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with Time Constraints

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Abstract

In this paper, we extend the classical Variable Size Bin Packing Problem (VS-BPP) by adding time features to both bins and items. Specifically, the bins act as machines that process the assigned batch of items with a fixed processing time. Hence, the items are available for processing at given times and are penalized for tardiness. Within this extension we also consider a stochastic variant, where the arrival times of the items have a discrete probability distribution. To solve these models, we build a Markov Chain Monte Carlo (MCMC) heuristic. We provide numerical tests to show the different decision making processes when time constraints and stochasticity are added to VS-BPP instances. The results show that these new models entail safer and higher cost solutions. We also compare the performance of the MCMC heuristic and an industrial solver to show the efficiency and the efficacy of our method.

Keywords: VS-BPP, Time Constraints, MCMC Heuristic, Stochasticity

1 Introduction

In the classical Bin Packing Problem (BPP) a set of items, each one with a certain weight, is allocated to a set of capacitated bins. The objective is to minimize the costs associated with the bins while packing all the items available. The Variable Size Bin Packing Problem (VS-BPP) handles the same decision problem but considers unequal bin sizes and costs.

In these problems time constraints are usually neglected, i.e. allocations are made without considering consequences on timings for bins and items. Therefore in this paper we are interested in extending the VS-BPP with a new class of problems which considers time issues, named Time Constrained VS-BPP (T-VS-BPP).

Basically, after the allocation, each bin processes its batch of items for a specific time, independent of the load (see Figure 1). Hence, the allocation is made considering:

- capacity constraints

- availabilities of the bins and arrival times of the items

- the different costs and the processing times of the bins

- deadlines of the items

When the arrivals of the items are affected by variability we consider a stochastic variant of the model defined as the Stochastic T-VS-BPP (S-T-VS-BPP).

Unlike the classical scheduling problems (parallel machine scheduling [4]), the sequence of the processed items is not considered since the items are processed in batches.

Typical applications of T-VS-BPP are found in factories whose machines process in batches, for example ovens, or in transportation systems, like mail delivery, truck packing and transportation of containers. In these examples, we can pack the different items over different transport means and the transportation time is generally independent of the loaded capacity. Moreover, it is realistic to assume that each item has a different arrival time, priority and deadline. For example, within the mail delivery problem, the arrival of the items is highly random.
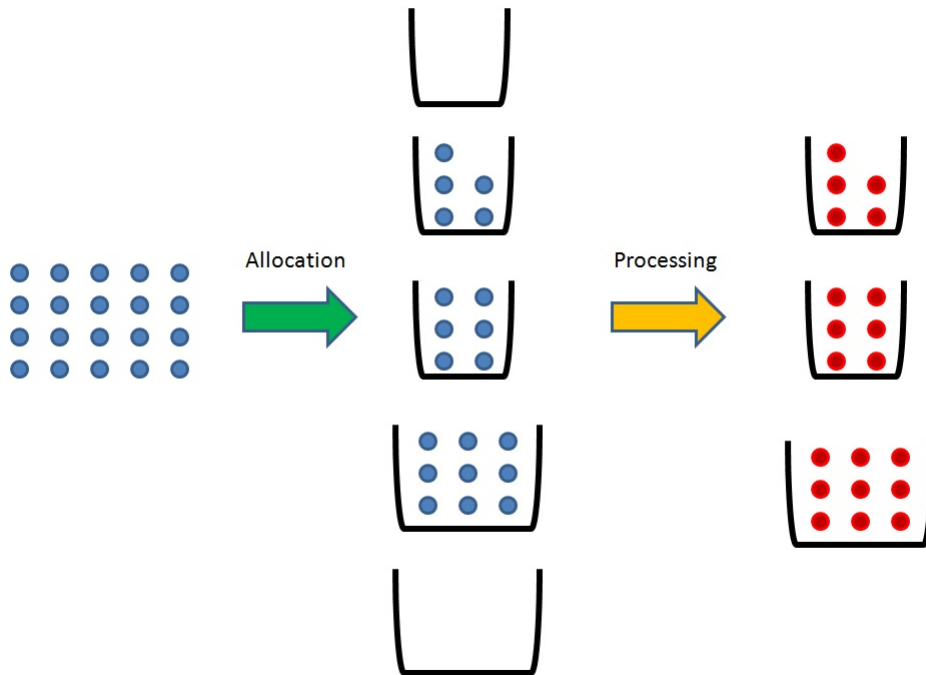

Figure 1: A generic setting for the T-VS-BPP.

In container terminals it is the everyday task of the logistic planner to allocate containers to different transport modes, facing the relevant trade-off of costs and service level. Within this supply chain, we may have the option to dispatch the containers either by

trucks (fast, little capacity, high price per container) or barges (slow, big capacity, lower price per container with high utilization). From a VS-BPP perspective it is evident that by allocating as many containers as possible to the barges, we reduce the number of bins needed and consequently realize economies of scale. On the other hand, within a T-VS-BPP perspective the allocation has to consider also the consequences of the time and therefore the use of trucks is necessarily taken into account to meet the deadlines. In the same form, as this practical case, we assume in the theoretical model that smaller bins can process items faster, but the cost per item processed is relatively high.

The classical Bin Packing Problem (BPP) has been treated extensively in the literature [17], [8] and several variants have been developed. The literature with regard to the VS-BPP is indeed relatively scarce. In [11], we can find precise definitions of VS-BPP and approximation algorithms, while in [5] several mathematical formulations are treated and processed on industrial solvers. Powerful heuristics have been also developed, see [18], [9], [6] and [14].

