

Dynamic activity-travel assignment in multi-state supernetworks

Citation for published version (APA):

Liu, P., Liao, F., Huang, H.-J., & Timmermans, H. J. P. (2015). Dynamic activity-travel assignment in multi-state supernetworks. *Transportation Research Procedia*, 7, 24-43. <https://doi.org/10.1016/j.trpro.2015.06.002>

DOI:

[10.1016/j.trpro.2015.06.002](https://doi.org/10.1016/j.trpro.2015.06.002)

Document status and date:

Published: 01/01/2015

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

21st International Symposium on Transportation and Traffic Theory

Dynamic Activity-Travel Assignment in Multi-State Supernetworks

Peng Liu^{a,b}, Feixiong Liao^b, Hai-Jun Huang^{a,*}, Harry Timmermans^b

^a*School of Economics and Management, Beihang University, Beijing 100191, PR China*

^b*Urban Planning Group, Eindhoven University of Technology, Eindhoven PO Box 513, 5600 MB, the Netherlands*

Abstract

The integration of activity-based modeling and dynamic traffic assignment for travel demand analysis has recently attracted ever-increasing attention. However, related studies have limitations either on the integration structure or the number of choice facets being captured. This paper proposes a formulation of dynamic activity-travel assignment (DATA) in the framework of multi-state supernetworks, in which any path through a personalized supernetwork represents a particular activity-travel pattern (ATP) at a high level of spatial and temporal detail. DATA is formulated as a discrete-time dynamic user equilibrium (DUE) problem, which is reformulated as an equivalent variational inequality (VI) problem. A generalized dynamic link disutility function is established with the accommodation of different characteristics of the links in the supernetworks. Flow constraints and non-uniqueness of equilibria are also investigated. In the proposed formulation, the choice of departure time, route, mode, activity sequence, activity and parking location are all unified into one time-dependent ATP choice. As a result, the interdependences among all these choice facets can be readily captured. A solution algorithm based on the route-swapping mechanism is adopted to find the user equilibrium. A numerical example with simulated scenarios is provided to demonstrate the advantages of the proposed approach.

© 2015 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Selection and peer-review under responsibility of Kobe University

Keywords: dynamic activity-travel assignment; multi-state supernetwork; activity-travel link/path disutility

1. Introduction

It has been widely recognized in transportation research that travel is the demand derived from conducting activities at the destinations. Many issues in transportation planning and traffic management require prediction of

* Corresponding author. Tel.: +86-10-82313411; fax: +86-10-82317785.
E-mail address: haijunhuang@buaa.edu.cn

travel demand for various social and economic activities as input. Thus, instead of focusing only on trips and transport networks, travel demand analysis should also examine why, where and when people engage in activities, and how activity engagement is related to the spatial and institutional organization of an urban area (Huang and Lam 2005; Rasouli and Timmermans, 2014). Because the activity-based modeling (ABM) paradigm of travel demand analysis better complies with this condition, ABM has become the dominant approach to travel demand forecasting in academic research (Axhausen and Gärling, 1992; Kitamura et al., 1996; McNally, 2000; Shiftan and Ben-Akiva, 2011; Pinjari and Bhat, 2011). Over the past decades, a great number of activity-based models have been proposed to address specific choice facets of travel behavior, such as mode choice, route choice and location choice, etc., (e.g., Bos, 2004; Prato, 2009; Horni et al., 2009; Arentze and Molin, 2013). More importantly, comprehensive activity-based travel demand forecasting systems have been developed to avoid weaknesses of conventional trip-based (4-step) models (e.g., Bowman and Ben-Akiva, 2001; TASHA, Miller and Roodra, 2003; ALBATROSS, Arentze and Timmermans, 2004a; CEMDAP, Bhat et al., 2004; MATSIM, Balmer et al., 2006). However, these systems need external traffic assignment models to converge to a state that no traveler wants to change his/her routes unilaterally nor change his/her activity-travel schedule. Recently, Lin et al., (2008) and Auld and Mohammadian (2009) have suggested feedback mechanisms between an activity-based model and a traffic assignment model. At a series of time intervals, the time-dependent activity-travel demand is fed into a traffic flow model. The predicted travel times are then returned to the activity-based model, which will reschedule activities. This process is repeated until a state of equilibrium is reached. Although this integration structure has improved the classic combination of these models, a fundamental limitation of such coupling is that behavioral principles underlying the activity-based model of travel demand are not necessarily consistent with assumptions underlying the assignment algorithms.

The shift of analysis from trips to activities has also been witnessed in a few traffic assignment models that combined multiple choice facets of implementing activities. These models offer a more realistic depiction of dynamic travel choice behavior and more compatible solutions than the traditional dynamic traffic assignment (DTA) models. Lam and Yin (2001) were the first to present a conceptual activity-based and time-dependent traffic assignment model. In their model, activity location choice is presented as a multinomial logit model and route choice is described as a dynamic user equilibrium (DUE) condition. Abdelghany and Mahmassani (2003) developed a stochastic DUE model to capture departure time, activity sequence and path choice. Lam and Huang (2002, 2003) proposed a combined activity/travel choice model for capacity-constrained networks, in which choice of route and departure time is modeled simultaneously. Zhang et al. (2005) established an equilibrium model considering path choice and congestion state based on the bottleneck model; however, the dynamic link travel time was simplified. As to the representation of travel behavior, these studies have only looked at specific combinations of choice facets but with less focus on behavioral realism. Overall, ABM and DTA are loosely coupled in the literature: in particular, (1) assignment models mostly focus on traffic and largely ignore or simplify activity program implementation; (2) the level of choice detail in time and/or space is low; (3) time window constraints at activity locations are often ignored in DTA; and (4) activity durations are generally assigned rather than treated as a choice.

In recent years, supernetwork representations have sparked interests in modeling multimodal travel choice problems in an integrated framework (e.g., Nagurney et al., 2002, 2003; Carlier et al., 2003). Although there is no essential difference between a supernetwork and a normal traffic network, supernetworks are capable of representing the transition and interactions between multiple modes. Different choices are turned to indifferent path choice through the constructed supernetwork (Nagurney et al., 2003). Therefore, supernetwork representations offer a natural basis for activity-travel assignment, which has the potential to solve the first two limitations aforementioned by default. Meanwhile, in the temporal dimension, time window constraints and activity duration also can be modeled by treating time as a continuum along the full ATPs. This feature allows addressing the latter two limitations in a consistent manner. However, integrated time-dependent activity-travel assignment models based on supernetworks are infrequent in the literature. Ramadurai and Ukkusuri (2010, 2011) proposed a unified framework termed *activity-travel networks* to model activity location, activity duration, and route choices simultaneously. In their model, dynamic traffic flow is captured by a cell-based transmission model, and an efficient algorithm for DUE assignment is proposed. Ouyang et al. (2011) proposed a model for solving the daily activity-travel pattern (ATP) scheduling problem by constructing an expanded time-space network. Fu and Lam (2014) purposed an activity-based network equilibrium model under uncertainties for scheduling daily activity-travel patterns in multi-modal transit networks. Nevertheless, most of these studies ignored transfer behaviors between

private vehicles and public transport; besides, time-dependent travel demand, activity sequence and duration are more or less exogenous.

Whereas, a multi-state supernetwork framework (Arentze and Timmermans, 2004b; Liao et al., 2010, 2011, 2012, 2013) has been advanced recently for modeling individual activity-travel scheduling decisions because multiple choice facets can be modeled simultaneously at a high level of detail and in a unified fashion. In the supernetworks, nodes denote real locations in space and any link is either a travel link, which always causes a change of location, or a transition link, which never causes a change of location but a change of mode or activity state. Each link represents a traveler's specific action such as walking, cycling, driving, parking or picking-up a car, boarding or alighting a bus or train, conducting a specific activity, etc. It should be noted that public transport network also can be presented in a normal network, either in the form of time-dependent network or time-expanded network (Pyrga et al., 2008). Multi-state supernetworks allow representing multi-modal travel consistently because the choice of parking is explicitly captured. Considering the start and end point of a supernetwork as a virtual O-D pair, any path (from O to D) represents a feasible ATP, expressing the choice of mode, route, parking and activity location, and sequence of activities. The detailed representation of ATPs provides a potential avenue for dynamic activity-travel assignment at a high level of detail with the accommodation of interdependences along the trip chains.

Therefore, the purpose of this study is to investigate the activity-travel behavior including departure time/traffic route/transport mode/activity location/activity sequence choices, and thus to understand these dynamic effects in the multi-state supernetworks accordingly. Essentially, the ultimate aim of this research is to predict the temporal and spatial distribution activity-travel patterns for transportation planning and traffic management purposes. Following this direction, in this paper, we present a dynamic activity-travel assignment (DATA) model in the multi-state supernetwork framework, which combines ABM and DTA in the strongest sense. The DATA model as an integrated formulation provides a viable channel for modeling dynamic equilibrium with multiple dimensional choices in the ABM paradigm.

To that end, the remainder of this paper is organized as follows. Section 2 presents the basic considerations and the formulation of DATA using DUE conditions in multi-state supernetworks. Subsequently, the equilibrium conditions are transformed into an equivalent path-based VI problem (all relevant notations are listed below). Section 3 discusses the mechanisms to calculate the link travel times in a dynamic environment. It also formulates the activity-travel link disutility functions with a focus on the properties of various links in the supernetworks. Section 4 presents essential flow constraints and adopts a modified iterative algorithm to find the equilibrium state. Section 5 illustrates the proposed model and algorithm with a numerical example of simulated scenarios. Finally, a discussion and some concluding remarks are provided.

