A general compound multirate method for circuit simulation problems

A General Compound Multirate Method for Circuit Simulation Problems

A. Verhoeven[1], A. El Guennouni[2], E.J.W. Ter Maten[3] and R.M.M. Mattheij[1]

[1] Technische Universiteit Eindhoven. e-mail: averhoev@win.tue.nl
[2] Yacht Technology and Philips Reserarch Laboratories, ED&T / AS
[3] Technische Universiteit Eindhoven and Philips Reserarch Laboratories, ED&T / AS

Transient analysis of electrical circuits

Electrical circuits can be modelled by the following Differential Algebraic Equation

$$\frac{d}{dt}\mathbf{q}(t,x)+\mathbf{j}(t,x)=0\,.$$

During the design of Integrated Circuits several times a transient simulation is performed. These simulations give a lot of information, but are also rather expensive. In analog sub-circuits, the exact values of the voltages and currents are important, because they correspond with physical signals. In digital sub-circuits only the logical states are important because all information is stored in digital form. Standard electrical circuits consist 90% of digital sub-circuits. Often the digital sub-circuits are temporarily latent, which means that nearly nothing happens. It would be attractive to use different timegrids, such that the large latent or low-frequent subcircuits need much less work.

Multirate approach

Assume that the describing Differential-Algebraic Equation can be written like

(1) $$\frac{d}{dt}\mathbf{q}_A(y,z)+\mathbf{j}_A(y,z)=0$$

(2) $$\frac{d}{dt}\mathbf{q}_L(y,z)+\mathbf{j}_L(y,z)=0$$

such that $\mathbf{y},\mathbf{z}$ correspond to the active and latent part of $\mathbf{x}$, respectively. A multirate method would integrate (1) and (2) separately with different stepsize H and h. In [1, ?] a lot of possibilities have been summarised. Two well-known multirate methods are the "Fastest first" and the "Slowest first" methods, which use inter- and extrapolation. If the parts (1) and (2) are closely coupled. This can destroy the stability of the multirate method. The stability of the "Slowest first" Euler Backward method can be improved by adding a "compound step". This "General Compound Strategy" has been described in [?]. Instead of only the slow part (2), in algorithm 1 a larger system, consisting of equation (3) and (4), is solved for $\alpha>0$ and $q=\frac{H}{h}$.

ALGORITHM 1. A General Compound (G.C.) Strategy

(3) $$q_A(y_{n+\alpha q},z_n+\alpha(z_{n+q}-z_n))-q_A(y_n,z_n)+\alpha Hj_A(y_{n+\alpha q},z_n+\alpha(z_{n+q}-z_n))=0$$

(4) $$q_L(y_n+\frac{1}{\alpha}(y_{n+\alpha q}-y_n),z_{n+q})-q_L(y_n,z_n)+Hj_L(y_n+\frac{1}{\alpha}(y_{n+\alpha q}-y_n),z_{n+q})=0$$

(5) $$q_A(y_{n+j+1},\hat{z}_{n+j+1})-q_A(y_{n+j},\hat{z}_{n+j})+hj_A(y_{n+j+1},\hat{z}_{n+j+1})=0$$

If $\alpha=1$, the "compound step" is just the result of Euler Backward with a large step H, which is easy to implement. If $\alpha=\frac{1}{q}$, the solutions z_{n+q} and y_{n+1} are simultaneously calculated. This option correspond to the multirate method described in [1].

