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The price of reverse factoring : financing rates vs. payment delays

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The price of reverse factoring: Financing rates vs. payment delays

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ABSTRACT

Reverse factoring—a financial arrangement where a corporation facilitates early payment of its trade credit obligations to suppliers—is increasingly popular in industry. Many firms use the scheme to induce their suppliers to grant them more lenient payment terms. By means of a periodic review base stock model that includes alternative sources of financing, we explore the following question: what extensions of payment terms allow the supplier to benefit from reverse factoring? We obtain solutions by means of simulation optimisation. We find that an extension of payment terms induces a non-linear financing cost for the supplier, beyond the opportunity cost of carrying additional receivables. Furthermore, we find that the size of the payment term extension that a supplier can accommodate depends on demand uncertainty and the cost structure of the supplier. Overall, our results show that the financial implications of an extension of payment terms need careful assessment in stochastic settings.

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1. Introduction

Trade credit is a short-term loan between firms that is linked both in terms of timing and value to the exchange of goods between them (Ferris, 1981). Recent estimates suggest that around 80–90 percent of the world trade is facilitated by trade credit (Williams, 2008). In the manufacturing sector, accounts receivable make up 20–25 percent of the total assets of firms (Fewings, 1992; Mian & Smith, 1992). The role of trade credit in our economy is extensive, and it has consequently been the topic of investigation of many studies (Seifert, Seifert, & Protopappa-Sieke, 2013). These studies link the provision of trade credit to information asymmetry, transaction costs, hedging, moral hazard, quality assurance, and many other motives and market phenomena.

Regardless of the motive, the provision of trade credit is considered to be an investment on the microeconomic level. Its terms should therefore account for the opportunity cost of tying up capital in an asset, i.e., the receivable. Financial management practices conventionally consider the provider's cost of capital as the basis for the opportunity cost (see, e.g., Brealey, Myers, & Allen, 2011), but recent developments challenge this perspective. Indeed, revelation of information about the risk of a specific asset can be the basis for improved financing terms, which reflect the risk of the asset concerned as opposed to that of the firm in general. Pfohl and Gomm (2009) show how this results from an inter-company approach to financing, which they call 'Supply Chain Finance' (SCF). The term SCF is also increasingly used by financial institutions to denote payables financing or early payment services (Casterman, 2013). Among these, 'reverse factoring' is a prime example and has received considerable recent interest from the business and research community (Tanrisever, Cetinay, Reindorp, & Fransoo, 2012; Wuttke, Blome, Foerstl, & Henke, 2013). It is essentially a development of conventional factoring arrangements. In the latter, a firm independently sells one or more of its receivables to a financier—the factor—against a premium (Soufani, 2002). In reverse factoring, the firm's client is also involved: the client makes an explicit guarantee to the factor that the payment obligation will be met (Klapper, 2006). This guarantee entails that the factor can offer financing at a rate as low as when the client itself would apply for funds. Investment grade firms can therefore use reverse factoring to realise a significant reduction in cost of credit for their suppliers.

According to Hurtrez and Salvadori (2010), recent technological advances allow reverse factoring to be offered efficiently, and challenging economic conditions have accelerated adoption. Specifically, the credit crisis increased the spread of short-term capital costs between large corporations and their SME suppliers; in some cases, the latter even saw their access to short-term capital cut. A study initiated by the Bank of England concludes that reverse factoring offers significant opportunities to rejuvenate lending to SME firms (Association of Corporate Treasurers, 2010). Nonetheless, many buyers also see reverse factoring as a means to reduce their own working capital costs: by offering competitively priced early payment options, they induce their suppliers to offer longer payment terms. From a survey among executives, Seifert and Seifert (2011) find that buyers managed to reduce net working capital by 13 percent on average through reverse factoring. While literature suggests that payment terms can be reconfigured in a collaborative spirit, the approach of some buyers appears

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to neglect this perspective. For instance, Milne (2009) reports that a large corporation introduced reverse factoring as a 'sweetener' to an unpopular decision to move its payment terms to suppliers from 45 to 90 days. Wuttke et al. (2013) cite an executive of a major chemical firm: "We would say to our supplier, we will extend payment terms anyway. It is up to you to take our SCF[1] offer or leave it." Such examples, together with Aberdeen's survey findings that 17 percent of its respondents experienced 'pressure' from trading partners to adopt supply chain finance, suggest that the benefit of a reverse factoring arrangement for suppliers may in some cases be open to question (Pezza, 2011).

Even if there is extension in *contractual* payment terms, the supplier can use reverse factoring to obtain early payment cheaply. Consequently, a trade-off between 'longer' and 'cheaper' arises. This provides the central motivation for our study. Adjusting payment terms on the basis of an adjustment in financing rates assumes that the net effect of these changes on the financing costs of a firm can be assessed. The assessment is often made by considering the cost of capital in conjunction with the average value of outstanding receivables and/or payables (Hofmann & Kotzab, 2010; Randall & Farris, 2009). This approach presumes that the configuration of trade credit can be made independently from operations; but some studies show that lot-size or inventory decisions can interact with the receipt and/or provision of trade credit (Gupta & Wang, 2009; Protopappa-Sieke & Seifert, 2010; Song & Tong, 2012). On account of the possibility for interaction between financial and operational terms, we explore the costs and benefits of reverse factoring in the context of stochastic inventory management. We take a discrete-time, infinite-horizon, base-stock inventory model of a supplier firm and incorporate financial dimensions in the state description. Initially, we assume that the firm has only access to conventional short-term financing sources; we then extend this with the option to sell receivables through reverse factoring. Further, since a firm's opportunity cost rate for carrying receivables may influence its use of reverse factoring, we consider relevant alternative implementations for the latter: manual or auto discounting.[2] In all cases our objective is the minimisation of average cost per period, defined as the sum of inventory and financing costs.

Within the operations management literature, our work contributes first of all to the relatively young research line in the area of supply chain finance (Pfohl & Gomm, 2009; Randall & Farris, 2009; Wuttke et al., 2013). In particular, our work complements that of Tanrisever et al. (2012), who obtain analytical insights from a single period model of reverse factoring: we examine the conditions under which reverse factoring is economically viable in a multi-period setting. We find that manual and auto discounting are to be treated as different types of system with different accompanying trade-offs. While auto discounting allows for making a trade-off independent of inventory operations, manual discounting involves a more complex trade-off which is conditioned on demand uncertainty and the supplier firm's cost structure. These parameters affect the discounting cost, but they also impact the expected volume of receivables. The overall impact on the payment term decision may consequently be difficult to predict. Furthermore, we show that the ability to extend payment terms in an economically justified fashion with manual discounting may be restricted to settings where the opportunity cost rate for holding receivables is low. In an extensive numerical study, we find a maximum opportunity cost rate of 0.5 percent per year for most of our settings. This corresponds to the short-term borrowing cost of investment-grade firms, which are unlikely to be the supplier in a reverse factoring arrangement.

[1] In trade publications, the general term Supply Chain Finance (SCF) is frequently used to refer to reverse factoring in particular.

[2] Practitioners use the term "discounting" to describe the sale of a receivable by supplier in a reverse factoring system, presumably because the cash that the supplier receives is a only a fraction of the face value of the receivable.

We contribute also to an emerging research area that considers interactions between inventory and financing in a multi-period stochastic setting (Babich, 2010; Gupta & Wang, 2009; Hu & Sobel, 2007; Luo & Shang, 2013; Maddah, Jaber, & Abboud, 2004; Protopappa-Sieke & Seifert, 2010; Song & Tong, 2012). Our experiments suggest that the cash retention level required to finance a base stock operation increases asymptotically in the payment term. The value of retained cash is therefore decreasing in the payment term. Viewing a payment term as lead time, this finding conforms to an intuition of Goldberg, Katz-Rogozhnikov, Lu, Sharma, and Squillante (2012): when lead time is very long, the system is subject to so much randomness between an event and its consequence that 'being smarter' provides almost no benefit.

The remainder of this article is structured as follows. In Section 2 we discuss our research questions and literature relevant to our problem. In Section 3 we describe the models we implement in our simulations. In Section 4 we discuss the design of our experiments. In Section 5 we present the results from the experiments. In Section 6 we summarise our findings and draw final conclusions.

