Comparative assessment of harmonic, random, swept sine and shock excitation methods for the identification of machine tool structures with rotating spindles

COMPARATIVE ASSESSMENT OF HARMONIC, RANDOM, SWEPT SINE AND SHOCK EXCITATION METHODS FOR THE IDENTIFICATION OF MACHINE TOOL STRUCTURES WITH ROTATING SPINDLES

dr. ir. H. VAN BRUSSEL, Katholieke Universiteit Leuven, Belgium
Submitted by: Prof. J. PETERS

The different excitation methods are compared together with their respective advantages and drawbacks and illustrated with experimental data.

Special consideration will be given to shock excitation of rotating machine-tool spindles. This technique is very promising by its simplicity but severe problems arise due to the low signal to noise ratio. Some techniques are discussed to improve this behaviour.

1. INTRODUCTION

The analysis of the dynamic behaviour of machine tools has already contributed significantly to the improvement of their dynamic stability against chatter.

Until very recently, the harmonic analysis was the only reliable analysis method available. Its application is widespread in industrial research labs. For the solution of more complex problems, where several response curves are involved simultaneously e.g. for predicting the chatter behaviour of machine tools using dynamic cutting coefficients, the method proves to be less flexible. This is mainly due to the analog nature of the instruments involved, making them less suitable for utilization in connection with a digital computer.

The explosive evolution of digital techniques in the last few years and the availability of digital instruments at reasonable prices have opened the door for the implementation of other identification techniques which were already known in theory, but of which a scientist could only dream when it came to practical application.

The harmonic analysis has been so successful by the mere fact that sinusoïdal signals are so easily obtained and measured with general purpose analog measuring instruments. In theory, however, for measuring a transfer function, the form of the excitation signals is of no importance. Indeed, it can be proven that the transfer function $H(\omega)$ of a linear system is defined as the ratio of the Fourier transform $G(\omega)$ of the output signal $g(t)$ to the Fourier transform $F(\omega)$ of the input signal $f(t)$. For a linear system, a harmonic input causes a harmonic output at the same frequency, the ratio of both giving $H(\omega)$ at that frequency. By changing the frequency, $H(\omega)$ can be obtained over a frequency range.

The main difficulty in applying other excitation signals was calculating the Fouriertransforms of in-and output. However, presently, digital spectrum analyzers are able to do that very fast and accurate by hardware or software, using the wellknown Fast-Fourier-Transform algorithm.

The aim of this paper is to report on practical experiences with identification methods using other than harmonic excitation signals. The different signals used are : swept sine, pulse, random noise and pseudo-random noise. Each of them has definite advantages and drawbacks which will be discussed. Special emphasis is put on one particular situation of special practical interest, namely machine tools with rotating spindles.

For the experiments, a centerlathe with a high vibration level, due to inaccuracies in the spindle roller bearings and the gearbox, was used. The relative flexibility curve between workpiece and toolholder in the horizontal direction was measured with the different methods.

2. EXCITATION WITH CONTINUOUS SIGNALS

2.1. Harmonic analysis

As already mentioned in the introduction, the transfer function $H(\omega)$ of a linear system is obtained with harmonic analysis by applying subsequently harmonic signals of increasing frequency, at each frequency calculating the ratio of output and input Fourier transforms. When using analog instruments, this is done continuously by applying a slowly swept sine wave, enabling a continuous record of the Nyquist plot. In order to stay in a quasi steady-state condition, the sweep rate has to be kept very low.

When applying a digital Fourier-analyzer the frequency has to be kept constant at discrete values, during which time the transfer-function at that frequency is calculated as the ratio of two Fourier transforms, or of two power spectra (see Appendix). Because the same procedure has to be repeated at each frequency, it becomes very tedious to obtain the complete transfer function, especially for slightly damped structures, where the frequency resolution should be very small.

For the mentioned center lathe, the relative flexibility curve in horizontal direction was measured with harmonic analysis, for a spindle speed of 300 rev/min, using a contactless electromagnetic exciter, developed at the Katholieke Universiteit Leuven {1}. Using the H.P. Fourier Analyzer, it took two hours to obtain the response curve in dotted line of fig.5. Because all the input power is concentrated at one frequency (fig. 1a), the signal to noise ratio is very high and no averaging over several measurements is required.

