [1 - G(t)] \frac{(\lambda t)^i e^{-\lambda t}}{i!}\, dt$$

Here λ denotes the customer arrival rate. These values can explicitly be derived for various lead time distributions G(t), such as the uniform or the Erlang distribution.

The following two simple methods are generally used to estimate the mean stock level in an (R, S) inventory system:

Approximation of Silver and Peterson [5]:

$$\Psi \approx S - \left(\mu_L + \tfrac{1}{2}R\right) * \mu_D \qquad (2)$$

Linear approximation (e.g. used in Van der Heijden et al. [3]):

$$\Psi \approx \tfrac{1}{2}\left\{\psi_R + \psi_{L+R}\right\} \qquad (3)$$

with

$$\psi_t = E\left[\{S - D(t)\}^+\right] = S - E[D(t)] + E\left[\{D(t) - S\}^+\right], L \le t \le L+R \qquad (4)$$

So the linear approximation is simply the average of the expected stock at the start and at the end of a replenishment cycle. The terms $\psi_t = E\left[\{D(t) - S\}^+\right]$ can be calculated using a two-moment approximation of D(L+R) and D(L) respectively by a mixture of Erlang distributions, see Van der Heijden et al. [3] for details.

To obtain other simple, approximate methods, we observe the following:

1. From (4) we see that it is relatively easy to calculate the expected physical stock at any single time t within a replenishment cycle.
2. It is generally not easy to calculate the mean physical stock over the entire replenishment cycle, in fact being the integral of ψ_t over the values of t within a replenishment cycle.
3. The linear approximation (3) can be seen as a simple numerical integration of ψ_t using the trapezoidal rule.

The trapezoidal rule belongs to the class of Newton-Cotes formulas for numerical integration (cf. Stoer and Bulirsch [6]):

$$\Psi \approx \tfrac{1}{n}\sum_{i=0}^{n} \alpha_i \psi_{L+iR/n} \qquad (5)$$

For the trapezoidal rule we have that n=2 with weights $\alpha_1=\alpha_2=1$. This suggests a simple way to improve the approximation, namely using a higher order Newton-Cotes formula. In particular, for n=3 we obtain *Simpson's rule*:

$$\Psi \approx \tfrac{1}{6}\left\{\psi_R + 4\psi_{L+R/2} + \psi_{L+R}\right\} \qquad (6)$$

and for n=4 we have the so-called *3/8-rule*:

$$\Psi \approx \tfrac{1}{8}\left\{\psi_{R} + 3\psi_{L+R/3} + 3\psi_{L+2R/3} + \psi_{L+R}\right\} \tag{7}$$

We expect to improve the approximation accuracy in this simple way. In the next section, we shall examine the approximations (2), (3), (6) and (7) by comparison to exact results for compound Poisson demand according to (1).

4. Computational results

To compare the simple approximations to exact results, we use the following data set. The review period R equals 1 for all cases (normalisation of time) and the mean demand per period equals $\mu_D = 100$ for all cases (normalisation of quantity). The other parameters are varied as follows:

- the coefficient of variation of the period demand equals $c_D = 0.25, 0.50, 0.75$ or 1.0, where the coefficient of variation is defined as the ratio of the standard deviation to the mean (so $c_D = \sigma_D/\mu_D$)
- the mean replenishment order lead time equals $\mu_L = 0.2, 0.5, 1, 2$ or 5.
- the coefficient of variation of the lead time equals $c_L = 0$ (deterministic), 0.25 (uniform distribution) or 0.5 (Erlang-2 distribution).
- the target fill rate equals β =0.60, 0.70, 0.80, 0.90 0.95 or 0.99.

So totally 4*5*3*6 = 360 cases were tested. In each case, we first determined the order-up-to level S such, that the target fill rate is approximately attained by solving numerically the equation

$$1-\beta = \frac{E\left[\{D(L+R)-S\}^{+}\right] - E\left[\{D(L)-S\}^{+}\right]}{R\mu_D} \tag{8}$$

Note that we correct for the expected shortage at the start of a replenishment cycle, in contrast to the standard expression in Silver and Peterson [5]. This is necessary to avoid overestimation of the order-up-to level S if the target fill rate is low.

