

Mathematik Didaktik (teaching-learning mathematics) : an overview of the development of a web-based European module

Citation for published version (APA):

Hudson, B., Ongstad, S., Braathe, H. J., & Pepin, B. (1999). Mathematik Didaktik (teaching-learning mathematics) : an overview of the development of a web-based European module. In B. Hudson, F. Buchberger, & P. Kansanen (Eds.), *Didaktik/Fachdidaktik as science(-s) of the teaching profession?* (pp. 147-160). (TNTEE Publications; Vol. 2-1). TNTEE Publications.

Document status and date:

Published: 01/01/1999

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Brian Hudson, Sigmund Ongstad, Hans-Jorgen Braathe, Birgit Pepin
Sheffield Hallam University, UK
Oslo College, Norway
The Open University, UK

Mathematik Didaktik (Teaching-Learning Mathematics):
an overview of the development of a Web-based
European Module

Abstract

This paper outlines the development of the chapter (module) Mathematik Didaktik as a part of the electronic ‘text book’ Didaktik/Fachdidaktik developed by TNTEE Subnetwork E. The overall approach to the development of the module is based on a model of teaching-learning as an ‘integrative transformative science’ that pays due attention to the general aims of society as well as curricula, content and learning situations. As part of this perspective, teacher competence is broadly conceptualized in terms of “professional action structures” in contrast with the narrow emphasis on technical competence and on mechanistic conceptions of a ‘technology of teaching’ that currently prevail in some parts of Europe. The teaching-learning approach is based on problem-oriented, research-oriented and co-operative learning processes. Underpinning the development are particular ideas about the nature of mathematics itself. In particular the starting point for the development is around ‘big ideas’ in mathematics – in contrast to the fragmentation that is evident in the thinking of some policy makers at this time.

Rationale

The overall approach to the development of the module is based on the model of teaching-learning as an ‘integrative transformative science’ (Buchberger and Buchberger, 1999) that pays due attention to the ‘general aims of society’ as well as to curricula, content and learning situations. As part of this perspective, (beginning) teacher competence is broadly conceptualized in terms of “professional action structures” in contrast to the narrow emphasis on technical competence and on mechanistic conceptions of a ‘technology of teaching’ that currently prevails in some parts of Europe (Reynolds 1998). Such structures involve subject-related and “didactic” competence, methodological (teaching-learning) competence, management of learning groups, diagnostic competence, counselling competence, metacognitive competence, new media competence and co-operation. Accordingly the teaching-learning approach in Initial Teacher Education (ITE) is based on problem-oriented, research-oriented and co-operative learning processes.

The title of our chapter expresses implicitly at least two beliefs about mathematics education: firstly that there are three crucial elements involved, the mathematics, the teaching and the learning; or alternatively, the content, the teacher, and the learner. However these three elements only make

sense in a mutual triad where no aspect is given primacy. Pedagogies, though, tend to stress one aspect at the expense of others, for instance claiming that the most crucial question is “How to teach?”, “How to learn?”, or “What is mathematics?”. A central aim of this module is to develop a theoretical-practical platform for bridging the gap of this pedagogical triad. Secondly: practical schooling in mathematics has normally focused on the ‘what’ i.e. what the teacher teaches or what the learner learns or what the textbook describes. At the same time *language* and *communication* have tended to be treated as additional aspect to mathematics. By seeing teaching-learning fundamentally *as* communication, it becomes clear that an approach that is one-sidedly preoccupied with mathematics as spoken or written content, is accordingly ‘text-oriented’. Text is here taken in a broad sense. Unfortunately this ‘what-oriented’ perspective often brings with it a lack of understanding of the *non-text*, or the *context*. In communication there will always be an intimate interplay between what is said and what is not said, and the unsaid rests in the context. Hence, by focusing on teaching-learning *situations* in mathematics, we hope to problematize how mathematics education is contextualized or needs to be *recontextualized*. However, context is a notoriously difficult concept to grasp, theoretically and practically in its full sense, since logically and practically always there will be a context outside the context outside the context etc.

In the development of this module we will draw on some ‘didaktik models’ that are in use. The idea is not to give a scenario of representative models, but rather to rethink, on the basis of a communicational framework, what educational point of departure or *perspective* actually implies. The four co-authors of this paper have, for instance, different educational backgrounds, professional attitudes and practical experiences, which have forced upon us the question of whose preferences are most valid and relevant. We do not have a fixed answer, neither in this module nor in general, but we hope to enable student teachers of mathematics to rethink some principles, and consider what *they* think is most important, to clarify *their* own implicit perspectives, *their* ‘didaktik’ point of departure, without ending in pure *perspectivism*.

Also we need to reflect on the nature of mathematics itself and in particular to consider what might be the ‘Big Ideas’ (Faux 1998) in mathematics – in contrast to the fragmentation that is so evident in the Anglo-American tradition (Hudson 1999a and Pepin 1999). The concern about the atomization of subject matter based on the American tradition of ‘instruction’ was highlighted by Freudenthal (1978: 97) though at that time he held up the British ‘integrating interpretation of educational innovation’ as a model of good practice and saw pedagogues and general didacticians as part of the problem:

Indeed, atomization of subject matter is not merely a behaviouristic concern. It is the line of least resistance in technologising instruction. Pedagogues and general didacticians judge mathematics to be their most appropriate victim. Indeed in mathematics you can isolate and enumerate all concepts in order to have them trained systematically one by one, in pairs, in triples, as far as you want to go. It is a caricature of mathematics which is quite common. Therefore no subject is exposed to ruin by atomization as mathematics. It is too obvious that by atomistic instruction you cannot teach creativity in speaking and writing ... But mathematics seems to invite atomization, and so mathematics is hard to defend. Isolating, enumerating, exactly describing concepts and relations, growing them like cultures in vitro, and inoculating them by teaching – it is water to the mill of all people indoctrinated by atomism.