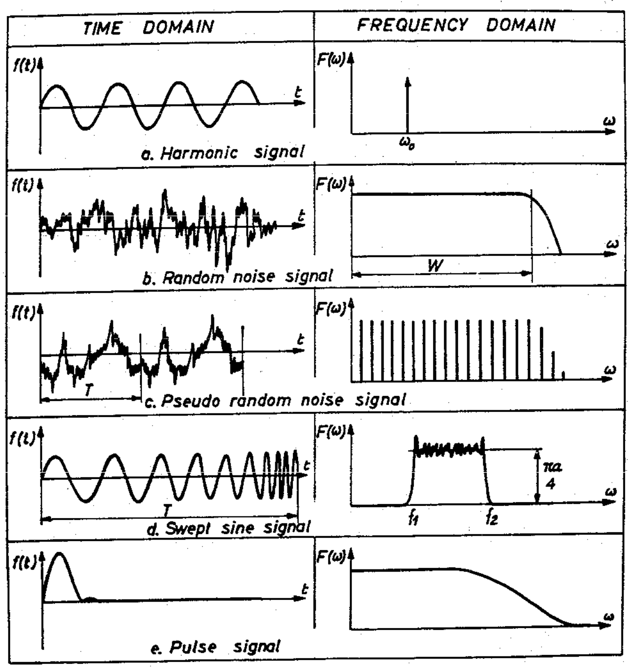


FIG. 1 - Excitation signals and their frequency content

Leakage is prevented by making the signal exactly periodic in the considered time window. This is accomplished by making the input signal period a multiple of the sampling interval, using a ratio-tuner.

2.2. Random noise excitation

Utilization of random noise as input signal shortens considerably the measuring time {7}. In a random noise signal, all frequencies in a limited bandwidth are available simultaneously (fig. 1b), which means that upon excitation with this signal, the system responds at all these frequencies, and the complete transfer function is obtained as the result of a single calculation.

Fig. 6 illustrates the response curve obtained on the same lathe using the same electromagnetic exciter with a random noise input signal after averaging in the frequency domain over 25 measurements. Averaging is necessary here because of the low power spectral density of the input signal.
Because of its very nature, the random input signal is in no way likely to be periodic in the considered time interval. Therefore, a Hanning window {6} has been applied to input and output signals, for reducing the leakage. A considerable improvement in the smoothness of the response curve can be seen from fig. 2, where the flexibility curves are compared with and without application of a Hanning window, for the lathe with stationary spindle.

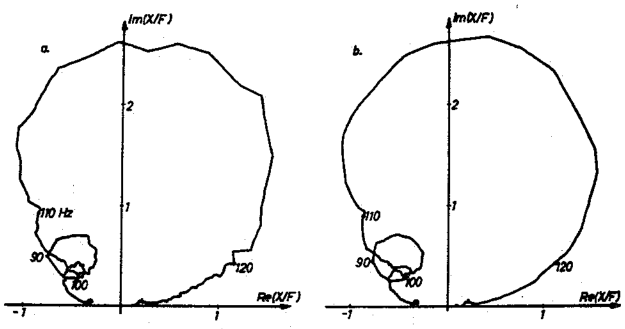


FIG. 2 - Random noise excitation a. without Hanning b. with Hanning

Averaging in the frequency domain takes a long time because for every measurement lengthy Fourier transform calculations have to be executed. Considerable time could be saved by applying time averaging. However, this requires exactly reproducible input signals (see Appendix). A solution has been found in using pseudo-random noise signals. These signals behave randomly in a certain time interval T, but are periodic with period T.

A very important consequence of this periodicity is that a pseudo-random signal has a discrete spectrum (fig. 1c). It has to be assured that the frequency spectrum resulting from taking the Fourier transform of the input-and output time domain signals exactly coincides with the discrete frequencies available in the pseudo random signal. Therefore, the AD-convertor of the Fourier Analyzer has to be sampled with the noise generator clock. Furthermore, the AD-convertor has to be triggered by the noise generator so that the sampling process starts at the starting instant of a new pseudo random noise period.

