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Effect of vapor bubbles on velocity fluctuations and dissipation rates in bubbly Rayleigh-Bénard convection

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Numerical results for kinetic and thermal energy dissipation rates in bubbly Rayleigh-Bénard convection are reported. Bubbles have a twofold effect on the flow: on the one hand, they absorb or release heat to the surrounding liquid phase, thus tending to decrease the temperature differences responsible for the convective motion; but on the other hand, the absorbed heat causes the bubbles to grow, thus increasing their buoyancy and enhancing turbulence (or, more properly, pseudoturbulence) by generating velocity fluctuations. This enhancement depends on the ratio of the sensible heat to the latent heat of the phase change, given by the Jakob number, which determines the dynamics of the bubble growth.

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I. INTRODUCTION

It is well known that boiling is a very efficient heat transfer process [1,2]. Bubbles growing near the heated surface absorb latent heat and cool it. Furthermore, the buoyant rise of the bubbles induces a turbulent, or pseudoturbulent, motion in the liquid which brings cooler liquid from the bulk closer to the heated wall and mixes it. The physics involved in this process is very rich and complex and still far from being completely understood. In this paper, which is a continuation of our earlier studies [3,4], we focus on the second aspect, namely the fluctuating motion induced by the bubbles and their thermal interaction with the surrounding liquid. For this purpose we consider a standard Rayleigh-Bénard setting in which a liquid undergoes natural convection in a cell, the base of which is warmer than the top. Vapor bubbles are introduced in this flow and their effect on it is studied numerically. Recent experimental results on a similar system also show an enhanced heat transport compared with single-phase natural convection [5].

Several papers exist on the buoyant rise of gas bubbles and their effect on the liquid motion [6–11]. What distinguishes the present work is that the mechanical coupling between the bubbles and the liquid is augmented and influenced by the thermal coupling, which causes the bubble volume to change. In the following, we first discuss briefly the mathematical formulation and numerical method and then focus on how bubbles promote turbulence and liquid velocity and temperature fluctuations in the Rayleigh-Bénard convection inside a cylindrical cell.

Details about the model used in this work can be found in [3]. The governing equations for the incompressible fluid flow under the Boussinesq approximation, augmented by the momentum and energy effects of the two-way coupled, pointlike bubbles, are [3,12]

\[ \mathbf{v} \cdot \mathbf{u} = 0, \]

\[ \frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \beta (T - T_{sat}) \mathbf{g} + \frac{1}{\rho} \sum_{i=1}^{N} \mathbf{f}_i \delta (\mathbf{x} - \mathbf{x}_i), \]

\[ \frac{DT}{Dt} = \frac{k}{\rho c_p} \nabla^2 T + \frac{1}{\rho c_p} \sum_{i=1}^{N} \mathbf{Q}_i \delta (\mathbf{x} - \mathbf{x}_i). \]

Here \( \mathbf{u}, \rho, \) and \( T \) are the liquid velocity, pressure, and temperature fields; \( \rho, \nu, \kappa, \) and \( c_p \) are the (constant) liquid density, kinematic viscosity, thermal conductivity, and specific heat; \( \beta \) is the isobaric thermal expansion coefficient; and \( N \) is the total number of bubbles. The mechanical and thermal forcings at the location \( \mathbf{x}_i \) of the \( i \)th bubble are given by \( \mathbf{f}_i = \frac{4}{3} \pi r_i^3 \rho (\frac{D\mathbf{u}}{Dt} |_{\mathbf{r}_i} - \mathbf{g}) \) and \( \mathbf{Q}_i = 4 \pi r_i^2 h_b (T_{sat} - T_i), \) respectively. Here \( h_b \) is the radius of the \( i \)th bubble, \( T_i \) the liquid temperature at the bubble location, and \( h_b \) the bubble convective heat transfer coefficient; \( T_{sat} \) is the saturation temperature at the system’s pressure.