2. Research questions and literature

A firm's weighted average cost of capital (WACC) is conventionally seen as the opportunity cost rate needed to analyse its cost of trade credit. The latter is calculated by multiplying the firm's average value of outstanding receivables by its WACC (Brealey et al., 2011). Average value of outstanding receivables is in turn the product of average daily volume of credit sales and average number of days until payment. With this approach, the cost of trade credit is a linear function of payment terms and independent of variability in demand. We hypothesise that variability in demand will influence the amount of financing necessitated by an extension of payment terms. It is well known from stochastic inventory theory that longer replenishment lead times require higher levels of safety stock, in order to hedge against intervening demand uncertainty (Zipkin, 2000). Viewing a payment term as lead time, we expect that a firm's financial position is exposed to more variability when extending payment terms. Additional delay in payment entails the possibility of incurring more cash outlays and receipts between the moment of selling goods and collecting payment. As cash flow uncertainty is associated with the need to borrow money and/or hold more cash (Opler, Pinkowitz, Stulz, & Williamson, 1999), financing cost may be a non-linear function of the payment term, regardless of the opportunity cost rate used for receivables. We thus formulate a first research question:

(RQ 1) What impact does extending payment terms have on the cost of managing a stochastic inventory operation?

The presumption that the financing needed to support physical flows is a linear function of payment terms also suggests that differences in the cost of short-term credit can be exploited in a straightforward fashion. Firms would benefit as long as the multiplier of the initial payment term is no greater than the inverse of the multiplier of the initial financing rate. If the initial financing rate were halved, for instance, the initial payment term could be doubled. Several studies analyse the potential savings from adjusting payment terms this way (Hofmann & Kotzab, 2010; Randall & Farris, 2009; Wuttke et al., 2013). In contrast, by considering explicitly the effect that demand uncertainty has on financial flows in a single period model, Tanrisever et al. (2012) find that this inverse-proportional relationship will generally underestimate the cost of an extension of payment terms. We explore this finding in a multi-period inventory setting, where firms conduct transactions in an ongoing manner.

A further qualification of the benefits of reverse factoring results from a firm's opportunity cost rate for holding receivables. The pertinence of this consideration is evident when we consider that there

are in practice two distinct ways in which firms use reverse factoring. Many choose an auto discounting policy, but some choose for manual discounting.[3] As the names suggest, with auto discounting the firm allows all receivables to be discounted automatically at the earliest possible moment, while with manual discounting the firm chooses when and which receivables to discount. That both approaches exist and are applied is evident from trade literature (cf. Dunn, 2011; Gustin, 2014), and both are included in a US Patent application for reverse factoring (Kramer, 2009). The very existence of manual discounting suggests that firms may assess the opportunity cost rate for receivables as equal to or lower than the discount rate. A bank overdraft embodies the same principle: it allows the firm to borrow money only when it needs it and pay interest only on the amount it borrows. In the context of reverse factoring, if the opportunity cost rate were higher than the discount rate, a firm should choose auto discounting. These considerations lead to our second research question:

(RQ 2) What payment term extension would allow a supplier to benefit from reverse factoring? Specifically, what is the maximum payment term extension when the receivables holding cost is:
  (a) zero?
  (b) equal to the cost of factoring?
  (c) positive but lower than the cost of factoring?

Looking further to relevant literature, we note first the relation of our study to the growing body of research the interface of operations and finance. Work in this area generally aims to identify conditions under which a tighter integration of the two disciplines creates value or allows improved risk management (Birge, Kouvelis, & Seppi, 2007). Imperfections in capital markets are often assumed, since an interaction between investment and financing decisions is only then possible (Modigliani & Miller, 1958). In our case, reverse factoring mitigates information asymmetry between financial intermediaries and firms, yielding an option to exploit cheaper credit. Specifically, the payment guarantee from buyer to factor entails that the supplier can discount receivables at a cheaper rate than would otherwise be possible.

Three other research topics are particularly relevant to our study: inventory incorporating payment schemes, trade credit policy, and cash management. For inventory theorists, even in deterministic settings, payment schemes undermine the conventional assumption that capital needs are related to average inventory levels. Beranek (1967) was among the first to study the implications of alternative payment practices on the economic lot size decision. Haley and Higgins (1973), Goyal (1985), and Rachamadugu (1989) further enrich this stream. Kim and Chung (1990) propose a model to combine the lot-size decision and the discount offered to customers for early payment.[4] Schiff and Lieber (1974) use control theory to study the relationship between inventory and accounts receivable policy. More recently, scholars have explored the significance of payment schemes by means of stochastic inventory models. For instance, Maddah et al. (2004) investigate the effect of receiving trade credit in a periodic review (s, S) inventory model. Gupta and Wang (2009) show that a base stock inventory policy continues to be optimal when a supplier gives trade credit, but requires adaptation of the base stock parameter.

Most research on trade credit itself is to be found in the economics literature. Given the existence of financial intermediaries, scholars have been interested to explain the role of trade credit (see Seifert et al., 2013, for an extensive review). In addition to this economic perspective, there is normative literature that explores the optimal credit policy to customers. Most of these studies consider the trade-off between lost sales when the policy is too tight and credit losses when policy is too easy. Davis (1966) is among the first to analyse trade credit in terms of marginal revenue and cost. Bierman and Hausman (1970) and Mehta (1970) formulate the credit decision respectively in a finite and infinite horizon framework. Fewings (1992) obtains closed-form solution for the value of granting credit and an upper bound on the acceptable default risk. Another series illustrates the nuances of correctly evaluating a credit policy (Atkins & Kim, 1977; Dyl, 1977; Kim & Atkins, 1978; Oh, 1976; Walia, 1977; Weston & Tuan, 1980). Nonetheless, this literature invariably assumes that inventory and/or procurement polices do not affect the trade credit decision.

Turning to the question of cash management, it is again the economics literature that describes the basic reasons for holding cash. Principally, these are: (1) transaction costs, (2) precaution to adverse shocks and/or costly access to capital markets, (3) taxes, and (4) agency problems (Bates, Kahle, & Stulz, 2009). Many models for optimal cash management exist, of which the ones by Baumol (1952) and Miller and Orr (1966) are seminal. Both models propose cash control policies to balance liquidity with the opportunity cost of holding cash. Recently, scholars have explored the significance of linking liquidity and/or cash management with inventory theory. For instance, Hu and Sobel (2007) show that an echelon base stock policy is sub-optimal in a serial supply chain with liquidity constraints. Luo and Shang (2013) show the value of centralised cash retention in a two-stage supply chain.

Few studies explicitly incorporate all aspects mentioned above: inventory control, trade credit, and cash management. Exceptions include Protopappa-Sieke and Seifert (2010), who optimise order quantities for a finite horizon model that includes working capital restrictions and payment delays. Song and Tong (2012) propose new accounting metrics that allow correction of classical inventory prescriptions for the influence of payment schemes and possible borrowing to cover cash shortages. Luo and Shang (2014) consider a model in which a firm can both receive and grant payment delays; they show that a working capital dependent base stock policy is optimal. Our work complements these studies by adding a further element to a firm's decision problem: the choice between conventional sources of capital and reverse factoring, given that the latter changes financing rates as well as payment delays.

3. Models

We develop a periodic review inventory model for a firm that sells to a creditworthy buyer who offers reverse factoring. Periods are indexed by the variable t and one period represents $l \in \mathbb{R}^+$ years. At the end of each period, the supplier firm will receive a stochastic demand $D_t \geq 0$ from the buyer. In order to have its products ready before demand is revealed, the supplier orders stock at start of each period. We assume that inventory is controlled by a base stock policy of I units. The supplier pays price c per unit for stock and sells at price $p > c$ per unit to the buyer. The ordered items are delivered immediately prior to the end of the period and the supplier pays for them upon delivery. Once demand is revealed, if it cannot be fully met from inventory, the unmet portion is back-ordered until the next period. Each backlogged unit entails a penalty cost b. For each unsold unit, the supplier incurs a storage cost $h < b$. The supplier grants the buyer a payment term of $k \in \mathbb{N}^+$ periods. The payment term starts to count from the moment that a demanded unit is met from inventory. Once revenue from prior demands are collected and costs are paid, the supplier may at the end of each period release cash to shareholders.

In the initial version of our model, the supplier meets periodic expenses with cash retained from previous periods or by borrowing from a bank. Borrowing only occurs to the extent that retained cash is insufficient. The annualised interest charge for borrowing is β per

[3] To our knowledge there is no published study on the relative prevalence of each type of discounting. From conversations with firm managers and trade finance professionals we have the impression that auto discounting is more widely applied.

[4] Despite some superficial similarity, note that a discount offered to the customer for early payment is conceptually distinct from 'discounting' in the context of reverse factoring. In the latter, the firm pays a time-dependent percentage of the value of the receivable to the factor, in order to receive advance payment. The customer still pays the full amount of the receivable at maturity.