Notation

SNK (N, L)	a supernetwork
N	the set of nodes in the supernetwork
L	the set of links in the supernetwork
H	the set of home locations
\underline{H}	the total number of home locations
A	the set of activities
\underline{A}	the total number of activities
I	the set of travelers' classes
\underline{I}	the total number of travelers' classes
K	the set of time intervals
\underline{K}	the total number of intervals
$[0, T]$	the interested time period
l	a link in supernetwork
h	a home location (node)
a	an activity
p, q	an ATP in supernetwork
k, k'	a time interval ($k, k' \in K$)

j, j'	a time instance ($j, j' \in [1, \underline{K}]$)
i	a class of travelers
P_h	the set of feasible ATPs from h
F^{hi}	the total number of travelers class i at h
$L_{a(l)}$	the set of links conducting the same activity as that of l
$f_p^{hi}(k)$	the inflow of i entering p from h during k
\mathbf{f}	a vector of all ATP inflows
$u_{lk}^{hip}(j)$	the inflow of i on l at j which departs from h entering p during k
$v_{lk}^{hip}(j)$	the outflow of i on l at j which departs from h entering p during k
$u_l(k)$	the inflow of l during k
\mathbf{u}_l	an inflow vector of l
$v_l(k)$	the outflow of l during k
$X_l(k)$	the number of travelers on l during k
$t_l^{iv}(j)$	the waiting time for i arriving at l at j
$t_l^{id}(j)$	the duration for i arriving at l at j
$t_l^{id}(k)$	the duration for i arriving at non-activity link l during k
$w_l^j(\cdot)$	the waiting disutility function for i on l
$g_l^j(\cdot)$	the (dis)utility function of actual duration for i on (non-)activity link l
$y_l^j(j)$	the (dis)utility of actual duration for i arriving at (non-)activity link l at j
y_l^j	the ideal (dis)utility of i on (non-)activity link l
$disU_p^{hi}(k)$	the disutility incurred by i entering p from h during k
$\mathbf{disU}(\mathbf{f})$	a vector of all ATP disutilities
$disU_{\min}^{hi}(k)$	$= \min \{ disU_p^{hi}(k) : \forall p \in P_h, k \in K \}$
PK^{hi}	$= \{ (p, k) : disU_p^{hi}(k) = disU_{\min}^{hi}(k), \forall p \in P_h, k \in K \}$
$disU_l^i(j)$	the disutility incurred by i arriving at l at j
$disU_l^{iw}(j)$	the waiting disutility incurred by i arriving at l at j
$disU_l^{id}(j)$	the duration disutility incurred by i arriving at l at j
d_l^i	the ideal duration of i on l
t_l^0	the free flow travel time for non-activity link l
c_l	the capacity of non-activity link l
n	$= \underline{L} \cdot \underline{K} \cdot \sum_{h \in H} P_h $
$[o_l, e_l]$	the time window of link l
β_l^{iw}	the disutility of unit waiting time for i on l
β_l^{id}	the (dis)utility of unit duration for i on (non-)activity link l
$\delta_{lk}^{hip}(j)$	the 0-1 indicator variable
η_l, γ_l	the parameters of travel time function for non-activity link l
ε	a convergence tolerance
μ, ζ	the parameters of the algorithm
ρ	a scale parameter
τ	an iteration index
ω	the interval length such that $\omega \cdot \underline{K} = T$
Ω	the set of vectors of feasible ATP inflows

2. Dynamic activity-travel assignment (DATA) model

2.1. Describing of the DATA problem

We first consider an activity-based multi-state supernetwork SNK (N, L) composed of a finite set of nodes, N , and a finite set of directed links, L . Let l denote a link in SNK, belonging to one of two basic link types, i.e., activity links (e.g., working and shopping) and non-activity links (traffic-related, such as parking, picking-up, private vehicle

road, public transport road) are provided. The set of home locations is represented by $H, H=\{h: h=1, \dots, H\}$ ($H \subset N$). Let a, A denote an activity and a set of activities respectively, i.e., $A=\{a: a=1, \dots, A\}$. Travelers can choose an activity sequence form A . Logically, any path from one point to another in SNK defines an activity-travel pattern, expressing the specific choice of traffic mode/route, parking/activity location, and activity sequence.

In order to formulate home-based daily activity-travel, a home location is both the origin and the destination of any feasible ATP. As shown in Fig. 1, h and h' denote an O-D pair, which refers to the same location, i.e., home. The ATP formed by the bold links shows that the traveler leaves home by car to conduct an activity at a_1 with parking at r_2 , then returns home and switches to bike to conduct another activity at a_2 with parking at r_4 , and finally returns home. PVNs and PTNs are personalized private vehicle networks and public transport networks, respectively. Therefore, the links interconnecting PVN and PTN represent parking/picking-up private vehicles, and those interconnecting PTN and PTN represent conducted activities (see Liao et al., 2010, 2013 for a detailed explanation). In this activity-based multi-state supernetwork, there exists at least one ATP from h to h' . Let p denote an ATP which is simply an ordered set of links in SNK, i.e., $\{l_1, l_2, \dots, l_m\}$. Let P_h denote the set of all feasible ATPs based on home h . We further assume that travelers are heterogeneous, and let $I, I=\{i: i=1, \dots, I\}$, be the set of travelers' classes. All travelers choose their optimal path from all feasible ATPs to maximize (minimize) the activity-travel (dis)utility. As a result, travelers are assigned in SNK. Similar to DTA in traffic network, DATA describes the temporal and spatial distribution of activity-travel flow in the dynamic supernetwork.

The time period T of interest is discretized to a finite set of time intervals, $K=\{k: k=1, \dots, K\}$. Let ω be the interval length such that $\omega \cdot K=T$. Furthermore, assume that the inflow to a specific ATP occurs during a time interval. Note that we do not deal with continuous time variation in this paper since the model will finally be implemented on computers on the basis of time slices. The study time period $[0, T]$ is chosen to be large enough such that it can cover all possible daily activity-travel patterns in SNK. On the other hand, the value of ω is chosen to be small enough so that the proposed discrete-time ATP/departure time choice model for DATA is close to its continuous-time counterpart.

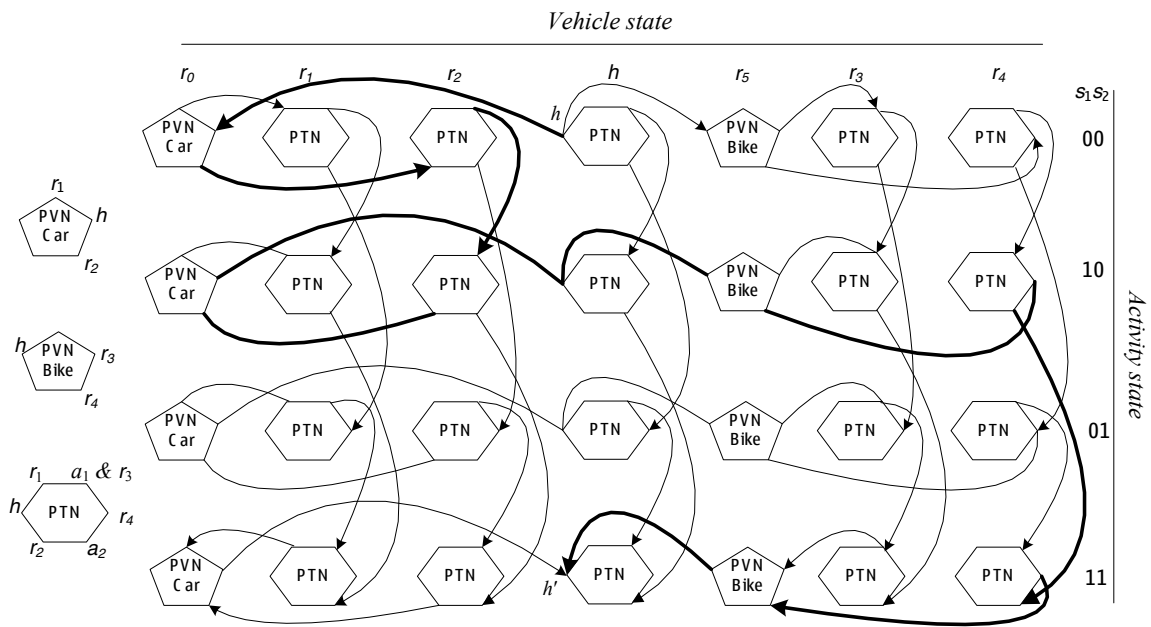


Fig. 1. Multi-state supernetwork representation.

The discrete-time DUE model for DATA problem can be expressed by finding $f_p^{hi*}(k)$ such that the following conditions hold:

$$disU_p^{hi*}(k) \begin{cases} = disU_{\min}^{hi}(k) & \text{if } f_p^{hi*}(k) > 0 \\ \geq disU_{\min}^{hi}(k) & \text{if } f_p^{hi*}(k) = 0 \end{cases} \quad \forall h \in H, i \in I, p \in P_h, k \in K, \quad (1)$$

$$\sum_{p \in P_h} \sum_{k \in K} f_p^{hi*}(k) = F^{hi} \quad \forall h \in H, i \in I, \quad (2)$$

$$f_p^{hi*}(k) \geq 0 \quad \forall h \in H, i \in I, p \in P_h, k \in K, \quad (3)$$

where $f_p^{hi*}(k)$ is the equilibrium inflow on ATP p of i that enters SNK from h during k , $disU_p^{hi*}(k)$ is the equilibrium activity-travel disutility incurred by i departing from h and choosing p during k , $disU_{\min}^{hi}(k)$ is the corresponding minimum disutility for i from h , and F^{hi} is the total number of i at h .

Eq. (1) is well known that, at an equilibrium state, for each i at h only those time-dependent ATPs (i.e., activity-travel path with specific departure time) that have the minimal disutility are used, and those time-dependent ATPs that are not used should have disutility that are higher than or equal to the minimal disutility. Eq. (2) represents the flow conservation constraints and Eq. (3) states the non-negativity conditions. Eqs. (1)-(3) establish an integrated model to formulate the DATA problem and obtain equilibrium across multiple dimensions of activity-travel choices.

2.2. Equivalent path-based VI model

The discrete-time DUE can be expressed as a path-based VI problem. Chen and Hsueh (1998) verified the equivalence between DUE conditions and VI formulation for route choice in the traffic network. Herein, we provide a rigorous proof for time-dependent ATP choice in the multi-state supernetwork framework. We derive below the necessary and sufficient conditions of the equivalence between equilibrium conditions Eqs. (1)-(3) and path-based VI problem separately.

