

Inventory redistribution for fashion products under demand parameter update

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Inventory redistribution for fashion products under demand parameter update

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Abstract

Demand for fashion products is usually highly uncertain. Often, there is only one possibility for procurement before the selling season. In order to improve the traditional newsvendor-type overage-underage trade-off we study a network of two expected profit maximizing retailers selling a fashion product where there is an additional opportunity for redistribution of stock during the selling season. We distinguish between the situation where redistribution is done at the moment when one of the retailers is running out of stock and the situation where the redistribution time is already determined and fixed before the selling season. We model the demand process at a retailer by a Poisson Process with an uncertain mean and use a Bayesian approach to update the distribution parameters before transshipments are done. In a numerical study we compare the different policies and show that timing flexibility and updating are especially beneficial in situations with low profit margins and high parameter uncertainty. Further, we show that depending on the instance, an optimal predetermined transshipment timing depends on the problem parameters and may be between the middle and the end of the selling season.

Key words: Fashion retail, inventory, transshipments, Bayesian updates

1 Introduction

Though inventory and information management for fashion products have a long tradition, new concepts and strategies are still at the core of the supply chain management research agenda. Since customer requirements are increasing, the use of efficient logistics and innovative supply chain management strategies is necessary for fashion retailers competitiveness. Besides cost reduction, a high product availability is essential to meet customer demands, improve customer service, and ensure customer satisfaction, which is a key success factor for any business.

However, striving for excellent operational performance in a fashion supply chain is a challenging and notoriously difficult task, since fashion products are usually characterized by long replenishment lead times, short selling seasons, and nearly unpredictable demand and therefore, inaccurate forecasts. In extreme cases the replenishment lead times can be longer than the selling season and often, retailers can only place a single order long before the selling season starts and there are no additional replenishment opportunities during the selling season. Further, highly uncertain demand triggers the optimization problem of balancing the trade-off between providing too many or too few items associated with excess stock, which has to be sold with a markdown and often below procurement cost at the end of the season, or lost sales and unsatisfied customers due to inventory stockouts.

Facing this challenge, several innovative business practices were initiated, investigated, and implemented to create more flexibility in fashion supply chains. A broadly used strategy for improvement, that has attracted considerable attention in recent years, is Quick Response (Hammond (1990)), aiming at lead time reduction and enabling order placements closer to the start of the selling season. The postponement of ordering decisions after initial demand signals have been received leads to better demand forecasts. In a multi-product environment, a smart allocation of production capacity offers the option to accurately respond to highly uncertain demands as proposed by Fisher et al. (1997). For multiple retailers an operational strategy to reduce uncertainty is to pool risks by sharing inventories (Tagaras (1999)). This can be achieved by a delayed allocation of (some) central stock to retailers or the opportunity to redistribute inventories. Compared to a pure push approach with a single initial supply only, the latter provides an efficient means of salvaging excess inventories besides clearance pricing.

Our research was motivated by a fashion retailer buying and selling luxury lingerie at several outlet stores in Europe. Recent advances in information technology enable the use of point-of-sales data to manage supply chain processes efficiently. We investigate a supply chain management strategy combining risk-pooling with the possibility to improve forecasts using actual sales data. Retailers can share their inventories through lateral transshipments and information gathered regarding sales until the moment of transshipment is used to optimize an inventory rebalancing decision. The option to rebalance inventories serves as the operational enabler for demand learning by (partly) postponing the final inventory allocation. However, there is a trade-off between the value of improved demand information by further postponing the time of rebalancing and the opportunity to make use of it as later rebalancing reduces the time to sell redistributed items.

The use of proactive transshipments by having the opportunity to rebalance inventory has

not been investigated as an enabler to unlock the benefits of demand parameter learning. Our research question and contribution is to identify the benefits of the transshipment opportunity in the presence of demand parameter uncertainty, to provide insights on the time instant for rebalancing and how this impacts initial inventory investments. In this paper we use a newsvendor-type, single-item, single-period stochastic inventory model under the assumption that unsatisfied demand is lost and we allow for an additional opportunity during the selling season for redistributing stocks between two retailers after early sales have been observed and demand parameters have been updated according to a Bayesian approach. Our goal is to determine the optimal initial inventory levels that maximize the expected profits of the entire selling season. Generally, there is no benefit from rebalancing as long as both retailers have positive inventory. Therefore, in the absence of costs and lead times for redistribution, the optimal time for a single redistribution is the time of the first stockout of a retailer. However, due to other operational constraints, there might be a necessity to fix the time of redistribution beforehand. In this case we optimize the timing of the redistribution, too, which provides insights into how to resolve the trade-off between the value of demand parameter learning and the possibility of making use of it.

The remainder of the paper is structured as follows. In Section 2 we review the existing literature related to our research topic. In Section 3 we develop a mathematical model to determine inventory levels and the optimal timing for rebalancing under different demand learning and rebalancing timing strategies. Section 4 reports numerical results and provides insights on potential improvements by demand learning and inventory rebalancing. The paper concludes with a summary of the main contribution and an outlook of further research.

2 Literature Review

Two streams of research are closely related to our work, i) risk-pooling through lateral transshipments and ii) newsvendor models with two replenishment possibilities and/or demand parameter updates.

2.1 Risk-pooling through lateral transshipments

In retail supply chains mainly two different risk-pooling strategies are applied to hedge against uncertainty. In the first approach, only a portion of the goods received by the central warehouse is shipped to the retail outlets at the beginning of the selling season while the remaining part is kept centrally at the warehouse. During the selling season there exist one or several opportunities for the warehouse to allocate and distribute the retained central stock to the retailers in order to balance their inventories. Optimal initial inventory levels, optimal allocation policies, and the optimal timing of withdrawals from warehouse stock are studied in many papers, e.g., Jackson (1988), Jackson and Muckstadt (1989), McGavin et al. (1993), McGavin et al. (1997), Cao and Silver (2005). The second approach, also followed in this paper, is based on a complete PUSH-strategy where all goods are shipped to the retail outlets at the beginning of the selling season and retailers

may share their inventories using lateral transshipments.