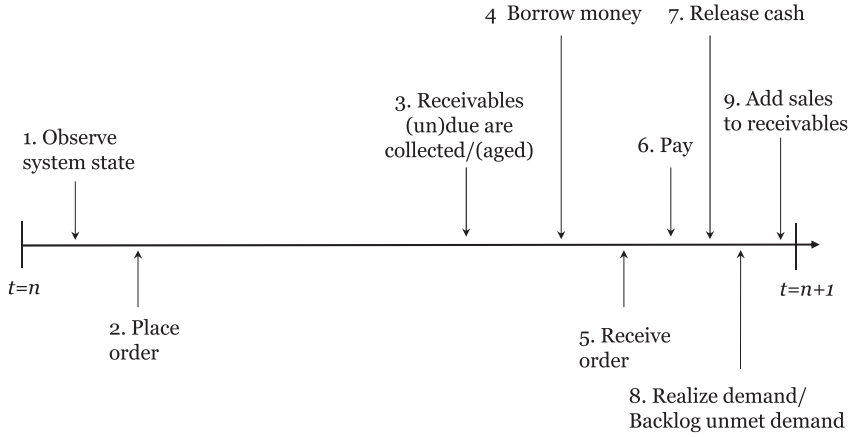


Fig. 1. The sequence of events within a single period.

monetary unit. Cash retention is governed by a constant threshold policy: cash is released whenever it exceeds a threshold level $T \geq 0$, but only to the extent that the cash level is returned to T. As shareholders could have invested retained cash elsewhere, an annualised opportunity cost rate of α is assessed on each monetary unit retained. In a perfect capital market we should expect $\alpha = \beta$, but we assume that capital market frictions may entail $\alpha < \beta$ or $\alpha > \beta$ (Myers & Majluf, 1984)

While we do not theoretically demonstrate a constant threshold policy to be the optimal form of cash management for our model, analogy to a base stock policy appears warranted by the opposition between α and β, which constitute respectively holding and shortage costs for cash. (Cf. discussion of Song & Tong, 2012, where a "base cash" policy is also used.) Cash management policies used in practice may be more complex—e.g., the dual threshold model of Stone (1972)—but contextual factors that motivate these policies, such as net balance requirements of banking agreements, are absent from our model. We assume unlimited borrowing capacity. The model can impose a credit limit, but this forces much of our focus to lie on default events instead of purely on the change in financing needs that results from a payment period extension.

In every period the supplier in our model receives the money from the sales realised k periods ago. The supplier's total periodic payment is P_t, which includes a fixed cost f and variable expenses for the replenishment of its stock, inventory (holding and shortage) costs, and interest for debt outstanding during the period t. Furthermore, analogous to the opportunity cost rate for holding cash, an opportunity cost rate of η per year is assessed on each monetary unit of accounts receivable that result from the payment term. We assume $\eta < \alpha$ as the risk of investing in an account receivable is lower than the risk of investing in the firm itself. Indeed, while settlement of the account receivable is due after a known delay, the timing of cash dividends from the firm depends on demand and realised profits, and is consequently uncertain.

The system state at the start of period t is $S_t = (x_t, m_t, \boldsymbol{r}_t)$. The scalar x_t represents the inventory position and the scalar m_t represents the cash position. A tangible cash balance or a bank overdraft is represented by $m_t > 0$ or $m_t < 0$ respectively. The k-dimensional vector $\boldsymbol{r}_t$ with components $r_{t,i}$ for $i = 1, 2, \ldots, k$ represents the outstanding accounts receivable, i.e., the payments to be collected at the end of periods $t, t+1, \ldots, t+k-1$. The vector S_t conveys all information needed to implement the ordering and cash retention policies in period t: how many units needed to reach base stock, the associated cash payment and the amount of cash that will be received.

Fig. 1 summarises the sequence of events in a period. At the start of period S_t is observed (1) and the order is placed (2). At the end of the period, cash is collected from the oldest accounts receivable and the position of others is decremented (3). Money is borrowed if needed (4), the units ordered are received (5) and the cash payment is made (6). If policy allows, cash is be released from the firm (7). Finally, demand is received and met to the extent that inventory allows. This creates a new account receivable (9).

Since the initial version of our model includes only a conventional source of short-term financing, bank borrowing, we henceforth refer to this as the 'conventional financing' model (CF). The mathematical formulation of this model is given in Section 3.1 below. In Section 3.2 we describe extensions to CF that model the application of reverse factoring. These extensions represent, respectively, the 'manual discounting' model (MD), where discounting only occurs when cash deficits arise (Section 3.2.1), and the 'auto discounting' model (AD), where discounting is always applied as soon as possible (Section 3.2.2).

3.1. Conventional financing model (CF)

When the firm finances its operations solely from internal cash and conventionally borrowing, the transition equations for inventory, cash, and receivables are as follows:

$$x_{t+1} = I - D_t \tag{1}$$

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + \min\{I, D_t\})p & i = k \\ r_{t,i+1} & i = 1, \ldots, k-1 \end{cases} \tag{2}$$

$$m_{t+1} = \min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\} \tag{3}$$

where

$$P_t(I, x_t, m_t) = f + (I - x_t)c + h(x_t)^+ + b(-x_t)^+ + \beta(-m_t)^+ \text{ and}$$
$$(a)^+ = \max\{0, a\}.$$

Equation (1) specifies the inventory position at the start of period $t+1$ to be the base stock level minus the demand from period t. Equation (2) describes the payments to be collected in the next k periods. For $i = k$ payment consists of revenue from demand that was in backlog at the start of period t, plus revenue from demand that occurs and is satisfied in period t. For $i \leq k-1$, record-keeping for payments due from demands prior to period t is updated. Equation (3) specifies the cash position at the start of period $t+1$ to be the cash position at the start of period t, plus the payment collected in period t, minus the expenses made in period t. If the cash position at the end of period t exceeds T, the firm releases exactly the amount of cash needed to bring the firm's cash position down to T.

For a specific joint base stock and cash management policy $Z = (I, T)$, we define $G_{CF}(Z)$ to be the long-run average cost per period.

$$G_{CF}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} \Big[h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - r_{t,1})^+ + \alpha(\min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\})^+ + \eta \sum_{i=1}^{i=k} r_{t,i} \Big]. \quad (4)$$

The definition includes direct costs for inventory and borrowing and opportunity costs assessed on cash management and receivables. We wish to find the policy $Z^* = (I^*, T^*)$ that minimises $G_{CF}(Z)$.

Note that the cash management cost is linked to uncertainty in the match between incoming and outgoing cash flows. If demand were constant, the firm would always be able to match these flows and would not need to borrow money and/or retain cash. Furthermore, there is an interaction between the base stock level I and the cash retention level T. The replenishment cost in period t depends on I and D_{t-1}, the demand of the preceding period. The cash available to meet the replenishment cost depends on T and the size of the demand met in period $t-k$. Even when the payment term is only one period a deficit can arise, since backlogged demand is included in the immediate replenishment cost but revenue is delayed.

3.2. Reverse factoring model extensions

Like factoring, reverse factoring allows a firm to discount a receivable, i.e., receive cash now instead of waiting until the agreed payment delay has elapsed. We define the scalar $\gamma \in (0, 1)$ to be the annualised fraction of face value that the firm must pay to discount a receivable. For example, if $\gamma = 4$ percent, then reverse factoring gives the firm the opportunity to receive immediately 99 percent of the face value of a receivable that would otherwise result in cash payment in three months ($100 - 4 \times 3/12 = 99$). The remaining 1 percent constitutes the financial cost of the transaction (and revenue for the factor).

For the reverse factoring model we set $\gamma < \beta$, so cash from discounting is preferred over cash from borrowing. This lower financing rate is a key characteristic of reverse factoring. Credit risk in the transaction is low, since there is an explicit guarantee from the firm's customer to the factor that payment will be made on the account receivable, and the customer is typically a large, creditworthy corporation. In conventional factoring, the customer is not necessarily credit-worthy and gives no such payment guarantee. So the transaction is risker for the factor and γ is typically higher than the cost of a bank loan for a firm (Soufani, 2002). A further key characteristic of reverse factoring is the possible extension of the agreed payment delay. Even though the supplier can discount receivables at a lower rate, the cost of a discount transaction is increasing in the agreed payment delay. The trade-off between discount rate and agreed payment delay, as identified in our second research question, is consequently critical.[5]

The static k-dimensional vector $\boldsymbol{\gamma}$ with components γ_j represents the rates applicable for discounting receivables that are otherwise due j periods from the beginning of the current period. We set $\gamma_1 = 0$ since receivable $r_{t,1}$ is due anyway at the end of period t. For $j > 1$ we set $\gamma_j = \gamma_{j-1} + l\gamma$ (recall l is the length of one period in years). The discount γ_j is applied at the moment the receivable is discounted. The discount increases in the due date of the receivable, so the firm discounts receivables in order of decreasing age, i.e., first the ones are due soonest. If the firm discounts all receivables but still needs more cash, conventional borrowing is used. The sequence of events with reverse factoring is the same as in Fig. 1, except that at (4) the firm discounts receivables as needed and available, before resorting to borrowing.