This method corresponds to a simultaneous excitation with a number of discrete harmonic signals. The energy input per frequency is higher than in the case of random noise excitation. Because the time window T is taken equal to the pseudo-random signal period, there is no leakage at all.

Application to the lathe experiment yielded very poor results as can be seen from the dotted line in fig. 7, where the measured points are scattered randomly.

2.3. Swept sine excitation

In the harmonic analysis it is necessary to keep the sweep rate of the input signal very low in order to stay in a quasi steady-state condition.

It has been shown {2} that the power spectrum of a linearly swept sine wave f(t) defined by:

$$f(t) = \sin(at + b)t \quad \text{with } 0<t<T \qquad (1)$$

where $a = \pi(f_2-f_1)/T$

$b = 2\pi f_1$

f_1 and f_2 are resp. the higher and lower sweep limit

T = sweep time,

is continuous between f_1 and f_2 with an average value of $\pi/4a$ and two peaks at f_1 and f_2 (fig.1d).
This means that application of a swept sine wave is equivalent to using random noise, except that here the signal is completely deterministic and can be obtained with any general purpose sweep generator. Time averaging is easily performed by triggering the AD-convertor with the starting pulse of the sweep generator.

The dotted line in fig.8 gives the response curve for the lathe experiment, obtained with a swept sine input signal, after averaging over 25 measurements in the frequency domain. The much faster time averaging method yielded the dotted line of fig. 9. Interesting to remark here is that the coherence function was identical to one over the whole frequency range in both cases.

3. SHOCK EXCITATION METHOD

3.1. Technique

The methods described up to here all require a dynamic force exciter and for exciting rotating spindles a contactless electromagnetic exciter. In many cases, serious problems arise with the mounting of the exciter on the structure. Moreover, the exciter mass changes the dynamic behaviour of the tested structure.

An interesting method, not requiring any special exciter, is the use of a force impulse as input signal.
A Dirac impulse would be ideal because it has a flat power spectrum, containing all frequencies. Practically, a hammer is used, with a load cell mounted between the striking head and the hammer body. The pulse shape and thus its frequency content (fig. 1e), is determinded by the material of the hammer head. The harder the head, the sharper the pulse and the higher frequencies are generated. For the lathe experiment, a very soft plastic head was used for concentrating all the input power at frequencies below 250 Hz.

An essential difference with the use of an electromagnetic exciter is that the hammer excitation is absolute while when using an exciter, a relative force is applied between two points of the structure. This is no problem however, as a relative response curve can be obtained as the difference between two absolute response curves.

With non-rotating spindle, satisfactory results are obtained after averaging over 10 to 20 measurements. With the spindle rotating at 300 rev/min, an acceptable curve is only obtained after 50 measurements (fig.10), while the coherence is very low. The reason is the very high noise/signal ratio when the machine is rotating. Besides the random noise, there is a harmonic disturbance due to the eccentricity of the excited workpiece in the chuck. Fig. 3a illustrates the response due to the hammer pulse and fig.3b its Fourier transform. The single spectral line due to the workpiece eccentricity dominates.

Clearing this line results in the spectrum of fig.3c. After inverse transformation to the timedomain, fig.3d is obtained. It clearly indicates the decaying response, but also the remaining noise.