Such a view of mathematics is one that Fruedenthal (1978: 96) considers ‘every mathematician will detest from the depths of his/her heart’. However mathematicians and mathematics educationalists have been unable to resist the technologising force of the bureaucrats and politicians on the national curriculum of schools in England and Wales, which has also more recently been applied to teacher

education itself. These issues highlight the need also to consider epistemologies of mathematics and mathematics education as a component of a module on the teaching-learning of mathematics. A full discussion of the ways in which epistemologies and educational traditions ‘permeate through to teachers’ pedagogies in schools’ can be found in Pepin (1999).

Teaching-Learning Mathematics

Mathematics education as methodologism

Preparation for performance has often been developed into a separate art form in important fields of skills and knowledge as a general method and almost as a field in itself. For example *rhetoric* was developed over centuries and across cultures to handle different communicational situations. However, in more recent times, this *metier* faded out, although some of its traditions were (tacitly) carried on into European school systems. Thus the main idea, or perspective, was retained, i.e. content can be handled by more or less general *methods*. This implicit standpoint gave the practical and theoretical premise for a *general didaktik*. From this perspective *content* was seen as relatively unproblematic.

As recently as the 1960s teacher education in most European countries had a *methodological* orientation. Hence to *teach* mathematics was considered as a practical activity. One started from the textbook, in which referred knowledge was seen as more or less given, and which was to be kept in line with prescriptions in the written curricula of the national state. Student teachers were stimulated by teachers in *pedagogy* to think about which pedagogical principle(s) might be relevant to use in preparing lessons in the school disciplines: Was it Kerschensteiner’s work-school principle, or Dewey’s learning by doing, or Maslow’s hierarchy of basic needs? In Norway student teachers had to record beforehand *what* they would teach and an *aim* or a *purpose* for each lesson. This tradition was accordingly partly indebted to Tyler’s rationale, but perhaps without understanding the differences in *context* between the two cultures in question, the US and Norway (Strand and Kvernbekk 1998).

Critical alternatives to the methodological tradition

In many European countries teacher education in the 1960s consisted of pedagogy, disciplines and praxis, with ‘methods’ added more or less as a topic to one of these elements. However many teachers in pedagogy were critical of what went on in practice, and wanted a more holistic approach. *Didaktik* and not *metodik* was the discipline which should create more reflexive understanding and wholeness.

In the early 1970s a progressive movement within the disciplines in schools led to the development of a new discipline in many teacher education colleges in Europe – that of *Fagdidaktik*. This was intended to extend the practice of a limited *Fagmetodik* to a mutual combination of the discipline and its *didaktik*. This new ‘discipline’ was soon captured by the terms *what*, *how* and *why*. These terms were not intended to be understood as three separate elements. The conscious and conscientious student teacher, the democratic written curriculum and the progressive textbooks were intended to treat this as a set, as three dynamic *relationships*, the *what-how*, the *what-why*, and the *how-why*. This was the general intention. However the different disciplines adopted this new perspective in quite different ways.

For example in books on mathematics *didaktik* in the 1980s this understanding received somewhat different interpretations. For example, Solvang’s (1986) *Matematikkdidaktikk*, a much used Norwegian book in teacher education for upper secondary education, includes a chapter called *Main elements in math teaching and learning*, which is concerned with the planning of teaching and the delimitation of *didaktik*:

In chapter 2 we dealt with bits of the field which traditionally has been called the didaktik aspect of the discipline. There we looked at problems concerning the selection of [subject] matter, organization of [subject] matter and the goal for our math teaching. In addition to this one has mentioned the methodological aspect of math teaching, often called math methodology or math [fagmetodikk]. (...) If we look at those/these two aspects of teaching math as a school discipline based on research over the last 20 years, it will be difficult to keep them separate as suggested above. This has led to the use of math didaktik among writers as a collective concept. The intention with the matter we will discuss in this section, is to enable the teacher to make systematic reflections on how she can:

- prepare teaching*
- carry out teaching*
- analyse the accomplished teaching with possible improvement in mind*

(...) When the disciplinary and the general goals are made ready, we can start the framing of the actual plan:

- WHAT is going to be done*
- THE PURPOSE in doing this*
- HOW to carry out this*
- WHY carry out this in such a way*

In addition to these four points the planning will include choice of means and control of to which extent the disciplinary goals are achieved/reached. (Solvang 1986: 41–44)

Solvang’s claims that mathematics didaktik could be seen as a compound of general didaktik and mathematical methodology, is not in line with what happened in Mother Tongue Education (MTE), which took a more independent direction by developing the new *fagdidaktik* more directly from the discipline (Ongstad 1999). Hence the *why* has had a stronger position in MTE. Solvang, being closer to general didaktization and methodologism, accordingly operates with a special variant of the what-how-why triad. The why is weakened and reduced to a question of defending the selection of methods. The more basic reason for this may be that mathematics as a subject may be seen as relatively unproblematic. The why is cut off from having a critical function. Hence the why has not grown from the discipline as such, but from the heritage of methodologism within the discipline. We should underline that there is nothing morally or professionally wrong with such a perspective, and that Solvang has elsewhere touched upon the more critical aspects.

