On the connection between different noise structures for LPV-SS models

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On the Connection Between Different Noise Structures for LPV-SS Models*

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1 Introduction

Many chemical or physical processes exhibit parameter variations due to non-stationary or nonlinear behaviour, often dependent on measurable exogenous variables or measurable process states. These nonlinear parameter variations can be captured in the linear parameter-varying (LPV) modelling paradigm, which originates from the need of finding model structures, that are linear and low in complexity, but still allow to represent the nonlinear aspects of systems during control design. To achieve this, the LPV model class assumes a linear relation between the inputs and outputs of the system, however, the parameters of this relation are functions of a measurable, time-varying signal, the scheduling variable \( p \), which expresses the current operating conditions, time-conditions, or nonlinearities of the plant.

2 LPV-SS models with general noise structure

Recasting nonlinear system descriptions as LPV representations can be accomplished in various forms, such as state-space (SS), impulse response, kernel, or input-output representations. For control purposes, LPV-SS representation forms are preferable, particularly with static and affine dependence on the scheduling signal. However, generally speaking, not all phenomena affecting the physical system at hand are captured in the original nonlinear model equations, therefore, a disturbance term is usually added to the state equation and output signal to represent unmodelled dynamics, environmental effects, measurement disturbances, etc. Hence, the LPV model should equivalently be equipped with an appropriate noise structure. Such an LPV-SS representation with general noise structure is given by

\[
\begin{align*}
    x_{t+1} &= A(p_t)x_t + B(p_t)u_t + F(p_t)w_t, \quad (1a) \\
    y_t &= C(p_t)x_t + D(p_t)u_t + H(p_t)v_t, \quad (1b)
\end{align*}
\]

where subscript \( t \) is the discrete time, \( x \) is the state variable, \( y \) is the measured output signal, \( u \) denotes the input signal, \( w, v \) are assumed to be zero-mean noise processes satisfying

\[
\begin{bmatrix} w_t \\ v_t \end{bmatrix} \sim \mathcal{N}(0, \begin{bmatrix} W & S \\ S^\top & R \end{bmatrix})
\]

with covariance matrices \( W, S, R \), and \( A, \ldots, H \) are affine functions in the scheduling signal, i.e., \( A(p_t) = A_0 + \sum_{i=1}^{n_p} A_ip_i^t \) with \( p_i^t \) the \( i \)-th element in \( p \). The system description (1) is capable of representing a large generality of noise scenarios, like its LTI or time-varying counterparts [1].

3 Connection to the innovation noise structure

In practice, the coefficients \( \{A_i, \ldots, H_i\}_{i=0}^{n_p} \) of the model (1) are estimated from data, as their values are unknown. However, state-of-the-art LPV-SS identification approaches are designed for an innovation type of noise structure, e.g. [2]:

\[
\begin{align*}
    \dot{x}_t &= A(p_t)x_t + B(p_t)u_t + K(p_t)\xi_t, \quad (2a) \\
    y_t &= C(p_t)x_t + D(p_t)u_t + \xi_t, \quad (2b)
\end{align*}
\]

where \( \xi_t \) is a zero-mean noise process satisfying \( \xi_t \sim \mathcal{N}(0, \Omega) \) with covariance matrix \( \Omega \), and \( K \) is an affine function in the scheduling signal, similar parameterized as \( A, \ldots, H \) in (1). In the LTI case, i.e., for \( A(\cdot) = A, \ldots, H(\cdot) = H, K(\cdot) = K \), the innovation noise structure (2) is the optimal linear filter given (1) [1]. Hence, in this case, the innovation noise structure can represent the noise scenarios expressed by (1). As many LPV identification schemes are extensions from their LTI counterpart, the innovation noise structure is an obvious and popular choice.

Therefore, we will analyse the stochastic properties of (1)-(2) and their connection. We will show that: i) the matrix functions \( K, \Omega \) should have rational and dynamic dependence on the scheduling signal for an equivalence between (1) and (2) governed by the Kalman filter equations; ii) in practical situations \( K(p_t, \ldots, p_{t-n}) \) can be a good approximation due to the asymptotic convergence of the underlying filter. In addition, if it is assumed that this approximation is equal to the data-generating system then, for some cases, this system has an equivalent LPV-SS representation with affine and static dependency on the scheduling signal at the cost of additional states \( x \), as will be demonstrated by an example. Hence, system (1) can possibly be captured in the innovation form (2), but at the cost of a non-state-minimal system. Consequently, LPV subspace identification schemes might lose their rank revealing properties.

References


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