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Preferential states of rotating turbulent flows in a square container with a step topography

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The self-organization of confined, quasi-two-dimensional turbulent flows in a rotating square container with a step-like topography is investigated by means of laboratory experiments and numerical simulations based on a rigid lid, shallow-water formulation. The domain is divided by a bottom discontinuity into two rectangular regions, one being shallow and the other deep. The existence of a preferential vorticity distribution in the long-term evolution of the decaying flow is discussed. Initially, the turbulent flow organizes into larger structures. After a few rotation periods, a continuous jet-like flow is consistently observed along the step, with the shallow region at its right. This flow is associated with the adjustment of the fluid to equilibrium over a bottom discontinuity in an anti-clockwise rotating system. At the end of the step, two persistent structures are formed due to the collision of this jet with the vertical wall: a cyclonic circulation cell in the deep region, while an anticyclonic cell occurs in the shallow part of the domain. The laboratory experiments are well-reproduced by the simulations. Due to bottom friction effects, the fluid motion is halted before a complete organization of the flow is accomplished. In order to study the full process, additional numerical simulations were performed with zero Ekman friction. Same principal features are observed as in the experiments, but now a complete organization of the flow into four vortices is obtained: in the deep part of the flow domain, a cyclone-anticyclone pair is observed that fills up the entire region, and the mirrored double cell structure occurs on the shallow side. Such a disposition of the vortices is directly associated with the interaction of the flow along the step and the downstream wall at which it collides, as observed in the experiments. It is shown that this arrangement is systematically obtained in simulations with very different initial conditions. The existence of a preferential vorticity distribution induced by a topographic step is further discussed in terms of the aspect ratio of the domain.

I. INTRODUCTION

Most oceanic and atmospheric flows are turbulent. In a typical three-dimensional (3D) turbulent flow, a wide range of scales of motion can be found, in which the kinetic energy is transferred from large to smaller scales, at which it is dissipated. In contrast, the atmosphere and oceans are essentially very thin layers of fluid, mostly vertically stratified and under the influence of background rotation. Due to these physical restrictions, geophysical flows are often considered as quasi-two-dimensional and, as a consequence, the properties of two-dimensional turbulence are often invoked as the underlying mechanisms affecting the geostrophic turbulence in the oceans and the atmosphere. Two-dimensional (2D) turbulence presents a self-organization property, known as the inverse energy cascade, where small-scale structures tend to merge and give place to more coherent and less dissipative eddies. This property has been widely studied and discussed since several decades ago.1–5
The evolution of 2D turbulent flows confined within a closed domain has recently been studied both numerically and experimentally. The self-organization process strongly depends on the domain geometry and on the boundary conditions at the lateral walls, which play a fundamental role in the flow organization as they act as sources of vorticity and net angular momentum. For instance, van de Konijnenberg et al. and Clercx et al. found that the final state of the self-organization process in a square container tends to be a single vortex with a size comparable to the domain. Maassen et al. studied the self-organization in rectangular containers and found a cell pattern of alternate-sign vortices along the main axis. A similar configuration is found during the spin-up process in rectangular domains.

Another important topic is the role of the bottom topography in the establishment of these quasi-stationary final states of decaying 2D turbulent flows. The existence of a well-organized flow field due to topographic features is a robust result from which the final quasi-steady states can be predicted a priori using minimum-enstrophy arguments. Bretherton and Haidvogel performed a numerical study of the inverse energy cascade of a quasi-geostrophic, decaying turbulent flow over random topography. They found that the flow tends towards a stationary state aligned with the topography, with cyclonic (anticyclonic) circulations around depressions (bumps). In a recent work, using random topography and a less restrictive shallow water dynamics, Zavala Sansón et al. described the evolution and decay of a rotating homogeneous flow, and found a quasi-steady state characterized by a nearly linear relationship between potential vorticity and transport function. They found that vorticity production due to no-slip walls contributes to a slight disorganization of the flow, and that global energy decays faster for topographies with shorter horizontal length scales due to more effective viscous dissipation. Statistical mechanics of two-dimensional and geophysical flows have been used to study the self-organization of turbulent flows (see Ref. 14). The results provided by statistical mechanics concepts on the explanation of different phenomena, such as the prediction of preferential states of flow systems in equilibrium, have encouraged the development of the theory in order to extend its validity.