In Uljens (1997) the logic relationship between general pedagogy, general didaktik and fagdidaktik is inclusion:

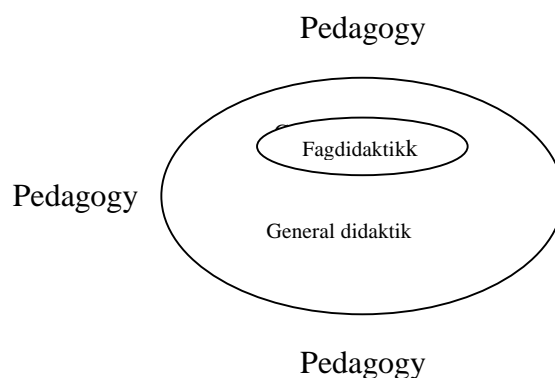


Figure 1 Relationship between general pedagogy, general didaktik and fagdidaktik (Uljens, 1997)

According to Gjone (1998) this is the preferred *perspective* in pedagogy. He presents this understanding with respect, but adds:

However fagdidaktikk connected to central school disciplines had an independent development, so that even if one uses general pedagogical methods (methods from educational science), there exist for many fagdidaktizians a distance to general didaktik and pedagogy. Since the end of the 1950s there has grown up a strong international consciousness about mathematics didaktik as a separate discipline or research field. Based on a fagdidakik perspective the following diagram would be more relevant for a 'positioning' in relation to other fields:

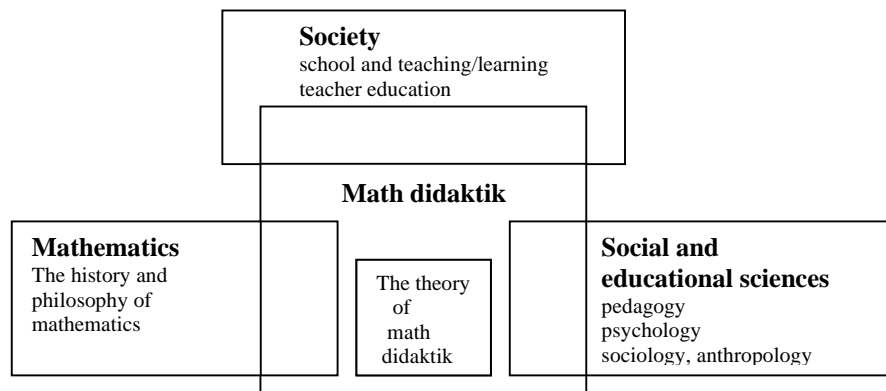


Figure 2 'Positioning' of mathematics didaktik in relation to other fields (From Gjone 1998: 84)

Hence mathematics didaktik for Gjone includes aspects of such central elements as a theory of science, pedagogy/didaktik, psychology (learning), discipline (mathematics), methods (practical), language (communicational) and critique (social). (Gjone 1998: 85–89) However he points also to other approaches such as Biehler *et al.* (1994).

Further, even scholars within general pedagogy were critical about the lack of broad and systemic understanding of didaktik. At the end of the 1970s and beginning of the 1980s many new models were coined to try to grasp the new complexity for didaktik as a whole.

The Didactic Relation Model or Relational Curriculum Design

The discussion in this section is based on Bjørndal and Lieberg (1978), Imsen (1997) and Strand and Kvernbekk (1998).

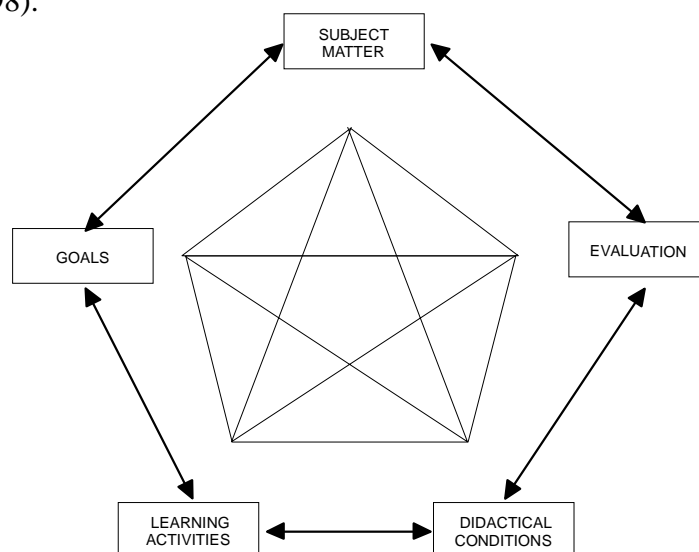


Figure 3 Relational Curriculum Design (Bjørndal and Lieberg 1978, Strand and Kvernbekk 1998)

The model in Figure 3 represents a much used Nordic approach and is termed “relational curriculum design” (didaktisk relasjonstenkning). It is meant to help student teachers and practising teachers to plan, process and evaluate teaching (Bjørndal and Lieberg 1978: 132–133). The model emphasizes the internal relationship between variables such as didactical conditions, subject matter, goals/ aims, learning activities and forms of evaluation. Deliberately no starting points or directions between the factors are given. Thus considerations may start from any point. According to Bjørndal and Lieberg (1978: 44) the intention in a broader sense is to create points of departure for processes of teaching and upbringing.

The model is closely linked to an educational system in which the national curriculum states the goals for education and describes subject matter, themes and to some degree teaching methods. The national curriculum is therefore supposed to represent the start for every teacher’s curriculum design, but cannot itself function as a direct guide to teaching. Hence the supposed need for a model. (For a specific critique of the use of models in the field of didaktik, see Strand and Kvernbekk 1998.)