Lateral transshipments have received increasing attention over the last years and many contributions demonstrate their benefits in centralized and decentralized distribution environments. Lateral transshipments are often applied in spare parts systems where they occur in response to stockouts. Another area of application are distribution networks of retailers. Contributions are devoted to centrally controlled systems and to the coordination impact of transshipments in decentralized systems using game theoretic methods, e.g. Rudi et al. (2001) for the two-retailer case and Wee and Dada (2005) for n retailers. However, the majority of all these studies assumes that transshipments are only possible at the end of a period where locations with excess stock transship to locations with excess demand. This implies a pure reactive approach for clearing excessive stock and excessive demands. For further literature we refer to the review presented in Wong et al. (2006). However, in case of fashion goods it is very unlikely that customers that arrive within the season are willing to wait until the end of the selling season and therefore, a proactive transshipment policy is required.

A few studies devoted to periodic inventory models allow redistribution of stock between order moments. Mostly, only one redistribution opportunity is considered and the time instant is assumed to be exogenously predetermined. Jönsson and Silver (1987) argue that backorders are likely to occur at the end of a replenishment cycle and therefore, stock transfers should only take place close to the end of a cycle. They fix the time for redistribution at one subperiod before the end of the order cycle motivated by high required service levels and therefore high inventory levels. Similar arguments are used in Tagaras and Vlachos (2002). However, an optimal point in time for redistribution is not determined. Das (1975) and Lee and Whang (2002) do not comment at all on the choice of the redistribution instant. While in the last named articles replenishment and redistribution decisions are jointly optimized, Allen (1958) focusses on optimal lateral transshipment decisions under given initial inventory levels. The joint optimization of timing and rebalancing decisions for given initial inventory levels is studied in Agrawal et al. (2004). Based on their investigations they conclude that rebalancing of stock tends to take place later during the period.

2.2 Newsvendor models and information update

Newsvendor models are a frequently used tool for the determination of order quantities for fashion products (see Khouja (1999) for a review). The economic trade-off between providing too many or too few items, associated with overage and underage costs, is optimized. However, the traditional newsvendor model only allows goods to be produced (or ordered) once before the selling season starts. Lau and Lau (1998) and Li et al. (2009) analyze the benefit of a second order opportunity during the selling season. While a second order opportunity can lead to lower inventory costs and fewer lost sales at the end of the selling season, additional set-up costs and higher procurement unit costs might occur. Under the assumption of independently identically normally distributed demands and constant cost parameters Lau and Lau (1998) illustrate for a pre-determined second order time instant that with increasing demand uncertainty and with decreasing product's profitability (measured by the newsvendor ratio) the value of a second order moment

increases. If demand uncertainty increases linearly with expected demand, the second order moment should be set at the time when the average demand of the first subperiod is equal to 75% of the total expected demand.

A second order moment can be even more beneficial using modern technologies and point-of-sales data to update demand forecasts before the second order decision. Bradford and Sugure (1990) divide the selling season in two equal time-periods and apply a Bayesian approach to optimize the two ordering decisions when demand follows a Poisson process with an unknown Gamma-distributed parameter λ . A dynamic programming formulation is presented in Murray and Silver (1966) to optimize the profit for a selling season with a finite number of possible acquisition times. The underlying demand model assumes N potential customers, willing to buy the product with probability p , where the numerical value of p is unknown and updated based on information gathered during the selling season. The selling season is also divided into several sub-periods in Eppen and Iyer (1997). However, purchasing is only possible at the beginning of the first period, while dumping is possible at the beginning of all future sub-periods. The sell-off price is decreasing with time and Bayesian updates are applied to improve demand forecasts. One of their main conclusions is that updating is always beneficial and under certain circumstances can yield significant improvements.

While Bradford and Sugure (1990), Murray and Silver (1966), and Eppen and Iyer (1997) use point-of-sales data to update the demand parameters and allow for additional ordering or dumping decisions during the selling season, another stream of literature assumes that all decisions are made before the selling season. Only one purchasing opportunity is considered in Iyer and Bergen (1997), who focus on the benefit of a Quick Response strategy, providing shorter lead times, which enables data collection of related items to be used to decrease forecast errors for the item being ordered. Iyer and Bergen (1997) apply information updates to an unknown mean and Choi et al. (2006) extend this to update both mean and variance of normally distributed demand.

A situation where a retailer has two instants to order a seasonal product and where the total order quantity arrives before the selling season is studied in Fisher and Raman (1996), Gurnani and Tang (1999), Choi et al. (2003), Miltenburg and Pong (2007a), and Miltenburg and Pong (2007b). These papers resolve the trade-off between early and late ordering. At the first order instant there is a high uncertainty about the expected demand and a low price uncertainty, while at the second order instant more information is available about demands, but the purchasing price is larger or subject to higher uncertainty. Donohue (2000) investigates a similar model with a focus on pricing schemes that coordinate the channel and maximize the total profit.

Although there are some similarities between existing approaches and this paper, there are two important differences. First, we jointly optimize the initial inventory levels, the redistribution quantities, and redistribution time instant. Second, our model allows for parameter updating using a Bayesian approach.