This paper describes the decay process and organization of a quasi-2D turbulent flow in a square container with discontinuous topography in a rotating system. The main objective is to describe the existence of a preferential, long-term state of the flow field in a square container due to the presence of a step-like topography. A step topography has been used in previous experimental studies to understand the role of an abrupt change of depth in the evolution of barotropic vortices and currents. A similar study was conducted by Tenreiro et al. in a recent work using a step-like topography dividing a rectangular domain in two square regions. The authors reported the existence of a critical value of the experimental parameters determined by the strength of the flow and the step height, after which both regions evolve almost independently. This separation strongly affected the flow organization. It was shown that the long-term evolution of the flow consisted of large-scale vortices occupying almost the entire shallow and deep regions. However, a particular flow configuration was not found, since the sign and even the number of vortices were found to depend on the initial condition. In contrast, the square geometry used in the present study implies the emergence of a well-defined arrangement of vortices after several rotation periods, even for very different initial conditions. In other words, the square geometry of the domain divided by a step topography makes the long-term evolution of the flow very predictable, in clear contrast with the rectangular domain. What are the physical mechanisms determining such a sharp change? In this paper, we try to answer this question by closely examining the combined effects of the step topography and the square geometry on the self-organization of the flow.

The motivation of this study is the emergence of mesoscale oceanic vortices in confined domains, including semi-enclosed regions such as the Gulf of California or the Gulf of Aden. For instance, during the summer period, the surface circulation of the Gulf of California is characterized by a linear array of counter-rotating mesoscale eddies placed along the main axis of this elongated basin. Therefore, it is of interest to investigate the combined effects of the domain geometry and the bottom topography on the formation and disposition of this arrangement of vortices.

The results to be reported were obtained from laboratory experiments and numerical simulations. The laboratory experiments, performed in a rotating tank, provide physical evidence of the decaying turbulence. However, the unavoidable presence of bottom friction hinders the long-term configuration
of the flow, obscuring the unique arrangement of vortices at late times. By turning off Ekman damping effects, the numerical simulations are useful in gaining a better understanding of the processes leading to the existence of a final vorticity distribution induced by the step and the square geometry after long times (several rotation periods of the system).

The paper is organized in four sections. In Sec. II, the experimental setup is described and the experimental results are discussed in terms of the decay process and the effect of the topographic step on the flow organization. Numerical simulations performed for the same flow configuration are presented in Sec. III. An additional set of simulations without bottom friction demonstrates the preferential vorticity distribution reached by the system. A general discussion of the results and the main conclusions are presented in Sec. IV.

II. LABORATORY EXPERIMENTS

A. Experimental setup and procedures

The laboratory experiments were performed in an $L \times L$ square, rotating tank filled with fresh water. The horizontal aspect ratio $\delta \equiv L/W$, with $W$ the width of the tank measured in the direction perpendicular to the step and with horizontal dimensions $L = 1$ m, is 1. A step-like topography was placed at the bottom of the tank dividing the domain in two rectangular regions with horizontal aspect ratio 2 and horizontal dimensions $L/2 \times L$. When the fluid is at rest, the height of the water column at the deep part was $H_0 = 0.2$ m. Two different step heights were used, $\Delta h/H_0 = 0.15$ and 0.25 (a schematic picture of the experimental setup is shown in Fig. 1). Hereafter, the former will be referred to as “low step,” and the latter as “high step.”

The system rotates around the vertical axis in anticlockwise direction with a fixed rotation rate $\Omega = 0.5$ rad s$^{-1}$. The flow decay is induced by both lateral and bottom friction effects, the latter implying an Ekman time scale $T_E = H_0/(\nu\Omega)^{1/2} \approx 280$ s, for $\nu = 10^{-6}$ m$^2$s$^{-1}$ (kinematic viscosity of water at 20$^\circ$C). The characteristic time scale associated with the Ekman decay is much longer than the rotation period, $T = 4\pi/\Omega \approx 12$ s, where $f = 2\Omega$ is the Coriolis parameter. In addition, the Ekman number $E = \nu f H_0^2$ is much smaller than unity (about $2.5 \times 10^{-5}$).

Before starting an experiment, the tank is rotating at a constant angular speed for about 45 min in order to ensure a state of solid-body rotation of the fluid. Due to this rotation, the free-surface has a parabolic shape (which produces a depth difference of about 0.3 cm between the center of the tank and the lateral walls). Nevertheless, the effects of the free-surface deformation on the flow evolution are ignored, assuming that changes due to the step (3 and 5 cm) are more important.

An initial flow field is generated by passing a grid of vertical bars through the fluid. The grid consists of 8 PVC bars with a rectangular cross-section with a width of $b = 4$ cm and a thickness of

![Fig. 1. Schematic top and side views of the experimental setup.](image-url)
were observed in all laboratory experiments performed. When using a high step (cyclone is formed in the deep region and an anticyclone at the shallow side. These two structures region at its right. When the flow approaches the vertical wall at the downstream end of the step, a distribution is forced by the mean flow along the topography, which maintains always the shallow shallow region, a strong anticyclonic circulation together with two smaller cyclones is found. This a distribution of vortices with alternate signs disposed along the main axis in the deep part. In the characteristic vorticity $\omega_0$ is estimated to be around $2 \text{s}^{-1}$, which corresponds to a Rossby number $Ro = \omega_0 T f \sim 2$.