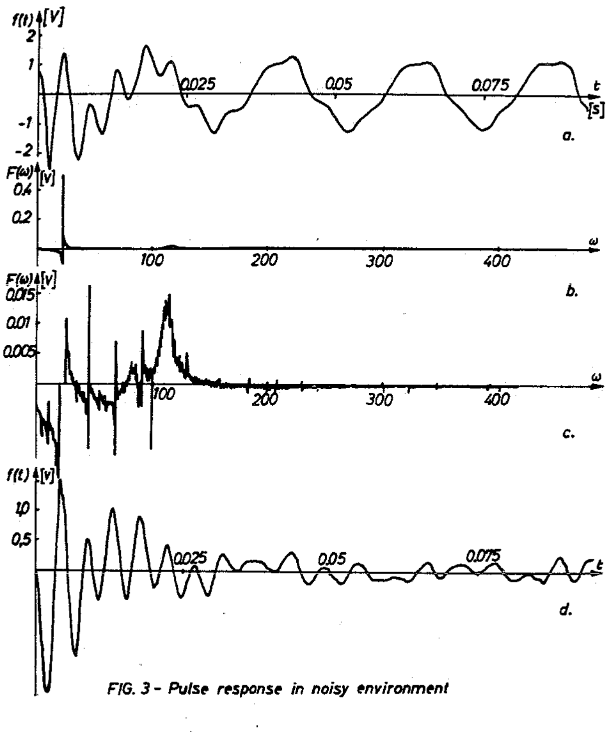


FIG. 3 - Pulse response in noisy environment

One is limited in increasing the hammer force by the structure and in the considered example by the stability of the workpiece in the chuck. The input energy is thus necessarily limited and remains small with respect to the noise level.

3.2. Noise eliminating techniques

Several techniques have been tried to eliminate the deteriorating effects of the noise.

3.2.1. Use of an exponential window

When, before calculating the transfer function $H(\omega)$, input signal $f(t)$ and output signal $g(t)$ are multiplied with an exponential time-window of the form e^{-at} the resulting transfer function $H'(\omega)$ is defined by:

$$g(t).e^{-at} = \int_{-\infty}^{\infty} f(\tau)e^{-a\tau}\,h'(t-\tau)d\tau \qquad (2)$$

$$\text{or } g(t) = \int_{-\infty}^{\infty} f(\tau).e^{a(t-\tau)}\,h'(t-\tau)d\tau \qquad (3)$$

$$\text{But also: } g(t) = \int_{-\infty}^{\infty} f(\tau)\;h(t-\tau)d\tau \qquad (4)$$

$$\text{Thus: } h(t)=h'(t)e^{at} \qquad (5)$$

$$\text{and: } H'(\omega) = H(\omega) * \frac{1}{j\omega+a} \quad (* \text{ means convolution}) \qquad (6)$$

Application of an exponential window has the following advantages:

- Truncation errors are eliminated for slightly damped structures where the response has not decayed to zero at the end of the sampling period.
- Noise is eliminated at the end of both input and output time domain signals.
- Smoothing is obtained due to the addition of artificial damping.
- More points define the response curves. This can be very important for slightly damped structures where, if i.e. Δf= 0,5 Hz, only three points define a complete resonance loop. Addition of artificial damping makes more points define the resonance loop. In the opposite way, for the same number of points, Δf can be increased, allowing to decrease T and thus also Δt for constant N. This allows to define the input pulse more accurately. Especially when using a hard hammer head, for obtaining high excitation frequencies, the hammer pulse is very sharp and narrow, requiring a small Δt.

Multiplication of input and output with e^{-at} gives rise to a very smooth $H'(\omega)$ -curve. However, this result has to be corrected as it contains artificial damping. This could be done by multiplying $h'(t)$ with e^{at} (formula 5) and then backtransform to obtain $H(\omega)$. This is practically not feasible, as for large t-values, the noise is amplified due to the rapidly increasing function e^{at}.

Good results have been obtained by clearing the noise down to the time instant where $h(t)$ has a negligible value, prior to backtransforming to $H(\omega)$. However, the main problem with this technique is that the resulting response curve is slightly changing depending on the time instant down to which has been cleared. No general rule can be put forward for stating how far noise should be cleared.