3 Model formulation

3.1 Assumptions and transshipment policies

We consider two retailers who order a single product from a supplier (manufacturer or distributor) with infinite supply and sell it over a selling season of length T . Retailers demand is stochastic and assumed to be independent between both retailers. Further, demand that cannot be filled from stock is considered lost. We assume that unit procurement costs c and unit sales revenues p are identical for both retailers. For further simplicity of presentation, the salvage value of units left over at the end of the selling season is zero. Replenishment lead times from the supplier may be positive, but retailers place their orders in advance such that products arrive at the beginning of the selling season.

Due to the short selling season there is no further opportunity for external resupply, but there is a single time instant where inventories can be redistributed. We assume that retailers are located close together such that transshipment lead times are negligible. The additional costs incurred by transshipments are assumed to be negligible too, e.g., due to a transportation contract with a logistics service provider. Since retailers can share their inventories, it has to be determined when and how many units to transship. As fashion customers are not willing to wait for a product in case of a stockout, we consider proactive transshipment policies. We assume that transshipments are only allowed once within the selling period and distinguish two policies with respect to the timing of redistribution.

For the FLEX-policy the transshipment instant is not fixed in advance. Since the transshipment time is assumed to be negligible the best theoretical instant for stock transfer is the first moment when a customer arrives at a retailer with zero stock. However, fashion retailers may not observe lost sales when shelves are empty, since not all customers are willing to contact a shop assistant. Therefore, we assume redistribution to take place when the inventory level of a retailer reaches zero. In this situation the transshipment instant is a random variable T_0 . If no redistribution takes place we set $T_0 = T$ and therefore, $T_0 \in [0, T]$. In this situation the decision process involves choosing the initial inventory levels S_1, S_2 and at T_0 how much inventory to redistribute.

One disadvantage of the FLEX policy is the uncertainty with respect to the timing of the transshipments that might result in planning difficulties if e.g. transportation capacities have to be reserved in advance. Therefore, retailers may prefer to know before the selling season when transshipments will take place. For this purpose we investigate the FIX-policy where the transshipment instant t_0 is fixed and known to the retailers prior to the selling season when initial ordering decisions take place. In this case we determine the optimal division of the selling season, because early stock transfers may lead to imbalance at the end of the selling season, resulting in lost sales, while late transshipments may lead to lost sales before redistribution takes place. The decision variables in this situation are the initial inventory levels S_1, S_2 , the redistribution time t_0 as well as the quantity to be redistributed in response to observed demand.

3.2 Demand model

Representative for fashion demand processes there are two sources of uncertainty. First, there is an inherent demand uncertainty, meaning that even if the mean demand is known, there are unpredictable fluctuations. This is reflected by the assumption that demand at retailer i follows a Poisson Process with parameter λ_i if mean demand is known. Further, there is uncertainty with respect to the value of the parameter λ_i . We assume that the prior information regarding λ_i follows a gamma distribution with parameters $\alpha_i > 0$ and $\beta_i > 0$ and density

$$f_{\alpha_i, \beta_i}(x) = \frac{\beta_i^{\alpha_i} x^{\alpha_i-1} e^{-\beta_i x}}{\Gamma(\alpha_i)} \quad (1)$$

where $\Gamma(\alpha)$ is the Gamma function defined as $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$. Thus, the expected value $E[\lambda_i]$ and variance $VAR[\lambda_i]$ are given as:

$$E[\lambda_i] = \frac{\alpha_i}{\beta_i}, \quad VAR[\lambda_i] = \frac{\alpha_i}{\beta_i^2}. \quad (2)$$

It follows that unconditional demand $D_i(t)$ during a time interval with length t is negative binomial with parameters α_i and $\frac{\beta_i}{\beta_i+t}$, and corresponding expected value and variance are

$$E[D_i(t)] = \frac{\alpha_i}{\beta_i} t, \quad VAR[D_i(t)] = \frac{\alpha_i}{\beta_i} t + \frac{\alpha_i}{\beta_i^2} t^2. \quad (3)$$

The first term of the variance in (3) reflects the variance with respect to the inherent demand uncertainty while the second term is induced by the uncertainty about the demand parameter. While the inherent demand uncertainty is exogenous to our model and cannot be influenced, we study the situation where retailers collect information and use actual point-of-sales data to update the probability distributions for the redistribution decision at time t_0 . A retailer i observing demand d_i can update his probability distribution following a Bayesian approach. The resulting posterior distribution for the parameter λ_i is again a gamma but with new parameters $\alpha_i + d_i$ and $\beta_i + t_0$ and the unconditional demand for the remaining time of the selling season is distributed according to a negative binomial distribution with parameters $\alpha_i + d_i$ and $\frac{\beta_i+t_0}{\beta_i+T}$.

3.3 Objective functions

The objective is to maximize the expected profit during the selling season for the total supply chain under centralized information and decision making. Decisions take place on two stages. For a given transshipment policy the initial inventory levels S_i have to be determined for both retailers at the first stage. At the second stage the optimal rebalancing of remaining inventories, as to maximize the expected revenues over the remaining selling season, has to be identified.

In order to formulate the expected profit we use the following notation. Let S_i be the initial inventory level of retailer i and $r_i(m)$ the share retailer i gets from redistributing a total of m units at t_0 . Further, let $D_i^1 = D_i(t_0 | \lambda_i)$ and $D_i^2 = D_i(T - t_0 | \lambda_i)$ denote

retailer i demand for a given value of λ_i prior and after a given redistribution time instant t_0 respectively. The expectation taken over a random variable X is denoted as $E_X[\cdot]$ and $x^+ = \text{Max}(0, x)$. Further, for notational convenience, let $\lambda = (\lambda_1, \lambda_2)$ and $D^1 = (D_1^1, D_2^1)$.

