

Modal Identification for Multivariable Motion Systems

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Modal Identification for Multivariable Motion Systems: Applied to a Prototype Wafer-Stage

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1 Background

Accurate modal identification of multi-input multi-output (MIMO) models is vital for driving the future development of advanced mechatronic motion systems. These modal models offer interpretable, minimal-order representations [1] that enable effective control, provide design validation, and facilitate monitoring and diagnostics of machine dynamics.

2 Problem formulation

Mechanical systems can be modeled in a modal framework [2], where the system consists of rigid-body modes and flexible dynamic modes

$$\mathbf{G}(s, \rho) = \sum_{i=1}^{n_{\text{rbm}}} \frac{\phi_{l,i} \phi_{r,i}^{\top}}{s^2} + \sum_{i=1}^{n_{\text{flex}}} \frac{\phi_{l,i} \phi_{r,i}^{\top}}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}, \quad (1)$$

where the modal parameters are the resonance frequencies ω_i , the corresponding damping ratios ζ_i , and the left and right mode-shape vectors $\phi_{l,i}$, $\phi_{r,i}$. The modal parameters are jointly stored in the parameter vector ρ . The problem considered is to estimate modal models as described by (1) from data.

3 Approach

A two-stage approach is presented for estimating the modal parameters. First, an additive model is estimated, described by

$$\mathbf{P}(s, \beta) = \frac{\mathbf{M}}{s^2} + \sum_{i=1}^K \frac{\mathbf{R}_i}{a_{i,2}s^2 + a_{i,1}s + 1}, \quad (2)$$

with the parameters stored in the vector β and where the numerator terms \mathbf{M} and \mathbf{R}_i are full rank matrices. This model is obtained by minimizing the weighted least-squares criterion

$$\hat{\beta} = \arg \min_{\beta} \sum_{k=1}^N \|\mathbf{G}(\omega_k) - \mathbf{P}(j\omega_k, \beta)\|_F^2, \quad (3)$$

with $\|\cdot\|_F$ the Frobenius norm, for a given dataset of noisy FRF measurements $\mathbf{G}(\omega_k)$. The problem (3) is a non-linear and non-convex optimization problem, which is solved using a novel refined instrumental variable method for additive

systems. In the second stage, a modal model is obtained by optimally reducing the estimated full-rank $\hat{\mathbf{M}}$ and $\hat{\mathbf{R}}_i$ to rank-one matrices, enabling the extraction of the mode-shape vectors. Given a solution $\hat{\beta}$ from stage 1, along with the corresponding covariance $\Sigma_{\hat{\beta}}$, the parameter vector estimate $\hat{\rho}$ of the modal model is found by minimizing

$$\hat{\rho} = \arg \min_{\rho} \|\hat{\beta} - \mathbf{f}(\rho)\|_{\Sigma_{\hat{\beta}}}^2, \quad (4)$$

where $\mathbf{f}: \rho \mapsto \beta$ represents the mapping from the modal parameter vector to the additive parameter vector.

4 Results

The developed identification method is experimentally validated on a prototype wafer-stage setup, which features four sensors and thirteen actuators. A 40th-order model is estimated of the 4×13 plant consisting of three rigid-body modes and seventeen proportionally damped flexible modes. A subset of the identified modal model is presented in Figure 1, demonstrating close alignment with the FRF measurement across the considered frequency range.

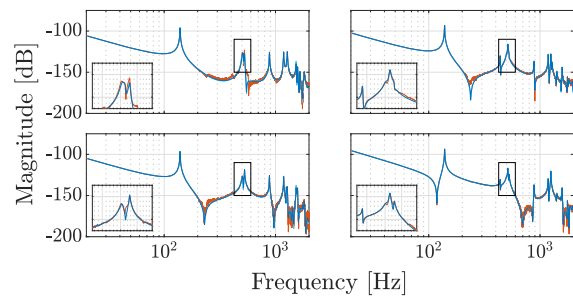


Figure 1: Bode magnitude plot of a subset of the plant with the FRF (—) and identified modal model, described by the eigenfrequencies, damping-ratios, and mode shapes (—).

References

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