However, to the best of our knowledge, a variant of VS-BPP with time constraints has never been treated in the literature.

The contributions of this paper are three-fold:

- We define a new class of packing problems with time constraints. The limited available literature offers opportunities for developments and extensions, considering also the wide applicability of the problem itself.

- We build a flexible heuristic based on a Markov Chain Monte Carlo method able to solve both the deterministic and the stochastic variant.

- We provide numerical examples to test the complexity of the problem and the performance of the algorithm. We analyze the different decision making process in VS-BPP, T-VS-BPP and S-T-VS-BPP instances. With respect to this, we observe that the stochastic model produces more reliable solutions at the expense of higher cost: more smaller bins are used in order to ensure a fast and on-time process. On the other hand, VS-BPP aims to reduce the number of bins used, in order to get economies of scale.

The paper is structured as follows: in Section 2, we introduce T-VS-BPP and S-T-VS-BPP with their reductions to the classical BPP and VSBPP for complexity issues, in Section 3, we describe the heuristic used to solve the instances. Finally Sections 4 and 5 close the paper with numerical results, computing analysis and conclusions.

2   The T-VS-BPP: Model, Features and Complexity

The T-VS-BPP is an extension of the VS-BPP formulation by adding time constraints (see Appendix for VS-BPP formulation). In this section we propose a single general formulation which can be applied to both the stochastic and the deterministic variant. The deterministic model is described as follows. Consider a set $N = \{1, \ldots, n\}$ of items with the ith with weight $w_i > 0$ and a set $R = \{1, \ldots, r\}$ of bins each one with a certain capacity

$B_j > 0$ and cost $C_j > 0$ with $j \in R$. The objective is to find the best allocation that satisfies capacity and time constraints, and minimizes a certain cost function related to time and usage of the bins. Henceforth, we keep the following index-set relation: i for the set of items N, j for the set of bins R, ω for the set of possible scenarios Z.

To characterize the allocation, let u_j, be a binary variable that keeps track whether a bin is used or not and let $X_{i,j}$ be the binary allocation matrix. The vector $A_{\omega,i}^N$ considers the arrival times of the items and the unique deterministic scenario ω occurs with probability $\pi_\omega = 1$. We define the deadlines D_i and a non-negative decision variable $\bar{d}_{\omega,i}$ that calculates the tardiness of item i for scenario ω. For each time unit of delay we pay a penalty p_i. The set of bins are available at given times, characterized by the vector A_j^R. In real-life, this can be related to maintenance, setup times, prior process etc. The decision variable $t_{\omega,j}$ keeps track of the times when the processes start for the scenario ω. If we use a bin j, then a cost C_j is incurred and a cost W for each stand-by time unit since its availability. We have the processing times P_j, independent of the number of loaded items and a setup time needed to prepare the bin and fill it with the objects. In particular we can define it equal to a constant value α (for instance, the time to reach the right temperature in an oven) plus a time factor $\bar{L}_j$ in direct proportion to the number of loaded items: $\bar{L}_j = L * \sum_{i=1}^n X_{i,j}$, where L is the time needed to load each item.

Therefore, the decision variables in this model are $X_{i,j}$, u_j, $t_{\omega,j}$ and $\bar{d}_{\omega,i}$.

We formulate the T-VS-BPP as follows:

$$Min \sum_{j \in R} u_j * C_j + \sum_{\omega \in Z} \pi_\omega * \sum_{j \in R} (t_{\omega,j} - A_j^R) * W + \sum_{\omega \in Z} \pi_\omega * \sum_{i \in N} p_i * \bar{d}_{\omega,i} \qquad (1)$$

Subject to:

$$\sum_{j \in R} X_{i,j} = 1 \qquad\qquad \forall i \in N \qquad (2)$$

$$\sum_{i \in N} w_i * X_{i,j} \leq B_j * u_j \qquad\qquad \forall j \in R \qquad (3)$$

$$t_{\omega,j} \geq A_j^R \qquad\qquad \forall j \in R; \forall \omega \in Z \qquad (4)$$

$$t_{\omega,j} \geq A_j^R + \alpha + L * \sum_{i \in N} X_{i,j} - (1 - X_{i,j}) * M \qquad \forall i \in N; \forall j \in R; \forall \omega \in Z \quad (5)$$

$$t_{\omega,j} \geq A_{\omega,i}^N + \alpha + L * \sum_{i \in N} X_{i,j} - (1 - X_{i,j}) * M \qquad \forall i \in N; \forall j \in R; \forall \omega \in Z \quad (6)$$

$$t_{\omega,j} + P_j \leq D_i + \bar{d}_{\omega,i} + (1 - X_{i,j}) * M \qquad \forall i \in N; \forall j \in R; \forall \omega \in Z \quad (7)$$

$$u_j \in (0,1) \qquad\qquad \forall j \in R \qquad (8)$$

$$X_{i,j} \in (0,1) \qquad\qquad \forall i \in N; \forall j \in R \qquad (9)$$

$$\bar{d}_{\omega,i} \geq 0 \qquad\qquad \forall i \in N; \forall \omega \in Z \qquad (10)$$

The objective function (1) minimizes the costs for usage, waiting times and tardiness. The classical VS-BPP constraints are (2) and (3). (2) ensures that each item i is packed, while (3) imposes that the capacities of the bins are not exceeded.