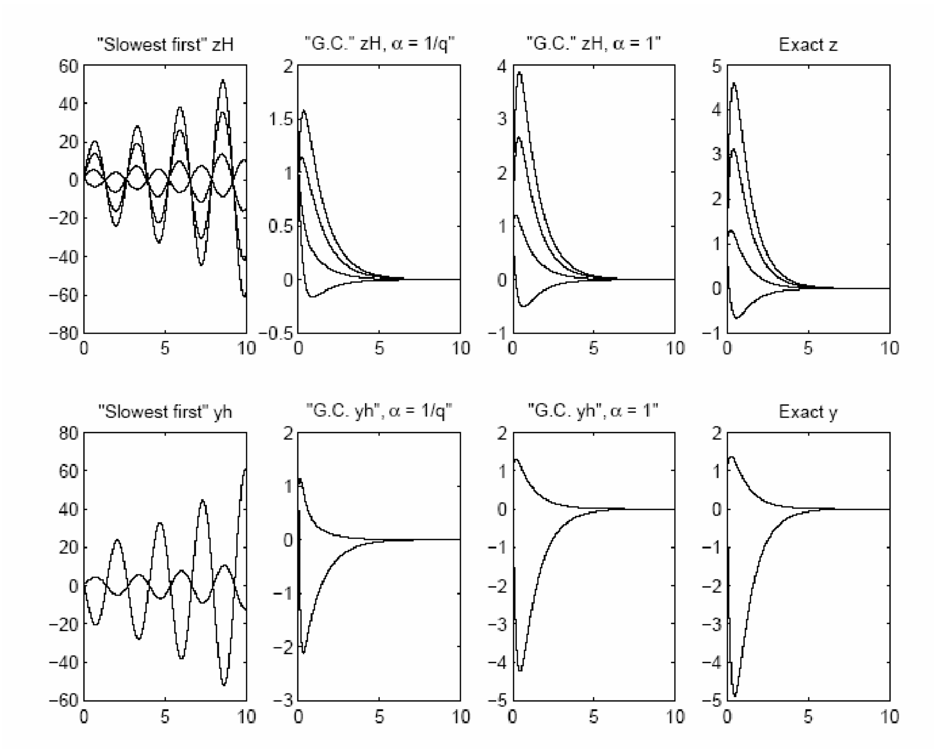

Figure 1. Comparison of stability of several multirate methods.

Stability analysis

For multirate methods, it is not sufficient t consider only the scalar test equation $\dot{y} = \lambda y$. Now, the next two dimensional test equation is considered.

(6)
$$\begin{pmatrix} \dot{y} \\ \dot{z} \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_{\mathbf{A}} \begin{pmatrix} y \\ z \end{pmatrix}$$

It appears that the stability of the multirate methods is not only dependent on the eigenvalues of A. In [2,?], one can find more background about this subject. We derive the following stability conditions for A:

	"Slowest first"	"G.C. (any α)"	"G.C. ($\alpha = 1$)"
Asymptotic:	$a_{12}a_{21} < a_{11}a_{22}$	$a_{12}a_{21} < a_{11}a_{22}$	$a_{12}a_{21} < a_{11}a_{22}$
Sufficient:	$a_{11} < 0$ $a_{22} < 0$ $\lvert a_{12}a_{21}\rvert < \lvert a_{11}a_{22}\rvert$	$a_{11} < 0$ A is stable $\alpha a_{11} + a_{22} < 0$ $-\alpha a_{11}^2 < a_{12}a_{21} < a_{11}a_{22}$	$a_{11} < 0$ A is stable $-a_{11}^2 < a_{12}a_{21} < a_{11}a_{22}$

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Symmetry analysis and exact invariant solutions for the drift-diffusion model of semiconductors.

V. Romano, J.M. Sellier and M. Torrisi.

Dipartimento di Matematica ed Informatica, Università di Catania, viale A.Doria 6, 95125, Catania, Italy. romano@dmi.unict.it; sellier@dmi.unict.it; torrisi@dmi.unict.it

Abstract.The symmetry classification of the drift-diffusion models for semiconductors is performed. Reduced systems and examples of exact invariant solutions are shown.

Introduction

Continuum models for the description of charge carrier transport in semiconductors have attracted in the last years the attention of applied mathematicians and engineers on account of their applications in the design of electron devices. Simple macroscopic models widely used in engineering applications are the drift-diffusion ones. They are based on the assumption of isothermal motion and are constituted by the balance equation for electron density and the Poisson equation for the electric potential. In the bipolar version there are two density balance equations, one for electrons and the other one for holes, coupled through generation-recombination terms.

In these models there is the presence of some arbitrary functions as the mobilities, whose expression is based on fitting of experimental data or Monte Carlo simulations.

A first symmetry analysis has already been performed in "V.Romano and M.Torrisi, Application of weak equivalence trasformations to a group analysis of a drift-diffusion model, J.Phys. A (1999) 32 7953-7963" for a simplified model with the use of weak equivalence classification. Here the most general drift-diffusion unipolar model is investigated for one dimensional problems. A symmetry classification is performed, giving the functional form of the constitutive functions, mobilities and