To obtain exact results for the mean physical stock, we have to choose the parameters of a compound Poisson demand process with Gamma distributed demand per customer. That is, we have to choose the customer arrival rate λ and two parameters r and α of the Gamma distribution such, that the mean and standard deviation coincide with μ_D and σ_D respectively. Because we have three parameters and two values μ_D and σ_D to match, we predetermine the customer arrival rate parameters as $\lambda=20$. Note that the mean physical stock appears to be quite insensitive to the choice of λ. A robustness test on all 360 cases indicated that values of λ varying from 5 to 30 yielded a range in the mean physical stock of 0.12% on average (maximum 0.54%). We refer to the appendix for details.

Fill rate	Silver & Peterson	Linear approximation	Simpson's rule	3/8 rule
0.60	47.6%	11.3%	0.45%	0.29%
0.70	23.9%	7.2%	0.16%	0.11%
0.80	10.4%	4.1%	0.16%	0.15%
0.90	3.1%	1.7%	0.17%	0.15%
0.95	1.1%	0.7%	0.11%	0.10%
0.98	0.3%	0.3%	0.05%	0.05%
ALL	**14.4%**	**4.2%**	**0.18%**	**0.14%**

Table 1. Mean percentage deviation of approximate stock levels

Fill rate	Silver & Peterson	Linear approximation	Simpson's rule	3/8 rule
0.60	72.9%	46.6%	3.4%	2.9%
0.70	38.0%	29.3%	1.1%	0.6%
0.80	17.7%	16.2%	1.0%	1.0%
0.90	6.0%	6.6%	1.5%	1.5%
0.95	2.4%	2.8%	1.0%	1.0%
0.98	0.8%	1.0%	0.4%	0.4%
ALL	**72.9%**	**46.6%**	**3.4%**	**2.9%**

Table 2. Maximum percentage deviation of approximate stock levels

The numerical results are shown in the Tables 1 and 2, sorted by the target fill rate, because this is the main discriminating factor in the approximation accuracy. Table 1 (2) shows the mean (maximum) relative deviation of the approximations from the exact mean physical stock based on a compound Poisson process. It is clear that all approximations perform well for high service levels, but also that the performance may deteriorate fastly with decreasing fill rate. Even for a target fill rate of β=0.90, both the Silver & Peterson and the Linear approximation may deviate 6% from the true value (see Table 2).

It appears that the Silver and Peterson expression (2) generally underestimates the mean physical stock. This can be explained by the fact that shortages are assumed to be negligible by Silver and Peterson, which is clearly true for high fill rates only. On the other hand, the linear approximation (3) generally overestimates the mean physical stock. The reason for this is the fact that the inventory process between replenishments shows a concave behaviour. This concavity gets stronger as the variability in demand increases and as the fill rate decreases.

The approximations using Simpson's rule (6) and the 3/8-rule (7) are clearly better. Both approximations do not structurally under- or overestimate the mean physical stock. Because the difference between the two methods is only small, we recommend approximating the mean physical stock using Simpson's rule. So the message is clear: Add only one supporting point halfway the replenishment cycle, and the major problems in approximation accuracy are solved. Such an approach can also be used to estimate mean stock levels in multi-echelon systems with periodic review.

Remark

These results are based on mixed Erlang approximations of the distribution of D(t), although a common approach is to use Normal approximations. However, such an approximation is somewhat less accurate because it does not prohibit negative lead time demand. Numerical experiments show that the Normal approximation causes an increase in the mean percentage deviation from 0.18% to 1.44%. for Simpson's rule. Also, the maximum percentage deviation increases from 3.4% to 4.0%.