Proof of necessity. We first rearrange the equilibrium condition Eq. (1) as follows:

$$\left(disU_p^{hi*}(k) - disU_{\min}^{hi}(k) \right) \cdot \left(f_p^{hi}(k) - f_p^{hi*}(k) \right) \geq 0 \quad \forall h \in H, i \in I, p \in P_h, k \in K. \quad (4)$$

Adding up the summation over h, i, p, k yields:

$$\sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} disU_p^{hi*}(k) \cdot \left(f_p^{hi}(k) - f_p^{hi*}(k) \right) \geq disU_{\min}^{hi}(k) \cdot \sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} \left(f_p^{hi}(k) - f_p^{hi*}(k) \right). \quad (5)$$

By making substituting $\sum_{p \in P_h} \sum_{k \in K} f_p^{hi}(k) = \sum_{p \in P_h} \sum_{k \in K} f_p^{hi*}(k) = F^{hi}$, the right hand side of the inequality equals to zero invariably, and the remaining results in the following VI formulation:

$$\sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} disU_p^{hi*}(k) \cdot \left(f_p^{hi}(k) - f_p^{hi*}(k) \right) \geq 0. \quad (6)$$

Eq. (6) is equivalent to finding a vector $\mathbf{f}^* \in \Omega$ such that the following finite-dimensional VI holds:

$$\mathbf{disU}(\mathbf{f}^*) \cdot (\mathbf{f} - \mathbf{f}^*) \geq 0 \quad \forall \mathbf{f} \in \Omega, \quad (7)$$

where \mathbf{f} is the vector of all time-dependent ATP inflows, $\mathbf{f}=(\dots, f_p^{hi}(k), \dots)^T$ with $n (=I \cdot \underline{K} \cdot \sum_{h \in H} |P_h|)$ elements, $\mathbf{disU}(\mathbf{f}^*)$ is the corresponding vector of all time-dependent ATP disutilities, $\mathbf{disU}(\mathbf{f}^*)=(\dots, disU_p^{hi^*}(k), \dots)$. Ω is a compact closed convex set ($\Omega \subset \mathbb{R}_+^n$), represented by:

$$\Omega = \left\{ \mathbf{f} : \sum_{p \in P_h} \sum_{k \in K} f_p^{hi}(k) = F^{hi} \text{ and } f_p^{hi}(k) \geq 0 \quad \forall h \in H, i \in I \right\}. \quad (8)$$

Proof of sufficiency. First, suppose \mathbf{f}^* is a solution of VI problem Eqs. (7)-(8). For any h, i , if $\underline{K} \cdot |P_h|=1$, there exists only one feasible path inflow; moreover, $f_p^{hi^*}(k) > 0$ and $disU_p^{hi^*}(k) = disU_{\min}^{hi}(k)$ always hold. Otherwise, if $\underline{K} \cdot |P_h| > 1$, there exists more than one feasible path inflow vector. Without loss of generality, we construct a feasible path inflow vector \mathbf{f} , which is identical to \mathbf{f}^* except for two elements, i.e., $f_p^{hi^*}(k)$ and $f_q^{hi^*}(k')$; thus, at least one of them is positive. Suppose $f_p^{hi^*}(k) > 0$, and take $f_p^{hi}(k) = f_p^{hi^*}(k) - \sigma$ and $f_q^{hi}(k') = f_q^{hi^*}(k') + \sigma$, with $0 < \sigma \leq f_p^{hi^*}(k)$. Substituting \mathbf{f} into Eq. (6) yields:

$$disU_p^{hi^*}(k) \cdot (f_p^{hi}(k) - f_p^{hi^*}(k)) + disU_q^{hi^*}(k') \cdot (f_q^{hi}(k') - f_q^{hi^*}(k')) \geq 0. \quad (9)$$

By using $f_p^{hi}(k) = f_p^{hi^*}(k) - \sigma$ and $f_q^{hi}(k') = f_q^{hi^*}(k') + \sigma$, we have:

$$disU_q^{hi^*}(k') \geq disU_p^{hi^*}(k). \quad (10)$$

If $f_q^{hi^*}(k') = 0$, repeating this procedure for any other h, i, p, k , we know that for any h, i , all unused time-dependent ATPs have disutilities that are higher than or equal to the minimal disutility.

If $f_q^{hi^*}(k') > 0$, take $f_p^{hi}(k) = f_p^{hi^*}(k) + \sigma$ and $f_q^{hi}(k') = f_q^{hi^*}(k') - \sigma$, with $0 < \sigma \leq f_q^{hi^*}(k')$. Similarly, we have:

$$disU_p^{hi^*}(k) \geq disU_q^{hi^*}(k'). \quad (11)$$

Synthesizing Eqs. (10) and (11), we have:

$$disU_p^{hi^*}(k) = disU_q^{hi^*}(k'). \quad (12)$$

Repeating this procedure for any other h, i, p, k , we know for any h, i , all used time-dependent ATPs have identical disutilities, which equal to the minimal disutility. Now, we have proven that a solution of the VI problem Eqs. (7)-(8) implies that the equilibrium conditions Eqs. (1)-(3) hold. This completes the proof. \square

3. Activity-travel link disutility

3.1. The disutility of waiting

In SNK, the activity-travel link disutility is assumed to be time-dependent and heterogeneous. *Time-dependent* means that the link disutility varies with the arrival time of the entry link; *heterogeneous* refers to the traveler's personalized preferences for different links. However, to the best of our knowledge, no consensus exists on the specification of the dynamic activity-travel link disutility in the literature. Existing studies tend to overlook one facet of time dimension in activity or non-activity links with the presence of time window constraints. During the traverse of a link, the times of arrival, entry and departure can be viewed as three different facets, arranged in order of time of occurrence. The gap between arrival and entry times is defined as waiting time caused by arriving earlier than the start of the link's time window. The gap between entry and departure times is described as duration. In this paper,

we propose a generalized function of activity-travel link disutility incurred by class i arriving at link l at time j as follows:

$$disU_l^i(j) = disU_l^{iw}(j) + disU_l^{id}(j) \quad \forall i \in I, l \in L, j \in [1, \underline{K}]. \tag{13}$$

As shown in Eq. (13), link disutility $disU_l^i(j)$ is divided into two parts i.e., $disU_l^{iw}(j)$ and $disU_l^{id}(j)$, which denote the disutility of waiting and duration, incurred by i arriving at l at j . Waiting disutility captures the adverse experience of waiting for entering a link. For activity link l , duration disutility describes the gap between the ideal utility (maximum utility that class i can gain from conducting the same activity with l) and the utility with the actual duration on l ; while for non-activity link l , it describes the gap between the ideal disutility (is set to zero) and the disutility with the actual duration on l . The purpose of this maneuver is to model activity and non-activity links in a unified fashion as conducting activity normally generates utility while travel produces disutility. It should be noted that the link inflows is not required during integral interval. Therefore, the link arrival time j can be a non-integer (due to the non-integral actual duration of previous links), and set $j \in [1, \underline{K}]$ without loss of generality.

Firstly, we define a generalized function to calculate the disutility of waiting as follows:

$$disU_l^{iw}(j) = w_l^i(t_l^{iw}(j)) \quad \forall i \in I, l \in L, j \in [1, \underline{K}], \tag{14}$$

where $t_l^{iw}(j)$ denotes the waiting time of i arriving at l at j , $w_l^i(x)$ is the waiting disutility function for i on l . Furthermore, we suppose $w_l^i(x)$ satisfies some basic properties:

- (i) $w_l^i(0) = 0$;
- (ii) $\lim_{x' \rightarrow x} w_l^i(x') = w_l^i(x)$;
- (iii) $\langle w_l^i(x') - w_l^i(x), x' - x \rangle > 0$.

The above properties signify that waiting disutility $disU_l^{iw}(j)$ is continuous and increasing with the length of waiting time $t_l^{iw}(j)$, and there is no waiting disutility when waiting time is zero.

Next, we model waiting time function. Above all, let $[o_l, e_l]$ denote a continuous time window of link l , satisfying $[o_l, e_l] \subseteq [1, \underline{K}]$. Logically, under the condition of rational traveler hypothesis, a traveler's arrival time j on link l cannot be later than e_l but may be earlier than o_l . If arrival time j is earlier than o_l , it must be associated with a waiting time i.e., $o_l - j$; otherwise, no waiting involved. For activity link l , time window $[o_l, e_l]$ represent its opening/closing hours. Other links (e.g., travel links of PVN) are assumed to have no restrictive time window, and travelers can traverse these links at any time during period $[1, \underline{K}]$ without waiting. Thus, we suppose the period $[1, \underline{K}]$ defines the time window for these links, i.e., $[o_l, e_l] = [1, \underline{K}]$. Consequently, the arrival time j at these non-activity links always satisfies $j \geq o_l$, and waiting time is zero. Without loss of generality, waiting time can be expressed as:

$$t_l^{iw}(j) = \max \{0, o_l - j\} \quad \forall i \in I, l \in L, j \in [1, \underline{K}]. \tag{15}$$

3.2. link duration disutility

As mentioned above, $disU_l^{id}(j)$ is defined as the gap between the ideal (dis)utility y_l^i and the (dis)utility of the actual duration for i arriving at (non-)activity link l at j , i.e., $y_l^i(j)$. Consequently, $disU_l^{id}(j)$ can be calculated by:

$$disU_l^{id}(j) = |y_l^i - y_l^i(j)| \quad \forall i \in I, l \in L, j \in [1, \underline{K}]. \tag{16}$$

We define a generalized function to formulate the actual (dis)utility $y_l^i(j)$ as follows:

$$y_l^i(j) = g_l^i(t_l^{id}(j)) \quad \forall i \in I, l \in L, j \in [1, \underline{K}], \tag{17}$$

where $t_l^{id}(j)$ denotes the actual duration of i arriving at l at j , $g_l^i(x)$ is the corresponding (dis)utility function of i on (non-)activity link l . Furthermore, we suppose $g_l^i(x)$ satisfies some basic properties, which are given below:

- (iv) $g_l^i(0) = 0$;
- (v) $\lim_{x' \rightarrow x} g_l^i(x') = g_l^i(x)$;
- (vi) $\langle g_l^i(x') - g_l^i(x), x' - x \rangle > 0$.

The above properties signify that there is zero (dis)utility when actual duration on (non-)activity link is zero, and $y_l^i(j)$ is continuous and increasing with the length of the actual duration.

On the other hand, for activity link l , the ideal utility y_l^i can be calculated by:

$$y_l^i = \max \left\{ y_m^i(j) : \forall m \in L_{a(l)}, j \in [1, \underline{K}] \right\} \quad \forall i \in I, \tag{18}$$

where $L_{a(l)}$ is the set of links conducting the same activity as that of l . Obviously, for activity link, actual utility is always no more than ideal utility, satisfying $y_l^i - y_l^i(j) \geq 0$. If l is non-activity link, ideal disutility satisfies $y_l^i = 0$. Therefore, for non-activity link, actual disutility is always more than ideal disutility, satisfying $y_l^i - y_l^i(j) < 0$.