Despite the moderate value of $Ro$, the flow was close to two-dimensional. This was evident from dye-visualization experiments, which revealed a strong columnar motion within a few rotation periods. Motivated by geophysical applications, we restrict our analysis to rotating fluid systems in which the flow is predominantly two-dimensional. Within this limit, the Rossby and the Ekman number do not significantly affect the principle flow evolution. This was verified both experimentally and numerically. For that reason, the Rossby and Ekman number are kept constant in this paper. Quantitative experiments were performed using passive tracers ($\sim 250 \text{ \mu m}$) floating on the surface. The flow field in both cases was recorded with a camera mounted in a co-rotating frame at some distance above the tank. Particle image velocimetry (PIV) was used to retrieve the horizontal velocity and relative vorticity fields from the quantitative experiments. By repeating experiments, it was found that the main results are clearly reproducible.

B. General features

Figure 2 shows the evolution of the vorticity field, together with velocity arrows and streamlines, for a typical experiment with a low step topography ($\Delta h/H_0 = 0.15$). Using cartesian coordinates $(x, y)$ in the horizontal plane, the velocity components are defined as $(u, v)$ and the vertical component of the relative vorticity is $\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. The black line at $x/L = 0.5$ indicates the step position, which divides the domain in a deep (left) and shallow (right) region. In the first two panels ($t/T = 5$ and 15, where $T$ is the rotation period of the system), a very disorganized flow field can be observed. In this stage, the flow is dominated by vortex interactions, where vortices of equal sign merge and generate larger structures, while different-sign vortices form self-propagating dipoles that interact with the topography and the lateral boundaries. Some of these dipolar structures are able to cross the topography from one side to the other, while others are reflected depending on their strength and size. At $t/T = 25$, a clear step signal can be identified as a flow along the topography with shallow water at its right, which is characterized by the patch of positive (negative) vorticity in the deep (shallow) region along the topography and by the orientation of the streamlines parallel to the step. From this time on, the flows in the two regions evolve almost independently: the transport across the step is reduced significantly (although not completely suppressed). At $t/T = 35$, there is a distribution of vortices with alternate signs disposed along the main axis in the deep part. In the shallow region, a strong anticyclonic circulation together with two smaller cyclones is found. This distribution is forced by the mean flow along the topography, which maintains always the shallow region at its right. When the flow approaches the vertical wall at the downstream end of the step, a cyclone is formed in the deep region and an anticyclone at the shallow side. These two structures were observed in all laboratory experiments performed. When using a high step ($\Delta h/H_0 = 0.25$), a similar configuration of the flow is found. The main difference is that the separation between the shallow and the deep regions occurs at earlier times (approximately at $t/T \sim 5$). This is further discussed below.

The mean current along the topography is associated with the geostrophic adjustment of the flow to the bottom topography, which is characterized by the flow pattern aligned along the topographic contours with the shallow water to the right$^{12,13}$ (in the Northern Hemisphere). This current is analyzed in order to quantify its influence on the flow organization due to the step presence. As stated above, after several rotation periods the step divides the domain in two nearly independent regions. The separation time strongly depends on the step height: the higher the step the faster the
FIG. 2. Sequence of relative vorticity surfaces, streamline contours, and velocity arrows from an experiment with a low step \((\Delta h/H_0 = 0.15)\). The black line at \(x/L = 0.5\) indicates the step position, which divides the domain into a deep (left) and a shallow (right) region. Grey (green) surfaces represent negative values of vorticity, and dark (red) surfaces represent positive values. The streamlines contour level increment is 0.005 m²s^{-1}. Maximum velocities are 0.02, 0.01, 0.005, and 0.003 ms⁻¹.

Figure 3 shows the velocity component along the step \(v(0.5, y, t)\) as a function of time for two typical experiments with low and high steps. The normalized velocity \(v/v_{max}\) along the step (vertical axis) is plotted for the full duration of the experiments (horizontal axis) with \(v_{max}\) the maximum absolute value of \(v\) for each time step. The flow along the step is clearly shown in both panels by the positive values. Panel (a) shows the low step case. At early times there are regions with positive and negative values of \(v/v_{max}\), which are associated with small-scale structures crossing the separation.
topography from one side to the other. The formation of the flow along the step can be noticed at $t/T \sim 5$ by the upward slope of regions of positive values. The flow along the low step is completely formed at $t/T \sim 20$. After this time all values are positive, indicating a flow along the step with the shallow region at its right. Panel (b) shows the high step case, where the flow along the step is already present at $t/T \sim 5$. Negative values at the lower part of the step ($y \sim 0.2$) for longer times (for instance, at $t/T \sim 15$ or $t/T \sim 33$) are directly associated with an anticyclonic structure present at this particular region.