3.2.2. Use of a correction function k(t)

In this second method, the input and output time domain signals are convolved with a function $k(t)$, which is defined in such a way that the convolution has the same effect as a multiplication with e^{-at}. Thus $k(t)$ should satisfy:

$$g(t) * k(t) = g(t).e^{-at} \qquad (7)$$

Transformation yields:

$$G(\omega).K(\omega) = \mathcal{F}\left[g(t).e^{-at}\right] \qquad (8)$$

$$\text{and: } K(\omega) = \mathcal{F}\left[g(t).e^{-at}\right]/G(\omega) \qquad (9)$$

The desired function $H(\omega)$ is then obtained by:

$$H(\omega) = \frac{G(\omega).K(\omega)}{F(\omega).K(\omega)} = \frac{G(\omega)}{F(\omega)} \qquad (10)$$

The great advantage of this method is that the curve $H(\omega)$ is obtained immediately, without any correction, while the noise reduction effect is theoretically the same as in the previous method.
Fig. 11 shows a result obtained with this method after averaging over 25 measurements. Except for some spikes, the curve is acceptable, while without application of this k-function, it was only a cluster of random points.

3.2.3. Analytical approximation

A new technique, for the analytical approximation of an experimentally obtained response curve, developed by VAN LOON {3}, can be applied here to obtain reliable results from shock excitation experiments on rotating spindles. Therefore, it is worthwhile to consider the technique first in some more detail. A curve fitting program, called DYNPAR, was developed based upon the following mathematical expression:

$$H(\omega) = \frac{X}{F}(\omega) = -\frac{1}{M'_{ij}\omega^2} + \sum_{k=1}^{n}\left[\frac{U_{ijk}+jV_{ijk}}{-\mu_k+j(\omega-\nu_k)} + \frac{U_{ijk}-jV_{ijk}}{-\mu_k+j(\omega+\nu_k)}\right] + S_{ij} \qquad (11)$$

This is a sum of complex modes, each mode being characterized by four parameters:

- the damped natural frequency ν
- the exponential decay rate μ_k
- the amplitude parameters U_{ijk} and V_{ijk}

Parameters μ_k and ν_k are characteristics of the system, while U_{ijk} and V_{ijk} are functions of the position of the excitation point and the position of the displacement transducer.
The modes, lying outside the considered frequency range are taken into account by two additional parameters:

- the lower modes by the effective mass M'_{ij}
- the higher modes by the residual flexibility S_{ij}

The curve fitting program DYNPAR calculates the 4n+2 parameters (for a n-degreeof freedom system) with a non-linear least-squares approximation, starting from initial values of μ and ν which are obtained from visual inspection of the measured response curves and the simple use of a chart {3} .
It can be shown that the analytical expression of a response curve which has been obtained from signals which are multiplied by an exponential window e^{-at} can be written as:

$$H'(\omega) = \frac{1}{M'_{ij}(j\omega-a)^2} + \sum_{k=1}^{n}\left[\frac{U_{ijk}+jV_{ijk}}{-(\mu_k+a)+j(\omega-\nu_k)} + \frac{U_{ijk}-jV_{ijk}}{-(\mu_k+a)+j(\omega+\nu_k)}\right] + S'_{ij} \qquad (12)$$

The damping parameters μ_k are increased by the exponent a of the exponential window. The modal parameters of the original $H(\omega)$-curve are thus easily obtained by subtracting a from the decay parameters (μ_k+a), the other parameters remaining the same. With the plotting routine of the DYNPAR program, the original $H(\omega)$-curve can then be plotted.

4. DISCUSSION OF THE EXPERIMENTAL RESULTS

In a first series of experiments, all described methods have been applied for determining the transfer function $\ddot{X}/F$ of a simple one degree of freedom system. The tested structure was a CIRP-testrig used for cooperative chatter research. The obtained curves are illustrated in fig. 4. Completely identical results are obtained with all methods.