Next we discuss further model extensions that accommodate the two ways of applying reverse factoring: manual discounting or auto discounting.

3.2.1. Manual discounting (MD)

In this case the supplier prefers to discount receivables rather than borrow to cover a cash deficit, but does not discount receivables if enjoying a cash surplus. This entails changes to the transition equations for receivables (2) and cash (3). For receivables we have

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + \min\{I, D_t\})p & i = k \\ (1 - \varphi_{t,i}) r_{t,i+1} & i = 1, \ldots, k-1 \end{cases} \quad (5)$$

where

$$\varphi_{t,i} = \frac{\min\{(1-\gamma_n) r_{t,i+1}, (P_t(I, x_t, m_t) - m_t - \sum_{n=1}^{n=i}(1-\gamma_n) r_{t,n})^+\}}{(1-\gamma_n) r_{t,i+1}}$$

is the fraction of $r_{t,i+1}$ that needs to be discounted to meet the cash need. For cash we have

$$m_{t+1} = \begin{cases} \min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\} & m_t + r_{t,1} - P_t(I, x_t, m_t) \geq 0 \\ 0 & m_t + r_{t,1} - P_t(I, x_t, m_t) < 0 \ \wedge \\ & m_t + \sum_{n=1}^{n=k}(1-\gamma_n) r_{t,n} - P_t(I, x_t, m_t) \geq 0 \\ m_t + \sum_{n=1}^{n=k}(1-\gamma_n) r_{t,n} - P_t(I, x_t, m_t) & \text{otherwise.} \end{cases} \quad (6)$$

There are three possible outcomes for the cash position that are captured by the respective cases in (6):

(a) After paying P_t and without discounting any receivables, the firm's cash position is non-negative. If the firm's cash position less than or equal to T, no cash is released to shareholders; If it exceeds T, excess cash is released to shareholders and the cash position returns to T.
(b) The firm must discount some receivables to meet P_t. Receivables are discounted so that the cash position equals zero. No cash is released to shareholders.
(c) Even after discounting all receivables, the firm has insufficient cash to meet P_t. Borrowing occurs, so the cash position is negative. No cash is released to shareholders.

Again we wish to find the policy $Z^* = (I^*, T^*)$ that minimises the long run average cost per period, but the new objective function $G_{MD}(Z)$ includes factoring as well as conventional borrowing:

$$G_{MD}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} \Big[h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - r_{t,1})^+ + \alpha(\min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\})^+ + \sum_{i=1}^{k} (\gamma_i \varphi_{t,i} + \eta(1 - \varphi_{t,i})) r_{t,i} \Big]. \quad (7)$$

3.2.2. Auto discounting (AD)

In this case the supplier discounts the full value of any receivable as soon as it is possible to do so. Due to sequence of events (Fig. 1), holding costs for receivables are still incurred for one period. With auto discounting, the transition equations for receivables (2) and cash

[5] Our model could just as well represent a conventional factoring transaction, but as typically $\gamma \geq \beta$ in this case, borrowing would clearly always be preferable. The existence of the market for factoring services in practice (despite $\gamma \geq \beta$) is partly due that our baseline assumption of unlimited access to loans does not apply.

(3) are changed as follows.

$$r_{t+1,i} = \begin{cases} ((-x_t)^+ + \min\{I, D_t\})p & i = k \\ 0 & i = i, \ldots, k-1 \end{cases} \quad (8)$$

$$m_{t+1} = \min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\} \quad (9)$$

We aim to find the policy $Z^* = (I^*, T^*)$ that minimises the long run average cost per period, as defined by the objective function $G_{AD}(Z)$:

$$G_{AD}(Z) = \lim_{t\to\infty} \frac{1}{t} \sum_{t=1}^{\infty} \Big[h(x_t)^+ + b(-x_t)^+ + \beta(P_t(I, x_t, m_t) - m_t - \gamma_k r_{t,k})^+ + \alpha(\min\{m_t + r_{t,1} - P_t(I, x_t, m_t), T\})^+ + (\eta + \gamma_k) r_{t,k} \Big]. \quad (10)$$

4. Algorithm and experimental design

The objective function of a base stock policy in a pure inventory setting is convex (Porteus, 2002), but the inclusion of cash and receivables in the state description of our model precludes a comparable analytic insight. Besides the increased size of the state space, the interaction between the base stock parameter and cash retention parameter complicates analysis. In an initial exploration of the solution space by means of simulation, we find the objective function to exhibit convexity in both decision variables for all system configurations (CF, MD and AD) and a range of parameter values. Specifically, in all cases we find a unique policy Z^* that yields globally minimal average cost, and no policy that yields a local extreme point. Based on this insight, we utilise a 3-stage algorithm in our subsequent simulation experiments, in order to find the globally optimal policy efficiently. In this section we describe first this algorithm; see also Algorithm 1. Afterward we describe in detail the design of our experiments. The results of the experiments are given in Section 5.

4.1. Solution algorithm

In its first two stages, the algorithm determines a truncated search interval, $[I_l, I_u] \times [T_l, T_u]$, through iterative gradient estimations in each policy dimension. In the third stage, stochastic approximation is used to find the optimal policy. Stochastic approximation is an iterative scheme that attempts to find a zero of the gradient of the objective function. It has been widely studied since the pioneering works of Robbins and Monro (1951) and Kiefer and Wolfowitz (1952) (Broadie, Cicek, & Zeevi, 2009; Fu, 2006). We use a multidimensional version of the Kiefer-Wolfowitz algorithm, which was first introduced by Blum (1954). As algorithms of this type are prone to poor finite-time performance, we make two improvements to reach faster convergence, following the proposals of Broadie et al. (2009). First, we use different tuning sequences in each dimension, in order to adapt better to the different convexity characteristics of each. Second, to avoid long oscillatory periods, we check in each iteration whether the next policy would be located within the truncated search interval; if it goes outside, the tuning sequence is amended to ensure that the next policy lies again within the search interval.

Algorithm 1 Stochastic approximation algorithm for determination of Z^*

Step 0: Choose algorithm parameters

- initial step sizes a_0^k for $k = 1, 2$; default values $a_0^1 = 1, a_0^2 = 2$;
- initial policy $Z_0 = (I_0, T_0)$; default value $Z_0 = (\mu_D, 0)$;
- stopping condition υ; default value $\upsilon = 1 \times 10^{-6}$;

Step 1: Localise $[I_l, I_u]$, the search interval for the base stock parameter

Set $Z_n = (\mu_D + n a_0^1, 0)$ for $n \in \mathbb{N}^+$. Evaluate iteratively the gradient estimation $\tilde{G}'(n) = (\tilde{G}(Z_{n+1}) - \tilde{G}(Z_n))/a_0^1$. In each iteration, increase the number of replications dynamically until the confidence interval of the estimation, $[\tilde{G}_{LB}, \tilde{G}_{UB}]$, indicates a statistically significant direction, i.e., $\tilde{G}'_{LB}(n) > 0$ or $\tilde{G}'_{UB}(n) < 0$. If $\tilde{G}'_{UB}(n) < 0$, then $\{n\} \leftarrow \{n+1\}$; if $\tilde{G}'_{LB}(n) > 0$, store values as indicated below and move to step 2.

- initial policy for step 2: $\{Z_0\} \leftarrow \{Z_n\}$;
- base stock search interval for step 3: $[I_l, I_u] = [I_n - a_0^1, I_n + a_0^1]$.

Step 2: Localise $[T_l, T_u]$, the search interval for the cash retention parameter

Starting at policy Z_0, evaluate iteratively the gradient $\tilde{G}'(n)$ with $Z_n = (I_0, 0 + n a_0^2)$ until $\tilde{G}'_{LB}(n) > 0$ or $\tilde{G}'_{UB}(n) < 0$. If $\tilde{G}'_{UB}(n) < 0$, then $\{n\} \leftarrow \{n+1\}$; if $\tilde{G}'_{LB}(n) > 0$ store values as indicated below and move to step 3.