Inequalities (7) state that each item i is completed with tardiness $\bar{d}_{\omega,i}$. With inequalities

(6) the process for machine j ($t_{\omega,j}$) starts after each assigned items has arrived and with (5) after the availability time of the machine itself. They also include the necessary setup time. With constraints (4) we make sure that the second addend of the objective function is equal to zero in case no jobs are assigned to machine j.

We can reduce this formulation to the VS-BPP. In particular let the waiting cost W and the penalties for delay p_i be equal to 0. For that reduction, the model is completely independent of the variables $t_{\omega,j}$ and $d_{\omega,i}$ and, as a consequence, constraints (4), (5), (6) and (7) are trivial. Hence we get the reduction to the NP-hard VS-BPP and therefore (S-)T-VS-BPP can be also classified as NP-hard (see appendix for VS-BPP complexity).

Stochasticity within S-T-VS-BPP

For the stochastic model, we consider a set $Z = \{1, \ldots, z\}$, with $z > 1$, of possible scenarios for the arrival times of the items ($A^N_{\omega,i}$). Each scenario occurs with probability $\pi_\omega > 0$ with $\sum_{\omega \in Z} \pi_\omega = 1$. In this variant, we face a more complex problem where perfect information is not available till the realization of the stochastic parameters.

Before the realization, we can be obliged to set some decision variables which cannot be changed after the forthcoming scenario (non adaptive variables). For instance, the allocation of the items $X_{i,j}$ and consequently u_j, are critical to change at the very last moment since it is usually hard to organize in a different way a pre-scheduled set of machines, especially due to setup issues. On the other hand $t_{\omega,j}$ and $\bar{d}_{\omega,i}$ are adaptive and can be changed after the realization of $A^N_{\omega,i}$. In fact, they are the only decision variables depending on the realized scenario ω.

Such model is known in the literature as recourse model [15] since when uncertain data become available we have an opportunity to adjust or adapt some of our decisions to the information that is received. We refer to this adaptation as "recourse" and to decision variables that are permitted to vary according to the realized scenario as "recourse variables".

3 An MCMC Heuristic to solve (S-)TC-VSBPP

The Markov Chain Monte Carlo algorithms (MCMC), as presented in [13] and [16], are effective and efficient optimization tools. The general method has its roots in physics, where the earliest uses go back to the 1950's. It was later developed in other areas as image analysis and Bayesian statistics [13].

The main idea of MCMC is as follows. Suppose we have an irreducible and aperiodic Markov Chain with π as stationary distribution. If we run the chain starting from an arbitrary position, then the Markov Chain convergence theorem (Theorem 5.2 in [13]) guarantees that the distribution of the chain at time K converges to π as K goes to infinity.

To sample the stationary distribution, the Markov Chain Monte Carlo methods, as the Metropolis method, construct irreducible and aperiodic chains with reversible distribution π. According to Theorem 6.1 in [13], a reversible distribution is also a stationary distribution. A possible method to obtain such chain is to build a *Metropolis* algorithm [20]. The Metropolis method was named after one of the author of [20], but the theoretical contributions were mostly due to the physicist Marshall N. Rosenbluth. The algorithm formed the basis for Monte Carlo statistical mechanics simulations of atomic and molecular systems. We give a theoretical introduction in section 3.1.

3.1 An MCMC algorithm using a Metropolis chain

Assume that our problem has the following form: in a discrete and finite state space $S = \{s_1, \ldots, s_n\}$ we want to find the state s which minimizes a given function $f(s)$.
We define a connected graph $\mathcal{G}(S, E)$ and we denote with $d(s)$ the degree of the vertex s. Then we define the following transition probabilities

$$
P_{s,s'} = \begin{cases}
0 & \text{if } (s, s') \notin E \\
\frac{1}{d(s)} & \text{if } (s, s') \in E \quad \text{and } \frac{d(s)}{d(s')}e^{-\frac{f(s')-f(s)}{T}} \geq 1 \\
\frac{1}{d(s')}e^{-\frac{f(s')-f(s)}{T}} & \text{if } (s, s') \in E \quad \text{and } \frac{d(s)}{d(s')}e^{-\frac{f(s')-f(s)}{T}} < 1 \\
1 - \sum_{t \neq s} P_{s,t} & \text{if } s' = s
\end{cases} \tag{11}
$$

where $T > 0$ is a fixed parameter called temperature. It is a standard task (see [13]) to show that the Boltzmann distribution:

$$
\pi_{f,T}(s) = \frac{1}{Z_{f,T}} \exp\left(\frac{-f(s)}{T}\right) \tag{12}
$$

where

$$
Z_{f,T} = \sum_{s \in S} \exp\left(\frac{-f(s)}{T}\right)
$$

is reversible for the Markov chain defined by equation (11). Therefore, it is also its unique stationary measure, because for connected graphs $\mathcal{G}(S, E)$ the chain is evidently irreducible and aperiodic. Moreover, according to Theorem 13.1 in [13], the stationary measure $\pi_{f,T}(s)$ places most of its probability on the element(s) s that minimize(s) $f(s)$, when T is small enough.

If we run the chain for a "sufficiently long" time K, due to Theorem 5.2 in [13], our final state is sampled from a probability distribution that converges to the stationary probability (equation (12)) as K goes to infinity.

In the literature it is common to see a schedule of decreasing temperatures T_i, $i = (1, ..., K_K)$, called Simulated Annealing (see [19] and [22]), and a sequence $K_1, K_2, \ldots, K_K$ where K_i is the number of iterations we want our chain to run with temperature T_i. Our approach is different, because we will use a single temperature that will be drawn after some experiments regarding time and solution goodness. These experiments are described in section 4.3.1.