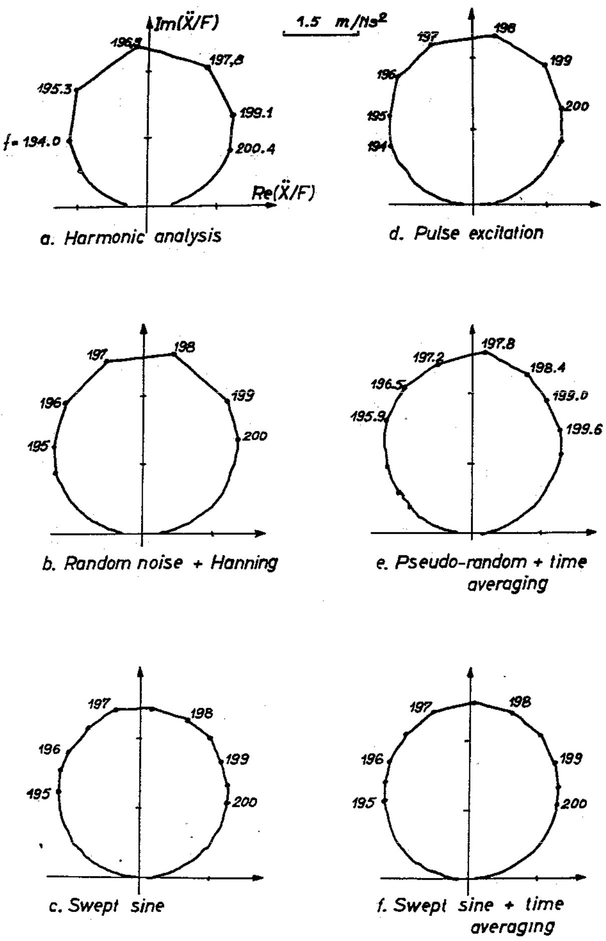
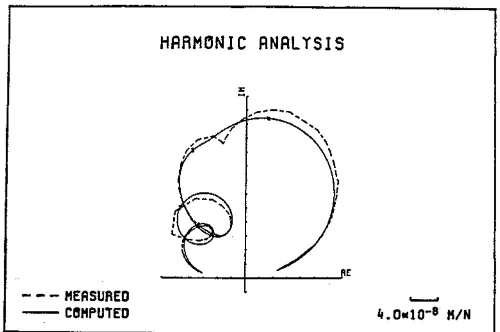


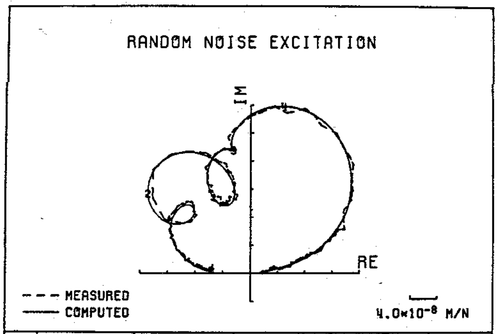
FIG. 4 -



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	517.519	-39.760	-9.407	31.942	0.077
2	559.580	-7.433	-5.785	2.823	0.013
3	680.679	-8.860	0.162	2.384	0.013
4	704.961	-32.653	1.780	69.469	0.046

EFF. MASS -4.5672×10^{2} KG RES. FLEX. 3.9913×10^{-9} M/N

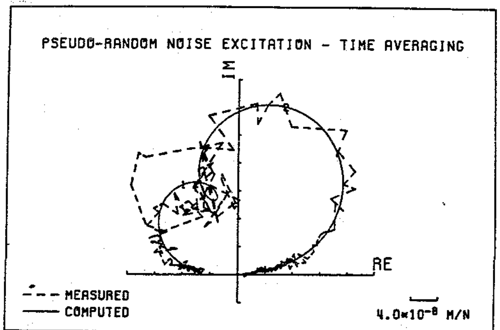
FIG. 5



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	545.500	-52.700	-10.686	64.501	0.096
2	563.100	-7.700	-8.714	-1.559	0.014
3	693.899	-11.090	6.096	0.442	0.016
4	721.099	-31.600	5.682	74.282	0.044

EFF. MASS 8.6332×10^{3} KG RES. FLEX. 1.6251×10^{-9} M/N

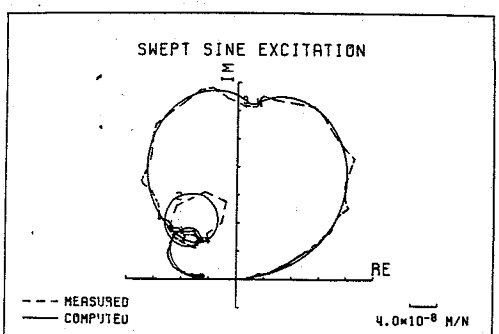
FIG.6



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	464.000	-53.400	-14.457	59.569	0.114
2	592.200	-26.600	11.590	56.956	0.045