- the initial policy for step 3: $\{Z_0\} \leftarrow \{Z_n\}$;
- the cash retention search interval $[T_l, T_u] = [T_n - a_0^2, T_n + a_0^2]$.

Step 3: Determine the joint optimal policy: Z^*

- Set $\{a_0^k\} \leftarrow \{0.1 a_0^k\}$, $\{\tau^k\} \leftarrow \{0.1 a_0^k\}$, and $\{\lambda^k\} \leftarrow \{0\}$ for $k = 1, 2$.
- Evaluate $\tilde{G}' = (\tilde{G}(I_0 + a_0^1, T_0) - \tilde{G}(Z_0))/a_0^1$ and set $\{\theta^1\} \leftarrow \{1/|\tilde{G}'_1|\}$.
- Evaluate $\tilde{G}' = (\tilde{G}(I_0, T_0 + a_0^2) - \tilde{G}(Z_0))/a_0^2$ and set $\{\theta^2\} \leftarrow \{1/|\tilde{G}'_2|\}$.

Use the following recursion to calculate Z_{n+1}:

$$Z_{n+1} = Z_n - \left(a_n^1 \frac{\tilde{G}(Z_n + c_n^1) - \tilde{G}(Z_n)}{c_n^1}, a_n^2 \frac{\tilde{G}(Z_n + c_n^2) - \tilde{G}(Z_n)}{c_n^2} \right),$$

where

- $\boldsymbol{c}_n$ with $c_n^k = \tau^k / n^{\frac{1}{4}}$ for $k = 1, 2$ is the sequence of finite difference widths,
- $\boldsymbol{a}_n$ with $a_n^k = \theta^k / (n + \lambda^k)$ for $k = 1, 2$ is the sequence of step sizes.

In each iteration, check:

- if $|\tilde{G}(Z_{n+1}) - \tilde{G}(Z_n)| < \upsilon$, return Z_{n+1} and terminate search;
- if $I_{n+1} < I_l$ or $I_{n+1} > I_u$, adapt λ^1 such that $I_l \le I_{n+1} \le I_u$;
- if $T_{n+1} < T_l$ or $T_{n+1} > T_u$, adapt λ^2 such that $T_l \le T_{n+1} \le T_u$.

4.2. Experimental design and parameter settings

We answer our research questions by means of four sets of simulation experiments: Experiment 1 and Experiment 2(a)–(c), corresponding to the numbering of the research questions in Section 2. In each experiment, we explore 3×3 basic settings: all combinations of three possible levels for the expected net profit margin, $\omega = (\mu_D(p - c) - f)/\mu_D p$, and three possible levels for operating leverage, $\psi = f/(\mu_D c + f)$. Operating leverage is a measure of the relationship between fixed cost and total cost for a firm (Brealey et al., 2011). The specific values of p, c and f that underlie the nine basic settings are shown in Table 1.

In all experiments we take demand to be log-normally distributed with mean $\mu_D = 10$ and coefficient of variation $c.v. = \mu_D/\sigma_D$ equal to either 0.25 or 0.50. Full detail of demand and cost parameters for each experiment appears in Table 2. In all experiments, one period corresponds to one week. Table 2 shows annual financing rates, which are converted to weekly rates under the assumption of simple interest.

Table 1
Values of unit selling price p, unit cost c, and total fixed cost f underlying the experimental settings of net profit margin ω and operating leverage ψ.

$\psi \backslash \omega$	0.1	0.2	0.3
0	10, 9, 0	10, 8, 0	10, 7, 0
0.3	10, 6.3, 27	10, 5.6, 24	10, 4.9, 21
0.6	10, 3.6, 54	10, 3.2, 48	10, 2.8, 42

The next paragraphs describe explicitly our four experiments. Although we determine the optimal policy Z^* for every experimental instance, we are generally most interested to compare policies or the performance of the system across different payment terms. Consequently, in order to facilitate the presentation, we use $Z^*(k) = (I^*(k), T^*(k))$ to denote the optimal policy for payment term k, and we rewrite the objective functions as $G_{(.)}(k)$, suppressing the immediate dependence on Z^*.

Experiment 1: *The impact of payment terms with conventional financing and no opportunity cost rate for holding receivables.* Here we explore how payment terms impact total financing cost for the supplier firm when the opportunity cost rate for receivables is neglected, i.e., $\eta = 0$ percent. In addition to the experimental settings shown in Table 1, we test the sensitivity of our findings to changes in other factors: the relative magnitude of inventory backlog cost b to inventory holding cost h, and the relative magnitude of cash opportunity cost rate α to conventional financing cost rate β.

Experiment 2(a): *Maximum payment term extension with no opportunity cost rate for holding receivables.* We explore the trade-off between cheaper credit and extended payment terms in reverse factoring when there is no opportunity cost rate for holding a receivable, i.e., $\eta = 0$ percent. For each initial payment term k_0, we determine $Z^*(k_0)$ when only conventional financing at rate β is used. Then, with reverse factoring at rate $\gamma \leq \beta$ also available, we determine the maximum extended payment term $k_e \geq k_0$ such that $G_{\mathrm{MD}}(k_e) \leq G_{\mathrm{CF}}(k_0)$.

Experiment 2(b): *Maximum payment term extension with greatest opportunity cost rate for holding receivables.* We set the opportunity cost rate for holding receivables equal to the cost rate for factoring, $\eta = \gamma$, but otherwise explore the same trade-off as in Experiment 2(a). Accordingly, we seek the maximum extended payment term $k_e \geq k_0$ for which $G_{\mathrm{AD}}(k_e) \leq G_{\mathrm{CF}}(k_0)$.

Experiment 2(c): *Maximum payment term extension with intermediate opportunity cost rate for holding receivables.* Again we explore the same basic trade-off as in Experiment 2(a), but now the opportunity cost rate for holding receivables is less than reverse factoring rate, $0 < \eta < \gamma$. We determine the maximum extended payment term $k_e \geq k_0$ such that $G_{\mathrm{MD}}(k_e) \leq G_{\mathrm{CF}}(k_0)$.

In all experiments we let the system start with zero cash, zero inventory, and zero receivables. We begin to assess performance after a warm-up of 500 periods, which is determined based on Welch's procedure (Law & Kelton, 2000; Welch, 1983). We calculate 95 percent confidence intervals from 30 independent replications, each with total run-length of 20,000 periods (including warmup). Relative error is approximately 0.5 percent (Law & Kelton, 2000). After some initial global calibration, we were able to locate the optimal policy and cost for each setting on an ordinary personal computer within two or three minutes.

5. Numerical results

Here we present and discuss the results from each experiment. Section 5.1 covers Experiment 1, the impact of a payment term extension on the firm's financing cost, and the accompanying sensitivity analysis for changes in inventory and cash management cost parameters. Sections 5.2–5.4 cover respectively Experiments 2(a)–(c), the trade-off between payment term extension and reverse factoring for the three different scenarios of opportunity cost rate for holding receivables.

5.1. The impact of payment terms with conventional financing and no opportunity cost rate for holding receivables

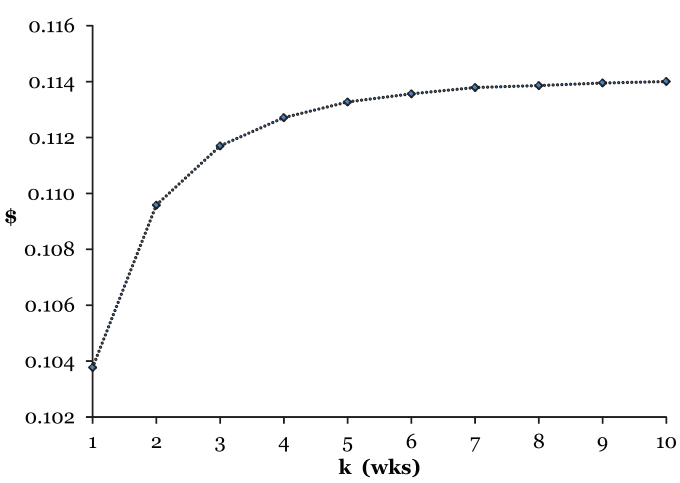
In all configurations of this experiment, we observe the following general relationship between financing cost and payment term:

> *The optimal cost $G_{CF}(k)$ increases asymptotically in the payment term k.*

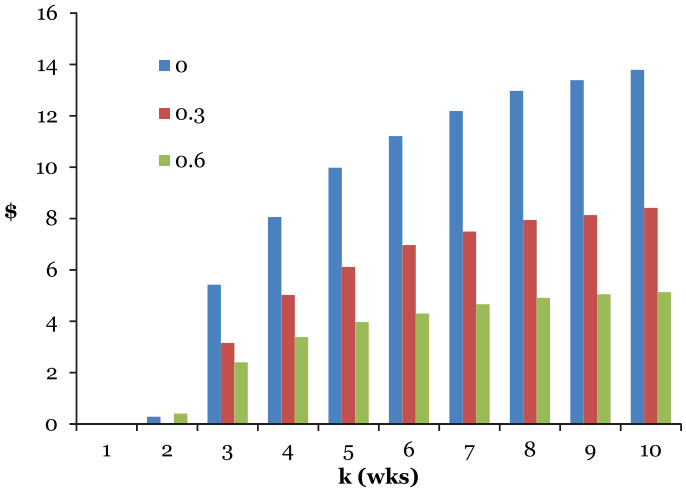
Fig. 2a illustrates this finding. While the optimal cash retention level $T^*(k)$ increases asymptotically, the optimal base stock level $I^*(k)$ decreases slightly or remains constant. Changes in the base stock level occur because a backlog event delays the receipt of cash, which may entail a financing need. The relative impact of this is greater when the payment term is short, as the base stock then tends to be higher. Despite changes in the base stock level, changes in inventory cost appear statistically insignificant across the different payment term settings. The increase in cost from a payment term extension can thus be entirely attributed to greater variability in cash flow. As there is no opportunity cost rate for holding receivables in this experiment ($\eta = 0$), we conclude that a payment term extension entails greater financial costs than such opportunity costs alone.

Table 2
Demand and cost parameter settings.

Experiment	1	2(a)	2(b)	2(c)
Scenario	$\mathrm{CF}_{\eta=0\text{ percent}}$	$\mathrm{CF}_{\eta=0\text{ percent}}$ vs. $\mathrm{MD}_{\eta=0\text{ percent}}$	$\mathrm{CF}_{\eta=8\text{ percent}}$ vs. $\mathrm{AD}_{\eta=\gamma\text{ percent}}$	$\mathrm{CF}_{\eta>0\text{ percent}}$ vs. $\mathrm{MD}_{\eta>0\text{ percent}}$
μ_D	10	10	10	10
$c.v.$	0.25, 0.5	0.25, 0.5	0.25, 0.5	0.25
h	0.02	0.02	0.02	0.02
b	0.1,0.2,0.4	0.2	0.2	0.2
α (in percent)	4,8,12	8	8	8
β (in percent)	8	8	8	8
η (in percent)	0	0	8	0.5,1,2,4
k	1–10	n.a.	n.a.	n.a.
k_0	n.a.	2,4	2,4	2,4
k_e	n.a.	2–12,4–14	2–12,4–14	2–12,4–14



(a) $G_{CF}(k)$ with fixed parameters $c.v. = 0.25, \omega = 0.2, \psi = 0.3$



(b) $T^*(k)$ with $c.v. = 0.25, \omega = 0.2$, and varing levels of operating leverage ψ.

Fig. 2. The impact of trade credit on system cost and cash retention.

The apparent concavity of the objective function implies that the relative cost of extending payment terms decreases with the pre-existing payment term. This makes intuitive sense. In a system with arbitrarily long payment terms, the incoming and outgoing cash flows become essentially independent. Additional delay in cash receipts resulting from a payment term extension should have negligible impact. The optimal cash retention level thus increases asymptotically. Being 'smarter' with cash provides little benefit when payment terms are very long. This same argument has already been used to explain the asymptotic optimality of constant-order policies for lost sales inventory models with large lead times (Goldberg et al., 2012).

Since we set $\alpha = \beta$ in this experiment, the cash retention level is the result of a trade-off that minimises the amount of capital needed for running the base stock operation. According to conventional finance literature, capital market frictions form the main motivation to retain and/or optimise cash (Bates et al., 2009; Myers & Majluf, 1984). Frictions can cause $\alpha \neq \beta$. In the last part of this section we therefore present a sensitivity study on the impact of these frictions.

Turning to the basic parameters that define our experimental scenarios, we examine their effect on the relative cost of a payment term extension. Specifically, if $G_{CF}(k)$ is the cost of an initial payment term k, then $\Delta G(k) \equiv (G_{CF}(k+1) - G_{CF}(k))/G_{CF}(k)$ is the relative cost of extending the payment term by one week. The results of Experiment 1 then support the following assertion:

> *The firm's relative cost of a payment term extension $\Delta G(k)$ is increasing in the coefficient of variation for demand, but decreasing in the initial payment term k, the net profit margin ω, and the operating leverage ψ.*

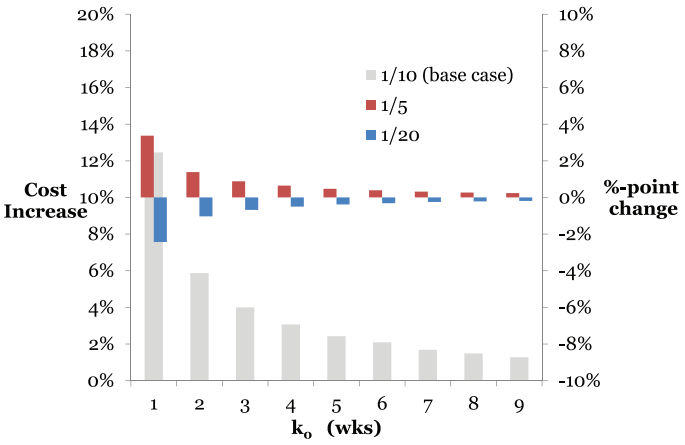
Sample paths show that a higher demand uncertainty causes higher uncertainty in the incoming and outgoing cash flows, exacerbating the impact of the payment term extension. The cash deficits or excesses accumulated will each tend to be greater in magnitude. A lower net profit margin or a lower operating leverage also increases the firm's sensitivity to an extension of payment terms. The effect of a higher net profit margin is intuitively reasonable, since it provides a greater buffer against the potential deficits that arise from a mismatch between incoming and outgoing cash flows. The effect of operating leverage is less obvious, since fixed cost are often considered to be burdensome. Firms with high operating leverage are even considered to more risky by investors (Brealey et al., 2011). While we indeed find that a higher operating leverage may imply a higher absolute cost, higher operating leverage makes the firm less sensitive to payment term extension. This appears to result from the relative stability of the outgoing cash flows for firm with higher operating leverage. Moreover, the optimal cash retention level decreases with the operating leverage. Fig. 2b illustrates this. Firms that rely more heavily on external purchases need to keep more cash to competitively sustain their payment terms to customers than firms that rely more on internal production with fixed costs. While a variable cost structure may be an attractive way to handle lower demand realisations in an uncertain market, it has negative implications for the ability to match incoming and outgoing cash flows when demand is more stable.

5.1.1. Sensitivity study for other parameters

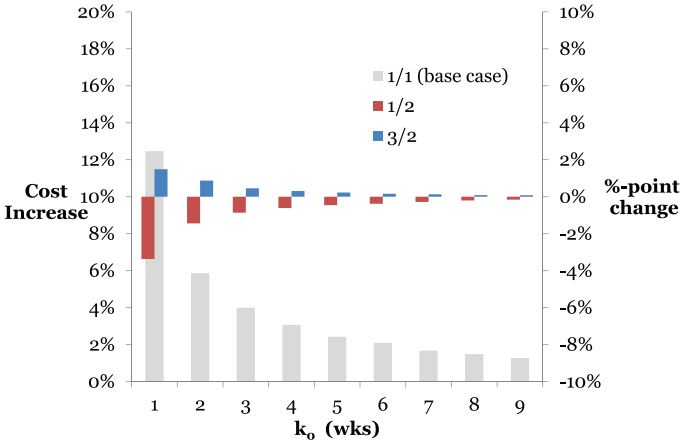
As the cost of a payment term extension is the basis of exploration in our subsequent experiments, we explore the sensitivity of our results to changes in the relative magnitude of inventory costs, h and b, and the relative magnitude of financing costs, α and β. These studies support the following assertion:

> *The relative cost of a payment term extension is increasing in the ratio h/b and in the ratio α/β.*

Changes in the inventory or financing cost ratios have a significant effect on the optimal base stock and cash retention level, but a minor effect on the increase in financing cost that results from a payment term extension. Fig. 3 provides illustration. Fig. 3a shows the impact of varying the inventory cost ratio. As the value h/b increases, the sensitivity of costs to an increase in payment terms also increases. An increase in h/b entails a decrease in the optimal base stock level, which increases the probability of backlog. As explained earlier, backlog increases the probability of incurring a cash deficit, as it simultaneously delays cash receipts while additional cash is needed for stock replenishment. Fig. 3b shows the impact of varying the financing cost ratio. As the value α/β increases, the relative cost of a payment term extension also increases. An increase in α/β means that the cost of holding cash becomes relatively expensive in comparison to borrowing, which limits the ability to use retained cash as a protection against cash flow uncertainty. If α/β is large, the firm may completely stop holding cash, i.e., $T^* = 0$ becomes the optimal cash retention threshold.