The choice of T is a very delicate question since too low temperatures could lead to local minima, though the convergence to these minima is really fast. This problem is well-known in the literature as the metastability problem (see [2],[3],[10]) and this term refers to the particular property of the Markov chain to reach a pseudo-equilibrium and to remain effectively in a restricted subset of the state space.

On the other hand, high values of T would make the chain following a distribution becoming uniform over the solution space as T increases; this becomes clear when we consider equation (12) as probability distribution and set $T \to \infty$ (see [7] and [19]).

The Metropolis Algorithm

A general Metropolis algorithm works as follows. We start from a feasible configuration Z_ξ and we try a new solution Z'_ξ, built on Z_ξ according to some predefined rules; then we accept it as new starting Z_ξ with a certain probability proportional to its goodness.

The aim is to find the best possible configuration Z^*_ξ such that the function $f(Z^*_\xi)$ is as small as possible.

Therefore, we switch to Z'_ξ with probability $P(T) = \min\left\{\exp(\frac{f(Z_\xi) - f(Z'_\xi)}{T}), 1\right\}$, and stay in permutation Z_ξ for another time unit with complementary probability $1 - P(T)$.

This is the transition mechanism of the Markov chain that has the Boltzmann distribution as a reversible distribution, by the general Metropolis chain theory discussed previously. We now show its application for the (S-)T-VS-BPP.

3.2 Solving (S-)T-VS-BPP with MCMC heuristic

We now present a MCMC heuristic to solve both the T-VS-BPP and S-T-VS-BPP. The algorithm starts with a simple greedy procedure to obtain an initial feasible solution Z_{ξ_0}. Specifically, we consider two non-ordered lists of bins (R) and items (N) and we try to place the first item n_i available in the first bin r_j. If it is infeasible we try the next bin. When an insertion is successful, we consider the next item and bin. When we reach the last bin, we restart.

After the initial solution is built, we launch the MCMC algorithm.

Within the algorithm we use two different neighborhoods to switch to a new solution. We refer to them as single switch and double switch. Starting from configuration Z_ξ, we find Z'_ξ either switching the allocation of a single item to a different bin (single) or swapping two objects belonging to different bins (double) (see Figure 2). The items are chosen according to an uniform distribution. The single switch ensures the possibility to change the number

Algorithm 1 Greedy Algorithm

$i = 0; j = 0$
$n_i \in N; \ r_j \in R$
while $i \neq |N|$ **do**
  if $\sum_{i \in N}(w_i * X_{i,r_j}) + w_{n_i} \leq B_j$ **then**
    $X_{n_i,r_j} = 1$
    i=i+1; j=j+1
    if $j = |R|$ **then**
      $j = 0$
    end if
  else
    $j = j + 1$
    if $j = |R|$ **then**
      $j = 0$
    end if
  end if
end while
Initial Solution $\rightarrow Z_{\xi_0}$

---

of bins used and this is necessary since the initial solution is meant to distribute all the items in all the available bins. In each iteration, the algorithm chooses with a certain probability x (we fix 50% each) which method of search should be used.

This search technique is frequently used and is known in the literature as Variable Neighbor Search metaheuristics (VNS) [23], [21]. The main idea of VNS is to explore either at random or systematically a set of neighborhoods to escape from local optima.
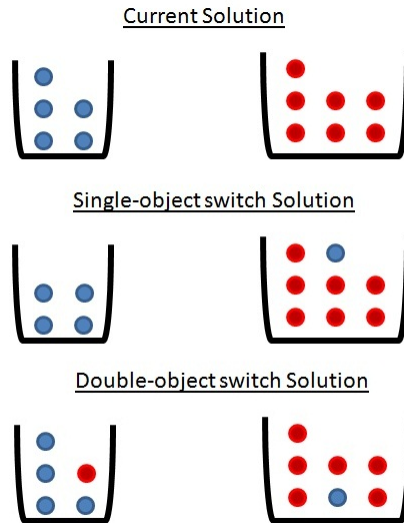

Figure 2: Single and double switch transitions

Algorithm 2 MCMC Algorithm for (S-)T-VS-BPP
***

$Z_\xi \leftarrow Z_{\xi_0}$

Input T, x, K, M

$k \leftarrow 0$

$Z_{\xi_0} \leftarrow Z_\xi*$

while $k \neq K$ **do**

  if $random(0,1) < x$ **then**

    single switch

  else

    double switch

  end if

  Build Z'_ξ

  if Z'_ξ *unfeasible* **then**

    $\rightarrow f(Z'_\xi) = f(Z'_\xi) + M$

  end if

  $Z'_\xi \leftarrow neighbour(s)$

  if $P(T) \geq random(0,1)$ **then**

    $Z'_\xi \leftarrow Z_\xi$

  end if

  if $f(Z'_\xi) < f(Z_\xi*)$ **then**

    $Z_\xi \leftarrow Z_\xi*$

  end if

  $k \leftarrow k + 1$

end while

return $Z_\xi*$
***

Note that only feasible states are allowed in the search. Therefore, in case an unfeasible configuration Z'_ξ is found (for an overflow of the capacity), we add a big penalty (M) to $f(Z'_\xi)$ such that the probability to switch is very close to 0. Finally, the algorithm runs for a fixed number of iterations K which is decided according to efficiency. In fact, we do not let our algorithm run more than 5 minutes for the final run and few seconds for the experimentations regarding the temperature.