C. Results based on ensemble averages

In order to quantify the flow organization due to the step signal, an ensemble average based on 32 experiments is performed (14 for the low step and 18 for the high step case). The differences between experiments are associated with small perturbations on the initial conditions as the grid passes along the domain: even though the same grid speed is used in all cases, there are always
unavoidable, small differences between the resulting initial conditions. Although the detailed flow evolution differs between individual experiments, some robust features, such as the flow along the step, were observed in all cases.

In Figure 4, the mean spatial flow fields are shown at $t/T = 35$ for the low (panel a) and high (panel b) step experiments. The step signal is easily noticed by the positive (negative) vorticity values aligned along the step in the deep (shallow) region. These are directly associated with the structure of the mean flow along the topography with shallow water at its right. When this flow interacts with the downstream wall, it gives rise to a positive (negative) relative vorticity patch at the deep (shallow) region, as described for individual cases in Fig. 2. In general, the final configuration is an irregular pattern of vortex-like structures in the deep region, alternately disposed, and a large patch of negative vorticity at the shallow side. In fact, the anticyclonic vortex formed next to the downstream wall in the shallow part is a rather robust and persistent feature. The streamlines and velocity vectors are also shown, and corroborate these arrangements.

In Figure 5, the time evolution of the mean normalized velocity component along the step, defined as

$$V_s(t) = \frac{1}{v_{max}L} \int_0^L v(0.5, y, t) dy,$$  

is plotted for each experiment. For the low step case (panel a), strong variations of $V_s$ values can be noticed for $t/T \sim 0 - 15$. After $t/T \sim 15$, all experiments reveal a positive value of $V_s$, and lower amplitudes of oscillation, which is directly associated with the flow along the step. In panel (b), for the high step, there is a strong tendency towards positive mean values of $V_s$ just after $t/T \sim 5$. The high step forces an earlier separation of the domain in two subregions and the oscillation amplitude is smaller. These measurements show that the mean flow along the step is clearly identified in the ensemble of experiments.
Taking advantage of the relatively large number of experiments, some global quantities of the ensemble are calculated, which are the total kinetic energy and the enstrophy:

\[ E = \frac{1}{2} \int (u^2 + v^2) \, dx \, dy, \]

\[ Z = \frac{1}{2} \int \omega^2 \, dx \, dy. \]

Figure 6 shows the time evolution of the averaged kinetic energy and the averaged enstrophy for the two step heights. A very similar decay in both quantities is observed until \( t/T \sim 35 \). After this time, the inverse energy cascade is halted by viscous effects that start to dominate the decay process. In addition, velocity measurements are less accurate for very weak motions. As a result, the energy and enstrophy reach a constant value, perhaps also due to a weak influence of wind-induced motions at the free-surface. Thus, these measurements indicate that the experimental results are useful up to 35 to 40 rotation periods.

### III. NUMERICAL SIMULATIONS

In this section, numerical simulations of decaying quasi-2D turbulence with discontinuous topography are presented. A barotropic, rigid-lid, shallow-water model in the \( \omega - \psi \) formulation with Ekman friction is solved with a finite differences scheme (see, e.g., Ref. 24). The model is derived by considering the horizontal velocity components as depth independent, \( u = u(x, y, t) \) and \( v = v(x, y, t) \), and integrating the continuity equation from the solid bottom to the surface. In order to derive a two-dimensional system, a rigid-lid is considered where fluid depth changes associated with the free-surface are assumed to be small compared with those caused by the topography. An additional effect on rotating fluid experiments is the adjustment of the flow in the thin Ekman layers. The bottom boundary layer is produced by the no-slip boundary condition at the solid bottom. The circulation and horizontal divergence within this layer provide the mechanism for bottom friction effects. The smallness of the secondary Ekman circulations from and to the Ekman layer allows its incorporation in a 2D physical model. Linear and nonlinear Ekman effects are formulated in a quasi-2D model with variable topography as described by Zavala Sansón and van Heijst (2002).24
The evolution equation for the relative vorticity $\omega$ is

\[
\frac{\partial \omega}{\partial t} + J(q, \psi) - \frac{\delta E}{2h} \nabla \psi \cdot \nabla q = \nu \nabla^2 \omega - \frac{\delta E}{2h} \omega (\omega + f),
\]

and it is verified that

\[
\omega = -\frac{1}{h} \nabla^2 \psi + \frac{1}{h^2} \nabla h \cdot \nabla \psi + \frac{\delta E}{2h} \frac{2}{h^2} J(h, \psi),
\]

where $q = (\omega + f)/h$ is the potential vorticity, with $h(x, y)$ the fluid depth (which is time independent, according with the rigid-lid approximation), $\delta E = (2\nu f)^{1/2}$ is the thickness of the bottom Ekman layer, $J$ the Jacobian operator and $\psi$ is a transport function defined as

\[
h \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y} \left( h u - \frac{1}{2} \delta E v \right),
\]

\[
h \frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x} \left( h u + \frac{1}{2} \delta E u \right).
\]

The model includes linear and nonlinear Ekman friction terms (proportional to $\delta E$), which provide a good representation of bottom friction effects. Note that from this formulation, some other purely 2D models with and without Ekman friction can be recovered. The results suggest that such a 2D
formulation from the shallow-water equations with rigid-lid represents satisfactory the laboratory experiments simulating both inviscid and viscous topography effects correctly.