5. Conclusions

In this note we showed that traditional methods to approximate the mean physical stock in simple periodic review inventory systems yield inaccurate results for low service levels. This can easily be corrected by including an estimate for the expected physical stock halfway the replenishment cycle. Applying Simpson's rule for numerical integration (see equation (6)), the approximation accuracy improves drastically. Further improvement does not seem to be sensible. Extension of this approach to the estimation of stock levels in multi-echelon (R, S) inventory systems is straightforward. Probably a similar simple approximate method can be applied to continuous review inventory systems as well.

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Appendix

In this appendix we give the results on the sensitivity of the mean physical stock to the choice of the arrival rate λ of the compound Poisson process. Table 3 shows the minimum and maximum physical stock according to the exact expression (1) for 5≤λ≤30. The range is defined as $\frac{\max. - \min.}{(\min. + \max.)/2} * 100\%$. Note that for cD=0.25 a compound Poisson process cannot be fitted for λ<17, so the range only covers 17≤λ≤30 in these cases.

			$c_D = 0.25$			$c_D = 0.5$			$c_D = 0.75$			$c_D = 1.00$		
μ_L	c_L	β	min.	max.	range	min.	max.	range	min.	max.	range	min.	max.	range
0.2	0	0.60	20.75	20.76	0.05%	30.48	30.53	0.19%	45.24	45.42	0.41%	64.52	64.76	0.37%
0.2	0	0.70	28.26	28.27	0.01%	42.12	42.24	0.29%	63.38	63.41	0.05%	90.71	90.75	0.05%
0.2	0	0.80	38.17	38.19	0.06%	58.75	59.01	0.45%	89.66	89.87	0.23%	129.03	129.15	0.10%
0.2	0	0.90	53.42	53.46	0.06%	87.03	87.35	0.36%	135.56	135.84	0.21%	196.41	196.63	0.11%
0.2	0	0.95	66.98	67.01	0.05%	114.49	114.74	0.22%	181.34	181.58	0.13%	264.37	264.56	0.07%
0.2	0	0.98	82.98	83.00	0.02%	149.23	149.37	0.10%	240.82	240.96	0.06%	353.67	353.80	0.04%
0.2	0.25	0.60	20.89	20.90	0.05%	30.57	30.63	0.19%	45.26	45.44	0.41%	64.52	64.76	0.38%
0.2	0.25	0.70	28.45	28.45	0.01%	42.26	42.38	0.28%	63.43	63.45	0.04%	90.72	90.77	0.05%
0.2	0.25	0.80	38.43	38.45	0.05%	58.96	59.22	0.45%	89.78	89.98	0.23%	129.08	129.21	0.10%
0.2	0.25	0.90	53.86	53.89	0.06%	87.36	87.67	0.36%	135.80	136.08	0.21%	196.57	196.78	0.11%
0.2	0.25	0.95	67.60	67.63	0.05%	114.93	115.18	0.22%	181.74	181.97	0.13%	264.64	264.84	0.07%