The bubbles are tracked by solving the following dynamical equation in which their mass is neglected (see, e.g., [13]):

\[ C_A \rho \left( \frac{4}{3} \pi r_b^3 \left( \frac{D\mathbf{u}}{Dt} - \frac{dV}{dt} \right) + (\mathbf{u} - \mathbf{V}) \frac{d}{dt} \left( \frac{4}{3} \pi r_b^3 \right) \right) \]

\[ - \frac{1}{2} C_D \rho \frac{2}{r_b} |\mathbf{u} - (\mathbf{V} - \mathbf{u})| \mathbf{u} + \frac{4}{3} \pi r_b^3 \rho \frac{D\mathbf{u}}{Dt} \]

\[ + C_L \frac{4}{3} \pi r_b^3 \rho (\nabla \times \mathbf{u}) \times (\nabla \mathbf{u}) - \frac{4}{3} \pi r_b^3 \rho g = 0 \]
in which \( L \) is the latent heat of the liquid and \( \rho_v \) the vapor density.

The heat transfer coefficient of each bubble is expressed in terms of the Nusselt number,

\[
Nu_b = \frac{2r_b h_b}{\kappa},
\]

by a semiempirical formula which interpolates between the value \( Nu_b = 2 \) at very low Jakob numbers and small relative velocities and

\[
Nu_b = 2\sqrt{\frac{2r_b |V - u|}{\pi \kappa}}
\]

at large relative velocities; see [3] for details.

Our use of the point particle approximation is justified by the facts that the Kolmogorov length scale is always larger than the bubble mean radius and, second, that the volume fraction for the bubble phase is at most only 0.2\% (see, e.g., [18]).

The numerical treatment of the equations follows that described in [19,20] and in [3]. Briefly, the field equations are discretized in space by a second-order finite difference scheme based on the projection method on a nonuniform staggered grid with 33 \( \times \) 25 \( \times \) 65 points in the azimuthal, radial, and axial directions, respectively. Time is advanced by the third-order Runge-Kutta scheme with a constant nondimensional time step of \( 2 \times 10^{-4} \) dimensionless time units. The adequacy of this spatial and temporal discretization was demonstrated in [3] and a further test is described later.

In each computational cell the forces exerted by the bubbles on the liquid are replaced by an equivalent system of forces at the grid nodes constructed in such a way as to produce the same net resultant and couple on the liquid as the original forces (see [3]). An analogous strategy is followed for the bubble-related sources in the right-hand side of the energy equation (3) so that the total amount of heat that each bubble exchanges with the liquid is preserved. The interpolations used in these steps are second-order accurate and therefore consistent with the overall spatial accuracy of the discretization.

Bubbles are removed when they reach the top plate of the cell and reinjected with their lower surface tangent to the bottom plate with the local fluid velocity and an initial radius of 12.5 \( \mu \text{m} \). The equations were solved for fixed Rayleigh numbers, \( Ra \), and \( T_C \) were taken as \( T_H = T_{sat} + \frac{1}{2} \Delta T \) and \( T_C = T_{sat} - \frac{1}{2} \Delta T \), respectively, with \( \Delta T = 0.25 \text{ K} \). This temperature difference would not correspond to a realistic boiling situation, but it enables us to focus on the physics of the bubble-induced flow at an affordable computational cost. The fluid properties used are those of water at 100 \( ^\circ \text{C} \).

The simulations were conducted in a cylindrical cell with an aspect (diameter-to-height) ratio of \( d/H = 1/2 \); the height was 17.9 mm. The equations were solved for fixed Rayleigh number, \( Ra = g\beta(T_H - T_C)H^3/\nu \kappa \), and Prandtl number, \( Pr = \nu/\kappa \). The thermal energy balance of the bubble with the
TABLE I. Summary of length and velocity scales for the present simulations for different Jakob numbers. $H$ is the cell height, $\eta$ the Kolmogorov length scale, $\lambda$ the Taylor length scale, $r_b$ the bubble radius averaged over time and over all the bubbles, $\langle u' \rangle_{i,V}$ the time- and volume-averaged rms liquid velocity fluctuation, $\nabla_r$ the mean bubble-liquid vertical relative velocity, and $Re_i$ the Taylor Reynolds number.