The numerical domain has the same dimensions as the experimental tank. Likewise, the same flow parameters are used as in the experiments. The domain represents an \( L \times L \) square tank, with \( L = 1 \). The topography consists of a discontinuity (step-like topography) dividing the domain in two equal rectangular regions (the discontinuity is actually a very narrow, steep slope due to the spatial discretization). As in the experiments, in all simulations the water column in the deep part of the domain is \( H_0 = 0.2 \). Two step heights are used, \( \Delta h_1 = 0 \) (low step) and \( \Delta h_2 = 0.25 \) (high step). The rotation period around the vertical axis is \( T = 4\pi f \), with \( f = 1 \) being the Coriolis parameter. The flow decay is induced by frictional effects, where the kinematic viscosity is \( \nu = 10^{-6} \).

No-slip boundary conditions are imposed at the solid boundaries in order to represent the lateral tank walls. The spatial discretization consists of 257 \( \times \) 257 grid points and the time step used is fixed at \( \Delta t = 10^{-3} \).

A. Comparison with the laboratory experiments

In order to compare the quasi-2D simulations with the laboratory results, it is convenient to use the experimental flow fields as the initial conditions. Therefore, the vorticity field is taken from the first few seconds of each experiment, interpolated onto a 257 \( \times \) 257 numerical grid, and used as the initial vorticity distribution in the simulations.

Figure 7 shows the evolution of the relative vorticity surfaces for a low step in a typical simulation using lateral no-slip boundary conditions and bottom friction. The black line at \( x/L = 0.5 \) indicates the position of the step, which divides the domain in a deep (left) and a shallow (right) region, as in the laboratory experiments. The calculated relative vorticity distributions can be compared with the experimental case presented in Figure 2. It is verified that the most important features in the experiment are clearly reproduced by the simulation: strong vortex-vortex interactions with same-sign vortices merging at initial stages, a step signal clearly seen by the aligned positive (negative) vorticity along the step at the deep (shallow) region, and the formation of larger structures at later times. For \( t/T = 25 \), near the downstream wall the existence of a cyclone (anticyclone) in the deep (shallow) region of the domain is observed. These vortices are associated with the interactions of the flow along the step with the vertical wall at the downstream end of the step. Near the upstream wall, the process tends to be the inverse: an anticyclone (cyclone) is formed in the deep (shallow) region.

Some of the processes just described play a key role in the late evolution of the flow. In particular, the mean flow along the step and the generation of counter-rotating vortices next to the downstream wall, which are clearly reproduced numerically. In order to reinforce this notion, Figure 8 shows the mean spatial distribution of vorticity from an ensemble of 32 simulations with different initial conditions at \( t/T = 35 \). These average distributions show that the flow along the step and the large vortices at the downstream wall are very robust features in the whole ensemble of simulations. A comparison can be made with the ensemble of experiments in Figure 4. There is a general qualitative agreement, specially for the large anticyclone over the shallow region and the persistent flow along the step.

Figure 9 shows the time evolutions of energy \( E \) and the enstrophy \( Z \) for both the numerical simulations and the laboratory experiments. Despite the relative differences of up to 60% (low step) and 50% (high step) between simulations and experiments, the computed and measured time evolutions generally agree within an order of magnitude. Recall that the experimental values are useful up to 35 to 40 rotation periods, since at larger times they reach a spurious steady value, as mentioned in Sec. II C. In contrast, the decay of the simulated energy and enstrophy values continue as time progresses. The differences might be associated to several factors, either numerical (artificial viscosity) or experimental (inherent errors in the PIV measurements). Nevertheless, the numerical model captures the essential features of the observed process, and will therefore be adopted in Sec. III B to examine the long-term flow evolution without Ekman decay in a qualitative sense.
FIG. 7. Sequence of relative vorticity surfaces from a simulation with a low step and no-slip boundary conditions. The black line at $x/L = 0.5$ indicates the step position, which divides the domain into a deep (left) and shallow (right) regions. Grey (green) surfaces represent negative values of vorticity, and dark (red) surfaces represent positive values.

B. Results without bottom friction

In this section, we examine the self-organization process in the absence of bottom friction. It will be shown that there is a preferred distribution of vorticity induced by the step, i.e., a geometrical arrangement of the vortices at late times due to the step topography. Such a final configuration consists of four vortices covering the whole domain, two in each region. This arrangement is suggested by the experimental and numerical results presented previously, specially regarding the cyclone-anticyclone pair next to the downstream wall. However, bottom friction effects drain the
FIG. 8. Numerically calculated mean spatial vorticity distribution at $t/T = 35$ for simulations with (a) a low step and (b) a high step. Color surfaces as in Fig. 4. The ensembles are based on 32 simulations with slightly different initial conditions for each step height.