0.2	0.25	0.98	83.86	83.88	0.02%	149.82	149.96	0.10%	241.44	241.58	0.06%	354.12	354.25	0.04%
0.2	0.5	0.60	21.31	21.32	0.04%	30.85	30.91	0.21%	45.31	45.50	0.42%	64.53	64.77	0.38%
0.2	0.5	0.70	28.99	29.00	0.02%	42.67	42.78	0.26%	63.57	63.59	0.03%	90.77	90.82	0.06%
0.2	0.5	0.80	39.22	39.24	0.05%	59.56	59.82	0.43%	90.10	90.29	0.22%	129.25	129.37	0.09%
0.2	0.5	0.90	55.16	55.20	0.06%	88.32	88.63	0.34%	136.52	136.79	0.20%	197.02	197.23	0.11%
0.2	0.5	0.95	69.48	69.51	0.04%	116.25	116.49	0.21%	182.92	183.15	0.13%	265.45	265.64	0.07%
0.2	0.5	0.98	86.51	86.53	0.02%	151.60	151.74	0.09%	243.29	243.43	0.06%	355.45	355.58	0.04%
0.5	0	0.60	21.90	21.90	0.01%	34.27	34.29	0.07%	51.52	51.63	0.23%	72.39	72.59	0.28%
0.5	0	0.70	29.73	29.74	0.04%	46.95	47.14	0.42%	71.32	71.43	0.15%	100.81	100.82	0.01%
0.5	0	0.80	40.21	40.24	0.07%	65.09	65.43	0.52%	99.90	100.18	0.28%	142.07	142.25	0.13%
0.5	0	0.90	56.56	56.60	0.07%	95.85	96.21	0.38%	149.36	149.70	0.22%	214.16	214.43	0.13%
0.5	0	0.95	71.19	71.23	0.05%	125.57	125.85	0.22%	198.35	198.62	0.14%	286.42	286.65	0.08%
0.5	0	0.98	88.51	88.53	0.02%	163.01	163.17	0.09%	261.61	261.77	0.06%	380.92	381.07	0.04%
0.5	0.25	0.60	22.78	22.78	0.01%	34.90	34.91	0.05%	51.78	51.91	0.24%	72.50	72.71	0.28%
0.5	0.25	0.70	30.87	30.89	0.04%	47.81	48.00	0.38%	71.75	71.84	0.13%	101.02	101.03	0.01%
0.5	0.25	0.80	41.82	41.85	0.07%	66.31	66.63	0.48%	100.59	100.86	0.27%	142.45	142.63	0.13%
0.5	0.25	0.90	59.11	59.15	0.06%	97.66	98.01	0.35%	150.55	150.87	0.22%	214.94	215.21	0.12%
0.5	0.25	0.95	74.76	74.79	0.04%	127.98	128.24	0.21%	200.05	200.32	0.13%	287.65	287.88	0.08%
0.5	0.25	0.98	93.42	93.44	0.02%	166.19	166.34	0.09%	264.02	264.17	0.06%	382.80	382.95	0.04%
0.5	0.5	0.60	25.00	25.00	0.01%	36.32	36.35	0.07%	52.50	52.63	0.26%	72.84	73.06	0.29%
0.5	0.5	0.70	34.02	34.03	0.04%	50.00	50.15	0.30%	72.93	73.01	0.10%	101.61	101.62	0.01%
0.5	0.5	0.80	46.42	46.44	0.05%	69.62	69.90	0.40%	102.53	102.77	0.24%	143.56	143.72	0.12%
0.5	0.5	0.90	66.41	66.44	0.04%	103.03	103.33	0.30%	153.95	154.26	0.20%	217.16	217.41	0.12%
0.5	0.5	0.95	84.87	84.89	0.03%	135.47	135.71	0.17%	205.02	205.27	0.12%	291.16	291.38	0.08%
0.5	0.5	0.98	107.26	107.27	0.01%	176.51	176.65	0.08%	271.08	271.24	0.06%	388.14	388.28	0.04%

Table 3 (first part). Sensitivity of the physical stock to the customer arrival rate