The choice of K is indeed crucial because for an algorithm to be efficient it has to run for a number of iterations much smaller than the size of the problem. Moreover, the analytical evaluation of the number of steps needed to sample π within an acceptable error (mixing time of the chain [16]), may be a hard problem (fast mixing times are indicators of the quality of the chain). Usually in these heuristic applications, K is usually chosen by empirical observation of the Markov Chain, or by an appeal to combinatorial or physical intuition. This means that no precise claim can be made about the distribution of the samples, and consequently no performance guarantee can be given for the associated approximation algorithms. This observation holds for almost all existing Monte Carlo experiments in physics, and for almost all MCMC applications in combinatorial optimization. It is a considerable challenge for theoretical computer science to analyze the mixing time in such applications, and hence to place these algorithms on a firm foundation [16].

4 Numerical Experiments

In this section, we provide numerical experiments on 10 instances, for the VS-BPP and its time-dependent variants. The number of items goes from 10 to 100 (N). The aim is to generate some insights on the complexity of the problem and to analyze the different decision making processes. We then describe the experiments necessary for the search of an effective and efficient temperature, to be set in the algorithm.

All experiments have been run on an Intel(R)Core(TM)2 DUO machine with 2.93 GhZ and 4.00 RAM memory and the algorithm is coded in C++.

4.1 Instances Generation

The instances are generated according to the parameters in Table 1. The penalty costs p_i and the weights w_i for late deliveries are generated according to a Rayleigh distribution respectively with parameter $\sigma = 0.4$ and $\sigma = 3$. The deadlines are uniformly generated: for the first bundle the range is $[8, 28]$, for the second $[16, 36]$.

Parameter	Description	Value
C^s	Cost for small bins	3
C^b	Cost for big bins	8
B^s	Capacity small bins	6
B^b	Capacity big bins	20
P^s	Processing time small bins	3
P^b	Processing time big bins	10
W	Waiting cost	0.1
α	setup time	1
L	loading time	0.1
R^s	number of small bins available	0.5*N
R^b	number of big bins available	0.25*N
A_j^R	availability of the bins	$0 \quad \forall j \in R$

Table 1: Input parameters to generate the instances

With regards to the arrival times A_i^N of the items, for the T-VS-BPP instances, we consider two bundles of items arriving at times 4 and 12. For the S-T-VS-BPP we consider a discrete probability distribution.

Variability in the S-T-VS-BPP

We model the variability in the arrivals $A_{\omega,i}^N$ using discrete distributions with values as described in Table 2. In this distribution we have 50% probability (expected value) that our items will arrive on time (first bundle at 4, second bundle at 12), with 40% we have 4 hours of delay and finally with 10% we have 1 hour of early arrival.

The assumption that the items arrive in at most 2 or 3 batches is necessary to consider, because otherwise the number of combinations increases exponentially with the number of batches Y: 3^Y. For instance, if all items arrive in 1 batch, then we have just 3 scenarios.

Scenario	Arrival time	Distribution	Values(Bundle 1, Bundle 2)
On-Time (OT)	E[x]	50%	4,12
Late (L)	E[x]+4	40%	8,16
Early (E)	E[x]-1	10%	3,11

Table 2: Probability Distribution for the availability of the items

If the items arrive within 2 batches then the number of possible scenarios increases to 9 as reported in Table 3.

Scenario	Arrival time of the 1st batch	Arrival time of the 2nd batch	Probability
1	On-Time	Early	5%
2	On-Time	On-Time	25%
3	On-Time	Late	20%
4	Early	Early	1%
5	Early	On-Time	5%
6	Early	Late	4%
7	Late	Early	4%
8	Late	On-Time	20%
9	Late	Late	16%

Table 3: Possible Scenarios when 2 independent bundles arrive

4.2 Solving the VS-BPP

To open the discussion on the tests and the different results of the models, we first show the experiments on VS-BPP. In the 10 instances we set p_i and W equal to 0. In such case it is natural that an optimal solution would try to get economies of scale allocating most of the items in the big bins (R^b).

We show the results for VS-BPP in table 4.

		Industrial Solver	MCMC	Solution features	
Instance	N	Value	Value	R^s	R^b
1	10	**22**	**22**	2/5	2/2
2	15	**27**	**27**	1/7	3/4
3	20	**32**	**32**	0/10	4/5
4	21	**35**	**35**	1/10	4/5
5	22	**35**	**35**	1/11	4/5
6	23	**35**	**35**	1/11	4/5
7	40	**54**	**54**	2/20	6/10
8	50	**72**	**72**	0/25	9/12
9	75	**107**	**107**	1/37	13/18
10	100	**136**	**136**	0/50	17/25

Table 4: Results for VS-BPP.

The results are all obtained within 60 seconds for the MCMC heuristic and always within 1 second for the industrial solver.

4.3 Numerical Tests and Performances Solving the (S-)T-VS-BPP

We now show the results of the experimentations on the T-VS-BPP and the S-T-VS-BPP. Firstly, we describe an experimental procedure to find good temperature values for the algorithm. Secondly, we solve the stochastic and deterministic instances and generate insights based on the solutions. Finally, we solve again every instance with an average temperature to look more deeply at the behavior of the heuristic and its dependence on the parameter.