There are different ways of criticizing a model that claims to be complete (Imsen 1997: 336). One is fundamental and illustrates a deep scepticism of the very use of models. Another is to attack the precision of the concepts. Finally the model’s ability to grasp relevant aspects may be questioned, which pinpoints the criticism to a question of purpose. This particular model has been criticized for not being able to highlight hidden structures (Imsen 1997: 37) in schools and classrooms (“the hidden curriculum”) and for underestimating the importance of organizational aspects in general. (Imsen 1997: 336)

From the perspective of mathematics it could be asked whether the model is specific enough to catch the very nature of mathematical education. As far as we know no Norwegian book that might be termed ‘mathematics didaktik’, refers to this model. Solvang (1986), Breteig and Venheim (1998), Nygaard *et al.* (1998), Tufteland (ed. 1998) and Herbjørnsen (1998) covering different educational levels, do not mention this model. Thus most student teachers of mathematics education in Norway during the last decade will have met, or will meet, at least two different, non-related kinds of curricular thinking. One tries to develop general, relational thinking, based on pedagogy, whilst the other focuses on the different specific elements that make the teaching and learning of mathematics significantly different from other disciplines. The intention of this module on Matematik Didaktik is to help to bridge the gap resulting from this divide, which is a general educational problem in Europe.

Teaching-learning as communication

Unenge and Wyndhamn (1986: 107) claim that teaching-learning simplistically can be described as a triangular drama between student, teacher and (subject) matter. The sides in such a triangle will signify the communications (in plural) that will take place.

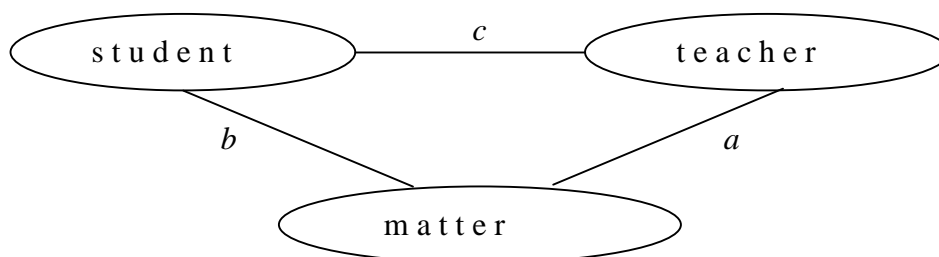


Figure 4 Triangular drama between student, teacher and (subject) matter (Unenge and Wyndhamn, 1986: 107)

Side *a* is the teacher's knowledge about the (subject) matter, his or her didactic theory. Side *b* is the student's communication with the matter, what s/he is going to learn. Side *c* represents the communication between the teacher and the student. The result of this drama is supposed to be that the student actually learns. However, according to Unenge and Wyndhamn, rows of research results show that this does not happen as simply as the figure might suggest. They claim that side *b* does not exist, since the student has a *pre-configuration* of the 'matter'. Instead they present an extended model with five corners where *experiences* play an important role (Unenge and Wyndhamn 1986: 108).

However Unenge and Wyndhamn have *not* really addressed the question of communication in spite of the fact that it is their explicit point of departure. The (subject) matters do not appear for us as 'matter' they arrive as 'utterances': from mathematicians in the past, from textbook writers, from educational agents such as the government, school authorities or teachers. Further an utterance arrives in a *context*, formed by *discourses* and/or *genres* of the discipline, school and everyday life. And how we understand these utterances depends on which communicational *aspect* we tend to perceive as important or *dominant*. Let us look at the student-matter line. If this matter is a mathematics problem, the basic process in the student's head is *thinking*. Accordingly we see matter as *content*. *Logic* thus becomes important. In our research we seek for a sensible didaktik to find support for a *cognitive* preference. The work of Piaget and the field of cognitive psychology will be seen as highly relevant.

However the utterance can also be seen as an *act*. Someone tries to 'force' you as a student to *do* something. The following kind of utterance, in the *context* of school and mathematics invites, forces, teases, you to *answer*, or in other words, to *perform*. It is seen as a communicational genre known and recognized as a *task*: e.g. find x when $3x = 45$. Of course one needs thinking, knowledge and logic to *solve* the 'problem', but seen from the *perspective* of action, the utterance asks social questions such as why is this mathematics? Do I have to do it? Why should I do it? Whose parents have given their children a good start and motivation for this kind of 'pursuit'? What do you need to make sense out of a sport like that? These 'impertinent' or critical questions may lead us in different directions. Utterances and accordingly mathematics can be seen as *activity* (Leontjev 1978 and Davydov and Markova, 1982–83). Alternatively mathematics and mathematical utterances can be seen as socially dependent on *culture*. Hence Vygotskyian (1962) approaches seem valid and relevant, focusing on the social or cultural (Bishop, 1991) conditions for the development of thinking. Further, mathematics can be related to questions of power and politics (Mellin-Olsen, 1987).

Utterances have a further *third* dimension in addition to content and function/action. Its most immediate and direct aspect is its appearance, its *form*, depending on the medium or the channel through which it is brought to us. However this third dimension is often the forgotten dimension. Mathematics is not only logic and culture but also it has a profound aesthetic aspect, recognised by many. Mathematics is hated and loved, it is awful and beautiful, it is clear and unclear, negatively frustrating and positively challenging.

Thus an utterance or a sign or a word or a text or a text element is triadic. Metaphorically, three traditional word classes can help to illustrate the nature of the main aspects involved: a noun for cognitive content and reference, a verb for social process and action and an adjective for emotional reactions to form and structure. Or, to put it differently, the noun helps us to categorize, to see the focused phenomenon as a delineated thing, as an object in the 'real' world that can be conceptualized in the mind. Categorization, nominalization and conceptualization help us to keep what we learn stable for a moment, but at the same time we easily lose sight of the process, the social relation the phenomena are part of, not to mention the emotional qualities of the 'thing' in the learner's mind/body.

