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Optimized switching control for fast motion systems with nanometer accuracy

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Summary. For a class of switching motion control systems, optimal values of the switching parameters are obtained through a machine-in-the-loop optimization approach. The optimal parameters provide the means to balance servo performances. High-gain feedback is switched on to suppress large amplitude oscillations and is switched off to avoid amplification of small amplitude noises. Stability and servo performance are studied using a fast motion system which requires nanometer accuracy.

Introduction

The motion control industry can significantly benefit from control designs that deal with the trade-off between disturbance rejection and measurement noise sensitivity (known as the waterbed effect). Consider, for example, the wafer scanner industry where switching control is used to improve performance under high-gain feedback when being exposed to large-amplitude disturbances induced by its scanning set-points [1, 2]. In the absence of such disturbances, low-gain feedback preserves a small noise response and (at the same time) induces favorable robustness properties.

Consider Lur'e type switching control systems of the form:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}_1 u + \mathbf{b}_2 v \\ y &= \mathbf{c}^T \mathbf{x} \\ u &= -\phi(y)y,\end{aligned}\quad (1)$$

with state vector $\mathbf{x} = \mathbf{x}(t) \in \mathbb{R}^m$, $\mathbf{A} \in \mathbb{R}^{m \times m}$, $\mathbf{b}_1, \mathbf{b}_2, \mathbf{c} \in \mathbb{R}^m$, disturbances $v : |v(t)| \leq \gamma$ with $\gamma \geq 0$, and a saturation-based switching function ϕ defined by

$$\phi(y) = \begin{cases} \alpha, & \text{if } |y| \leq \delta \\ \frac{\alpha\delta}{|y|}, & \text{otherwise,} \end{cases}\quad (2)$$

with $0 \leq \alpha \leq \alpha_{\max}$ the gain and $0 < \delta < \delta_{\max}$ the switching length. Stability of system (1) with stable linear part and (sector-bounded) switching function (2) follows from the circle criterion if

$$\Re \{ \mathbf{c}^T (j\omega \mathbf{I} - \mathbf{A})^{-1} \mathbf{b}_1 \} \geq -\frac{1}{\alpha}.\quad (3)$$

So the choice of switching gain α is bounded by circle criterion evaluation and strongly relates to performance and robustness of the closed-loop system. The larger the input y (where $|y| > \delta$) the smaller the effective gain, hence favorable robustness properties. The choice of switching length δ is stability-invariant and thus strictly performance driven. It is the aim of this work to derive the optimal values for both δ and α through an extremum seeking and iterative procedure.

Dynamics, stability, and extremum seeking of the switching system

For system (1) consider the discrete-time representation

$$\begin{aligned}\mathbf{x}(n+1) &= \mathbf{A}_d \mathbf{x}(n) + \mathbf{b}_{d,1} u(n) + \mathbf{b}_{d,2} v(n) \\ y(n) &= \mathbf{c}_d^T \mathbf{x}(n),\end{aligned}\quad (4)$$

at sampling instances $n > 0$ and $\mathbf{A}_d, \mathbf{b}_{d,1}, \mathbf{b}_{d,2}, \mathbf{c}_d^T$ of appropriate dimensions. System (4) can be put into lifted form:

$$\begin{bmatrix} y(2) \\ \vdots \\ y(n+1) \end{bmatrix} = \overbrace{\begin{bmatrix} \mathbf{c}_d^T \mathbf{b}_{d,1} & \mathbf{0} \\ \vdots & \ddots \\ \mathbf{c}_d^T \mathbf{A}_d^{n-1} \mathbf{b}_{d,1} & \cdots & \mathbf{c}_d^T \mathbf{b}_{d,1} \end{bmatrix}}^{\mathbf{S}_c} \begin{bmatrix} u(1) \\ \vdots \\ u(n) \end{bmatrix} + \overbrace{\begin{bmatrix} \mathbf{c}_d^T \mathbf{b}_{d,2} & \mathbf{0} \\ \vdots & \ddots \\ \mathbf{c}_d^T \mathbf{A}_d^{n-1} \mathbf{b}_{d,2} & \cdots & \mathbf{c}_d^T \mathbf{b}_{d,2} \end{bmatrix}}^{\mathbf{S}} \begin{bmatrix} v(1) \\ \vdots \\ v(n) \end{bmatrix} + \begin{bmatrix} \mathbf{c}_d^T \mathbf{A}_d \\ \vdots \\ \mathbf{c}_d^T \mathbf{A}_d^n \end{bmatrix} \mathbf{x}(1),\quad (5)$$