For the situation with a fixed transshipment time instant t_0 the expected profit for given initial inventory levels S_i and a rebalancing policy $r_i(m)$ is stated in the following.

$$\Pi_{FIX} = -c \sum_{i=1}^2 S_i + p E_{\lambda} \left[\sum_{i=1}^2 E_{D_i^1, D_i^2} \left[\min\{D_i^1, S_i\} + \min \left\{ D_i^2; r_i \left(\sum_{j=1}^2 (S_j - D_j^1)^+ \right) \right\} \right] \right] \quad (4)$$

Let $\pi_i(x, t)$ denote the expected sales quantity of retailer i during a period of length t with an initial inventory level of x . Then the expected profit can be characterized by initial procurement costs and sales price times expected sales quantities before and after the redistribution of stock.

$$\Pi_{FIX} = -c \sum_{i=1}^2 S_i + p \sum_{i=1}^2 \pi_i(S_i, t_0) + p E_{D^1} \left[\sum_{i=1}^2 \pi_i \left(r_i \left(\sum_{j=1}^2 (S_j - D_j^1)^+ \right), T - t_0 \right) \right]$$

Expected sales of retailer i before the redistribution can easily be computed, since demand before the transshipment moment t_0 is negative binomially distributed with parameters α_i and $\frac{\beta_i}{\beta_i + t_0}$

$$\pi_i(S_i, t_0) = \sum_{d=0}^{\infty} \min\{d; S_i\} \binom{\alpha_i + d - 1}{d} \left(\frac{\beta_i}{\beta_i + t_0} \right)^{\alpha_i} \left(\frac{t_0}{\beta_i + t_0} \right)^d. \quad (5)$$

Note that expected sales of retailer i after the redistribution of stock are dependent on the demand process of both retailers before redistribution. For given first period demand d_i of retailer i we get

$$\begin{aligned} \pi_i \left(r_i \left(\sum_{j=1}^2 (S_j - d_j)^+ \right), T - t_0 \right) = \\ \sum_{d=0}^{\infty} \min \left\{ d; r_i \left(\sum_{j=1}^2 (S_j - d_j)^+ \right) \right\} P(D_i(T - t_0) = d | d_i) \end{aligned} \quad (6)$$

where

$$P(D_i(T - t_0) = d | d_i) = \binom{d + d_i + \alpha_i - 1}{d} \left(\frac{\beta_i + t_0}{\beta_i + T} \right)^{d_i + \alpha_i} \left(\frac{T - t_0}{\beta_i + T} \right)^d \quad (7)$$

which is a negative binomial distribution with parameters $\alpha_i + d_i$ and $\frac{\beta_i + t_0}{\beta_i + T}$. Then, the unconditional expected sales of retailer i become

$$\sum_{d_1=0}^{\infty} \sum_{d_2=0}^{\infty} \pi_i \left(r_i \left(\sum_{j=1}^2 (S_j - d_j)^+ \right), T - t_0 \right) P(D_1(t_0) = d_1) P(D_2(t_0) = d_2).$$

If there is no possibility to gather information or sales data are not used to update demand parameters, i.e., the demand probabilities in (6) are not conditioned on observed demands d_i , the decision maker will use a negative binomial distribution with parameters α_i and $\frac{\beta_i}{\beta_i + T - t_0}$ instead of (7) to compute the expected profit.

$$P(D_i(T - t_0) = d) = \binom{\alpha_i + d - 1}{d} \left(\frac{\beta_i}{\beta_i + T - t_0} \right)^{\alpha_i} \left(\frac{T - t_0}{\beta_i + T - t_0} \right)^d. \quad (8)$$

Under the transshipment policy FLEX the redistribution instant is determined by the demand process and therefore uncertain. In this situation the expected profit is formulated as:

$$\begin{aligned} \Pi_{FLEX} = & -c \sum_{i=1}^2 S_i \\ & + p \sum_{i=1}^2 E_{\lambda_i} \left[E_{t_0} \left[E_{D_i^1, D_i^2} \left[\min\{D_i^1, S_i\} + \min \left\{ D_i^2; r_i \left(\sum_{j=1}^2 (S_j - D_j^1)^+ \right) \right\} \right] \right] \right] \end{aligned} \quad (9)$$

with only the two initial inventory levels S_i as decision variables.

3.4 Optimal decisions

The optimal redistribution decision at t_0 only depends on the remaining time to sell products and the available total inventory and is the same for policy FLEX and FIX. Let m denote the total inventory in the system at t_0 available for redistribution. We are interested in the optimal distribution of the m units among the two retailers to maximize the expected revenues in the remaining selling season of length $T - t_0$. The optimal transshipment quantities are the solution of the following problem:

$$\begin{aligned} \max_{r_1(m), r_2(m)} & p \left(\pi_1(r_1(m), T - t_0) + \pi_2(r_2(m), T - t_0) \right) \\ \text{s.t.} & r_1(m) + r_2(m) = m \end{aligned} \quad (10)$$

The optimal redistribution decisions are characterized in the following proposition.