In the following part, suppose that each class of travelers has a heterogeneous ideal duration for each link. Let d_l^i be the ideal duration of i on l . We formulate duration on activity links and non-activity links, respectively. Firstly, duration on activity link is flexible. If a traveler arriving at l at j , the entry time on l is $\max\{o_l, j\}$ and the total length of feasible duration is $e_l - \max\{o_l, j\}$. If feasible duration is long enough (not shorter than the ideal duration), behaviorally, traveler will choose ideal duration d_l^i as the actual duration; otherwise, when feasible duration fell short of ideal duration, traveler will choose the one as long as possible to complete the activity, i.e., $e_l - \max\{o_l, j\}$. Let $t_l^{id}(j)$ denote the actual duration of i arriving at l at j . Consequently, actual duration $t_l^{id}(j)$ can be calculated by:

$$t_l^{id}(j) = \min \left\{ e_l - \max\{o_l, j\}, d_l^i \right\} \quad \forall i \in I, j \in [1, \underline{K}]. \tag{19}$$

Secondly, since waiting time of non-activity link is assumed to be zero, the travel time is indicated by the difference between the arrival times of the exit node and the entry node. In this sense, there is no essential difference as to duration between non-activity links and activity links. Therefore, we still use $t_l^{id}(j)$ to denote the duration of i arriving at non-activity link l at j . In addition, suppose ideal duration d_l^i for non-activity link is zero, because for these links (e.g., parking, driving and picking-up), travelers always hope travel time the shorter the better. For non-activity link links, travelers are incapable of choosing duration freely, because the travel times depend on the flow of the links.

The approaches of travel time formulations for PVN can be classified to be based on the queue representations: point queue (e.g., Huang and Lam, 2002) and physical queue (e.g., Han et al., 2011; Ukkusuri et al., 2012). However, non-activity links in SNK not only represent driving on PVN, but also can be parking, picking up or public transit link and so on. Up to now, there is no consensus on the travel time functions of non-activity links, but some widely accepted traffic properties need to be guaranteed, which are given below:

- (vii) $\lim_{\mathbf{u}_l' \rightarrow \mathbf{u}_l} t_l^{id}(j, \mathbf{u}_l') = t_l^{id}(j, \mathbf{u}_l)$;
- (viii) $t_l^{id}(j)$ only depends on $u_l(1), u_l(2), \dots, u_l([j])$;
- (ix) if $j > j'$, then $j + t_l^{id}(j) \geq j' + t_l^{id}(j')$,

where $u_l(k)$ denotes the inflow of l during k and $[j] = \min\{m: j \leq m \text{ and } m \in \mathbb{Z}\}$, \mathbf{u}_l denotes inflows vector of link l , i.e., $\mathbf{u}_l = (\dots, u_l(k), \dots)^T$ with \underline{K} elements. Property (vii) signifies that the non-activity link travel time is continuous with respect to the link inflows vector, respectively. Properties (viii) and (ix) show weak causality and First-in first-out (FIFO), respectively. Weak causality means that travel time for traveler having arrival time j only depends on the inflows during and earlier than interval $[j]$, but not on inflow later than $[j]$. Obviously, weak causality in discrete-time is similar to the causality (suggested by Carey et al., 2003; Szeto and Lo, 2005, et al.) in continuous-

time counterpart, when the value of interval is chosen to be small enough. FIFO (suggested by Huang and Lam, 2002; Szeto and Lo, 2006, et al.) means that the traveler who arrives at a non-activity link earlier will not exit later.

In this paper, for a non-activity link, the travel time (i.e., link actual duration) associated with integral arrival time k is calculated by the following iterative function:

$$t_l^{id}(k) = \max \left\{ t_l^{id}(k-1) + \eta_l (u_l(k)/c_l)^{\gamma_l} - 1, t_l^0 \right\} \quad \forall i \in I, k \in K, \tag{20}$$

where t_l^0 is the free flow travel time of link l , c_l is the capacity of link l , and η_l, γ_l are the corresponding parameters resembling BPR function. Furthermore, we have $t_l^{id}(0) = t_l^0$, and parameters t_l^0, c_l, η_l and γ_l are non-negative.

In this equation, the departure time $k + t_l^{id}(k)$ associated with integral arrival time k is developed to represent the aggregate effects of the departure time associated with arrival time $k-1$, i.e., $k-1 + t_l^{id}(k-1)$ plus delay time due to congestion from the inflow during interval k , i.e., $\eta_l \cdot (u_l(k)/c_l)^{\gamma_l}$. As a supplement, departure time $k + t_l^{id}(k)$ should be no earlier than $k + t_l^0$. Specifically, when η_l and γ_l are both equal to 1, Eq. (20) is equivalent to the bottleneck point-queue travel time function proposed by Huang and Lam (2002). In their formulation, link travel time function is expressed as $t_l^{id}(k) = t_l^0 + X_l(k)/c_l$. Queue $X_l(k)$ is computed by $\max\{X_l(k-1) + u_l(k) - c_l, 0\}$. By rearranging link travel time function, $X_l(k) = c_l \cdot (t_l^{id}(k) - t_l^0)$ and $X_l(k-1) = c_l \cdot (t_l^{id}(k-1) - t_l^0)$ hold. Combining the two equations and queue function, we obtain $t_l^{id}(k) = \max\{t_l^{id}(k-1) + u_l(k)/c_l - 1, t_l^0\}$. Consequently, Eq. (20) is a generalized form of the bottleneck point-queue travel time function. Furthermore, when η_l is close to 0, it implies that there is almost no delay time due to congestion, i.e., public transport link; when γ_l is not equal to 1, it implies that delay time is non-linear.

As mentioned above, the arrival time of a link may not be an integer. For any non-activity link l , if arrival time $j \in [k-1, k]$, we use linear interpolation of adjacent intervals $k-1$ and k to approximate the link travel time as formulated by Long et al. (2013), which can be expressed as follows:

$$t_l^{id}(j) = (k-j) \cdot t_l^{id}(k-1) + (j-k+1) \cdot t_l^{id}(k) \quad \forall i \in I, j \in [1, \underline{K}]. \tag{21}$$

First, we prove that the proposed non-activity link travel time Eqs. (20)-(21) ensure properties (vii) and (viii) hold. According to Eq. (20), $t_l^{id}(k)$ is only associated and continuous with variables $u_l(k)$ and $t_l^{id}(k-1)$, and $t_l^{id}(k-1)$ is only associated and continuous with variables $u_l(k-1)$ and $t_l^{id}(k-2)$, therefore, $t_l^{id}(k)$ is only associated and continuous with variables $u_l(k), u_l(k-1)$, and $t_l^{id}(k-2)$. Repeating this association from interval $k-2$ descending to 1, we know $t_l^{id}(k)$ is only associated and continuous with variables $u_l(1), u_l(2), \dots, u_l(k)$. From Eq. (21), $t_l^{id}(j)$ is only associated and continuous with variables $t_l^{id}(k)$ and $t_l^{id}(k-1)$. Summing up the above, we know that $t_l^{id}(j)$ is associated and continuous with variables $u_l(1), u_l(2), \dots, u_l(k)$, so property (vii) holds. In addition, since variables $u_l(1), u_l(2), \dots, u_l(k)$ is the first k elements of vector \mathbf{u} , property (viii) holds.

Next, we can also prove that the proposed non-activity link travel time Eqs. (20)-(21) ensure property (ix) holds. Suppose $j \in [k-1, k]$ and $j' \in [k'-1, k']$, and then under assumption $j > j'$, we have $k \geq k'$, obviously. If $k = k'$, according to Eqs. (20)-(21), we have:

$$j + t_l^{id}(j) - (j' + t_l^{id}(j')) = (j - j') \cdot (t_l^{id}(k) - t_l^{id}(k-1) + 1) \geq 0. \tag{22}$$

Eq. (22) implies that under assumptions of $j > j'$ and $k = k'$, $j + t_l^{id}(j) \geq j' + t_l^{id}(j')$ holds. If $k > k'$, we have $k-1 \geq k'$. According to Eqs. (20) and (22), we know that:

$$j + t_l^{id}(j) \geq k-1 + t_l^{id}(k-1) \geq k' + t_l^{id}(k') \geq j' + t_l^{id}(j'). \tag{23}$$

Eq. (23) implies that under assumption of $j > j'$ and $k > k'$, $j + t_l^{id}(j) \geq j' + t_l^{id}(j')$ holds. In summary, for any arrival times j and j' , if $j > j'$, $j + t_l^{id}(j) \geq j' + t_l^{id}(j')$ holds.

4. Flow constraints and algorithm

4.1. Flow constraints

In order to formulate how path inflows spread through the dynamic supernetwork, we need to formulate the essential flow constraints. To start with, let $u_{lk}^{hip}(j)$ represent the inflow of i on l at j which departs from h entering p during k . It can be expressed as:

$$u_{lk}^{hip}(j) = \delta_{lk}^{hip}(j) \cdot f_p^{hi}(k) \quad \forall h \in H, i \in I, p \in P_h, l \in L, k \in K, j \in [1, \underline{K}]. \tag{24}$$

It must be emphasized that indicator variables $\delta_{lk}^{hip}(j)$ is equal to 1, if existing i entering p from h during k and arriving at l at j ; otherwise, 0. This is detailed as:

$$\delta_{lk}^{hip}(j) = \begin{cases} 1 & \text{if } k + t_{l_1}^{iw} + t_{l_1}^{id} + t_{l_2}^{iw} + t_{l_2}^{id} \cdots + t_{l_{b-1}}^{iw} + t_{l_{b-1}}^{id} = j \\ 0 & \text{otherwise} \end{cases} \quad \forall l_b \in p, \tag{25}$$

where $t_{l_1}^{iw} = t_{l_1}^{iw}(k)$, $t_{l_1}^{id} = t_{l_1}^{id}(k)$, $t_{l_2}^{iw} = t_{l_2}^{iw}(k + t_{l_1}^{iw}(k) + t_{l_1}^{id}(k))$, $t_{l_2}^{id} = t_{l_2}^{id}(k + t_{l_1}^{iw}(k) + t_{l_1}^{id}(k))$, ..., for short. Eqs. (24)-(25) describe the flow propagation along ATP through the using of the indicator variables.