The mixture of these basic elements goes on mutually and continuously. The challenge for teaching-learning is that we cannot know what is most relevant. We have to know the situations, the context. A general understanding of logic, or insights into children’s social background, does not help the teacher much if the child in question hates mathematics. Even mathematik-didaktik has to relate explicitly to such traditional triads as *feelings, thought, will*, or *beauty, truth, goodness*, or *experience, understanding, action*. They can all be put into a triadic understanding of didaktik, which has an explicit communicative foundation. This didaktik will have three main inseparable (= reciprocal) aspects:

Figure 5 Triadic understandings of didaktik

| Aesthetics | Epistemology | Ethics |
|------------|--------------|----------|
| student | matter | teacher |
| teacher | matter | student |
| form | content | use |
| structure | reference | action |
| feelings | thought | will |
| beauty | truth | goodness |
| adjective | noun | verb |
| heart | head | hand |

The most important source for the first (left) column is the *inner nature* of the utterer/receiver (*self* in the broadest sense) (Habermas 1981, 1988). Therefore expertise perspectives in this corner tend to be dominated by *psychology*. Similarly, the most important source for the middle column is *outer nature* (*world* in the broadest sense), and here disciplines which help us to explore ‘matters’ will be relevant. Finally the third column’s most important source is our relationship to *others* (or *society* in the broadest sense), understood, however, not as a social place, but as dynamic processes and relations embodied in people who reproduce it and change it only when acting, by uttering (Giddens 1984 and Dewey 1916).

We touched on the question of *context* above. Generally this concept has been taken too literally despite the warnings from theorists in sociology and psychology (Bourdieu 1977, Bateson 1972). If we start from a teaching situation in mathematics, this ‘situation’ will be heavily influenced by the *genres* by which the utterances are framed. Let us say there is a mathematical genre that is taught, for instance triangles. It is taught in a certain classroom genre we all know – a mixture of blackboard presentation and individual calculation, combined with the teacher circulating in the classroom, helping students. Contexts then are mixtures of more or less specified and conscious genres.

These genres, which are resources for ways of communicating, may *add* elements to each other. Thus a ‘class’ is a lesson where a bell is ringing, then the teacher starts explaining graphs, then students ask or are being asked, then students work on tasks, then a bell is ringing, and then the class is over. Or genres are *intertwined* or form *families*, for instance as a mathematical progression in textbooks from basic presentation of the coordinate system, via parables, to a general advanced understanding of conic sections. Or they are *encapsulated*, like *Chinese boxes*: triangles, in trigonometry, in tasks, in exams, in schools. However this mixture of genres is not untidy for a person who is enculturated to this growing system of genres. It is precisely our capacity to move rapidly ‘in’ and ‘out’ of different genres that make us able to communicate in and with contexts. Solomon and O’Neill (1998) provide further discussion on the notion of mathematics as genres. However there are different kind of genres. Some are shaped for description and reference, such as

definitions, photographs, patents etc. Alternatively, others are shaped for action and performance, like commands, instructions and tasks, or for seemingly aesthetic purposes, like paintings. However they all run the risk of being interpreted dysfunctionally. This is the reason why cognitivists tend to try to increase the refinement of their methods and approaches in explaining how students can or cannot learn mathematics. This is the reason why action researchers fail to change school. This is the reason why student oriented teachers keep a good relation to their students, but may fail to teach them mathematics. All this suggests that the interpretation of the context for mathematics teaching and learning is deeply rooted in a lack of communicational understanding. Triadic thinking of this kind will not necessarily help teachers to avoid misunderstanding. It is primarily a tool for becoming *aware* of what is *not* thought of in the moment of utterance and in the moment of interpreting. *Didaktik* is such a tool, and therefore a sound understanding of communication is basic for any teaching-learning situation.

Such a ‘didaktik’ point of departure is consistent with, but also adds to, that offered by the work of Gattegno (1987) who has been an influential figure on practice, if not policy, in mathematics education in England and Wales, principally through the profound influence of his thinking on the work of the Association of Teachers of Mathematics. In reflecting on his contribution Tahta (1988) comments that “Gattegno’s proposal is that shared awareness is an appropriate basis for a science”. He suggests the need to enlarge our notion of science and argues that all sciences begin with a new awareness – “of light, or sound, or, in the case of mathematics, of relations as such”. He argues further that the science of education “is concerned with the awareness of awareness itself”. An important role for the teacher according to Gattegno is in “forcing awareness”. This has echoes of the role of the teacher in Vygotsky’s (1962) Zone of Proximal Development (ZPD). Tahta also discusses “ways of knowing” and gives the example of “intuition” which is illustrated in relation to the use of geoboards, Cuisenaire rods and mathematical films. He argues that intuition “demands the whole of one’s self” and that this is what is required when one meets and tries “to maintain complexity”. He argues that it operates in “precisely the opposite way to the ‘focusing’ traditionally stressed in Western thought and education”. A further parallel can be found in the work of Jaworski and her use of what is described as the ‘Teaching Triad’ (Jaworski and Potari, 1998) which is composed of three domains: Management of Learning, Sensitivity to Students and Mathematical Challenge. It is seen as a framework ‘to capture the essential elements of the complexity involved’. All these aspects are seen to be consistent with a ‘didaktik’ point of departure.

The What and Why of mathematics

As indicated earlier, we also need to reflect on the nature of mathematics itself and in particular to consider what are the ‘Big Ideas’ in mathematics – in contrast to the fragmentation of the National Curriculum for England and Wales and also of the ‘Standards’ of the Teacher Training Agency (Hudson, 1999b). We need also to consider epistemologies of mathematics and mathematics education and the way in which these and educational traditions permeate through to teachers’ pedagogies in schools. Further we need to ask the question “why mathematics?”