μ_L	c_L	β	$c_D = 0.25$ min.	max.	range	$c_D = 0.5$ min.	max.	range	$c_D = 0.75$ min.	max.	range	$c_D = 1.00$ min.	max.	range
1	0	0.60	23.80	23.80	0.03%	39.69	39.76	0.17%	60.34	60.39	0.08%	83.04	83.19	0.18%
1	0	0.70	32.16	32.19	0.08%	54.02	54.29	0.49%	82.86	83.04	0.22%	115.41	115.49	0.07%
1	0	0.80	43.51	43.55	0.09%	74.41	74.81	0.54%	115.15	115.50	0.30%	162.47	162.73	0.16%
1	0	0.90	61.43	61.48	0.08%	108.68	109.09	0.38%	170.33	170.72	0.23%	244.64	244.97	0.13%
1	0	0.95	77.60	77.64	0.05%	141.55	141.85	0.22%	224.33	224.63	0.13%	326.88	327.15	0.08%
1	0	0.98	96.78	96.81	0.02%	182.66	182.83	0.09%	293.36	293.53	0.06%	434.25	434.40	0.04%
1	0.25	0.60	26.94	26.94	0.01%	41.59	41.62	0.08%	61.46	61.53	0.10%	83.67	83.83	0.19%
1	0.25	0.70	36.38	36.40	0.05%	56.73	56.95	0.38%	84.64	84.80	0.18%	116.28	116.34	0.05%
1	0.25	0.80	49.40	49.43	0.06%	78.35	78.70	0.45%	117.94	118.26	0.27%	163.76	164.00	0.15%
1	0.25	0.90	70.47	70.50	0.05%	114.88	115.24	0.32%	175.00	175.37	0.21%	246.84	247.16	0.13%
1	0.25	0.95	89.91	89.94	0.03%	150.05	150.33	0.18%	230.95	231.24	0.12%	330.14	330.40	0.08%
1	0.25	0.98	113.43	113.44	0.02%	194.20	194.35	0.08%	302.55	302.72	0.06%	438.99	439.14	0.04%
1	0.5	0.60	33.53	33.54	0.01%	45.89	45.91	0.04%	63.68	63.76	0.13%	85.47	85.64	0.20%
1	0.5	0.70	46.04	46.06	0.03%	63.16	63.32	0.24%	88.55	88.65	0.12%	118.75	118.79	0.03%
1	0.5	0.80	63.64	63.65	0.03%	88.03	88.28	0.28%	124.70	124.96	0.21%	167.40	167.60	0.12%
1	0.5	0.90	92.77	92.79	0.02%	130.46	130.72	0.20%	187.55	187.86	0.16%	252.91	253.20	0.11%
1	0.5	0.95	120.35	120.36	0.01%	171.74	171.94	0.12%	249.97	250.21	0.10%	338.96	339.20	0.07%
1	0.5	0.98	154.51	154.52	0.01%	224.03	224.15	0.05%	330.66	330.80	0.04%	451.67	451.82	0.03%
2	0	0.60	27.32	27.34	0.06%	48.60	48.73	0.25%	74.13	74.15	0.03%	101.49	101.56	0.07%
2	0	0.70	36.74	36.78	0.11%	65.75	66.08	0.51%	101.09	101.35	0.25%	139.76	139.93	0.12%
2	0	0.80	49.63	49.68	0.11%	89.92	90.40	0.53%	139.45	139.87	0.30%	194.86	195.21	0.18%
2	0	0.90	70.19	70.24	0.08%	130.15	130.61	0.36%	204.16	204.60	0.22%	289.54	289.93	0.13%
2	0	0.95	88.87	88.91	0.05%	168.32	168.66	0.20%	266.64	266.97	0.13%	382.79	383.09	0.08%
2	0	0.98	111.11	111.13	0.02%	215.61	215.80	0.09%	345.50	345.70	0.06%	502.76	502.94	0.04%
2	0.25	0.60	37.44	37.44	0.00%	54.54	54.58	0.06%	77.52	77.54	0.03%	104.48	104.57	0.09%
2	0.25	0.70	50.31	50.32	0.02%	74.15	74.35	0.27%	106.07	106.26	0.17%	144.13	144.26	0.09%
2	0.25	0.80	68.37	68.39	0.03%	102.08	102.41	0.32%	147.02	147.37	0.23%	201.27	201.57	0.15%