According to the principle of flow propagation, link inflows and outflows can be substituted for each other naturally. Therefore, the outflow of i on l at $j + t_l^{iw}(j) + t_l^{id}(j)$ which departs from h entering p during k can be expressed as:

$$v_{lk}^{hip}(j + t_l^{iw}(j) + t_l^{id}(j)) = u_{lk}^{hip}(j) \quad \forall h \in H, i \in I, p \in P_h, l \in L, k \in K, j \in [1, \underline{K}]. \tag{26}$$

Eq. (26) depicts the one-to-one relationship among link-specific inflow, outflow and travel time. The similar requirement as that was widely adopted in previous studies (e.g., Chen and Hsueh, 1998; Lam and Yin, 2001). Furthermore, Eq. (26) can guarantee that when travelers belong to the same class, live in same home location and make exactly the same travel choices (i.e., choosing identical time-dependent ATP), they will have the same activity-travel time-space trajectory. Base on $u_{lk}^{hip}(j)$ and $v_{lk}^{hip}(j)$, adding up the summation over h, i, p, k , the inflow and outflow on l during k' , i.e., $u_l(k')$ and $v_l(k')$, can be expressed as:

$$u_l(k') = \sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} \sum_{j \in [k'-1, k']} u_{lk}^{hip}(j) \quad \forall l \in L, k' \in K, \tag{27}$$

$$v_l(k') = \sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} \sum_{j \in [k'-1, k']} v_{lk}^{hip}(j) \quad \forall l \in L, k' \in K. \tag{28}$$

Besides the link inflow and link outflow, the number of travelers on link can be deemed as the third significant variables for describing the states of link flow. It can be calculated by:

$$X_l(k') = \sum_{k \leq k'} u_l(k) - \sum_{k \leq k'} v_l(k) \quad \forall l \in L, k' \in K. \tag{29}$$

Overall, flow constraints are summarized as follows: Eq. (2) represents flow conservation constraint; Eq. (3) represents non-negativity constraint; Eqs. (24)-(25) represent the flow propagation constraints; Eq. (26) represents time-flow consistency constraint; and Eqs. (27)-(29) represent the definitional constraints.

We further assume that time-dependent ATP disutilities are calculated by adding up the actual disutilities on these links along that ATP. Therefore, $disU_p^{hi}(k)$ can be calculated by the following function:

$$disU_p^{hi}(k) = \sum_{l \in p} \sum_{j \in [1, K]} \delta_{lk}^{hip}(j) \cdot disU_l^i(j) \quad \forall h \in H, i \in I, p \in P_h, k \in K. \tag{30}$$

4.2. Solution algorithm

At the beginning of this subsection, we first analyze the continuity and monotonicity of time-dependent path disutilities in terms of \mathbf{f} . This property is very important for discussing the existence and uniqueness of DUE, and can guarantee reasonable iterative adjustments on the path inflows \mathbf{f} in the solution algorithm for DUE.

From Eq. (8), we know Ω is a compact closed convex set with $\Omega \subset \mathbb{R}_+^n$. On the other hand, $\mathbf{disU}(\mathbf{f})$ can be regarded as an n -dimensional vector valued function, which satisfies $\mathbf{disU}(\mathbf{f}): \Omega \rightarrow \mathbb{R}_+^n$. Therefore, if $\mathbf{disU}(\mathbf{f})$ is continuous on Ω , the solution of VI problem Eqs. (7)-(8) will exist (Nagurney, 1998). Continuity is denoted by:

$$\lim_{\mathbf{f}' \rightarrow \mathbf{f}} \mathbf{disU}(\mathbf{f}') \rightarrow \mathbf{disU}(\mathbf{f}) \quad \forall \mathbf{f} \in \Omega. \tag{31}$$

According to Properties (ii), (v), (vii) and Eq. (30), ATP disutilities are continuous with link inflow vectors. If interval length ω is infinite close to zero (i.e., continuous-time), the link inflows will be continuous with path inflows vector \mathbf{f} , and then the existence of the DUE solution can be guaranteed. Unfortunately, the DUE model is formulated and computed on the basis of time slices (ω maybe not small enough), and it cannot undertake that path disutilities $\mathbf{disU}(\mathbf{f})$ are continuous with path inflows vector \mathbf{f} . Therefore, the value of ω should be small enough so that it can ensure the existence of the proposed discrete-time DUE; or at least ensure the computing result by the following algorithm of discrete-time model is very close to the corresponding solution of continuous-time DUE (i.e., within an acceptable tolerance).

If $\mathbf{disU}(\mathbf{f})$ is strictly monotonic on Ω , the solution of VI problem Eqs. (7)-(8) will be unique, if one exists (Nagurney, 1998). Strictly monotonicity is denoted by:

$$\langle \mathbf{disU}(\mathbf{f}') - \mathbf{disU}(\mathbf{f}), \mathbf{f}' - \mathbf{f} \rangle > 0 \quad \forall \mathbf{f}, \mathbf{f}' \in \Omega. \tag{32}$$

Due to the inclusion of 0-1 integer variables $\delta_{lk}^{hip}(j)$, the ATP disutilities calculated by Eq. (30) are non-linear and non-convex. In addition, strict monotonicity of the ATP disutility is highly dependent on the structure of SNK that makes Eq. (32) invalid in general. Therefore, path-based VI problem, Eqs. (7)-(8), may have multiple local solutions. In other words, we cannot guarantee that an algorithm will converge to the same equilibrium starting from different initial points or using different iterative sequences of scale parameters.

In the previous studies of solving DUE problems, Huang and Lam (2002) proposed a algorithm considering departure time choices, based on the route-swapping process suggested by Smith and Wisten (1995) and the route choice adjustment process introduced by Nagurney and Zhang (1997). Szeto and Lo (2006) developed the heuristic route-swapping algorithm to find the solution of tolerance-based DUE problem. This algorithm was adapted to solve activity-travel choices problem by Ramadurai and Ukkusuri (2010). In this paper, we adopts a modified algorithm to solve DATA problem, i.e., the proposed VI model Eqs. (7)-(8).

The basic procedure of the proposed algorithm is: for each traveler class i at home location h , to satisfy the DUE conditions in Eqs. (1)-(3), the inflows of non-minimum disutility time-dependent ATP are swapped to the minimum disutility paths; the swapped volume is proportional to the gap between the time-dependent path disutility and the minimum one; the process is iterated until the stopping criterion is met. The detailed algorithm is presented as follows.

Step 0. Initialization.

Set parameters $\rho > 0, \mu > 0$ and convergence tolerance $\varepsilon > 0$;

choose scale parameters sequence $\{\rho_\tau : \tau = 0, 1, 2, \dots\}$, where $\rho_\tau = \rho/[(\tau+1)\mu]$;
 set iteration index $\tau=0$ and let \mathbf{f}_0 be any feasible inflows vector.

Step 1. Compute ATP disutility.

Compute link waiting time $t_i^{hw}(j)_\tau$ by Eq. (15), and link waiting disutility $disU_i^{hw}(j)_\tau$ by Eq. (14);
 compute link duration $t_i^{id}(j)_\tau$ for activity links by Eq. (19) and for non-activity links by Eqs. (20)-(21);
 compute link duration disutility $disU_i^{id}(j)_\tau$ by Eq. (16);
 compute $disU_i^{id}(j)_\tau$ by Eq. (13) and $disU_p^{hi}(k)_\tau$ by Eq. (30).

Step 2. Check the stopping criterion.

The iteration terminates if

$$\frac{\sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} f_p^{hi}(k)_\tau \cdot (disU_p^{hi}(k)_\tau - disU_{\min}^{hi}(k)_\tau)}{\sum_{h \in H} \sum_{i \in I} \sum_{p \in P_h} \sum_{k \in K} f_p^{hi}(k)_\tau \cdot disU_{\min}^{hi}(k)_\tau} < \varepsilon. \tag{33}$$

Step 3. Update ATP inflow.

$\forall h \in H, i \in I$, find $PK_\tau^{hi} = \{(p, k) : disU_p^{hi}(k)_\tau = disU_{\min}^{hi}(k)_\tau, \forall p \in P_h, k \in K\}$ and then compute

$$f_p^{hi}(k)_{\tau+1} = \begin{cases} \max \{0, f_p^{hi}(k)_\tau - \rho_\tau \cdot f_p^{hi}(k)_\tau \cdot (disU_p^{hi}(k)_\tau - disU_{\min}^{hi}(k)_\tau)\} & \text{if } (p, k) \notin PK_\tau^{hi} \\ f_p^{hi}(k)_\tau + \sum_{(p, k) \notin PK_{hi}} (f_p^{hi}(k)_\tau - f_p^{hi}(k)_{\tau+1}) / |PK_{hi}| & \text{if } (p, k) \in PK_\tau^{hi} \end{cases}, \tag{34}$$

let $\tau = \tau + 1$, and return to step 1.

Generally, the scale parameter sequence $\{\rho_\tau\}$ should be small enough to prevent rebound of convergence indicator, and big enough to avoid the stagnant of the swapping process. Basically, appropriateness of the sequence depends on the structure of the studied networks, and it is difficult to find a general setting. In this algorithm, we take $\rho_\tau = \rho/[(\tau+1)\mu]$. It is apparent that $\lim_{\tau \rightarrow \infty} \rho_\tau = 0, \rho_\tau \cdot f_p^{hi}(k)_\tau = 0$ and $\sum_{\tau} \rho_\tau \cdot f_p^{hi}(k)_\tau = \infty$ hold for all h, i, p and k . Therefore, sequence $\{\mathbf{f}_\tau\}$ generated by this algorithm will converge to a flow pattern even if the ATP disutilities are not monotonic, proven by Huang and Lam (2002). For any h, i , choose $f_p^{hi}(k)_0 = F^{hi} / (K \cdot P_h)$ as the initial ATP inflows in Step 0. This initial inflow vector is simply averaged on all time-dependent ATP, which is also feasible.

In previous studies, Huang and Lam (2002) found that flows assigned discretely to time intervals can lead to the unrealistic fluctuation of flows between time intervals. Szeto and Lo (2006), Ramadurai and Ukkusuri (2010) reported that (dis)utility functions with sudden change would increase the complexity of the algorithm convergence. However, in Step 1, when we calculate time-dependent ATP disutilities, link arrival time j is allowed to be a non-integer, and for traffic links, linear interpolation is utilized to approximate the link travel time for non-integral arrival times. These techniques make transitions of various variables smoother between time intervals, which make the value of variables harder to jump and fall suddenly.

In Step 2, the left hand side of stopping criterion is a gap function (i.e., convergence indicator) measuring how close a solution of the τ -th iteration is to the equilibrium condition Eqs. (1)-(3). If stopping criterion Eq. (33) is met, terminate the procedure and obtain the final solution. Step 3 generates new flow pattern $\mathbf{f}_{\tau+1}$ which deducts flows from the paths with higher disutility than the minimum disutility time-dependent paths and adds these flows onto the minimum time-dependent paths. The sum of the deductions is equal to the sum of the additions so as to ensure the validity of the solution from the swapping procedure. Meanwhile, this shift guarantees that inflows are non-negative.