The article by Faux (1998: 12–18) is very relevant here; he draws on the thinking of Gattegno and Freudenthal in particular. He suggests the following list of ‘Big Ideas’ in mathematics:

- Numbers are ordered and well structured
- Mathematics is shot through with infinity
- A lot for a little
- Equivalence
- Inverse
- Transformation

He illustrates the idea of ‘a lot for a little’ with reference to working on a ‘100 square’ i.e. a 10 by 10 square grid containing the numbers 1 to 100 in ordered rows of 10. Using the example of asking the question: ‘What is the sum of the numbers 1 to 100?’, he illustrates how he extends the activity by asking: ‘What can we now do because we have solved that problem?’ and: ‘what is now available to us? The sum of the first 20 numbers: is that directly available?’ He uses this example to show how he can ‘gain a lot for a little’ and proceeds to elaborate on his own philosophy of mathematics:

For me it's an important idea. It was what attracted me to mathematics when I was in school. A lot of subjects were dense with things that I first had to learn in disconnected ways – French words, history dates. In mathematics I could get started with very little and get on and get success; there were no disconnected facts to learn. That was important for me.

These words will speak powerfully to those with an appreciation of the nature of mathematics and will no doubt fall on barren ground for the ‘atomisers’ and ‘technologists’. However these principles will inform the development of the module Mathematik Didaktik.

Others aspects (not exhaustive) which have been identified for development include:

- Mathematics as logic, language and semiotics
- Mathematics in contexts
- History of mathematics
- Critical mathematics
- Realistic mathematics

Evaluating the Teaching-Learning of Mathematics

Research underlines the provisionality of knowledge. Teaching, at every level, is vulnerable if it does not acknowledge that error is a realistic intellectual achievement and failure a practical achievement, for a critical appreciation of error and failure is a necessary foundation for improvement. Research which disciplines curiosity and calls certainty into question, is a proper basis for teaching. (Rudduck, and Hopkins 1985)

Stenhouse (in Rudduck, and Hopkins 1985), in reflecting the crucial role of the university in teacher education, argues that the knowledge taught in universities is won through research and that such knowledge cannot be taught correctly except through some form of research-based teaching. ‘Knowledge’ that is represented as authoritative, and established independently of scholarly warrant, he argues “cannot be knowledge. It is faith”. He argues further that what is unquestionable is unverifiable and unfalsifiable. In contrast our knowledge is questionable, verifiable and differentially secure. He highlights the point that unless our students understand that what they take from their experience is in error: the error that research yields established authoritative knowledge that cannot be questioned. Speaking at his inaugural lecture in 1979, his words seem prophetic: “That this error is widespread must be apparent to anyone who has listened to the questions asked of academics by laymen on television. And if we educate teachers who will transmit this error to their pupils, the error will continue to be widespread. We shall support by our teaching the idea that faith in authority is an acceptable substitute for grasp of the grounds of knowledge, even perhaps a substitute for faith in God ... Once the Lord spoke to man: now scientists tell us that”.

Research is seen as a strategy that is applicable not only to the humanistic and scientific, but also to the professional, disciplines. So that just as research in history or literature or chemistry can provide stepping stones for teaching about those subjects, so educational research can provide stepping

stones for teaching and learning about teaching. Such an approach, in contrast to the constituent disciplines approach, treats education itself – teaching, learning, running schools and educational systems – as the subject of research.

Problems are selected because of their importance as educational problems – for their significance in the context of professional practice. Research and development guided by such problems will contribute to the understanding of educational action. Therefore this provides the rationale for educational action research with the aim of developing thoughtful reflection in order to strengthen the professional judgement of teachers. Imsen's (1999) model for the interpretation of class activities and the learning circle is consistent with such an approach. It also reflects the position of the teaching-learning process in the wider societal context:

- Classroom level
- School level (organization, leadership)
- Local environment (home, local culture, municipal conditions etc.)
- Central level (state, national authorities)

In turn, this approach is resonant with that of Gattegno. As Tahta (1988) observes “the science of education uses aspects of watchfulness as its tools and a process of continuous feedback as its verification”. These ideas are developed further by Mason (1994) through what he refers to as ‘the discipline of noticing’. This approach works on ideas of developing awareness ‘in the moment’ – and has been developed in the specific context of mathematics education.

This rationale will underpin the development of that section of the module relating to the evaluation of teaching-learning situations.

Structure and components of the module

As indicated earlier, a central aim of this module is to develop a theoretical-practical platform for bridging the gap of the pedagogical triad: the mathematics, the teaching and the learning, or alternatively, the content, the teacher, and the learner. This paper outlines some of the major aspects of theoretical underpinning this development. However this project is ‘work in progress’ and the practical side of the theoretical-practical platform is the second stage in this process. In terms of the stage of development of our current thinking, Figure 6 encapsulates an overview of the structure and contents of the module.