4.3.1 Finding the Temperature

In order to find a good T, we first launch some runs (4-5) of the algorithm with different temperatures and with a relatively small K value (5 million $\sim$ 3-10 secs). We then keep the temperature which on average provides better solutions. On the set of instances tested we notice that on average T should not be greater than 1; in fact, for $T > 1$ the algorithm is very inconsistent and frequently selects poor solutions. With $T < 0.1$, we notice that the algorithm reaches a local minimum very quickly and then it becomes impossible to make a "jump" toward other minima since the procedure is too cautious in accepting worse solutions (metastability). Therefore, in a graph where we put the temperatures versus the solution values, we observe a u-curve where the left side is flatter and lower then the right side. In Figure 3 a typical behavior for a "40 items" instance is shown.
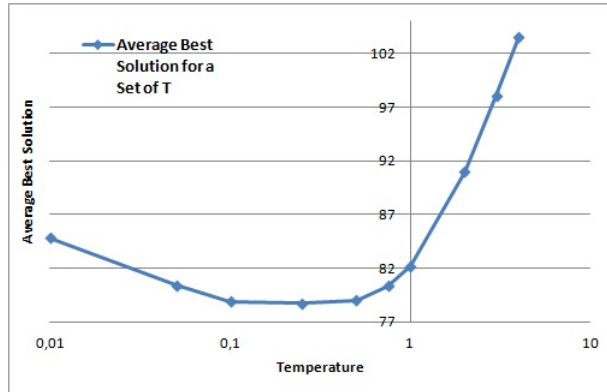
Figure 3: Average trend of the temperatures for the 40 items instance. The set of temperatures used is: 0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 2, 3, 4. For each temperature we draw the average of the best solutions of 4 runs with 5000000 iterations. In this example $T = 0.25$ is the best choice.

This behavior is even more visible if we look at the time series of the temperatures along the runs. In Figure 4, the algorithm with a temperature of 4 is highly variable and does not follow a path toward "good" local minima. With temperature 0.01 we reach a local minima but we cannot make the "jump" toward others; in fact we get a straight line throughout the run because the algorithm is stuck in that solution. Similarly we have temperature 0.1 but its behavior is less extreme and it finds luckily the optima. Then the right temperature is a good trade-off and according to this chart we would choose between $T = 0.5$ and $T = 0.25$. These graphs recall to some extent the experimentation made in [22] where a cooling schedule is applied to a single run. These authors in [22] show that higher temperatures imply a faster and inaccurate sampling in a coarse grained fashion. On the other hand, as the temperature decreases the search is much more refined.

4.3.2 Numerical Tests on (S-)T-VS-BPP instances

We test the two models with the same 10 instances used for VS-BPP, except for the values of p_i and W_j, described in 4.1. The industrial solver is able to find the optima for up to 22 items for the deterministic variant and up to 20 for the stochastic one. When the exact number of bins is defined a priori (*) those limits increase: the exact number of bins arises from the MCMC solutions. For larger sizes the industrial solver goes out of memory (OoM) and cannot provide the optimal solution.
In Tables 5 and 6 we show the main results from the experimentations.
From these results, we make the following observations:

- The stochastic solutions always have higher cost than the deterministic counterparts. The difference among these values is known in literature as value of the stochastic solution (VSS) [1]; this quantity is the cost of ignoring uncertainty in choosing a decision. It is also evident that plugging the deterministic solution in a stochastic environment produces on average higher costs. This is because deterministic solutions are optimal only when expected values realize, but could be detrimental when other scenarios happen.
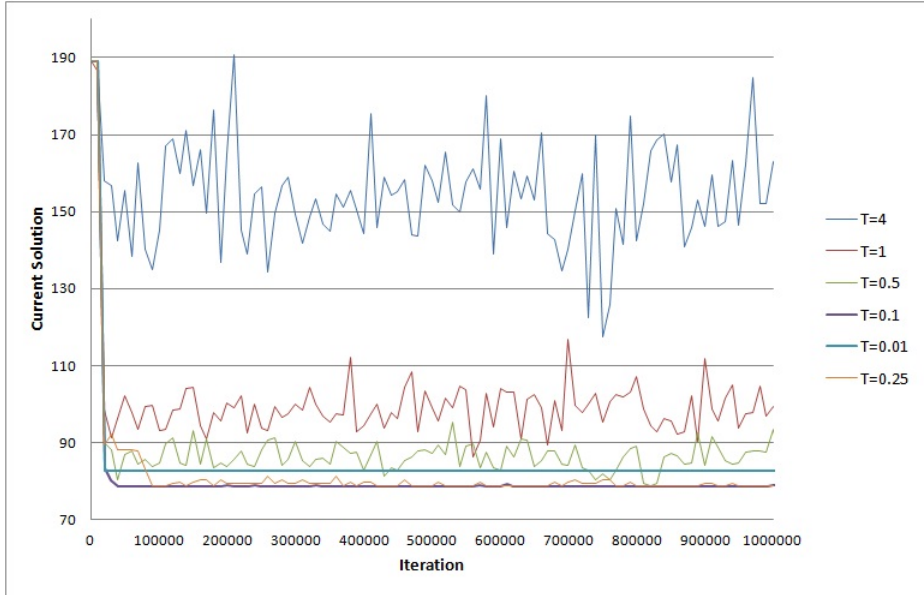
13

Figure 4: Solutions trend for some temperatures for the 40 items instance.

- The number of small bins used (R^s) in the stochastic solutions is always greater or at most equal than in the deterministic ones. This is because the small bins are faster and can handle better the variability in the arrival times of the items in order to process them on time. So the model works in safety when it allocates critical items to such bins.