5. Numerical example

5.1. Basic setting

In this section, we illustrate the proposed DATA model using a simple transportation network depicted in Fig. 2a. It shows that travelers live at home location h , and work and shop in the center of a delimited region. The working place w is connected with h by three directed private vehicle links (links 1 and 2 represent two independent routes from h to w , and link 3, which represents a route from w to h). Shopping locations s_1 and s_2 can be chosen by travelers, where s_1 is connected with w via walking links 4 and 5 having opposite directions; s_2 is connected with h via private vehicle links 6 and 7 having opposite directions. In addition, three parking places r_h, r_w and r_s are located at h, w and s_2 , respectively.

In this example, a traveler can choose the departure time from h (departure time choice), choose route link 1 or link 2 from h to w (route choice), choose shopping location s_1 or s_2 jointly with choosing to drive or walk (joint destination and transport mode choice), and choose the order of working and shopping (activity sequence choice). In a multi-state supernetwork, we can unify these different choices into a single time-dependent ATP choice. Although we only demonstrated the model and solution algorithm in this small-size network, it can be used to illustrate the expected theoretical properties. To illustrate the approach, we first generate all feasible ATPs. Synthesizing the traffic network and activities, we obtain the corresponding activity-based multi-state supernetwork as shown in Fig. 2b. In this SNK, besides traffic links 1-7, other links numbered 8-16 are defined. Links 8, 9 and 10 represent working, shopping at s_1 and shopping at s_2 , respectively; links 10-16 represent picking up or parking at r_h, r_w or r_s , respectively. In order to simplify the formulation and save computing time, $w_l^i(\cdot)$ and $g_l^i(\cdot)$ can be expressed as linear functions without loss of generality. Externalizing Eq. (13), we adopt the following function for formulating activity-travel link disutility:

$$disU_l^i(j) = \beta_l^{iw} \cdot t_l^{iw}(j) + |y_l^i - \beta_l^{id} \cdot t_l^{id}(j)| \quad \forall i \in I, l \in L, j \in [1, \underline{K}], \tag{35}$$

where parameter β_l^{iw} is the disutility of unit waiting time for i on l , and $w_l^i(j) = \beta_l^{iw} \cdot t_l^{iw}(j)$ satisfies properties (i)-(iii), obviously. Parameter β_l^{id} is the (dis)utility of unit duration for i on (non-)activity link l , and $g_l^i(j) = \beta_l^{id} \cdot t_l^{id}(j)$ satisfies properties (iv)-(vi), obviously.

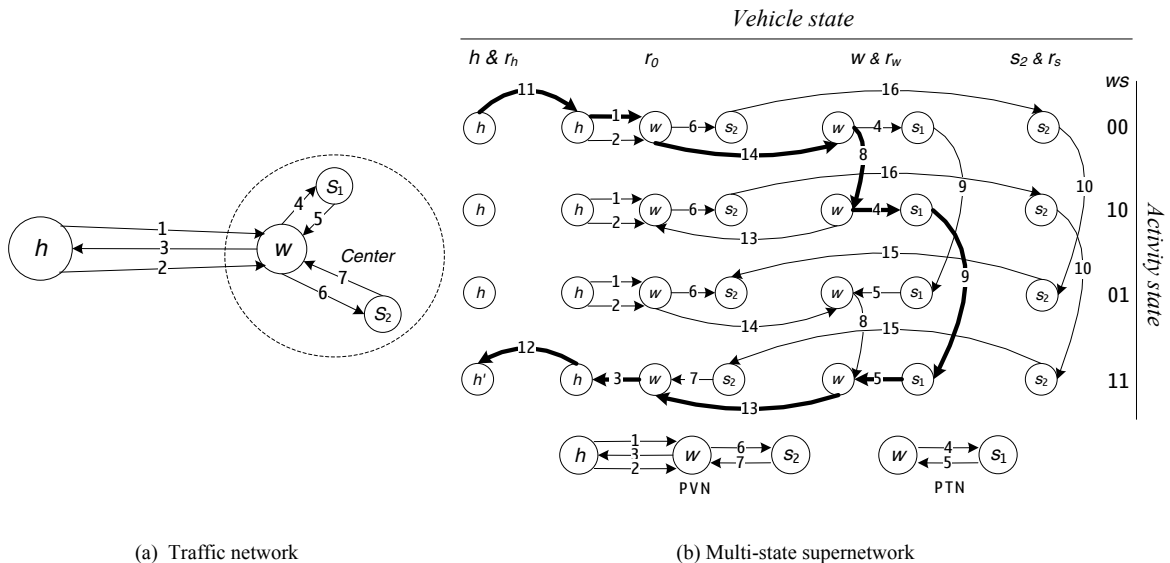


Fig. 2. Traffic network and corresponding multi-state supernetwork of example.

In this SNK, any route from h to h' is a feasible ATP. For example, a path denoted by the bold links shows that the traveler first picks up the car at home h and chooses link 1 to w for working and parks the car at r_w , after work, walks to s_1 for shopping, and finally goes back to pick up the car at r_w and drives home. Furthermore, this path can be represented as an ordered set $\{11, 1, 14, 8, 4, 9, 5, 13, 3, 12\}$. In fact, any path is a one to one mapping with a combined choice of route, mode, destination and sequence. We show the list of all feasible ATPs in Table 1.

Table 1. Feasible paths of example.

Number of path	Link order											
	1 st	2 nd	3 rd	4 th	5 th	6 th	7 th	8 th	9 th	10 th	11 th	12 th
path 1	11	1	14	8	4	9	5	13	3	12	/	/
path 2	11	1	14	8	13	6	16	10	15	7	3	12
path 3	11	1	14	4	9	5	8	13	3	12	/	/
path 4	11	1	6	16	10	15	7	14	8	13	3	12
path 5	11	2	14	8	4	9	5	13	3	12	/	/
path 6	11	2	14	8	13	6	16	10	15	7	3	12
path 7	11	2	14	4	9	5	8	13	3	12	/	/
path 8	11	2	6	16	10	15	7	14	8	13	3	12

The time horizon is taken from 6:00 to 22:00. Initially, there are two classes of travelers with different attributes. Each class has 1000 travelers. It is assumed that there is one-to-one correspondence between traveler and vehicle. Parameter settings for activity-travel links are shown in Table 2.

Table 2. Parameter settings.

Number of links	Link attribute				Traveler attribute		
	Time window (min)	t_i^0 (min)	η_i, γ_i	c_i (/min)	$\beta_i^{1w}, \beta_i^{2w}$ (/min)	d_i^1, d_i^2 (/min)	$\beta_i^{1d}, \beta_i^{2d}$ (/min)
link 1	[360, 1320]	20	1, 1	60	/	0, 0	0.1, 0.1
link 2	[360, 1320]	20	1, 1	50	/	0, 0	0.1, 0.1
link 3	[360, 1320]	20	1, 1	80	/	0, 0	0.1, 0.1
link 4, 5	[360, 1320]	10	0, /	/	/	0, 0	0.12, 0.12
link 6, 7	[360, 1320]	10	1, 1	80	/	0, 0	0.1, 0.1
link 8	[540, 1020]	/	/	/	0.06, 0.05	480, 420	0.3, 0.2
link 9, 10	[540, 1140]	/	/	/	0.06, 0.05	30, 40	0.5, 0.3
link 11-16	[360, 1320]	2	0, /	/	/	0, 0	0.15, 0.15

The symbol “/” means “not relevant”. Interval length was set to 2 minutes ($\omega=2$), implying that the time period $T=960$ minutes and the number of departure time choices interval $\underline{K}=480$. Consequently, there is a total of 3840 potential time-dependent paths (calculated by $\underline{K} \cdot |P_h|$) and some of them maybe not feasible) for each class of travelers. However, this number can be considerably reduced. For example, it covers all departure time choices interval by interval (one time interval equals to 2 minutes). In reality, travelers are likely to consider departure time choice at a less time-resolution, e.g., in every 10 minutes. If so, the number of potential paths is reduced to $3840/5=768$. Meanwhile, other factors (e.g., path overlapping and personalized paths) also contribute to reducing the path choice set. Finally, set parameters $\mu=1000, \rho=3\%, \varepsilon=10^{-5}$.

5.2. Results

The proposed algorithm was coded in MATLAB R2013b and run on a PC with an Intel Core(TM) Duo CPU with 3.16GHz and 4GB RAM. After approximate 130 seconds (1405 iterations), the stopping criterion is met Eq. (33). Consequently, a satisfactory solution of activity-travel flow pattern is obtained. It should be note that sensitivity analysis on the initial ATP inflows indicates that the solution is robust. The results are shown in the Fig. 3.

Fig. 3a demonstrates that a DUE solution for the time-dependent path inflow pattern between origin h and destination h' is obtained. It is illustrated that paths 1, 2, 5 and 6 for traveler class 1 and paths 1, 3, 5 and 7 for traveler class 2 have inflows. In this figure, we only intercept the time period 8:10-9:40, because all chosen departure times are assigned within this time period. The path disutilities between 8:10-9:40 are illustrated in Fig. 3b. Figs. 3a and 3b demonstrate that for each class of travelers, the disutilities are exactly equal and are the minimum for all chosen time-dependent paths.

Traveler class 1 who has fixed working time is concentrated around the time period 8:28-8:36 for departure via paths 1, 2, 5 and 6, which causes the morning peak. They depart so centralized that they can avoid arriving early or late for work. On the other hand, all travelers of class 1 choose working firstly and then shopping at s_1 or s_2 (represented by paths 1, 2, 5 and 6). It stems from the fact that shopping first (represented by paths 3, 4, 7 and 8) may cause travelers to arrive late for work, which is more difficult to tolerate for traveler class 1. A small number of travelers of class 2 who have flexible working time enters the SNK during 8:24-8:28 via paths 3 and 7. They depart so early that they can evade the concentrated departure of travelers of class 1 who prefer working on time. On the other hand, these travelers choose shopping at s_1 firstly and then working, because it can avoid nonsensical waiting for the opening of the office and higher disutility caused by driving to s_2 for shopping (represented by paths 2, 4, 6 and 8). The other travelers of class 2 enter the SNK during 8:36-9:24 via paths 1 and 5. They depart so late that they also can evade the concentrated departure of traveler class 1. On the other hand, these travelers choose working firstly and then shopping at s_2 , because it can avoid arriving late for working and higher disutility caused by driving to s_2 for shopping.