At the heart of the module is the overall focus of this module, which is that of:

- Preparing, realizing and evaluating the teaching-learning of mathematics

In turn this can be seen to be at the heart of a web of interconnecting components:

- Teaching-learning situations
- Theories and practices of teaching-learning
- The what and why of mathematics
- Preparing teaching-learning situations
- Evaluating teaching-learning situations
- Readings and other resources
- Aims, goals and use of the module

The structure and components of the module have been designed with regard to the overall approach to the module which, as indicated earlier, is based on problem-oriented, research-oriented and co-

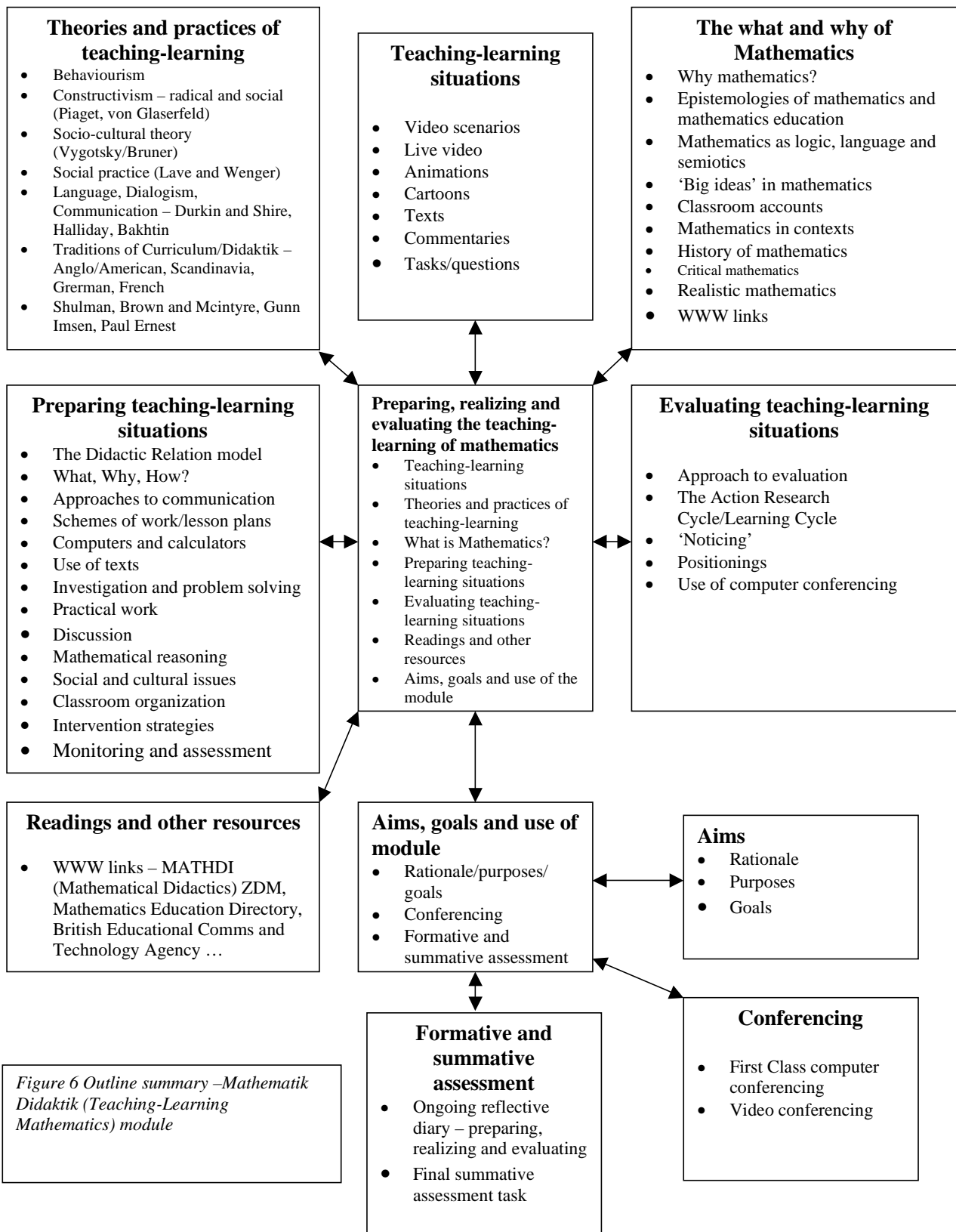


Figure 6 Outline summary –Mathematik Didaktik (Teaching-Learning Mathematics) module

operative learning processes. Accordingly scenarios of teaching-learning situations are to be developed using texts, animations and video etc. It is intended that these will set the contexts, present the problematic nature of the teaching-learning situation and act as a catalyst for raising problem questions with a view to fostering discussion, further research and background reading. Further research will be facilitated in the Web-based environment by the use, amongst other sites, of the MATHDI (MATHematical DIDactics) database. This is a highly comprehensive database of

research on mathematical didactics, developed by the Zentralblatt für Didaktik der Mathematik in co-operation with the European Mathematical Society. In addition further background study will be facilitated readings and guided readings around theories and practices of teaching-learning and the nature of mathematics. Discussion and co-operative learning processes will be fostered via the use of computer and video conferencing.