FIG. 9. Comparison between time evolution of kinetic energy $E$ and enstrophy $Z$ for the ensembles of experiments and simulations where the subscripts “Sim” and “Lab” refer to the numerical simulations and the laboratory experiments, respectively. The numerically calculated energy and enstrophy are obtained from an ensemble of 32 simulations for each step height.
FIG. 10. Numerically calculated mean spatial vorticity distribution at $t/T = 200$ for (a) low step and (b) high step simulations without bottom friction. Color surfaces as in Fig. 4. The ensembles are based on 8 simulations with slightly different initial conditions for each step height.

kinetic energy of the flow before fully reaching this final state. As a consequence, the four-vortices final distribution is not completely observed in the laboratory experiments. In order to avoid this, the Ekman terms in Eqs. (4) and (5) are dropped and an ensemble of new simulations is carried out.

A set of eight initial conditions similar to the ones used in Sec. III A are used again for the two step height cases. Now, the duration of the simulations can be longer due to the absence of Ekman damping. The mean spatial vorticity distribution of the ensemble at $t/T = 200$ is shown in Figure 10. Clearly, a well defined array of four vortices is observed in both cases. This unique distribution of the vortices is forced by the interaction of the flow along the step with the downstream wall, which results in two big cyclonic (deep) and anticyclonic (shallow) structures at the downstream wall. By geometry and due to the no-slip boundary condition, these persistent vortices force the inverse distribution at the upstream wall. The final arrangement of four vortices consists of two large cyclone-anticyclone pairs at each region. All vortices are surrounded by a shield of opposite vorticity, indicating the boundary layer at the lateral walls of the flow domain. Note that the disposition of the vortices is determined by the step, since the flow along the topography generates the vortices at the downstream wall.

In order to quantify the robustness of this step-induced vorticity distribution, a proper orthogonal decomposition (POD) technique is used. The POD analysis searches for structures that explain the maximum amount of changes in a 2D matrix. First, a space-realization array at a single time ($t/T = 200$) is used and the analysis seeks the maximum amount of changes or variations relative to the mean spatial distribution of vorticity of the ensemble. This mathematical procedure converts the set of observations into a new set called principal components, which gives the sign and the intensity of each linearly uncorrelated space-realization. Second, the evolution in time of the amount of changes relative to the mean spatial distribution is investigated.

In Figure 11, the first three modes for the low step are shown for $t/T = 200$. Panel (a) presents the first mode, which explains 81% of the variations relative to the mean spatial distribution of vorticity. In panel (b), the principal components relative to this mode show that all simulations have the vortices at the same regions, since all principal components have the same sign. A simulation with a negative value would indicate a similar vorticity distribution as the mean, but now with opposite
sign. Mode 2 represents much less variations (8%), which are associated with small oscillations of the final structures. In the third mode [panels (e) and (f)], as expected, the explained variations are even lower (3%); nevertheless, the step signal is identified by the spatial distribution of vorticity along the topography. For the high step case, the spatial distributions for each mode are very similar (not shown here), although the variations explained for each mode are different: 69% by the first, 10% by the second, and 8% by the third mode.

In order to investigate the temporal evolution of the amount of explained variations, the first three modes are calculated for each complete rotation of the system. The results are presented together for the low and high steps in Figure 12. In panel (a), the amount of explained variations

![Graphs showing temporal evolution of explained variations for different modes and steps.](image)

FIG. 11. Mean spatial vorticity distribution and respective principal components at $t/T = 200$ for the low step ensemble POD analysis: (a)-(b) first, (c)-(d) second, and (e)-(f) third modes.

![Graphs showing time evolution of explained changes for different modes.](image)

FIG. 12. Time evolution of explained changes relative to the mean spatial distribution of vorticity by the (a) first, (b) second, and (c) third modes. Continuous line for the low step and dashed lines for the high step.
by the first mode is shown. For the interval \( t/T = 0 - 5 \), both step heights present a similar amount of explained changes. For the period \( t/T = 5 - 40 \), the high step case presents a slightly larger percentage of explained changes due to a faster separation of the flow. After this period, both step cases present a similar percentage of explained changes. The amount of explained changes in the second and third modes decay as expected. These two modes represent small scale oscillations due to vortex interactions with the lateral boundaries and the topography.
C. Different initial conditions and geometries