EFF. MASS 1.6987×10^{4} KG RES. FLEX. 9.1059×10^{-10} M/N

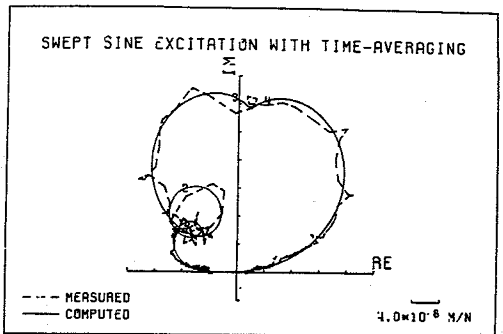
FIG.7



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	498.000	-40.000	-9.304	25.647	0.080
2	562.900	-5.200	-1.980	3.827	0.009
3	691.199	-18.200	83.770	7.865	0.026
4	698.799	-20.800	-80.068	76.937	0.030

EFF. MASS 1.9370×10^{3} KG RES. FLEX. 4.6694×10^{-9} M/N

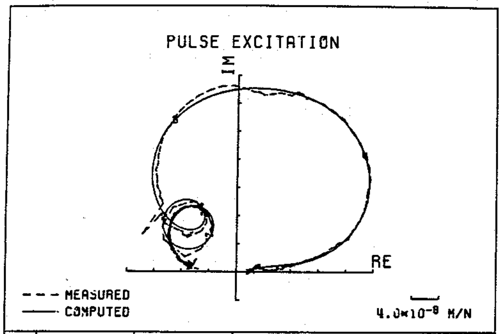
FIG.8



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	501.300	-41.600	-7.816	26.564	0.083
2	563.200	-5.600	-1.866	4.178	0.010
3	688.029	-19.100	42.305	28.428	0.028
4	703.099	-22.200	-39.124	56.171	0.032

EFF. MASS 3.6621×10^{3} KG RES. FLEX. 3.9296×10^{-9} M/N

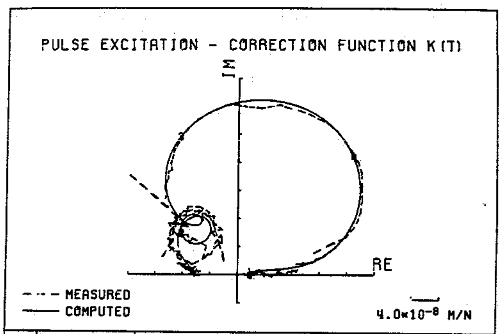
FIG.9



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	512.000	-58.680	13.558	60.551	0.114
2	563.900	-7.440	-5.751	2.011	0.013
3	673.299	-52.537	-68.014	71.464	0.078
4	741.769	-33.600	51.130	16.348	0.046

EFF. MASS 2.6487×10^{3} KG RES. FLEX. 2.3154×10^{-9} M/N

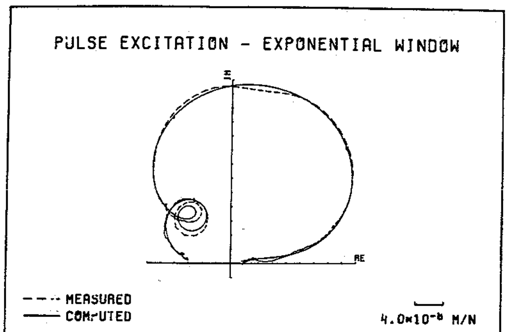
FIG.10



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	516.400	-86.400	5.573	81.334	0.165
2	561.360	-8.860	-4.358	-1.241	0.016
3	672.559	-54.120	-70.935	49.218	0.080
4	738.599	-39.230	60.780	20.888	0.053