Proposition 1: Suppose m units have to be allocated at time t_0 to two retailers. Then the optimal allocation, maximizing the expected revenues during the time interval of length $T - t_0$ as given in (10) has to satisfy the following inequalities:

$$P\left(D_1^2(T - t_0) \leq r_1^*(m)\right) \geq P\left(D_2^2(T - t_0) \leq r_2^*(m) - 1\right) \quad (11)$$

$$P\left(D_2^2(T - t_0) \leq r_2^*(m)\right) \geq P\left(D_1^2(T - t_0) \leq r_1^*(m) - 1\right) \quad (12)$$

Proof: Using $r_2(m) = m - r_1(m)$ we can express $\pi_2(r_2(m), T - t_0)$ as a function of $r_1(m)$

only:

$$\begin{aligned}
\Pi(r_1(m)) &:= p\pi_1(r_1(m), T - t_0) + p\pi_2(m - r_1(m), T - t_0) \\
&= pr_1(m)P(D_1^2(T - t_0) \geq r_1(m)) + p \sum_{j=0}^{r_1(m)-1} P(D_1^2(T - t_0) = j) \\
&\quad + p(m - r_1(m))P(D_2^2(T - t_0) \geq m - r_1(m)) + p \sum_{j=0}^{m-r_1(m)-1} P(D_2^2(T - t_0) = j)
\end{aligned} \tag{13}$$

An optimal inventory allocation $r_1^*(m)$ has to satisfy

$$\Pi(r_1^*(m)) - \Pi(r_1^*(m) + 1) \geq 0 \quad \text{and} \quad \Pi(r_1^*(m)) - \Pi(r_1^*(m) - 1) \geq 0 \tag{14}$$

Inserting (13) into (14) and simplifying terms yields (11) and (12).

Note that the inequalities (11) and (12) are the discrete equivalent to both retailers having equal non-stockout probabilities. In case of no parameter update the decision maker assumes that demand of the second subperiod with length $T - t_0$ is negative binomially distributed with parameters α_i and $\frac{\beta_i}{\beta_i + T - t_0}$, while in the other case the second parameter of the distribution will depend on the observed demand (7).

Since it is in general not possible to obtain analytical expressions for the optimal share $r_i^*(m)$, the optimization problems

$$\max_{S_1, S_2, t_0} \Pi_{FIX}(S_1, S_2, t_0) \quad \text{and} \quad \max_{S_1, S_2} \Pi_{FLEX}(S_1, S_2) \tag{15}$$

are solved numerically. For policy FIX we restrict ourselves to the optimization of the embedded version of the problem and only allow for discrete values of $t_0 \in M := \{0, t_1, t_2, \dots, T \mid 0 < t_i < T\}$. Then the optimal initial inventory levels ($S_1^*(t_i)$, $S_2^*(t_i)$) are determined for each value of the transshipment moment $t_i \in M$ and the overall optimum is computed using

$$\begin{aligned}
(S_1^*, S_2^*, t_0^*) &= \operatorname{argmax} \left\{ \Pi_{FIX}(S_1^*(t_i), S_2^*(t_i), t_i) \mid t_i \in M \right\} \\
\Pi_{FIX}^* &= \Pi_{FIX}(S_1^*, S_2^*, t_0^*)
\end{aligned}$$

In case of no updates for the FIX as well as for the FLEX policy the search domain can be bounded to $\mathcal{S} = \{(S_1, S_2) \mid 0 \leq S_1 \leq S_1^{news}, 0 \leq S_2 \leq S_2^{news}\}$. S_i^{news} denotes the optimal initial inventory level of retailer i if no redistribution takes place (news vendor solution). The optimization is done by complete enumeration.

In view of the characteristics of the objective function of the FLEX policy, we determine the expected profit for given inventory levels (S_1, S_2) by simulation. We use 150000 periods to obtain simulation statistics. All numerical results are obtained using MATLAB.

4 Computational experiments

In the previous section, we presented four different supply chain strategies depending on the choice of the transshipment strategy and the decision whether to update or not (see Table 1). Here we evaluate each of these across a variety of parameters.

		Information	
		Update	No update
Transshipment Strategy	FIX	I	II
	FLEX	III	IV

Table 1: Strategies

For the following comparisons, there is a difference between the objective functions used for obtaining the required policy parameters and for policy evaluation. The quality of a solution of initial inventory levels and a redistribution policy is evaluated by (4) for FIX and by (9) for FLEX for both information cases. However, depending on whether initial demands are used for an update or not, the initial inventory levels and the redistribution policy are either determined using (7) (update) or (8) (no update).

4.1 Experimental design

We employ a numerical study to investigate the impact of key parameters on the performance of the strategies, where the following parameter values are chosen. In all problem instances the sales revenue for an item is normalized to $p = 100$ and the planning horizon is normalized to $T = 1$. To create our test cases we distinguish between five different factors: newsvendor ratio $\gamma = \frac{p-c}{p}$, expected demand rate λ_i for each retailer, and coefficient of variation of demand parameter uncertainty for each retailer $c_v(\lambda) = \frac{\sqrt{VAR[\lambda]}}{E[\lambda]}$. For each factor we allow for three different levels (low, middle, high) shown in Table 2. We use a full factorial design and eliminate the symmetric examples leading to 135 problem instances for each strategy.

factor	low	middle	high
γ	0.2	0.5	0.8
λ_1	5	20	50
λ_2	5	20	50
$c_v(\lambda_1)$	0.1	0.5	1
$c_v(\lambda_2)$	0.1	0.5	1

Table 2: Parameter values

In the following we summarize the results and investigate the value of flexibility, the value of information, and the optimal time instant for redistribution.