In order to show that the step-induced final distribution of the vortices is independent of the initial flow, five simulations using different initial conditions (referred to as IC1 to IC5) for a high step \( \Delta H/H_0 = 0.25 \) were performed. Again, bottom friction is not considered. In Figure 13, the vorticity surfaces are plotted for \( t/T = 0 \) and \( t/T = 200 \) for each simulation. IC1 is a 16 × 16 array of cyclonic and anticyclonic Gaussian vortices with maximum vorticity \( |\omega| = 1 \) and diameter \( a = 0.05 \). This type of initial condition is similar to those obtained in laboratory experiments with electromagnetically forced vortices (see Ref. 26). The IC2 (IC3) represents a dipolar structure oriented towards the shallow (deep) side of the step topography. The diameter of the vortices is \( a = 0.1 \) and they are initially separated a distance \( L/2 \). Their peak vorticity is \( |\omega| = 1 \). IC4 consists of four vortices disposed as the expected final distribution described previously: the vortices are located at \((x, y) = (0.25, 0.25), (0.25, 0.75), (0.75, 0.25), \) and \((0.75, 0.75)\). Their size and strength are the same as in IC2 and IC3. IC5 is the inverse distribution.

As can be seen from the right panels, all different initial conditions result in a similar final distribution of four vortices: two cyclones and two anticyclones, arranged as in Figure 10. There are some differences, specially when using IC5, but in general there is a clear trend towards the expected configuration. Several other initial conditions have been used (not shown), generally resulting in the same final four-vortex configuration.

The simulation using IC1 is consistent with the experimental and numerical results presented in Secs. III A and III B: an initial array of several small-scale vortices evolves towards the expected preferential distribution. Simulations with IC2 and IC3 demonstrate that the same result is obtained when starting from a single pair of vortices. For IC4, it does not seem surprising that the long-term configuration is reached again. In contrast, it is remarkable that the initial condition IC5, which is exactly the opposite arrangement, evolves again towards the predicted four-vortex configuration.

In order to further investigate the existence of a preferential distribution of the vortices due to the step, different simulations were carried out for four different aspect ratios of the domain \( \delta = L/W = 3/2, 2, 5/2, \) and \( 3 \) maintaining in all cases the same step orientation. The initial conditions used are similar to IC1 with the number of vortices increasing with \( \delta \). In other words, the initial distribution are arrays of 16 × 16 vortices. In order to calculate ensemble averages, five different initial conditions are used for each \( \delta \), with the relative positions between adjacent vortices slightly and randomly changed.\textsuperscript{13}

In Figure 14, the mean vorticity surfaces for each aspect ratio are plotted at \( t/T = 200 \). The long-term flow configuration in all cases consists of an array of vortices along the rectangular, deep, and shallow subdomains. Considering the results presented in Secs. III A and III B, this is
the expected outcome, since the regions are effectively separated by the step. The final number of vortices depends on the aspect ratio of the subdomains.\textsuperscript{9,11} The important point to stress here is the sign of the vortices next to the downstream and upstream walls. The presence of a pair of structures at the downstream end of the step is clearly noticed in all cases, with a cyclonic structure in the deep part and an anticyclone in the shallow part. Again, these distributions of vorticity are due to the interaction between the flow along the step (that always maintains the shallow region on the right) and the upper wall. A remarkable result is that the distribution near the upstream wall seems to be also independent of $\delta$, always with an anticyclonic structure in the deep part and a cyclone in the shallow part (i.e., opposite to the pair at the downstream wall).

IV. DISCUSSION AND CONCLUSIONS

The self-organization of confined 2D turbulent flows in a square domain geometry with a step-like topography in a rotating system has been investigated by means of laboratory experiments and numerical simulations. The goal was to describe the flow evolution in the shallow and deep regions and to discuss the existence of a preferential final state of the flow induced by the topography.

The laboratory experiments were performed in a rotating square tank with horizontal aspect ratio $\delta = 1$. A step topography was used to divide the domain in two regions with different depths. A persistent flow along the topography with the shallow region at its right is observed in all cases. Such a mean current is consistent with the adjustment of a rotating fluid to equilibrium in the presence of variable topography, as discussed in previous studies on quasi-geostrophic\textsuperscript{12} and shallow-water\textsuperscript{13} dynamics. It has also been observed in previous experimental studies on step topographies.\textsuperscript{15, 16, 19, 20} Essentially, as the flow enters a more geostrophic regime, perturbations over the step propagate unidirectionally with the shallow water to the right.

The jet-like flow divides the domain in two nearly independent regions. The time required for the separation to be effective depends on the step height. The flow along the topography plays a fundamental role for the flow organization by its continuous interaction with the corresponding vertical wall perpendicular to the step. From this interaction, two vortices at the downstream end of the step are formed: cyclonic in the deep and anticyclonic in the shallow region. The presence of these two structures implies a continuous injection of vorticity into the flow interior. These results are consistent with previous experimental observations with different domain geometries.\textsuperscript{20} Numerical simulations based on a shallow water model and using no-slip boundary conditions showed the same principal features observed in the laboratory experiments.