- The use of small bins is mostly avoided in the VS-BPP to get economies of scale. In some cases this number reduces to 0 and the gap with the (S-)T-VS-BPP values is evident.

- The complexity clearly increases when time constraints are added. The industrial solver is not able to solve all the instances for the (S-)T-VS-BPP, as it is for the VS-BPP.

- The temperatures used for the final results mostly decrease as the number of items increase. This has an intuitive explanation: as the size of the instance increases and consequently its complexity the algorithm faces many more bad solutions. To avoid these paths the procedure needs a small temperature to hang on to the good solutions found and to build up new ones according to the good current structure.

4.3.3 Solving (S-)T-VS-BPP with an Average Temperature

In many industrial applications we face a fixed range of items and bins, therefore a smart way to apply an MCMC algorithm in these contexts would be to use a fixed temperature and avoid repeated experimentations. In these tests, we use the average of the temperatures used before and plug it for all instances. The results are shown in Table 7.

		Industrial Solver		MCMC			Solution features	
Instance	N	Value	Minutes	Value	T	Minutes	R^s	R^B
1	10	**27.65**	0.00	**27.65**	1	0.00	2/5	2/2
2	15	**37.34**	0.23	**37.34**	0.8	0.00	1/7	3/3
3	20	**43.8**	0.93	**43.8**	0.7	0.02	6/10	2/5
4	21	**45.02**	11.23	**45.02**	0.8	0.00	1/10	4/5
5	22	**47.52**	69.49	**47.52**	0.8	0.02	7/11	2/5
6	23	**OoM/51.15***	−/1.33	**51.15**	0.8	0.00	5/11	3/5
7	40	**OoM/OoM**	−/−	**78.73**	0.5	0.01	9/20	4/10
8	50	**OoM/OoM**	−/−	**102.7**	0.2	0.23	7/25	7/12
9	75	**OoM/OoM**	−/−	**149.38**	0.5	1.59	18/37	8/18
10	100	**OoM/OoM**	−/−	**190.4**	0.35	1.35	17/50	12/25

Table 5: Results for T-VS-BPP.

		Industrial Solver		MCMC			Solution features	
Instance	N	Value	Minutes	Value	T	Minutes	R^s	R^b
1	10	**29.12**	0.01	**29.12**	1	0.00	2/5	2/2
2	15	**41.18**	1.80	**41.18**	1	0.00	5/7	2/3
3	20	**48.062**	33.18	**48.062**	0.8	0.51	6/10	2/5
4	21	**OoM/49.57(*)**	-/12.83	**49.57**	0.8	0.00	4/10	3/5
5	22	**OoM/51.734(*)**	-/5.68	**51.734**	0.8	0.00	7/11	2/5
6	23	**OoM/57.132(*)**	-/2.31	**57.132**	0.8	0.00	8/11	2/5
7	40	**OoM/OoM**	−/−	**94.344**	0.5	0.30	13/20	3/10
8	50	**OoM/OoM**	−/−	**116.928**	0.45	1.08	10/25	6/12
9	75	**OoM/OoM**	−/−	**169.899**	0.5	0.52	18/37	8/18
10	100	**OoM/OoM**	−/−	**223.136**	0.4	6.09	20/50	11/25

Table 6: Results for S-T-VS-BPP.

		T-VS-BPP		Solution features		S-T-VS-BPP		Solution features	
Instance	N	T	Value	R^s	R^b	T	Value	R^s	R^b
1	10	0.645	**27.65**	2/5	2/2	0.705	**29.12**	2/5	2/2
2	15	0.645	**37.34**	1/7	3/3	0.705	**41.18**	5/7	2/3
3	20	0.645	**43.8**	6/10	2/5	0.705	**48.062**	6/10	2/5
4	21	0.645	**45.02**	1/10	4/5	0.705	**49.57**	4/10	3/5
5	22	0.645	**47.52**	7/11	2/5	0.705	**51.734**	7/11	2/5
6	23	0.645	**51.15**	5/11	3/5	0.705	**57.132**	8/11	2/5
7	40	0.645	79.92	13/20	3/10	0.705	94.368	13/20	3/10
8	50	0.645	105.38	10/25	6/12	0.705	120.108	11/25	6/12
9	75	0.645	151.21	18/37	8/18	0.705	174.222	22/37	7/18
10	100	0.645	201.92	22/50	11/25	0.705	233.419	28/50	9/25

Table 7: Results with an average temperature.

We see that with an average temperature the MCMC still reaches the optimum for smaller instances. When the size increases the temperature is too high to find good solutions and consequently we get worse performances.

5 Conclusions

In this study, a variant of the VS-BPP with time constraints has been considered. We proposed a deterministic and stochastic model. Due to its NP-hard property, we built an MCMC algorithm able to solve it and we provided comparisons with the performances of an industrial solver. The tests showed that the Industrial Solver can optimally solve the model up to a certain size. The heuristic can find the certified optima till this threshold with an excellent timing. The numerical tests confirmed also what we expect from the stochastic model, namely the optimal solutions are safer then the deterministic counterparts. In fact, in most of the instances, smaller and faster bins are preferred in order to meet the deadlines. As extreme, in the classical VS-BPP we noticed that the items are allocated in few high-capacitated bins, due to the lack of time issues in the objective function.

Future research on this work will be the application on real problems, in particular we are interested in the application within the container supply chains. Moreover, the problem is open to several extensions and it can be also studied under the point of view of multi-objective optimization. It is also interesting to find, a priori, an approximate value for the number of bins to be given as initial data. As we showed, the industrial solver can solve larger instances when this value is reduced.