4.2 The value of flexibility

To assess the value of flexibility we compare policy FLEX and FIX and we separately analyze the cases without updating (II and IV) and with updating (I and III). The relative difference Δ_i for the expected profit of policy FIX ($\Pi_{FIX,i}^*$) and policy FLEX ($\Pi_{FLEX,i}^*$)

under optimal decision making is computed for each instance i

$$\Delta_i := \frac{\Pi_{FLEX,i}^* - \Pi_{FIX,i}^*}{\Pi_{FIX,i}^*} \quad i = 1, 2, \dots, 135. \quad (16)$$

Table 3 summarizes the results for the minimum, average, and maximum relative cost difference over all problem instances.

$$\Delta_{min} = \min_i \{\Delta_i\}, \quad \Delta_{av} = \frac{1}{N} \sum_{i=1}^N \Delta_i, \quad \Delta_{max} = \max_i \{\Delta_i\}. \quad (17)$$

				no update			update		
factor	level	factor	level	Δ_{min}	Δ_{av}	Δ_{max}	Δ_{min}	Δ_{av}	Δ_{max}
		γ	low	0.16%	12.49%	115.90%	0.42%	3.94%	20.65%
			middle	0.00%	1.48%	4.79%	0.09%	1.53%	6.14%
			high	0.03%	0.44%	2.59%	0.01%	0.58%	2.39%
λ_1	low	λ_2	low	0.54%	11.43%	115.90%	1.22%	6.05%	20.65%
	low		middle	0.00%	4.13%	31.86%	0.28%	2.14%	9.34%
	low		high	0.01%	5.05%	56.78%	0.01%	0.89%	4.56%
	middle		middle	0.04%	3.70%	30.33%	0.09%	1.86%	4.74%
	middle		high	0.02%	2.91%	33.65%	0.02%	1.23%	5.30%
	high		high	0.03%	2.78%	20.66%	0.08%	0.82%	2.11%
$c_v(\lambda_1)$	low	$c_v(\lambda_2)$	low	0.09%	1.52%	8.19%	0.06%	1.54%	8.89%
	low		middle	0.00%	1.37%	8.23%	0.04%	1.33%	8.36%
	low		high	0.01%	2.90%	24.35%	0.01%	1.66%	9.26%
	middle		middle	0.03%	2.88%	11.53%	0.05%	2.18%	10.98%
	middle		high	0.02%	5.44%	27.70%	0.02%	2.15%	12.43%
	high		high	0.04%	17.07%	115.90%	0.22%	3.69%	20.65%

Table 3: Relative profit difference policy FLEX - policy FIX

While the average percentage improvement in maximal profit that results from flexibility is 4.1% in case of no parameter updates and 2.0% when information about sales data is used, the FLEX policy can be more beneficial in cases with low profit margins and high parameter or demand uncertainty. On the one hand this can be explained by low initial inventory levels due to low profit margins resulting in the necessity of a careful allocation of scarce inventory. On the other hand the variance of the redistribution time of the FLEX policy increases with increasing parameter uncertainty and, therefore, fixing the redistribution moment in advance is limiting the scope of actions and lowering profits. In general, sales can be increased and left over inventory can be decreased by applying a flexible transshipment policy. A comparison of the savings with respect to the use of information reveals smaller differences when parameters are updated before the redistribution decision is made.

We are also interested in how flexibility influences the ordering decisions. Therefore, we compute the relative difference of the optimal initial inventory levels as

$$\tilde{\Delta} := \frac{\left((S_{1,FLEX}^* + S_{2,FLEX}^*) - (S_{1,FIX}^* + S_{2,FIX}^*) \right)}{(S_{1,FIX}^* + S_{2,FIX}^*)}. \quad (18)$$

A comparison between the initial inventory levels of strategy I and III does not reveal a significant difference in the initial inventory levels (in 75% of the instances there is no change at all and the average relative difference is less than 1%). However, in case of no parameter update less inventory investment is needed if retailers apply a flexible transshipment policy (on average a reduction of 5% is achieved). We have to mention that these differences are not uniformly spread over all 135 examples, and they are larger in cases with high demand and/or parameter uncertainty. To demonstrate this effect we present the impact of the factors $(c_v(\lambda_1), c_v(\lambda_2))$ on the relative difference of the initial inventory levels in box and whisker plots, illustrating median, lower and upper quartile of the data. Figure 1 captures the effect for identical retailers while Figure 2 is devoted to situations with non-identical retailers.

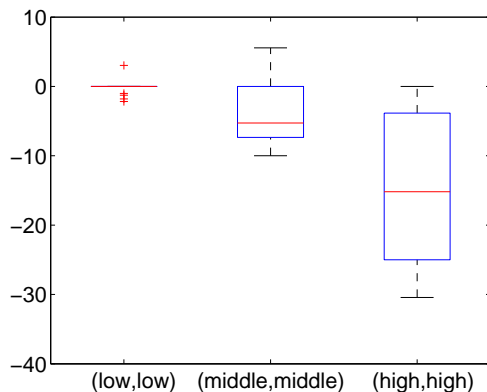


Figure 1: identical retailers
Impact of parameter uncertainty

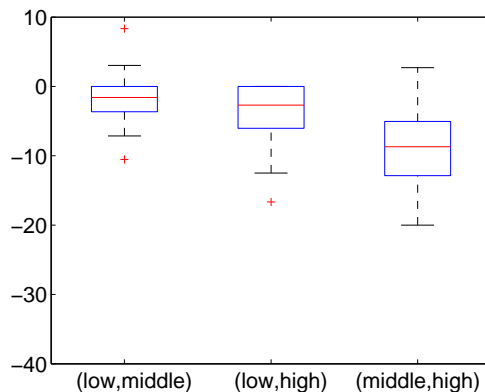


Figure 2: non-identical retailers
Impact of parameter uncertainty

Based on our numerical examples we conclude that substantial savings in inventory investments can be obtained by adding flexibility to the system in situations with high parameter uncertainty and no possibility of using additional demand information.

4.3 The value of information update

In order to quantify the value of information we compute for policy FIX (FLEX) the relative difference δ between the optimal profit in case of updates and when no parameter updates are used similar as in (16).

