(Un!)stable mixed finite element methods for viscoelastic fluid mechanics

(Un!)Stable Mixed Finite Element Methods for Viscoelastic Fluid Mechanics

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Introduction

Due to the nature of the governing equations, viscoelastic flow simulations require special numerical solution algorithms. Several stabilizing techniques have been proposed to overcome the loss of stability observed with increasing Weissenberg numbers. However for time dependent flows, the important question remains:

What are the temporal stability properties of the different mixed FEM under investigation?

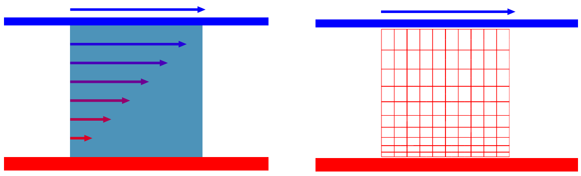

Figure 1: Plane Couette flow, periodic constraints are imposed on the in- and outflow of the computational domain.

Problem Definition

To assess the stability of a computational method, the inertia-less planar Couette flow (figure 1) of a Maxwell fluid is considered. The slowest decay rate of a spatially periodic disturbance $\varepsilon = \bar{\varepsilon}\exp(\sigma t)$ for this mathematically stable problem is [1]:

$$\text{real}(\sigma) \approx -\frac{1}{2\text{We}} \qquad \text{for} \quad \text{We} \gg 1$$

Direct time integration of the finite element equations evaluated about the steady base flow provides an indication of the temporal stability properties of a computational method.

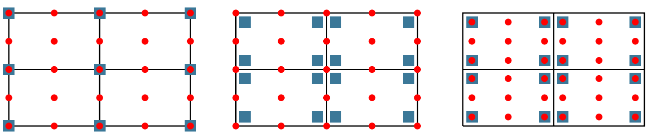

Figure 2: Spatial discretization, DAVSS-G/SUPG (*left*), DEVSS/DG extra stress variables are projected onto discontinuous subspaces (*middle*) and a fully discontinuous method where all variables are taken discontinuous over the element edges (*right*).

Computational Methods

Figure 2 schematically represents the mixed FE methods tested: the discontinuous Galerkin method DEVSS/DG of [2], a fully discontinuous method DG [after 3], the continuous DEVSS-G/SUPG [4] and the adaptive DAVSS-G/SUPG.

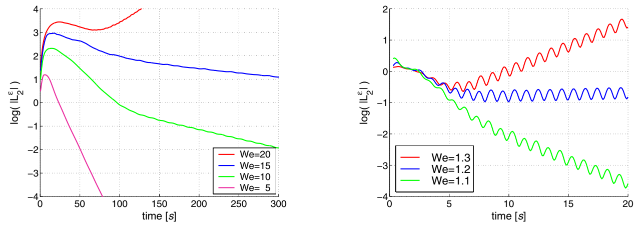

Figure 3: Temporal stability for DEVSS/DG (*left*) and DG (*right*).

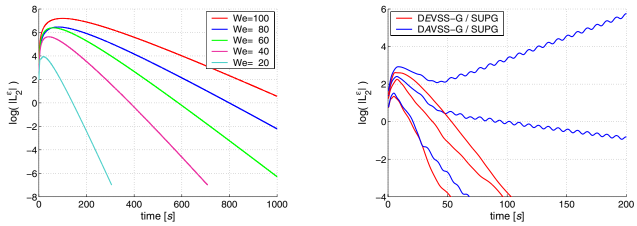

Figure 4: Temporal stability for DEVSS-G/SUPG (*left*) and DAVSS-G/SUPG at We = 5.0, 7.5 and 10.0 (*right*).

Results & Discussion

Figure 3 and 4 show the regular L_2 norm of a superimposed disturbance integrated in time. It is clear that:

- ▷ DG *unstable*
- ▷ DEVSS/DG *unstable*
- ▷ DAVSS-G/SUPG *stable* provided that A = E

One major drawback of the DEVSS-G/SUPG method involves the additional cost for solving the large system of equations. An attractive and stable alternative is the Θ-scheme which sequentially solves smaller subsets of equations.

References:

[1] Gorodtsov V.A., Leonov A.I.: Journal of Applied Mathematical Mechanics, **31** (1967) 310-319.

[2] Baaijens F.P.T. **et al.**: Journal of Non-Newtonian Fluid Mechanics **68** (1997), 173-203.

[3] Oden J.T., Babuska I., Baumann C.E.: Journal of Computational Physics **146** (1998), 491-519.

[4] Szady M.J. **et al.**: Journal of Non-Newtonian Fluid Mechanics **59** (1995), 215-243.