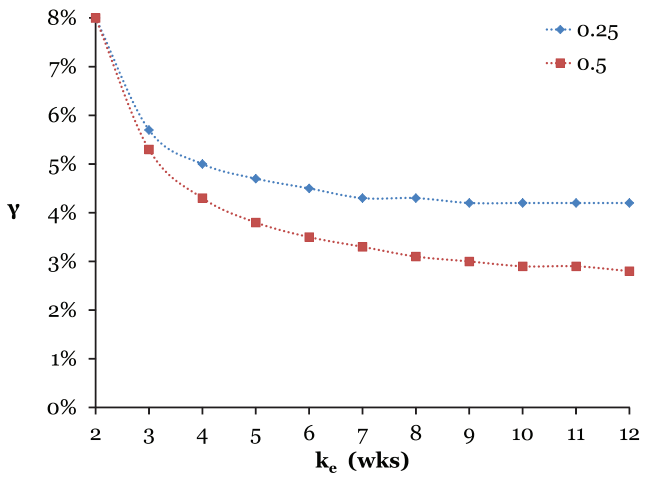


(a) Percentage cost increase from a payment term extension of 1 week for base case $h/b = 1/10$ (left axis) and incremental change for $h/b = 1/5$ and $h/b = 1/20$ (right axis)

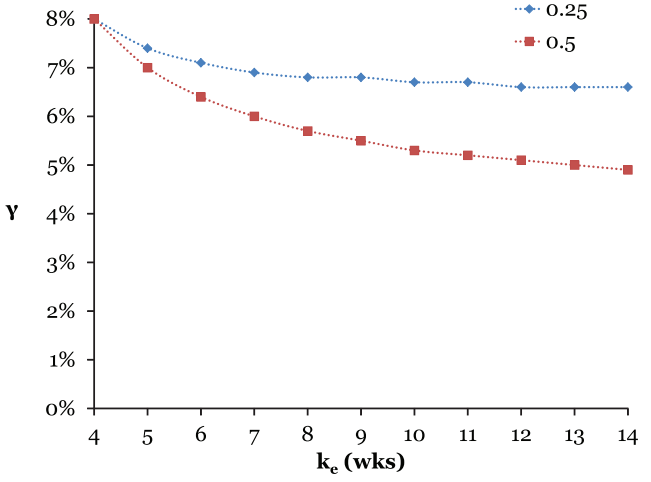


(b) Percentage cost increase from a payment term extension of 1 week for base case $\alpha/\beta = 1/1$ (left axis) and incremental change for $\alpha/\beta = 1/2$ and $\alpha/\beta = 3/2$ (right axis)

Fig. 3. Cost sensitivity to a payment term extension of 1 week for varying levels of inventory cost ratio h/b and financing cost ratio α/β. Fixed parameters $c.v. = 0.5, \omega = 0.1$, and $\psi = 0.5$.



(a) $k_0 = 2$



(b) $k_0 = 4$

Fig. 4. Trade-off between factoring rate γ and maximum extended payment term k_e with manual discounting. Fixed parameters $\eta = 0, \omega = 0.2, \psi = 0$; $c.v. = 0.25$ or 0.50.

5.2. Maximum payment term extension with no opportunity cost rate for holding receivables

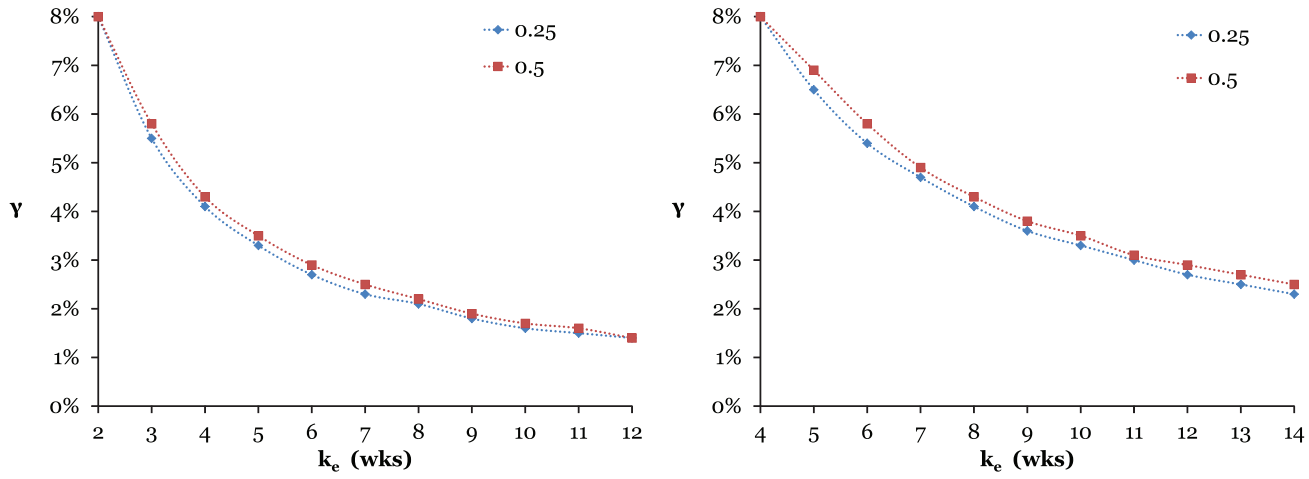
Building on the insights provided by Experiment 1, we explore the maximum payment term extension that allows a firm to benefit from reverse factoring when the opportunity cost rate for holding receivables is negligible. Although the payment term extension increases the total value of outstanding receivables, the firm only considers the direct cost of financing its inventory operation. We define k_e, the maximum payment term extension with reverse factoring, to be the longest payment term such that the firm no greater financing cost as it did with conventional financing and no payment term extension:

$$k_e \equiv \max k$$
$$\text{subject to } G_{MD}(k) - G_{CF}(k_0) \leq 0. \tag{11}$$

This experiment supports the following assertion:

> *When the opportunity cost rate for holding receivables is negligible and the firm uses manual discounting, the maximum payment term extension k_e for a given reverse factoring rate γ is decreasing in the coefficient of variation for demand, but increasing in the initial payment term k_0, the net profit margin ω, and the operating leverage ψ.*

The significance of our main experimental parameters for the maximum payment term extension with reverse factoring appears consistent with their significance for the relative cost of a payment term extension in the conventional financing setting. Where previously we saw greater relative costs for an extension, here we see a smaller maximum possible extension. Fig. 4 shows, for different values of initial payment term and demand uncertainty, the set of (γ, k_e) values for which the financing cost with reverse factoring and manual discounting case is equivalent to the initial cost with conventional financing. With reverse factoring and manual discounting, borrowing activity is generally reduced to negligible levels: factoring substitutes for borrowing. In some cases, most particularly when the experimental setting allows only a minimal payment term extension, borrowing may still occur. The optimal cash retention level may be positive in manual discounting ($T^* > 0$), but in contrast to the case with conventional financing, it decreases with the extended



(a) $k_0 = 2$ (b) $k_0 = 4$

Fig. 5. Trade-off between factoring rate γ and maximum extended payment term k_e with auto discounting. Fixed parameters $\eta = \gamma, \omega = 0.2, \psi = 0$; *c.v.* = 0.25 or 0.50.

payment term. As payment terms get longer the firm acquires enough receivables to finance operations without borrowing or retaining cash.

5.3. Maximum payment term extension with greatest opportunity cost rate for holding receivables

Here we explore the maximum payment term extension that allows a firm to benefit from reverse factoring when the opportunity cost rate for holding receivables η is equal to γ, the cost of discounting them. The definition of maximum payment term extension k_e in this experiment is analogous to (11), but with $G_{AD}(.)$ in place of $G_{MD}(.)$. When $\eta = \gamma$, the supplier is indifferent between manual and auto discounting. Although a greater opportunity cost rate for holding receivables is in principle possible, the choice for auto discounting is constant at this point and beyond. Our experiments support the following assertion.