Due to Ekman damping effects, the inverse energy cascade in the laboratory experiments and in the numerical simulations with bottom friction is halted at a finite time. As a result, some disorganization is still observed at long times, specially in the deep region. In order to study the full self-organization process, numerical simulations with zero Ekman friction were performed. For short times, these simulations show the same principal features as those with bottom friction. For longer times, a complete self-organization of the flow was reached, and it was found to be a unique pattern. Such a configuration consists of a well-defined, long-term distribution of four vortices alternately disposed: in the deep part, a cyclone-anticyclone pair that fills-up the entire region, and the mirror structures at the shallow side (see Fig. 10). All four structures are surrounded by a ring of opposite-signed vorticity. Looking along the step with shallow water to the right, the cyclones are disposed at the up-left and down-right corners and the anticyclones at the opposite regions (up-right and down-left corners). This vortex distribution was found to be unique and independent of the initial condition. This was shown by performing several simulations with different initial configurations and verifying that the four-vortex configuration was obtained after long times in all cases (Figure 13).

It is important to remark that despite the apparent symmetry of this distribution, the cyclonic vortices cannot occupy the positions of the anticyclones. In other words, the mirror configuration with cyclones instead of anticyclones and vice versa, is not possible. The reason is that the flow along the step breaks such a symmetry and therefore determines the position of the vortices. Indeed, when the flow along the step collides with the downstream wall, it induces the formation of a cyclone in the deep part and an anticyclone in the shallow region, as observed in the experiments.
Since this interaction is continuously occurring, it forces the formation of the other structures and, eventually, of the four-vortex configuration. The POD analysis confirms the persistence of a preferential distribution of the vortices due to the geometry induced by the discontinuity.

The persistence of the preferential states strongly suggests that there might be a general theory to explain them. A remarkable one is based on statistical mechanics ideas, for which a macrostate is composed by a set of microstates obeying specific conditions. A particularly suitable method for a fluid system is the use of conserved quantities, and the resulting functional relationship between vorticity and stream function (in two-dimensional flows) in order to find preferential, quasi-stationary states. Such a method has been applied by Bretherton and Haidvogel (1976), who proposed a linear relationship between potential vorticity and stream function of an arbitrary flow over random topography by using a variational, minimum-entrainment principle in the quasigeostrophic context. Their main prediction is that the flow evolves towards a state of minimum entrainment for a given total energy. The flow tendency is to evolve towards a quasi-steady state in which the flow is aligned along topographic contours (see also Zavala Sansón et al. (2010) in the shallow water context). In the present problem, the flow along the step with shallow water to the right (with \( f > 0 \)) is observed, which is in agreement with the mentioned theories. In both the deep and shallow regions, the flow evolves as predicted for a purely two-dimensional flow: small structures are organized in larger vortices covering the corresponding regions according with their aspect ratios. This is because, as we have seen, the step divides the two flat regions as if they were almost completely independent. However, it is not clear yet how to incorporate in a general theory the role of the wall at which the jet along the step collides, and which is fundamental in the final arrangement of the vortices, as it has been shown above. Further work is necessary on this regard.

For purely 2D flows in bounded domains, the evolution towards large structures filling the container is well-known, as it has been shown for square and rectangular boundaries. The preferential vorticity distribution in a square container divided by a step topography, however, is a remarkable feature. Indeed, when the geometry of the container and the step orientation are different, a well-defined long-term configuration might not be obtained. That is the case of a rectangular domain with aspect ratio 2, as shown by Tenreiro et al. In their experiments, the step divides the domain in two square regions, and a preferential state was not found. These results open new questions on the predictability of the flow evolution for different domain geometries with larger aspect ratios and step orientations. The emergence of a preferential state seems to depend on the length of the

![FIG. 15. Schematic representation of the long-term flow configurations in geometrical domains divided by a step topography.](image)

\( L_s = L_\delta \) is the length of the step, and \( L_\delta \) is the length of the long wall of the container (see text).
step \( L_s \) compared with the length of the long wall \( L_d \). This was shown in Figure 14, in which the vortex disposition in rectangular domains is uniquely determined by the step. In all these examples, the length of the step is always the same as that of the long wall \( (L_s/L_d = 1) \). In contrast, in the experiments of Tenreiro et al.\(^{20}\) \( L_s/L_d = 1/2 < 1 \). Figure 15 shows our present knowledge on the predictability of long-term flow configurations in square and rectangular domains divided by a step topography. Based on the present results, we postulate that the self-organization of a rotating flow in the presence of a step such that \( L_s = L_d \) leads to the preferential states indicated by the first two sketches. In contrast, the long-term flow configuration cannot be predicted \textit{a priori} when \( L_s/L_d = 1/2 \) (third sketch).\(^{20}\) In general, for \( L_s/L_d < 1 \) the flow organization after long times remains uncertain (last sketch). Further research is in progress in order to investigate these conclusions.

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