References

[1] Birge J.R. and Louveaux F., 2010. *Introduction to Stochastic Programming.* Springer.

[2] Bovier A., Eckhoff M., Gayrard V., Klein M., 2001. Metastability in stochastic dynamics of disordered mean-field models, *Probability Theory and Related Fields*, 119(1), pp. 99–161.

[3] Bovier A., Eckhoff M., Gayrard V., Klein M., 2002. Metastability and low-lying spectra in reversible Markov chains. *Communications in Mathematical Physics*, 228(2), pp. 219–255.

[4] Cheng T.C.E , Sin C.C.S., 1990. A state-of-the-art review of parallel-machine scheduling research.*European Journal of Operational Research*, 47(3),pp.271–292.

[5] Correia I. , Gouveia L., Saldanha-da-Gama F., 2006. Solving the variable size bin packing problem with discretized formulations. *Computers & Operations Research*, 35(6), pp.2103–2113.

[6] Csirik J., 1989. An on-line Algorithm for Variable-Sized Bin Packing. *Acta Informatica*, 26(8), pp.697–709.

[7] Dueck G., Scheue T., 1990. Threshold Accepting: A General Purpose Optimization Algorithm Appearing Superior to Simulated Annealing. *Journal of Computational Physics*, 90(1), pp. 161–175.

[8] Eilon S. , Christofides N., 1971. the Loading Problem, *Management Science*, 17(5), pp. 259–168.

[9] Falkenauer E., 1996. A Hybrid Grouping Genetic Algorithm for Bin Packing. *Journal of Heuristics*, 2(1), pp.5–30.

[10] Freidlin M.I. and Wentzell A.D., 1984. *Random perturbations of dynamical systems.* Springer.

[11] Friesen D.K. , Langston M.A., 1986. Variable Sized Bin Packing. *SIAM Journal on Computing*, 15(1), pp. 222–230.

[12] Garey M. R. and Johnson D. S., 1979. *Computers and Intractability: A Guide to the Theory of NP-Completeness*, W. H. Freeman and co.

[13] Häggström O., 2003. *Finite Markov Chain and Algorithmic Applications.* Cambridge University Press.

[14] Haouari M., Serairi M., 2009. Heuristics for the variable size bin-packing problem. *Computers and Operations Research*, 36(10), pp 2877–2884.

[15] Higle J. L., 2005. Stochastic Programming: Optimization When Uncertainty Matters, in *Tutorial on Operations Research*, Informs, pp.1–23.

[16] Jerrum M. and Sinclair A. , 1996. The Markov chain Monte Carlo method: An approach to approximate counting and integration. In *Approximation Algorithms for NP-hard Problems* (Dorit Hochbaum, ed.). PWS Publishing, pp.482–520.

[17] Johnson D. , 1974. Fast Algorithms for Bin Packing. *Journal of Computers and System Sciences*, 8(3), pp.272–314.

[18] Kang J., Park S., 2002. Algorithms for the variable sized bin packing problem. *European Journal of Operational Research*, 147(2), pp. 365–372.

[19] Kirkpatrick S., Gelatt C. D., Vecchi M. P., 1983. Optimization by Simulated Annealing. *Science*, 220(4598), pp.671–680.

[20] Metropolis N., Rosenbluth A.W., Rosenbluth M.N., Teller A.H, Teller E., 1953. Equations of State Calculations by Fast Computing Machines. *Journal of Chemical Physics*, 21(6), pp.1087–1091.

[21] Mladenovic M., Hansen P., 1997. Variable Neighborhood Search. *Computers and Operations Research*, 24(11), pp.1097–1100.

[22] Szu H., Hartley R., 1987. Fast Simulated Annealing. *Physics Letters A*, 122(3–4), pp. 157–162.

[23] Talbi E-G., 2009. *Metaheuristics: from design to implementation*. Wiley.

A   BPP and VS-BPP: Features and Complexity

A possible characterization of BPP is the following. Given a set $N = 1, \ldots, n$ of objects with the kth with weight $w_k > 0$ and a set $R = 1, \ldots, r$ of bins each one with a certain capacity $B_i > 0$ and cost, we need to fill all the n objects in the bins, without exceeding the capacities and in order to minimize the cost associated to such allocations; in particular let u_j, with $j \in R$, be a binary variable that keeps track whether a bin is used or not and let $X_{i,j}$, with $i \in N$ and $j \in R$, be the binary allocation matrix. Moreover let us assume that we have at our disposal an overall capacity bigger than the total weight of the items. A possible mathematical formulation for such problem is:

$$Min \quad \sum_{j=1}^{r} f(u_j) \tag{13}$$

Subject to:

$$\sum_{j=1}^{r} X_{i,j} = 1 \quad \forall i \in N \tag{14}$$

$$\sum_{i=1}^{n} w_i * X_{i,j} \leq B_j * u_j \quad \forall j \in R \tag{15}$$

$$u_j \in (0,1) \quad \forall j \in R$$

$$X_{i,j} \in (0,1) \quad \forall j \in R, \forall i \in N$$

The objective function (13) minimizes the cost associated with the bins. Constraint (14) ensures that each item i is packed, while (15) imposes that the capacities of the bins are not exceeded.

If the capacity of each bin is the same we face the classical NP-hard Bin Packing problem[12]; therefore the VS-BPP can be reduced to such case.

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