\begin{tabular}{cccccc}
Ja & $10^2 r_b/H$ & $10^2 \eta/H$ & $10^2 \lambda/H$ & $\langle u' \rangle_{i,V}/\nabla_r$ & $Re_i$
\hline
0.00 & 0.071 & 6.16 & 0.004 & \\
0.10 & 0.22 & 0.77 & 3.31 & 0.20 & 4.79 \\
0.15 & 0.24 & 0.68 & 3.43 & 0.21 & 6.49 \\
0.20 & 0.26 & 0.64 & 3.82 & 0.22 & 9.28 \\
0.25 & 0.27 & 0.60 & 3.89 & 0.23 & 10.76 \\
0.30 & 0.28 & 0.58 & 4.17 & 0.24 & 13.42 \\
0.40 & 0.31 & 0.54 & 4.39 & 0.24 & 17.15 \\
0.60 & 0.34 & 0.49 & 4.90 & 0.26 & 25.97 \\
0.80 & 0.37 & 0.46 & 5.34 & 0.27 & 35.57 \\
1.00 & 0.39 & 0.45 & 5.62 & 0.28 & 40.03 \\
\end{tabular}

The number of bubbles is kept fixed at $10^4$. The vapor surrounding fluid, Eq. (6), introduces a third parameter, the Jakob number, $Ja = \frac{\rho_c p(T_H - T_{sat})}{\rho_v L}$, which controls the bubble growth rate. For $Ja = 0$, the latent heat is infinite and bubbles cannot grow or shrink.

Velocities are nondimensionalized by the so-called free fall velocity $U = \sqrt{g \beta (T_H - T_{sat}) H}$, lengths by the cylinder height $H$, and times by $H/U$.

III. RESULTS

With the data specified above, the Jakob number for a water-vapor system would equal 1.3. However, in the simulations, we vary the Jakob number for fixed $Ra = 2 \times 10^5$ and $Pr = 1.75$ (the value for water at 100°C) to isolate the effect of bubble volume changes on the flow phenomena. The number of bubbles is kept fixed at 10,000. The vapor volume fraction calculated with the initial bubble radius is about $1.85 \times 10^{-5}$.

An additional check on the grid independence of the results was carried out for $Ja = 1.0$ by comparing the total heat transport for the standard grid with $33 \times 25 \times 65$ cells and a finer grid with $33 \times 40 \times 80$ cells. The statistical properties of the flow were within 0.6%, indicating that the resolution provided by the standard grid is sufficient and the flow scales are properly resolved.

The flow properties (Table I) were calculated once the system had reached a statistical stationary state as revealed by monitoring the total heat transport and the liquid velocity fluctuations. Columns 2, 3, and 4 in the table show the time- and volume-averaged bubble mean radius $r_b$, Kolmogorov length scale $\eta$, and Taylor length scale $\lambda = \langle u' \rangle_{i,V}/(\langle |\nabla u'| \rangle_{i,V})$, respectively. Here and elsewhere in this paper angle brackets denote averages over the variable(s) indicated by the subscripts $V$ for volume, $t$ for time, and, later, $A$ for the cross-sectional area; quantities averaged over all the bubbles and over time are indicated by an overline. The mean liquid velocity fluctuation $\langle u' \rangle_{i,V}$ is defined by

$$\langle u' \rangle = \left( \frac{1}{3} |u(x,t) - \langle u \rangle_{i,V}| \right)_{i,V}$$

and the mean liquid velocity gradient by

$$\langle |\nabla u' \rangle \rangle = \left( \frac{1}{3} |\nabla u(x,t) - \langle \nabla u \rangle_{i,V}| \right)_{i,V}.$$

With the present parameter values, the pattern of the single-phase flow in the cell is a roll with an approximately horizontal axis. For reasons explained below, the introduction of bubbles with $Ja = 0$, which forces them to maintain a fixed radius as noted before, modifies the flow pattern to a toroidal vortex with the descending stream near the axis and the ascending stream in the surrounding volume. As $Ja$ increases and the bubble volume is allowed to change and, in particular, to grow in the warmer liquid regions, a flow pattern qualitatively similar to the single-phase one but with higher velocities is gradually established. As they are introduced at the hot plate, bubbles are swept up by the annular roll, adding to its buoyancy and thereby increasing the intensity of the circulation. It is also found that in this regime the axis of the roll rotates in a horizontal plane, as in single-phase Rayleigh-Bénard flow.