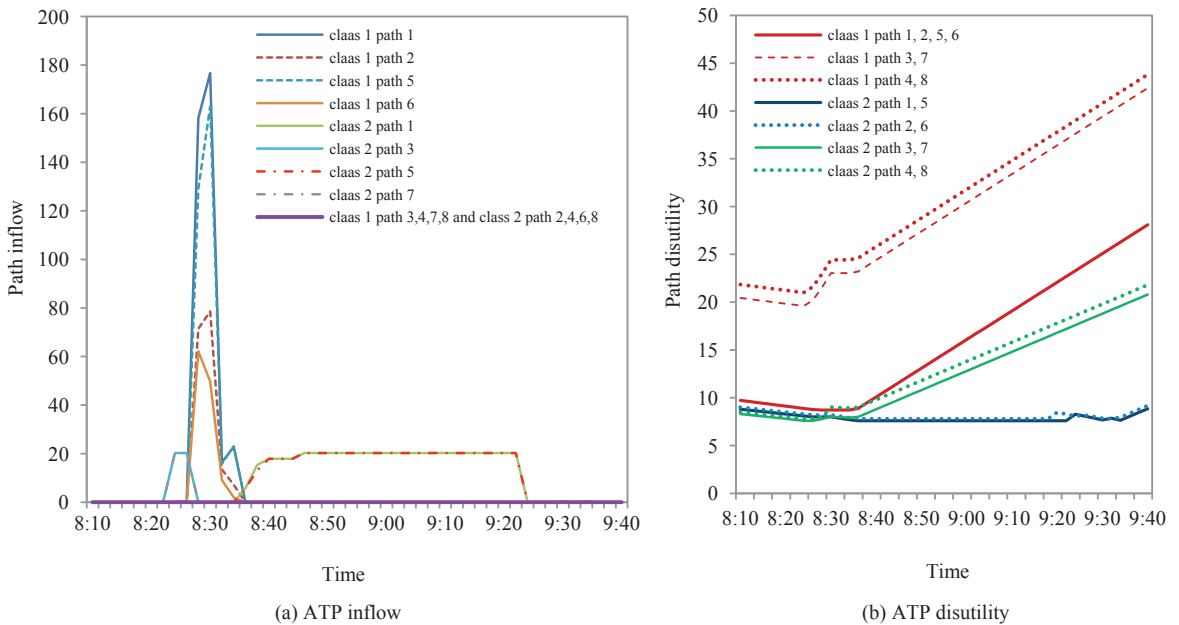


Fig. 3. The satisfactory solution.

In addition to Scenario 1, three more scenarios are formulated to allow a sensitivity analysis and policy evaluation. Parameter settings in these scenarios are exactly the same, except for the following: in scenario 2, $t_6^0 = t_7^0 = 9$; in scenario 3, $\beta_{10}^{1d} = 0.55, \beta_{10}^{2d} = 0.35$; and in scenario 4, $d_8^1(j) = 420$. Results are given below.

In Fig. 4, the horizontal axis denotes time period 8:00-20:00, and Figs. 4a, 4b, 4c and 4d represent the dynamic changes of the number of travelers in shopping locations in scenario 1, 2, 3 and 4, respectively. Comparison of Figs. 4a and 4b suggests that the number of travelers shopping at s_2 increases almost to the number observed for s_1 , when the free flow travel time between w and s_2 descends to 9 minutes. Particularly, traveler class 2 is influenced by the improvement of s_2 's accessibility. In addition, it leads to all feasible paths 1-8 being used by traveler class 2.

A comparison of Figs. 4a and 4c shows that the number of travelers shopping at s_2 increases to be much more than that at s_1 , when the unit time utility of shopping at s_2 for travelers class 1 and class 2 is increased. Especially, all members of traveler class 2 choose paths 2, 4, 6 and 8, so the used paths are reversed completely. Generally, traveler class 2 is more sensitive to parameter changes, which is mainly due to a greater freedom of choice brought about by the flexible work time. In scenario 4, traveler class 1 can choose arbitrary, but continuous 7 hours for work during the time window without penalty.

Comparing Figs. 4a and 4d indicates that all travelers choose s_1 for shopping but no one chooses s_2 , when the work ideal duration of traveler class 1 decreases to 7 hours. Especially, traveler class 1 changes their used paths to 1, 3, 5 and 7, which is the same as traveler class 2's path choice. Consequently, the number of travelers who choose shopping at s_1 in the morning first increases substantially.

Furthermore, the effects of flexible work time on the flows in SNK are shown in Fig. 5, where the horizontal axis denotes the time period 8:00-20:00. Figs. 5a and 5b represent the dynamic changes of the number of travelers in the transportation system (i.e., TS) including links 1-7 and links 11-16, working place (i.e., W) including link 8, shopping places (i.e., S) including links 9-10 and home (i.e., H).

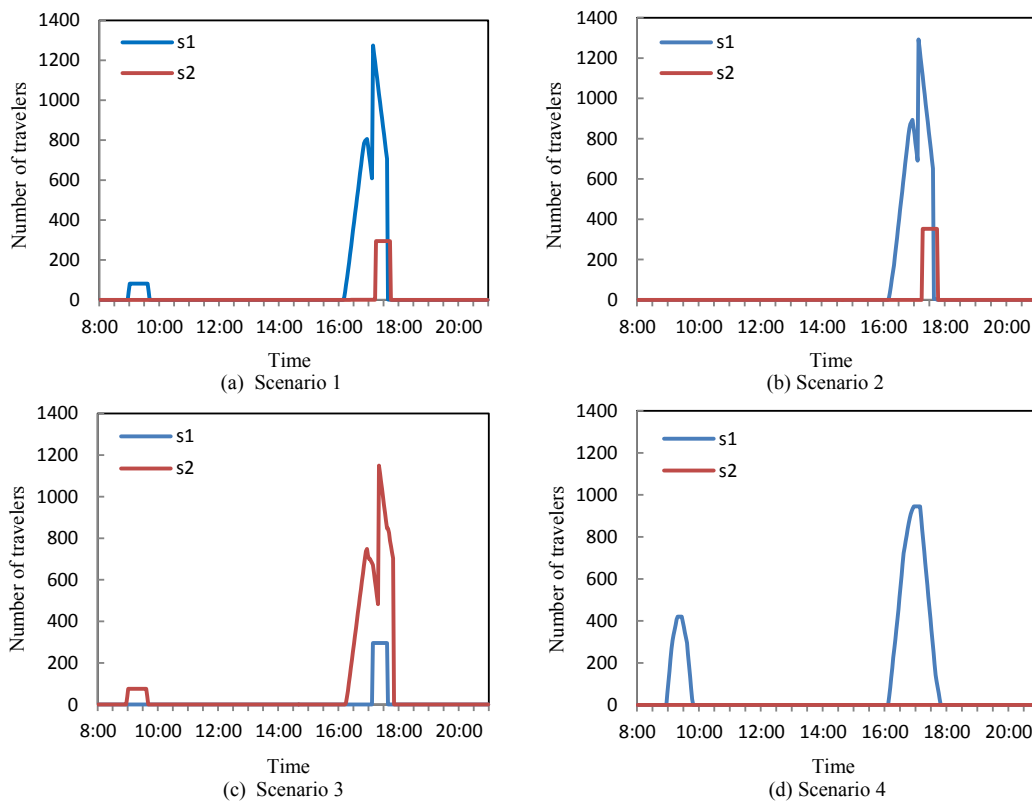


Fig. 4. Number of travelers in shopping locations for each of four scenarios.

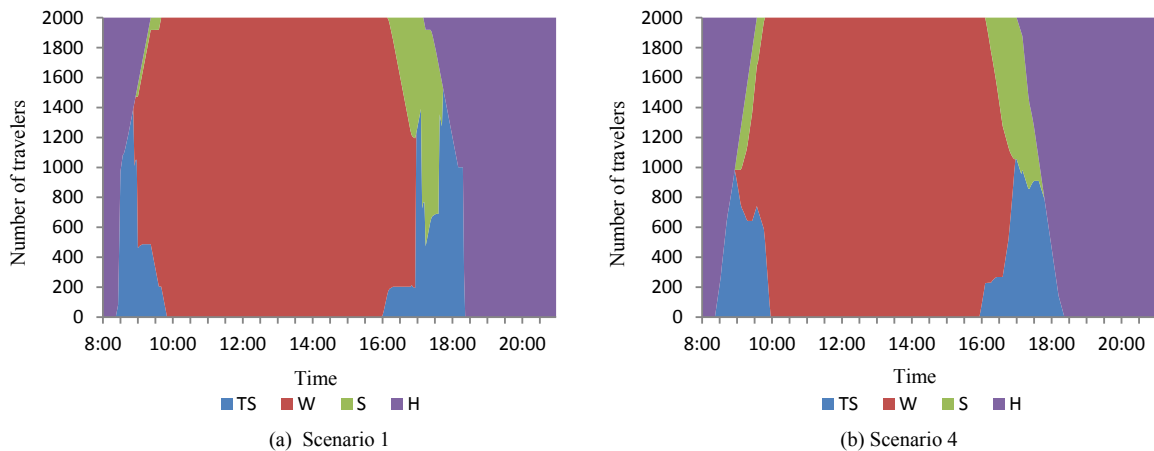


Fig. 5. Number of travelers in different types of links for scenarios 1 and 4.

Based on the number of travelers in the transportation system, the above two figures show that the morning and evening peak appear around 9:00 and 17:00, respectively. The maximum numbers reduce whether morning or evening peak in scenario 4 compared with scenario 1. The time-space distribution of traffic is gentler because traveler class 1 staggers departure time. Therefore, the policy of flexible working time relieves the traffic congestion.

Figs. 4 and 5 show that the time-dependent ATP inflows and the number of travelers on links are sensitive to the related parameters. Moreover, it is illustrated that OD travel demands in traditional DTA become very sensitive to link and traveler's attributes in DATA. Consequently, as an integrated consideration, DATA model can unify disparate choices and highlight the relationship between these choices. Utilizing the proposed model, some traffic policies such as road improvement and flexible working time can be assessed by changing of traveler disutility. In addition, even some business policies such as product or service improvement and time window optimization can be evaluated by changing of link inflow and activity duration.

