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# Relaxation Oscillation Dynamics in Semiconductor Diode Lasers With Optical Feedback

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**Abstract**—We theoretically investigate the stability of a single-mode semiconductor laser with weak optical feedback in the short external-delay regime. Although the laser is, in general, very sensitive to feedback-induced excitation of relaxation oscillations, we predict complete insensitivity for these oscillations when the product of oscillation frequency and external-delay time equals a small integer. This may form the basis for relaxation-oscillation-free laser design.

**Index Terms**—Delay systems, laser theory, optical feedback, relaxation oscillation, semiconductor lasers.

## I. INTRODUCTION

IN MANY applications Fabry-Pérot type semiconductor lasers use external optical feedback, or self-injection. The draw-back of such method is that feedback can introduce a sustained relaxation oscillation (RO), i.e. an intrinsic resonance between laser intensity and population inversion, which for a solitary laser is a damped oscillation, but which can easily be excited under influence of delayed feedback through a so-called Hopf-bifurcation [1], [2]. It was implicitly known that the occurrence of the RO is sensitive, among other things, to the applied settings of the phase of the feedback light [3], [4]. This was studied in [5], [6] with the help of electrically addressable phase shifters. It was also known that the minimum feedback level for onset of the relaxation oscillation is sensitive to the product of RO frequency times the external delay time [2], where it was predicted that this feedback-level shows maxima each time this product equals an integer, i.e. under resonance conditions. This prediction was based upon certain numerical and analytical considerations, but no simple explanation was given.

In this letter we focus on the particular role of the external-feedback delay time in the excitation of relaxation oscillations. With the help of the first-order differential delay equations, as first formulated by Lang and Kobayashi [4], we show that for conventional semiconductor lasers with weak optical feedback under RO-resonance condition, i.e. when the product of RO frequency  $\nu_{RO}$  and delay time  $\tau$  equals an integer, i.e.  $\nu_{RO}\tau \approx 0, 1, 2, \dots$ , the laser with feedback behaves with respect to the RO as if no feedback is present. Therefore, under the above-mentioned conditions the RO is damped and, in fact, will be suppressed. This prediction is valid in the regime of

weak feedback, such that the RO frequency is not deviating substantially from its value in the solitary laser.

## II. THEORY

The equations that describe the time evolution of small excursions from the steady-state solutions of phase, intensity and inversion in a single-longitudinal-mode semiconductor laser with weak feedback can be derived from the Lang-Kobayashi equations and are given by

$$\begin{aligned} \dot{\delta\varphi} &= -\gamma \cos(\beta) [\delta\varphi(t) - \delta\varphi(t - \tau)] \\ &\quad + (\gamma/2I_s) \sin(\beta) [\delta I(t) - \delta I(t - \tau)] + (1/2)\alpha\zeta\delta n(t) \end{aligned} \quad (1)$$

$$\begin{aligned} \dot{\delta I} &= -2\gamma I_s \sin(\beta) [\delta\varphi(t) - \delta\varphi(t - \tau)] \\ &\quad - \gamma \cos(\beta) [\delta I(t) - \delta I(t - \tau)] + \zeta I_s \delta n(t) \end{aligned} \quad (2)$$

$$\dot{\delta n} = -(1/T + \zeta I_s)\delta n(t) - \Gamma_0 \delta I(t). \quad (3)$$

Here  $\alpha$  is the linewidth-enhancement parameter,  $\zeta$  the linearized gain coefficient,  $\gamma$  the feedback rate,  $T$  the carrier recombination time and  $\Gamma_0$  the photon decay rate of the cavity;  $\beta = \omega_s \tau$  is the feedback phase with  $\omega_s$  the angular optical frequency;  $I_s$  and  $n_s$  are the steady-state values of intensity and inversion, all in the presence of feedback.

Eqs. (1) to (3) form a set of coupled linear differential equations with delay. They are analyzed for solutions with time dependence  $\exp(st)$  for complex-valued  $s$ . This leads to a system determinant with zeroes in the complex  $s$ -plane. The determinant has at least one trivial zero  $s = 0$ , which we divide out, and the result can be expressed as

$$\begin{aligned} \frac{D(s)}{s} &= \left( s + \frac{1}{T} + \zeta I_s \right) \\ &\quad \times \left( s + 2\gamma \cos \beta (1 - e^{-s\tau}) + \gamma^2 \frac{(1 - e^{-s\tau})^2}{s} \right) \\ &\quad + \zeta \Gamma_0 I_s \left( \frac{C}{\tau} \cos(\beta + \arctan \alpha) \frac{(1 - e^{-s\tau})}{s} + 1 \right) \end{aligned} \quad (4)$$

where  $C = \gamma \tau \sqrt{1 + \alpha^2}$  is the feedback coupling parameter. It has been shown in [7] that when  $C < 1$  only one single steady solution  $\{\omega_s, I_s, n_s\}$  exists. For this state to be stable, all zeroes in the complex  $s$ -plane of the rhs of (4) must have real parts  $< 0$ . This was investigated using the principle of the argument. Thus we were able to investigate the stability for each steady-state solution. We confine ourselves to situations of weak feedback, satisfying  $C < 1$ . In those cases the onset of instability is associated with the excitation of the RO.