For $Ja = 0$ the bubbles maintain the same radius with which they are injected and the vast majority of them rise relative to the liquid with a velocity very close to the terminal velocity of about 1.7 mm/s. To give an impression of the major effect of a nonzero Jakob number on the dynamics of the bubble-liquid interaction, we show in Fig. 1 isolines of the bubble probability density function in the radius-relative velocity plane for $Ja = 0.1$ [Fig. 1(a)] and $Ja = 1$ [Fig. 1(b)]. The solid black line in these figures is the terminal velocity as given by a balance of buoyancy and drag as computed from Eq. (5). As bubble growth and, with it, increased buoyancy are allowed, the spread of the bubbles’ velocity relative to that of the liquid increases considerably. This feature demonstrates the increasing importance of the additional forces present in the bubble equation of motion (4) with increasing bubble volume.

A measure of the turbulence intensity is given by the ratio $\langle |u' \rangle_{i,V}/\nabla_r \rangle$, where $\nabla_r$ is the mean bubble-liquid vertical relative velocity given by

$$\nabla_r = \left\langle \frac{1}{N} \sum_{i=1}^{N} |V_{z,i}(t) - u_{z,i}(t)| \right\rangle.$$

Column 5 of Table I shows that, in the present simulations, this quantity is small for $Ja = 0$ which, as noted above, corresponds to nongrowing bubbles, but becomes of the order of 20–30% as $Ja$ increases to 1. Note that here the turbulence intensity is larger than in the case of pseudoturbulence without heating and with periodic boundary conditions in all directions [9], where $\langle |u' \rangle_{i,V}/\nabla_r \rangle$ was found to be of the order of 6%. In that work [9], which was quantitatively confirmed by independent simulations by Calzavarini [22], the bubbles did not grow and the drag law was pure Stokes. We have repeated our simulations with $Ja = 0$ and the same drag law, still finding considerably larger velocity fluctuations than reported for the case of pseudoturbulence with periodic boundary conditions and no heating. Thus we conclude that the reason for this
difference lies in the nature of the two flows investigated. The work of [9] concerns the rise of fixed-radius bubbles in a quiescent liquid in a cube with periodic boundary conditions not only on the sides but also on the top and bottom. In that system the only liquid flow is induced by the bubbles and, in a frame of reference with zero mean flow, the bubbles rise while, by continuity, the liquid descends. The velocity of the two phases is therefore mostly parallel and the bubbles distribute statistically uniformly (but of course accumulate in vortices, which however are statistically uniformly distributed over the flow volume). In the present system, on the other hand, liquid flow is driven by buoyancy in addition to the bubbles, and the presence of impervious top and bottom boundaries causes the appearance of regions where the tendency of the bubbles to rise encounters nearly horizontal mean liquid streamlines. As noted before, bubbles are also swept up by the ascending convection current as they are introduced at the bottom boundary. All of these factors cause a much more complex situation and presumably marked nonuniformities in the vortex and thus also bubble distribution. We believe that this is the explanation for the difference between the present work and that of [9]. The velocity fluctuations of course increase substantially when the bubbles are allowed to grow.

It is also interesting to note that the velocity fluctuations are far from isotropic. Figure 2 shows the volume- and time-averaged liquid velocity fluctuations in the vertical and horizontal directions. The fluctuations become more and more anisotropic with increasing Jakob number. Velocity fluctuations for the single-phase case are basically zero as the flow is essentially laminar for the Rayleigh number used here.

The normalized time- and volume-averaged kinetic energy dissipation rate \( \langle \epsilon_K \rangle_{t,V} = v \langle |\nabla T|^2 \rangle_{t,V} \) is shown as a function of the Jakob number in Fig. 3. For comparison, the energy dissipation rate for single-phase convection is also shown by the continuous line. It can be observed that dissipation in the bubbly flow is generally higher than for the single-phase flow. However, for very small Jakob numbers, the opposite behavior is found as Fig. 3(b) shows: for these Jakob numbers energy dissipation is less, which implies that turbulence is attenuated. The explanation is that, for \( Ja = 0 \), the bubbles have an effectively infinite heat capacity and therefore they can absorb or release heat to the surrounding fluid while remaining at \( T_{sat} \), which also equals the mean temperature of the hot and the cold plates. The net effect is a tendency to “short-circuit” the temperature difference and, therefore, a tendency to the quenching of the natural convection. The same effect was found in [3]. This process is responsible for the transition mentioned earlier from the annular roll of the single-phase case to the lower-energy toroidal roll prevailing for small \( Ja \).