with $\mathbf{S}, \mathbf{S}_c \in \mathbb{R}^{n \times n}$ representing the sensitivity matrix and scaled complementary sensitivity matrix, respectively. Under the assumption that the initial state is zero: $\mathbf{x}(1) = \mathbf{0}$, (1) and (5) give rise to the following set of algebraic equations:

$$\begin{aligned}\mathbf{y}_k &= \mathbf{S}_c \mathbf{u}_k + \mathbf{S} \mathbf{v}_k \\ \mathbf{u}_k &= -\alpha_k \varphi(\mathbf{y}_k).\end{aligned}\quad (6)$$

At each trial k the data-sampled errors are given by $\mathbf{y}_k = [y(2) \dots y(n+1)]^T$. The data-sampled inputs are given by $\mathbf{v}_k = [v(1) \dots v(n)]^T \in \mathbb{R}^n$. The switching (and saturation-based) nonlinearities $\varphi(\mathbf{y}_k) \in \mathbb{R}^n$ satisfy

$$\varphi(\mathbf{y}_k) = \mathbf{y}_k - \varphi_1(\mathbf{y}_k) \mathbf{y}_k - \delta_k \varphi_2(\mathbf{y}_k),\quad (7)$$

with $\varphi_1(\mathbf{y}_k) \in \mathbb{R}^{n \times n}$ a positive semi-definite diagonal matrix and $\varphi_2(\mathbf{y}_k) \in \mathbb{R}^n$ a column:

$$\varphi_1(\mathbf{y}_k)[i, i] = \begin{cases} 0, & \text{if } |\mathbf{y}_k[i]| < \delta_k \\ 1, & \text{otherwise,} \end{cases} \quad \text{and } \varphi_2(\mathbf{y}_k)[i] = \begin{cases} 0, & \text{if } |\mathbf{y}_k[i]| < \delta_k \\ -\text{sign}(\mathbf{y}_k[i]), & \text{otherwise.} \end{cases} \quad (8)$$

Consider the objective function centered about the data-sampled and performance-relevant signals/intervals in \mathbf{y}_k :

$$V(\delta_k, \alpha_k) = \mathbf{y}_k^T(\delta_k, \alpha_k) \mathbf{y}_k(\delta_k, \alpha_k). \quad (9)$$

For system (6) and objective function (9) the aim is to find $\mathbf{p}_{\text{opt}} = [\delta_{\text{opt}} \ \alpha_{\text{opt}}]^T$ such that

$$\mathbf{p}_{\text{opt}} := \arg \min_{\mathbf{p}_k} V(\delta_k, \alpha_k), \quad \text{with } \mathbf{p}_k = [\delta_k \ \alpha_k]^T. \quad (10)$$

The optimal set of switching parameters \mathbf{p}_{opt} is found using the following Gauss-Newton based scheme

$$\mathbf{p}_{k+1} = \mathbf{p}_k - \beta \left(\frac{\partial \mathbf{y}_k^T}{\partial \mathbf{p}} \frac{\partial \mathbf{y}_k}{\partial \mathbf{p}} \right)^{-1} \frac{\partial \mathbf{y}_k^T}{\partial \mathbf{p}} \mathbf{y}_k, \quad (11)$$

with convergence parameter $0 < \beta < 1$ and gradients

$$\frac{\partial \mathbf{y}_k}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial \mathbf{y}_k}{\partial \delta} & \frac{\partial \mathbf{y}_k}{\partial \alpha} \end{bmatrix} = [\alpha_k \mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}_c \varphi_2(\mathbf{y}_k) \quad -\mathbf{A}_k^{-1}(\mathbf{y}_k) \mathbf{S}_c \varphi_1(\mathbf{y}_k)], \quad \mathbf{A}_k(\mathbf{y}_k) = \mathbf{I} + \alpha_k \mathbf{S}_c - \alpha_k \mathbf{S}_c \varphi_1(\mathbf{y}_k). \quad (12)$$