The first observation from Table 4 concerns the relative benefit of parameter updates and is in line with previous work. Using sales data to update demand parameters always pays. The average improvement in case of a fixed transshipment point is about 9% and for flexible transshipment moments 5.6%, because a less flexible system covers a wider scope for improvements resulting in higher benefits of additional information. The numerical results confirm what is intuitively expected: the value of information increases with increasing parameter uncertainty. Additionally, significant improvements can be observed in the low profit margin case where left over inventory as well as inventory investment decreases by using information. An illustration of the impact of the newsvendor ratio on

				FIX			FLEX		
		factor	level	δ_{min}	δ_{av}	δ_{max}	δ_{min}	δ_{av}	δ_{max}
		γ	low	0.00%	21.63%	128.82%	0.00%	11.53%	50.58%
			middle	0.03%	3.80%	14.33%	0.00%	3.85%	15.46%
			high	0.00%	1.31%	5.61%	0.00%	1.44%	6.23%
λ_1	low	λ_2	low	0.00%	11.69%	128.82%	0.34%	6.00%	27.87%
	low		middle	0.00%	8.34%	63.12%	0.00%	5.93%	39.51%
	low		high	0.00%	11.33%	99.23%	0.00%	5.98%	50.58%
	middle		middle	0.00%	7.90%	63.60%	0.14%	5.66%	31.47%
	middle		high	0.00%	6.94%	60.81%	0.00%	5.00%	26.70%
	high		high	0.00%	7.34%	53.45%	0.00%	5.03%	28.68%
$c_v(\lambda_1)$	low	$c_v(\lambda_2)$	low	0.00%	0.15%	1.31%	0.00%	0.16%	0.97%
	low		middle	0.06%	1.49%	8.61%	0.00%	1.45%	6.03%
	low		high	0.23%	7.23%	66.36%	0.43%	5.65%	37.37%
	middle		middle	0.69%	3.59%	9.30%	0.69%	2.87%	6.83%
	middle		high	0.10%	12.40%	84.42%	0.97%	8.47%	50.58%
	high		high	1.55%	31.43%	128.82%	1.66%	15.67%	35.25%

Table 4: Relative profit differences update - no-update

the relative percentage difference of inventory investment is shown in Figure 3 for policy FIX and in Figure 4 for the FLEX policy, respectively.

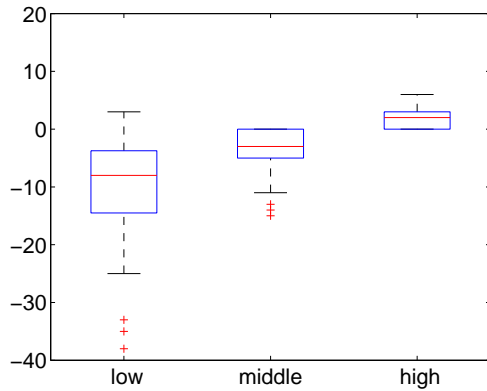


Figure 3: Policy FIX
Impact of newsvendor ratio

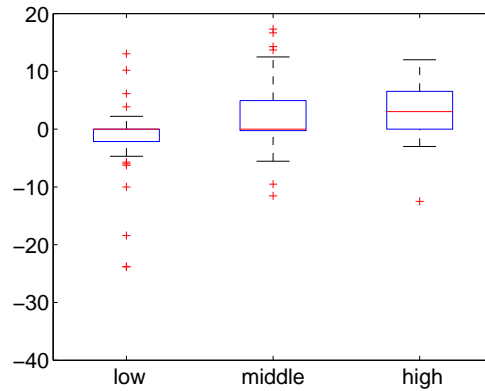


Figure 4: Policy FLEX
Impact of newsvendor ratio

For lower newsvendor ratios, the initial inventory levels can be reduced under parameter updates while for larger profit margins it is more likely that the inventory levels increase, because more sales are expected.

The optimal procurement decisions depend on the parameter uncertainty. The following Figures illustrate the relation between the parameter uncertainty and the relative difference of initial inventory levels induced by updating demand parameters during the selling season. Figure 5 and 6 show the results for a fixed transshipment moment.

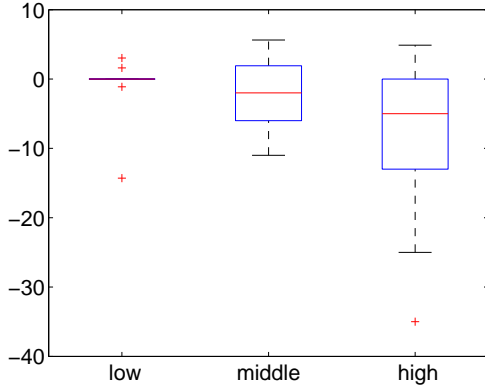


Figure 5: Identical retailers
Impact of parameter uncertainty

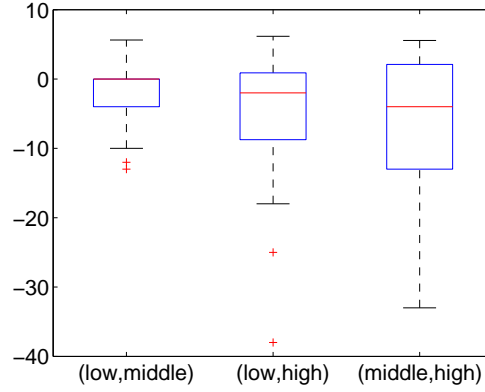


Figure 6: Non-identical retailers
Impact of parameter uncertainty

It can be seen that for a fixed transshipment time the use of point of sales data for the transshipment decisions result on average to reduced initial inventory levels compared to a strategy where no updates are done. This effect is increasing with increasing uncertainty. Parameter updates under a flexible transshipment policy increase replenishment order sizes as can be seen in Figure 7 and 8.