The tendency of the bubbles to reduce temperature differences is always present but, with increasing \( Ja \), it is overshadowed by the increased buoyancy provided to the two-phase medium. The bubbles rise faster and, by mechanically stirring the liquid phase, increase the kinetic energy dissipation rate.

The normalized time- and volume-averaged thermal energy dissipation rate \( \langle \epsilon_T \rangle_{t,V} = \kappa \langle |\nabla T|^2 \rangle_{t,V} \) is shown as a function of the Jakob number in Fig. 4, where the horizontal solid line is the single-phase result. Unlike the kinetic energy dissipation, even nongrowing bubbles with \( Ja = 0 \) slightly increase \( \langle \epsilon_T \rangle_{t,V} \). The reason lies in the fact that the bubble surface temperature is fixed, which causes a local cooling or heating of the surrounding liquid.

Figure 5 compares the normalized kinetic energy dissipation rate averaged over the azimuthal angle and time, \( H v \langle |\nabla T|^2 \rangle_{t,\theta}/U^3 \), in single-phase [Fig. 5(a)] and two-phase [Fig. 5(b)] flow with \( Ja = 0 \) [Fig. 5(b)] and \( Ja = 1 \) [Fig. 5(c)]; note the different (color) shaded scales in these three figures. The single-phase annular roll is characterized by a relatively
EFFECT OF VAPOR BUBBLES ON VELOCITY

FIG. 4. (Color online) (a) Normalized thermal energy dissipation rate as a function of the Jakob number $Ja$. (b) Energy dissipation rate for very low $Ja$, $10^{-8} \leq Ja \leq 10^{-2}$ on a log-log scale. In both graphs the horizontal solid line is the single-phase value for reference.

FIG. 5. (Color online) Angle-averaged kinetic energy dissipation rate $\langle \epsilon_k \rangle_{t,\theta}$ normalized by $U^3/H$ in the $r$-$z$ plane: (a) single phase, (b) $Ja = 0$ (nongrowing $10^4$ bubbles), and (c) $Ja = 1$ (growing $10^4$ bubbles). The color scale is logarithmic.

FIG. 6. (Color online) Angle-averaged thermal energy dissipation rate $\langle \epsilon_T \rangle_{t,\theta}$ normalized by $U \Delta T^2/H$ in the $r$-$z$ plane: (a) single phase, (b) $Ja = 0$ (nongrowing $10^4$ bubbles), and (c) $Ja = 1$ (growing $10^4$ bubbles). The (color) shaded scale is logarithmic.

FIG. 7. (Color online) (a) Vertical temperature profile averaged over the horizontal cross section for the single-phase case [solid (red) line], $Ja = 0$ [short-dashed (blue) line], $Ja = 0.1$ [long-dashed (green) line], and $Ja = 0.8$ [dotted (brown) line]. (b) Volume-averaged rms temperature fluctuation, $\langle T' \rangle$, as a function of the Jakob number, on a linear-log scale.
that, since the bubble temperature even for $Ja\rightarrow 0$ increases the thermal boundary layer thickness near both plates. Of course, the effect increases with $Ja$ and leads to a thinning of the boundary layers near the plate but are otherwise fairly constant and increase with $Ja$. The temperature fluctuations exhibit marked boundary layers near the plates due to the fact that, with their fixed interface temperature, bubbles behave as "cold spots" or "hot spots" near the bottom or top plates, thus inducing relatively strong temperature differences with respect to the adjacent liquid.

The present numerical investigation considered the kinetic and thermal energy dissipation rates in a two-phase Rayleigh-Bénard convection in a cylindrical cell, where $10^4$ saturated vapor bubbles are injected into the flow. Due to their fixed surface temperature, bubbles tend to smooth the liquid temperature differences by absorbing and releasing heat and add vertical momentum to the flow with their buoyancy. The balance between these competing effects depends on the ratio of the sensible heat to the latent heat of the liquid, as quantified by the Jakob number $Ja$ defined in Eq. (9). For very small $Ja$, the bubble volume change is small and the absorption or release of heat dominates over the buoyancy effect. The outcome is a reduction of the driving force of the circulation with a corresponding attenuation of the kinetic energy dissipation. The added buoyancy, however, starts becoming dominant already at small $Ja$ with a strong enhancement of the kinetic and thermal energy dissipations.

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[22] E. Calzavarini (private communication).