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### III. SUPPRESSION OF RELAXATION OSCILLATIONS

Inspection of eqs. (1)–(3) allows the following observation: if the laser exhibits a RO with frequency  $\nu_{RO}$  approximately satisfying the resonance condition  $\nu_{RO}\tau = 1, 2, \dots$ , eqs. (1) to (3) reduce in good approximation to the much simpler set

$$\left. \begin{aligned} \dot{\delta\varphi} &= (1/2)\alpha\xi\delta n(t) \\ \dot{\delta I} &= \xi I_s \delta n(t) \\ \dot{\delta n} &= -(1/T + \xi I_s)\delta n(t) - \Gamma_0 \delta I(t) \end{aligned} \right\} \quad (5)$$

which is equivalent to the dynamical equations for a solitary laser, i.e. a laser without feedback. This allows the conclusion that under the resonance condition the feedback tends to force the laser to behave as a solitary laser, in which case the RO is damped. Indeed, the last two equations in (5) lead to

$$\ddot{\delta n} + \left(\frac{1}{T} + \xi I_s\right) \dot{\delta n} + \xi \Gamma_0 I_s \delta n = 0 \quad (6)$$

i.e. a damped harmonic oscillator with frequency and damping given by

$$\nu_{RO} = \frac{1}{2\pi} \sqrt{\xi \Gamma_0 I_s}; \quad \lambda_{RO} = \frac{1}{T} + \xi I_s, \quad (7)$$

where we notice that under the weak-feedback condition considered the intensity is not very different from the solitary laser value. Hence, RO instabilities are predicted to disappear completely when the laser is pumped into a RO-resonance condition.

This qualitative prediction has been numerically confirmed for the first few resonances, as can be seen in the stability diagram in Fig.1. The parameter on the horizontal axis is the feedback phase  $\beta$ . Along the vertical axis stands the pump strength, defined as  $p = (J - J_{thr})/J_{thr}$  with  $J$  the injection current and  $J_{thr}$  the solitary laser threshold value. Also indicated is the corresponding value of the RO frequency,  $\nu_{RO} = (1/2\pi)\sqrt{\xi\Gamma_0 I_s} = (1/2\pi)\sqrt{\xi J_{thr} p}$ . For higher resonances, i.e. higher pump strengths, the stability bands tend to shrink and disappear.

One may wonder how the RO-suppression mechanism will affect the modulation properties of a laser with feedback. This has not been investigated here, but since the laser behaves under RO-suppression conditions similar to a single-mode laser without feedback, it is expected that the modulation properties around the RO-frequency are also similar. For frequencies different from the RO-frequency, we expect no special effect on the modulation properties.

In conclusion we have shown that a RO-instability for a bulk or quantum-well semiconductor laser with weak optical feedback can be suppressed when the laser is pumped in a region near the RO-resonance condition, that is, when the product of RO-frequency and delay time equals a small integer. This principle is currently used in the design of stable single-frequency integrated diode lasers.

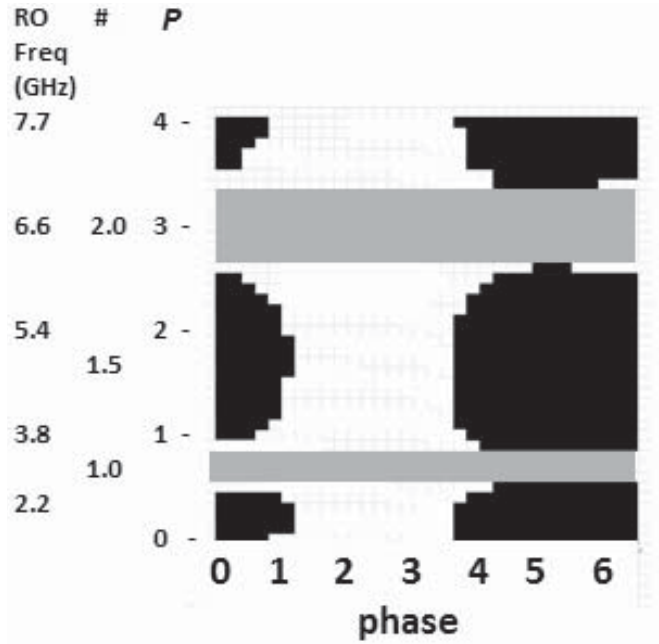


Fig. 1. Theoretical stability map for a single-mode semiconductor laser with weak optical feedback. Vertical is the pump strength and horizontal is the feedback phase. Black indicates the region where the steady state is unstable; white indicates stability. The bands of full phase-independent stability are indicated by grey. The transition from stable to unstable is characterized by excitation of the RO. The left-most column gives the RO frequency in GHz; the middle column gives the product of RO frequency and delay time. The two stability bands occur at the first and the second resonance condition. The bands have sufficiently wide openings in order for real applications. Parameters are:

$$\alpha = 2.6; \gamma = 1 \times 10^9 s^{-1}; \xi = 5800 s^{-1}; T = 1 \times 10^{-9} s; \Gamma_0 = 3 \times 10^{12} s^{-1}; \tau = 3 \times 10^{-10} s; J_{thr} = 10^{17} s^{-1}.$$

Quantum-dot lasers behave dynamically quite different from the above-described lasers and hence one should be reluctant when extending the conclusions made here to QD-lasers. Nevertheless, the qualitative reasoning leading to eqs. (5) is still applicable to QD-lasers, but it should be realized that the alpha parameter and hence the coupling constant  $C$  introduced below (4) strongly increases with output intensity.

Finally a few words on the potential applications of the RO suppression instability. The main application is in situations where weak filtered optical feedback is used in order to force the laser in single-mode operation, see e.g. [6]. In general this leads to stable operation only for certain values of the feedback phase  $\beta = \omega_s \tau$ , which is hard to control, even with the help of a phase-controlling element. Therefore, it would be most advantageous to operate the laser in the above-described pumping intervals of feedback-phase-independent stability.

### IV. ACKNOWLEDGMENT

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