Stability follows from the next argument. Consider $\zeta > 0$ and $0 < \xi < 1$ such that for all \mathbf{y}_k

$$\|\mathbf{y}_k\|_p = \sqrt{\mathbf{y}_k^T \mathbf{P}_k \mathbf{y}_k} \geq \eta \sqrt{\frac{1 + \zeta(1 + \beta)^2}{\beta(1 - \xi)}} : V(\delta_{k+1}) - V(\delta_k) \leq -\beta(1 - \beta) \|\mathbf{y}_k\|_p^2, \quad (13)$$

with $\mathbf{P}_k = (\partial \mathbf{y}_k / \partial \mathbf{p})(\partial \mathbf{y}_k^T / \partial \mathbf{p} \partial \mathbf{y}_k / \partial \mathbf{p})^{-1} \partial \mathbf{y}_k^T / \partial \mathbf{p}$. Given a positive Lyapunov function candidate (9) and a negative definite difference (13) convergence follows toward the invariant set given in (13). The proof follows (after some algebra) by substituting $\mathbf{y}_{k+1} = \mathbf{y}_k - \beta \mathbf{P}_k \mathbf{y}_k + \mathcal{O}_k$ with $\lim_{k \rightarrow \infty} \sup \|\mathcal{O}_k\| \leq \eta$ in the Lyapunov difference.

Machine-in-the-loop optimization

To illustrate the potential of the optimized switching controller, a machine-in-the-loop optimization is conducted on a fast moving stage system with nanometer accuracy [2]. The results are depicted in Figure 1 both in simulation (left part) and experiment (right part). For convergence parameter $\beta = 0.5$ and different sets of initial conditions, it can be seen (upper part) that δ and α converge to their optimal values in a limited number of iterations k . The lower part of the figure shows that switching control is able to reduce low-frequency oscillations in the first (acceleration) time-interval without substantially increasing the noise response in the second (scanning) time-interval. For the case of low-gain feedback ($\delta = 0$) keeping a small noise response comes at the cost of significant less suppression of low-frequency oscillations. Contrarily the case of high-gain feedback ($\delta = \delta_{\text{max}}, \alpha = \alpha_{\text{max}}$), which induces proper suppression, relates to poor noise response.

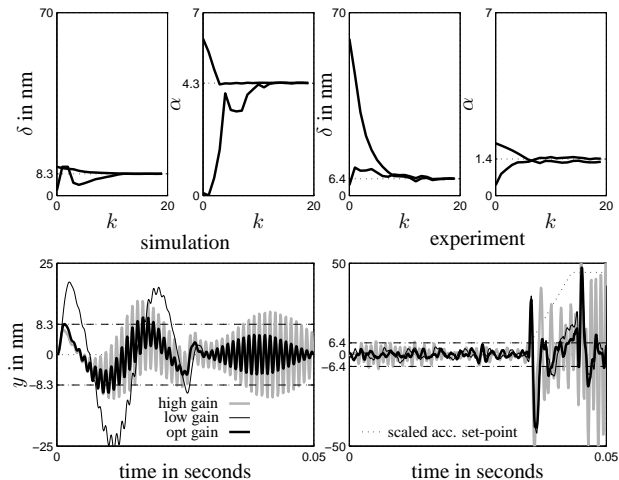


Figure 1: Time-domain performance after machine-in-the-loop optimization both in simulation (left part) and experiment (right part).

Conclusions

Through machine-dedicated calibration, optimized switching demonstrates improved motion control while keeping low complexity of the tuning and control design. The combined model/data-based approach is key in finding the gradients with respect to the parameters to be optimized: δ and α . The approach is strictly performance driven. The objective function contains performance-relevant signals obtained from relevant time intervals. Lyapunov arguments guarantee convergence to an invariant set. In simulation (and without noise) this set effectively reduces to zero. In experiment, it remains non-zero (though restricted to a fairly small bound) as expected from the Lyapunov stability argument.

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