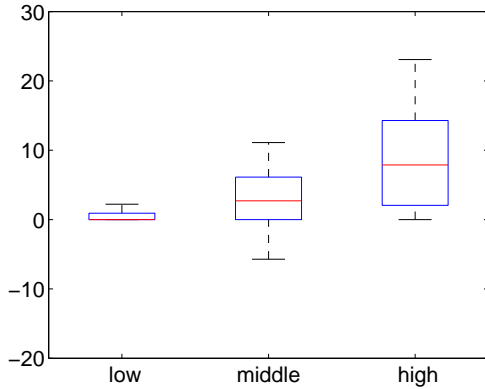


Figure 7: Identical retailers
Impact of parameter uncertainty

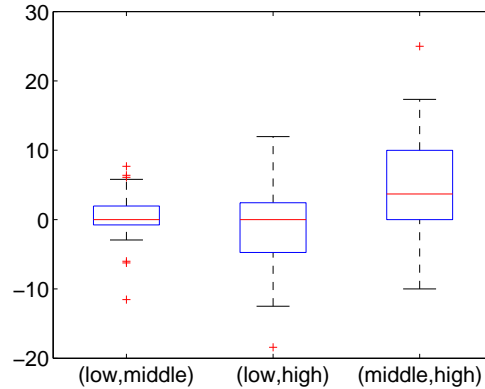


Figure 8: Non-identical retailers
Impact of parameter uncertainty

4.4 Optimal timing of redistribution

We determine the optimal rebalancing moment t_0 for policy FIX without parameter updates (Strategy II) as well as for Strategy I where information is gathered and used to compute the optimal redistribution of stocks. Table 5 reports on the average ($t_{0,av}$), the minimum ($t_{0,min}$), and the maximum ($t_{0,max}$) values over all instances.

For both strategies the optimal transshipment time is at least half of the selling season. Additionally, redistribution is done later if sales data is used to update demand parameters, because more time is needed to gather reliable information. While the opti-

				no-update			update		
		factor	level	$t_{0,min}$	$t_{0,av}$	$t_{0,max}$	$t_{0,min}$	$t_{0,av}$	$t_{0,max}$
		γ	low	0.50	0.59	0.75	0.50	0.62	0.80
			middle	0.50	0.61	0.80	0.50	0.65	0.85
			high	0.50	0.61	0.80	0.50	0.66	0.85
λ_1	low	λ_2	low	0.50	0.57	0.65	0.50	0.57	0.65
	low		middle	0.50	0.60	0.75	0.50	0.64	0.75
	low		high	0.50	0.60	0.75	0.50	0.69	0.85
	middle		middle	0.55	0.61	0.75	0.50	0.62	0.75
	middle		high	0.50	0.61	0.80	0.50	0.67	0.80
	high		high	0.55	0.62	0.80	0.50	0.64	0.80
$c_v(\lambda_1)$	low	$c_v(\lambda_2)$	low	0.55	0.73	0.80	0.55	0.74	0.85
	low		middle	0.55	0.64	0.75	0.55	0.72	0.80
	low		high	0.50	0.59	0.70	0.50	0.65	0.80
	middle		middle	0.55	0.58	0.60	0.55	0.63	0.75
	middle		high	0.50	0.55	0.60	0.50	0.58	0.70
	high		high	0.50	0.54	0.55	0.50	0.53	0.70

Table 5: Optimal redistribution time (policy FIX)

mal transshipment times for both strategies do not differ significantly in case of identical retailers, this does not hold for non-identical retailers.

The experiments also reveal that the optimal redistribution instant depends on the newsvendor ratio γ . With larger values of γ the optimal initial inventory levels are higher and the transshipment of goods can be postponed to a later moment in time. As a result the optimal value of t_0 is increasing with increasing γ .

Parameter uncertainty has a large impact on the optimal redistribution time. With increasing uncertainty the moment of the reallocation of stock should be set earlier independent of the policy used with respect to information. Since optimal initial inventory levels are robust against parameter uncertainty, they are not optimal for a specific demand pattern. Therefore, an early correction moment improves system performance.

5 Summary, Outlook and Extension

Quick response strategies with demand learning opportunities and inventory pooling through lateral transshipments are widely discussed and well analyzed supply chain management concepts to deal with highly uncertain fashion retail inventory management problems. In this paper we integrate these two research streams and investigate transshipments as an enabler of exploiting demand learning opportunities. We have illustrated the basic trade-offs and the numerical results give advice for practical rules of thumb on how to set the redistribution time instant and where parameter updating and redistribution timing flexibility pay off the most. Note that the reported benefits come on top of the general benefits of transshipments over independent newsvendor stocking policies.

The results show that an optimal redistribution will on average not take place at the middle of the selling season nor close to the end of it. Further, we saw that redistribution flexibility and parameter updating are somewhat substitute strategies, i.e., the presence of one instruments reduces the benefits of the other. The most promising improvements were achieved for instances with low newsvendor ratio, low demand values, and high parameter uncertainty. The differences in inventory levels between different strategies were only minor, implying that it is more important to do the right parameter update and redistribution.

Several simplifying assumptions need to be relaxed to generalize the model in future research. In the case of multiple (> 2) retailers, an exhaustive parameter search will no longer be applicable and therefore good approximations and effective heuristics for setting the inventory and timing parameters are required. Further, other demand models than the Poisson, e.g., the normal distribution, should be considered, however with the burden to have more than one uncertain parameter and requiring more complex parameter updating. Another limiting assumption which might not hold in general are negligible transshipment times and costs. Then, the option to withhold stock centrally for a later allocation instead of an immediate push of all units becomes a viable option to exploit the benefits of learning that needs to be traded-off with transshipments.

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