6. Conclusion and future work

The integration of ABM and DTA for travel demand analysis has recently attracted ever-increasing attention. However, related studies have limitations either on the integration structure or the number of choice facets being captured. This paper proposes a formulation of dynamic activity-travel assignment (DATA) in the framework of multi-state supernetworks. Under this framework, departure time/traffic route/traffic mode/activity or parking location/activity sequence choices are unified into the time-dependent ATP choice. A multi-class dynamic activity travel equilibrium problem is formulated as a discrete-time path-based VI model. Link disutility consists of waiting and duration gap with focuses on properties of various link incorporating endogenous duration and time window constraints. A solution algorithm based on path/time-swapping process is presented for solving the path-based VI problem. A numerical example with simulated scenarios demonstrates the power of the model to capture multi-modal and multi-activity trip chaining at equilibrium states and the sensitivity to policy interventions.

However, some important components have not been considered in the proposed model. First, effects on link disutility due to the varied number of travelers at capacity-constrained activity locations or public transport are not taken into account. Second, the suggested model may cause a problem that the number of travelers on links is greater than the capacity of the link. Therefore, spillback is not appropriately captured, which may not only exist in the in non-activity links, but also exists in the activity links. Third, as ATP choice and departure time choice are combined in the proposed model, a huge number of feasible time-dependent ATPs are generated, especially in a large-scale supernetwork. As a result, more efficient traffic assignment algorithms are needed. Fourth, the current model falls short of a general treatment on activity duration choice that travelers can choose any possible duration if time budget allows. Fifth, the DATA model assumes unrealistically that travelers have the complete information

about the activity-travel time and disutility; thus, an extension of DATA in an uncertain environment is also of great interest. These issues will be addressed in future research.

Acknowledgements

This research is jointly supported by the National Natural Science Foundation of China (No. 71361130011) and Dutch Science Foundation, and a grant from the National Basic Research Program of China (No. 2012CB725401). These supports are gratefully acknowledged.

References

- Abdelghany, A.F., Mahmassani, H.S., 2003. Temporal-spatial microassignment and sequencing of travel demand with activity-trip chains. *Transportation Research Record: Journal of the Transportation Research Board* 1831 (1), 89-97.
- Arentze, T.A., Timmermans, H.J.P., 2004a. A learning-based transportation oriented simulation system. *Transportation Research Part B* 38 (7), 613-633.
- Arentze, T.A., Timmermans, H.J.P., 2004b. A multi-state supernetwork approach to modelling multi-activity multi-modal trip chains. *International Journal of Geographical Information Science* 18 (7), 631-651.
- Arentze, T.A., Molin, E.J.E., 2013. Travelers' preferences in multimodal networks: design and results of a comprehensive series of choice experiments. *Transportation Research Part A* 58, 15-28.
- Auld, J., Mohammadian, A., 2009. Framework for the development of the agent-based dynamic activity planning and travel scheduling (ADAPTS) model. *Transportation Letters* 1 (3), 245-255.
- Axhausen, K.W., Gärling, T., 1992. Activity-based approaches to travel analysis: Conceptual Frameworks, Models and Research Problem. *Transport Reviews* 12 (4), 323-341.
- Balmer, M., Axhausen, K., Nagel, K., 2006. Agent-based demand modelling framework for large scale micro-simulations. *Transportation Research Record* 1985 (1), 125-134.
- Bhat, C.R., Guo, J.Y., Srinivasan, S., Sivakumar, A., 2004. A Comprehensive Econometric Microsimulator for Daily Activity-travel Patterns. *Transportation Research Record* 1894 (1), 57-66.
- Bos, D.M., 2004. Changing seat: a behavioural analysis of P&R use, *Trail Thesis Series*, The Netherlands.
- Bowman, J.L., Ben-Akiva, M., 2001. Activity-based disaggregate travel demand model system with activity schedules. *Transportation Research Part A* 35 (1), 1-28.
- Carey, M., Ge, Y. E., McCartney, M., 2003. A whole-link travel-time model with desirable properties. *Transportation science* 37 (1), 83-96.
- Carlier, K., Fiorenzo Catalano, S., Lindvel, C., Bovy, P., 2003. A supernetwork approach towards multimodal travel modelling. *Proceedings of the 82nd Annual Meeting of the Transportation Research Board*, Washington, D.C.
- Chen, H.K., Hsueh, C.F., 1998. A model and an algorithm for the dynamic user-optimal route choice problem. *Transportation Research Part B* 32 (97), 219-234.
- Fu, X., Lam, W.H., 2014. A network equilibrium approach for modelling activity-travel pattern scheduling problems in multi-modal transit networks with uncertainty. *Transportation* 41 (1), 37-55.
- Han, L., Ukkusuri, S.V., Doan, K., 2011. Complementarity formulations for the cell transmission model based dynamic user equilibrium with departure time choice, elastic demand and user heterogeneity. *Transportation Research Part B* 45(10), 749-1767.
- Horni, A., Scott, D.M., Balmer, M., Axhausen, K.W., 2009. Location choice modeling for shopping and leisure activities with MATSim. *Transportation Research Record* 2135 (1), 87-95.
- Huang, H.J., Lam, W.H., 2002. Modeling and solving the dynamic user equilibrium route and departure time choice problem in network with queues. *Transportation Research Part B* 36 (3), 253-273.
- Huang, H.J., Lam, W.H., 2005. A stochastic model for combined activity/destination/route choice problems. *Annals of Operations Research* 135, 111-125.
- Kitamura, R., Pas, E.I., Lula, C.V., Lawton, T.K., Benson, P.E., 1996. The Sequenced Activity Mobility Simulator (SAMS): An Integrated Approach to Modeling Transportation, Land-use and Air Quality. *Transportation* 23 (3), 267-291.
- Lam, W.H., Yin, Y., 2001. An activity-based time-dependent traffic assignment model, *Transportation Research Part B* 35(6), 549-574.
- Lam, W.H., Huang, H.J., 2002. A combined activity/travel choice model for congested road networks with queues. *Transportation* 29 (1), 5-29.
- Lam, W.H., Huang, H.J., 2003. Combined activity/travel choice models: time-dependent and dynamic versions. *Networks and Spatial Economics* 3 (3), 323-347.
- Liao, F., Arentze, T.A., Timmermans, H.J.P., 2010. Supernetwork approach for multi-modal and multi-activity travel planning. *Transportation Research Record* 2175 (1), 38-46.
- Liao, F., Arentze, T.A., Timmermans, H.J.P., 2011. Constructing personalized transportation network in multi-state supernetworks: A heuristic approach. *International Journal of Geographic Information Science* 25 (11), 1885-1903.
- Liao, F., Arentze, T.A. and Timmermans, H.J.P., 2012. Supernetwork approach for modeling traveler response to park-and-ride. *Transportation Research Record* 2323 (1), 10-17
- Liao, F., Arentze, T.A., Timmermans, H.J.P., 2013. Incorporating space-time constraints and activity-travel time profiles in a multi-state supernetwork approach to individual activity-travel scheduling. *Transportation Research Part B* 55, 41-58.

- Lin, D.Y., Eluru, N., Waller, S.T., Bhat, C.R., 2008. Integration of activity-based modeling and dynamic traffic assignment. *Transportation Research Record* 2076 (1), 52-61.
- Long, J., Huang, H.J., Gao, Z., 2013. Discretised route travel time models based on cumulative flows. *Journal of Advanced Transportation* 47 (1), 105-125.
- McNally, M.G., 2000. The activity-based approach. In: Hensher, D.A., Button, K.J. (Eds.), *Handbook of Transport Modeling*. Pergamon, Amsterdam, pp. 53-70.
- Miller, E. J., Roorda, M. J., 2003. Prototype model of household activity-travel scheduling. *Transportation Research Record* 1831 (1), 114-121.
- Nagurney, A., Zhang, D., 1997. Projected dynamical systems in the formulation, stability analysis, and computation of fixed-demand traffic network equilibria. *Transportation Science* 31 (2), 147-158.
- Nagurney, A., 1998. *Network economics: A variational inequality approach*. Springer, pp. 18.
- Nagurney A., Dong J., 2002. Urban location and transportation in the Information age: A multiclass, multicriteria network equilibrium perspective. *Environment and Planning B* 29 (1), 53-74.
- Nagurney A., Dong J., Mokhtarian P.L., 2003. A space-time network for telecommuting versus commuting decision making. *Regional Science* 82 (4), 451-473.
- Ouyang, L.Q., Lam, W.H., Li, Z.C., Huang, D., 2011. Network user equilibrium model for scheduling daily activity travel patterns in congested networks. *Transportation Research Record* 2254 (1), 131-139.
- Pinjari, A.R., Bhat, C.R., 2011. *Activity-based Travel Demand Analysis. A handbook of transport economics*, Edward Elgar Publishing, pp. 213-248.
- Prato C.G., 2009. Route Choice Modeling: Past, Present and Future Research Directions. *Journal of Choice modeling* 2 (1), 65-100.
- Pyrga, E., Schulz, F., Wagner, D., Zaroliagis, C., 2008. Efficient models for timetable information in public transportation systems. *Journal of Experimental Algorithmics* 12, 2-4.
- Ramadurai, G., Ukkusuri, S., 2010. Dynamic user equilibrium model for combined activity-travel choices using activity-travel super network representation. *Network and Spatial Economics* 10 (2), 273-292.
- Ramadurai, G., Ukkusuri, S., 2011. B-Dynamic: An efficient algorithm for the dynamic user equilibrium in activity travel networks. *Computer Aided Civil and Infrastructure Engineering* 26 (4), 254-269.
- Rasouli, S., Timmermans, H.J.P., 2014. Activity based models of travel demand: promises, progress and prospects. *The International Journal of Urban Sciences* 18 (1), 31-60.
- Shifan, Y., Ben-Akiva, M., 2011. A practical policy-sensitive activity-based travel demand model. *The Annals of Regional Science* 47 (3), 517-541.
- Smith, M.J., Wisten, M.B., 1995. A continuous day-to-day traffic assignment model and the existence of a continuous dynamic user equilibrium. *Annals of Operations Research* 60 (1), 59-79.
- Szeto, W.Y., Lo, H.K., 2005. Properties of dynamic traffic assignment with physical queues. *Journal of the Eastern Asia Society for Transportation Studies* 6, 2108-2123.
- Szeto, W.Y., Lo, H.K., 2006. Dynamic traffic assignment: properties and extensions. *Transportmetrica* 2 (1), 31-52.
- Ukkusuri, S.V., Han, L., Kien, D., 2012. Dynamic user equilibrium with a path based cell transmission model for general traffic networks. *Transportation Research Part B* 46 (10) 1657-1684.
- Zhang, X., Yang, H., Huang, H.J., Zhang, H.M., 2005. Integrated scheduling of daily work activities and morning-evening commutes with bottleneck congestion. *Transportation Research Part A* 39 (1), 41-60.