> *With reverse factoring and auto discounting, the maximum extended payment term k_e for a given reverse factoring rate γ is not affected by the coefficient of variation for demand, net profit margin, or operating leverage.*

Fig. 5 shows, for different values of initial payment term and demand uncertainty, the set of (γ, k_e) values for which the financing cost with reverse factoring and auto discounting case is equivalent to the initial cost with conventional financing. Note that the maximum payment term extension and the reverse factoring rate are inverse-proportionally related. Since the firm discounts all of its receivables in every period, periodic expenses can almost always be met. Borrowing activity is negligible in this setting, even when operating leverage and demand uncertainty are both high. When the factoring rate is equal to (or less than) the opportunity cost rate for holding a receivable, a decision-maker can evaluate a proposed reverse factoring arrangement independently of the stochastic and economic aspects of inventory operations.

5.4. Maximum payment term extension with intermediate opportunity cost rate for holding receivables

When the cost of receivables is higher than zero but below the cost of discounting, the cost of additional capital tied in receivables and the cost of additional capital required to fund cash deficits need to be accounted for in the trade-off. We wish to determine the maximum payment term extension that allows a supplier to benefit from reverse factoring. In this case the simulation results support the following assertion:

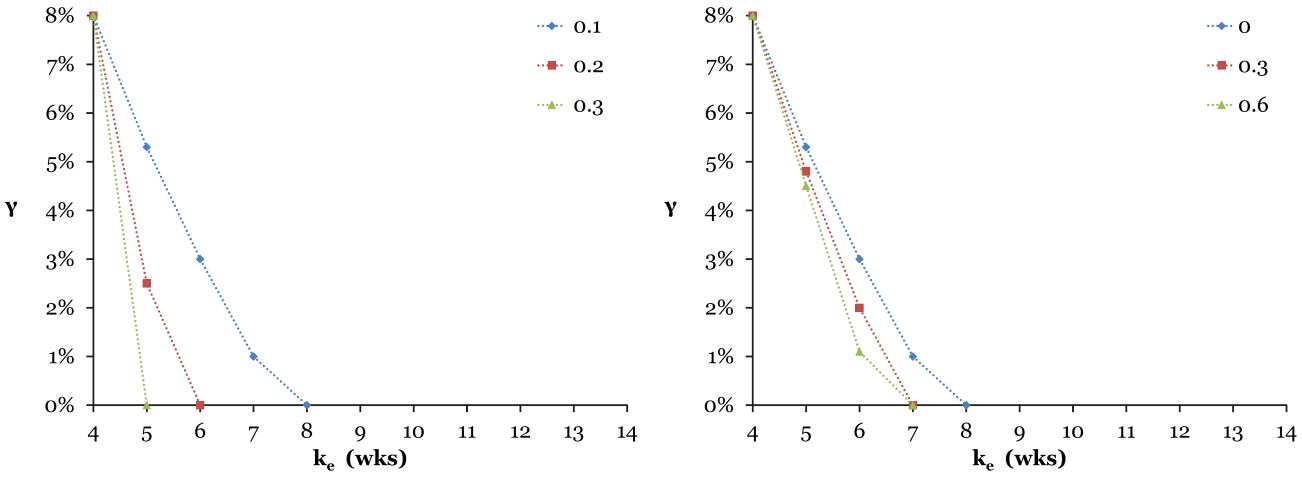
> *There exists an opportunity cost rate $\eta_{max} < \gamma$ such that no economically viable payment term extension is possible when $\eta_{max} < \eta < \gamma$. When $0 < \eta < \eta_{max}$, the maximum extended payment term k_e for reverse factoring is decreasing in η. The maximum extended payment term k_e is decreasing in the net profit margin ω and operating leverage ψ.*

Table 3
Maximum opportunity cost rate for holding receivables η_{max} (in percent) such that a payment term extension $k_e > k_0$ is possible.

$\psi \backslash \omega$	0.1	0.2	0.3
(a) $k_0 = 2$, *c.v.* = 0.25			
0	1	0.5	0.5
0.3	1	0.5	0.5
0.6	1	0.5	0.5
(b) $k_0 = 4$, *c.v.* = 0.25			
0	4	1	0.5
0.3	2	0.5	0.5
0.6	2	0.5	0.5

The maximum extended payment term appears to be highly sensitive to the opportunity cost rate for receivables. In this setting, the firm pays a dual premium for extended payment terms: the cost of carrying additional receivables and the cost of additional cash flow uncertainty. The possibility to extend payment terms and still realise a lower expected cost appears limited to settings where the opportunity cost rate for receivables is low. Table 3 shows the values of η_{max} that result from our basic experimental settings. In many cases, the opportunity cost rate for holding receivables must be below 0.5 percent if the firm is to extend payment terms and realise a reduction in financing cost. The presumption that a firm assesses its opportunity cost rate for holding receivables at a rate equal to or higher than the reverse factoring rate can therefore be deceiving in terms of value creation.

Even though no cost-effective payment term extension is feasible when $\eta_{max} < \eta < \gamma$, these cases may have practical relevance. As mentioned earlier, corporate customers sometimes unilaterally impose a payment term extension on their suppliers (e.g., Milne, 2009). Faced with such a situation, managers must still decide between auto discounting or manual discounting. Intuitively, since the cost of discounting any single receivable is greater than the opportunity cost,



(a) Different levels of ω; $\psi = 0$ fixed.

(b) Different levels of ψ; $\omega = 0.1$ fixed.

Fig. 6. Trade-off between γ and k_e with manual discounting. Fixed parameters $\eta = 0.005$, $k_0 = 4$, and $c.v. = 0.25$.

the selective approach of manual discounting should be preferred. Our experiments confirm this, showing that the relative advantage of manual discounting versus auto discounting increases in the opportunity cost of receivables and also in the size of the imposed payment term extension.

In contrast to the case of $\eta = 0$ examined in Experiment 2(a), the maximum extended payment term for reverse factoring appears to be *decreasing* in the net profit margin and operating leverage when $0 < \eta < \eta_{\max}$. Fig. 6 illustrates this. While a lower net profit margin or lower operating leverage make an extension of payment terms more costly in terms of cash flow variability, this same variability leads the firm to discount a greater proportion of its receivables. The average amount of outstanding receivables and the corresponding opportunity cost is thus ultimately lower. The latter effect tends to dominate when the opportunity cost rate for holding receivables increases toward $\eta_{\max}$, so lower net profit margin and lower operating leverage then both facilitate longer payment terms. As $\eta \to 0$ the cost of cash flow variability becomes more important, and direction of significance for net profit margin and operating leverage tends to reverse. This contrast shows that a good estimation of the opportunity cost rate for holding receivables is essential if managers are to evaluate any payment term extension proposed in a reverse factoring arrangement.

6. Conclusions

Despite growing business interest for supply chain finance, little is yet scientifically known about the optimal management and benefits of such innovative financing arrangements. Our study focuses on reverse factoring, an arrangement that promises improvement of working capital financing for investment-grade buyers and their suppliers. The buyer facilitates cheaper short term financing for the supplier, and the latter in return may be asked to grant longer payment terms. We couple a periodic review, infinite horizon base stock inventory model with financing by either conventional or reverse factoring, which allows us to explore the effect of a payment term extension.

The cost of a payment term is classically assessed purely in terms of the value of the associated receivables and the firm's weight average cost of capital. We show first that even without any opportunity cost rate, a payment term extension will generally entail greater financing needs. Additional capital is needed to cope with more variable cash flows. Introducing reverse factoring, we identify settings that allow a decision maker to make a payment term decision independently of inventory, and other settings where the maximum viable payment term extension depends on demand uncertainty, net profit margin, and operating leverage. We show that the significance of these parameters for financing costs may be complex and interrelated. Correspondingly, a decision about a payment term extension may be challenging.

Based on the data of a large provider of supply chain finance services, Klapper, Laeven, and Rajan (2012) find that creditworthy buyers receive contracts with the longest maturities from their smallest, least creditworthy suppliers. While it is known that large buyers may use their strong bargaining position in extending trade credit terms with their suppliers (Wilson & Summers, 2002), this finding can also be seen as an indication of how emerging services impacts the trade credit landscape. Features of reverse factoring suggest that the facilitation of trade credit has become easier: financing rates are low and receivables can be discounted any time during the trade credit period. Our results suggest, however, that making payment term decisions based on the expected working capital changes will not account for the dynamics of stochastic inventory operations and their interaction with financing requirements.

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