References

- BATESON, G. (1972) *Steps to an Ecology of the Mind*, New York: Ballantine Books.
- BIEHLER, R. et al. (eds.) (1994) *Didactics of Mathematics as a Scientific Discipline*, Dordrecht: Kluwer Academic Publishers.
- BISHOP, A. (1991) *Mathematical Enculturation. A Cultural Perspective on Mathematics Education*, Dordrecht: Kluwer Academic Publishers.
- BJØRNDAL, B. and S. LIEBERG (1978) *Nye veier i didaktikken?* Oslo: Aschehoug.
- BOURDIEU, P. (1977) *Outline of a Theory of Practice*, Cambridge: Cambridge University Press.
- BRETEIG, T. and R. VENHEIM (1998) *Matematikk for lærere*, Oslo: TANO Aschehoug.
- BUCHBERGER, F. and BUCHBERGER, I. (1999) Didaktik/Fachdidaktik as Science(-s) of the teaching profession? Chapter 5 of this volume.
- DAVYDOV, V. and A. MARKOVA (1982–83) A Concept of Educational Activity for Schoolchildren, *Soviet Psychology*, 21.
- DEWEY, J. (1916) *Democracy and Education*, New York: The Macmillan Comp.
- FAUX, G. (1998) What are the big ideas in mathematics? *Mathematics Teaching*, 163, June 1998, 12–18.
- FREUDENTHAL, H. (1978) *Weeding and Sowing: Preface to a Science of Mathematical Education*, D. Reidel Pubs, Dordrecht.
- GATTEGNO, C. (1987) *The Science of Education, part 1: theoretical considerations*, Educational Solutions, Reading.
- GIDDENS, A. (1984) *The Constitution of Society: Outline of the Theory of Structuration*, Berkeley, CA: University of California Press.
- GJONE, G. (1998) Innføring i fagdidaktikk (Introduction to Fagdidaktikk). In TUFTELAND, G. (ed.) 1998, *Matematikk1 for Allmennlærerutdanningen (Mathematics 1 for pre-service teachers)*, Universitetsforlaget AS Oslo.
- HABERMAS, J. (1981) *Theorie des kommunikativen Handelns*, (Vol. 2) Frankfurt am Main: Suhrkamp.
- HERBJØRNSSEN, O. (1998) *Rom, form og tall. Matematikkdidaktikk for barnetrinnet*, Oslo: Tano. Aschehoug.
- HUDSON, B. (1999a) Seeking connections between different perspectives on teacher education: in support of a science of the teaching profession. Chapter 3 of this volume.
- HUDSON, B. (1999b) Envisioning Social Justice in Teacher Education and Equalizing Opportunity in the Classroom, *Philosophy of Mathematics Education Journal*, March 1999, <http://www.exac.uk/~PERnest/>
- IMSEN, G. (1997) *Lærerenes verden. Innføring i generell didaktikk*, Oslo: Tano Aschehoug.
- IMSEN, G. (9) Reflection as a bridging concept between normative and descriptive approaches to didactics. Chapter 7 of this volume.
- JAWORSKI, B. and POTARI, D. (1998) Characterising mathematics teaching using the teaching triad, Proceedings of *PME 22: International Group for the Psychology of Mathematics Education Conference*, University of Stellenbosch, South Africa, July 1998, 3, 88–95.
- LEONT'EV, A. N. (1978) *Activity, Consciousness and Personality*, Englewood Cliffs, NJ: Prentice Hall.
- MELLIN-OLSEN, S. (1987) *The Politics of Mathematics Education*, D. Reidel Pubs, Dordrecht.
- NYGAARD, O. et al. (1998) *Aha. Matematikk og matematikkdidaktikk*, Kristiansand: HøgskoleForlaget.
- MASON, J. (1994) Researching from the inside in mathematics education – locating the I-You relationship, Proceedings of *PME 18: International Group for the Psychology of Mathematics Education Conference*, University of Lisbon, Portugal, July/August 1994, 1, 176–193.
- ONGSTAD, S. (1999) Sources of 'didaktization'. On defining disciplines and their '(fag-)didaktik' across borders illustrated with examples from Mother Tongue Education (MTE). Chapter 13 of this volume.
- PEPIN, B. (1999) Epistemologies, beliefs and conceptions of mathematics teaching and learning: the theory, and what is manifested in mathematics teachers' work in England, France and Germany. Chapter 10 of this volume.
- REYNOLDS, D. (1998) *Teacher Effectiveness: Better Teachers, Better Schools*, 1998 TTA Lecture, Teacher Training Agency.
- RUDDUCK, J. and HOPKINS, D. (1985) *Research as a Basis for Teaching – Readings from the Work of Lawrence Stenhouse*, Heinemann, London.
- SOLOMON, Y. and J. O'NEILL (1998) Math as Narrative, *Language and Education*, 12/3:210–221.
- SOLVANG, R. (1986) *Matematikkdidaktikk*, Oslo: NKI-forlaget.
- STRAND, T. and T. KVERNBEKK (1998) *Problems of Educational Models and their Use*, Paper at NFPF Conference, Lahti, Finland. March 1998.

TAHTA, D. (1988) The Science of Education, *Mathematics Teaching*, 125, Dec 1988, 10–13.

TUFTELAND, G. (ed.) (1998), *Matematikk1 for Allmennlærerutdanningen (Mathematics 1 for pre-service teachers)*, Universitetsforlaget AS Oslo.

ULJENS, M. (ed.) (1997) *Didaktik- teori, refleksjon og praktik*, Lund: Studentlitteratur.

UNENGE, J. and J. WYNDHAMN (1986) Matematikdidaktik och klasslärarutbildning. In F. MARTON (ed.) *Fackdidaktik. Volym III. Matematikk. Naturorienterande ämnen*, Lund: Studentlitteratur.

VYGOTSKY, L. S. (1962) *Thought and Language*, MIT Press.

Brian Hudson

School of Education
Sheffield Hallam University
Collegiate Campus
Sheffield S10 2BP, UK
Tel: +44 114 225 2346
Fax: +44 114 225 2339
Email: B.G.Hudson@shu.ac.uk

Sigmund Ongstad

Oslo College
Department of Education
Wergelandsveien 27, N-0167, Oslo
Tel: +47 22 45 21 82
Fax: +47 22 45 21 35
Email: sigmund.ongstad@lu.hioslo.no

Hans-Jorgen Braathe

Oslo College
Department of Education
Wergelandsveien 27, N-0167, Oslo
Tel: +47 22 45 21 74
Fax: +47 22 45 21 35
Email: hansjorgen.braathe@lu.hioslo.no

Birgit Pepin

Centre for Research in Teacher Education
School of Education
The Open University
Walton Hall
Milton Keynes, MK7 6AA, UK
Tel: +44 1908 652438
Fax: +44 1908 654111
Email: b.e.pepin@open.ac.uk