2	0.25	0.90	98.31	98.34	0.03%	149.20	149.56	0.23%	216.84	217.22	0.18%	299.65	300.01	0.12%
2	0.25	0.95	126.60	126.62	0.02%	194.51	194.78	0.14%	284.89	285.19	0.10%	396.70	396.99	0.07%
2	0.25	0.98	161.31	161.33	0.01%	251.20	251.35	0.06%	371.39	371.56	0.05%	521.70	521.87	0.03%
2	0.5	0.60	53.94	53.95	0.01%	66.83	66.87	0.05%	86.33	86.36	0.03%	110.50	110.60	0.09%
2	0.5	0.70	74.74	74.75	0.01%	92.55	92.67	0.13%	119.36	119.48	0.10%	154.13	154.22	0.05%
2	0.5	0.80	104.38	104.39	0.01%	129.66	129.83	0.14%	167.22	167.45	0.14%	217.65	217.88	0.11%
2	0.5	0.90	154.35	154.36	0.01%	193.15	193.32	0.09%	249.81	250.07	0.10%	328.12	328.41	0.09%
2	0.5	0.95	202.48	202.48	0.00%	255.12	255.25	0.05%	331.21	331.42	0.06%	437.97	438.19	0.05%
2	0.5	0.98	262.88	262.89	0.00%	333.83	333.91	0.02%	435.56	435.69	0.03%	580.18	580.32	0.02%
5	0	0.60	36.04	36.06	0.08%	68.34	68.52	0.26%	104.05	104.15	0.10%	141.86	141.89	0.02%
5	0	0.70	48.20	48.26	0.12%	91.86	92.26	0.44%	140.74	141.08	0.24%	193.21	193.49	0.14%
5	0	0.80	64.86	64.94	0.12%	124.63	125.18	0.44%	192.27	192.77	0.26%	266.01	266.46	0.17%
5	0	0.90	91.55	91.63	0.08%	178.35	178.88	0.29%	277.74	278.24	0.18%	388.33	388.80	0.12%
5	0	0.95	115.89	115.95	0.05%	228.57	228.96	0.17%	358.82	359.20	0.11%	506.06	506.42	0.07%
5	0	0.98	144.91	144.94	0.02%	289.92	290.14	0.07%	459.48	459.70	0.05%	654.33	654.54	0.03%
5	0.25	0.60	74.19	74.20	0.01%	92.99	93.01	0.02%	120.69	120.70	0.01%	153.75	153.79	0.02%
5	0.25	0.70	98.69	98.69	0.00%	125.51	125.60	0.07%	164.22	164.37	0.10%	210.37	210.53	0.08%
5	0.25	0.80	133.41	133.41	0.00%	171.81	172.01	0.12%	226.36	226.66	0.13%	291.51	291.84	0.11%
5	0.25	0.90	192.74	192.76	0.01%	250.23	250.51	0.11%	331.63	331.98	0.10%	429.79	430.16	0.09%
5	0.25	0.95	250.53	250.55	0.01%	325.86	326.09	0.07%	433.36	433.64	0.06%	564.48	564.78	0.05%
5	0.25	0.98	322.44	322.45	0.00%	420.32	420.45	0.03%	561.29	561.45	0.03%	735.54	735.72	0.02%
5	0.5	0.60	120.15	120.15	0.00%	132.63	132.65	0.01%	153.18	153.19	0.01%	177.65	177.69	0.02%
5	0.5	0.70	166.97	166.97	0.00%	184.55	184.61	0.03%	212.72	212.81	0.04%	248.62	248.71	0.04%
5	0.5	0.80	234.21	234.21	0.00%	259.64	259.72	0.03%	299.15	299.30	0.05%	352.94	353.12	0.05%
5	0.5	0.90	348.62	348.62	0.00%	388.60	388.69	0.02%	448.35	448.50	0.03%	535.89	536.10	0.04%
5	0.5	0.95	459.54	459.54	0.00%	514.88	514.95	0.01%	595.23	595.35	0.02%	718.85	719.02	0.02%
5	0.5	0.98	598.99	598.99	0.00%	675.47	675.51	0.01%	782.96	783.03	0.01%	956.38	956.48	0.01%

Table 3 (cont.). Sensitivity of the physical stock to the customer arrival rate λ