EFF. MASS 3.1980×10^{3} KG RES. FLEX. 2.8627×10^{-9} M/N

FIG.11



	NU RAD/SEC	MU RAD/SEC	$U \times 10^{-7}$ M/SEC/N	$V \times 10^{-7}$ M/SEC/N	ZETA
1	509.700	-65.360	7.376	69.000	0.127
2	562.100	-7.510	-5.053	0.282	0.013
3	675.109	-54.100	-81.451	62.159	0.080
4	740.179	-40.200	66.822	15.886	0.054

EFF. MASS 6.5625×10^{3} KG RES. FLEX. 2.9189×10^{-9} M/N

FIG.12

In these circumstances and provided that the necessary equipment is available, the fastest methods are thus the best ones. The pseudo-random and swept sine methods with time averaging areby far the fastest. The swept sine method has the additional advantage that a simple sweepgenerator can be used, and that no averaging is required to obtain a reliable result.

It is of course to be expected that excitation of the lathe spindle at 300 rev/min would yield less uniform results. Visual inspection of the response curves does not provide sufficient information to assess the different methods. A very good criterion for comparing quantitatively the different test results is provided by the analytical approximation program DYNPAR, discussed in section 3.2.3. The curves, computed with DYNPAR are superimposed in full line upon the dotted experimental curves (figs.5 to 12).
The 4n+2 modal parameters are also listed below the corresponding curves.

For comparison purposes, especially the damped natural frequencies ν_i (first column in figs.5 to 12, designated by NU), the decay rates μ_i (second column, designated by MU) and the damping ratio's ζ_i (last column, designated by ZETA), are important. The U and V values have a less direct physical meaning and are not further considered.

There is a good correspondence between the μ,ν and ζ -values for the harmonic and both swept sine methods, as can be seen from figs. 5,8 and 9. A maximum dispersion of 2,5 Hz is found in the damped natural frequencies.

Good agreement is found also between the μ,ν and ζ values for harmonic and random noise excitation (figs.5 and 6), except for the lowest mode where a deviation of 4,4 Hz is found in the damped natural frequency. Although very promising in noise free environments, only very poor curves could be obtained by pseudo random noise excitation with time averaging (fig.7). The program did not succeed in distinguishing four modes in the very shaky curve, although they can be vaguely distinguished visually.

There is a striking similarity of the parameters between the three curves obtained by shock excitation. The same result is thus obtained by averaging over 25 measurements and applying a correction function k(t) (fig.11) or an exponential window (fig.12).

A very amazing conclusion can be drawn when comparing the damped natural frequencies obtained by pulse testing, with those obtained by a harmonic, random or swept sine signal. The resonances are spread out over a larger frequency range in the case of pulse testing, as can be seen by comparing the NU-columns of figs. 5 and 10.
This is also reflected in the larger values of the damping ratio's ζ_i for each mode. The answer for this discrepancy is probably to be found in the absence of a static preload on the spindle when applying pulse excitation. Especially in the case of spindles, which behave non-linear due to clearance and Hertzian contact stresses, the different load conditions between harmonic and pulse excitation can give different response curves.

The non-linearities in the spindle bearings also account for the poor results obtained with pseudo-random excitation. The harmonics, generated by the non-linearities falsify the response-spectrum and the obtained response curve.
Contrarily, the swept sine methods and the random excitation methods do not suffer from non-linearities {2}, because of their continuous spectrum and thus their low power spectral density.

The observed discrepancies in the response curves, obtained with the different methods have to be assessed as a function of the influence they have in the applications for which the curves are used. For instance, if the curves are used for predicting the critical depth of cut at which chatter occurs, it is important to know to what extent errors in the response curves influence the predicted critical depth of cut. Another important point to consider here is that minor modifications in the machine-tool configuration e.g. shift of the cross-slide of the lathe, other lubrication conditions,...can have much influence on the response curve. Therefore, in many applications, average curves are to be considered.

5. CONCLUSIONS

In noise free environments, all the described methods prove to be very valuable and they yield identical results. The fastest methods are the pseudo-random and swept-sine methods with time averaging.

In industrial environments, of which the example considered in this paper is a special but very important case, pseudo-random excitation with time averaging proves to be unsatisfactory. The harmonic analysis yields very reliable results, but requires an electromagnetic exciter if rotating spindles are to be excited. Moreover, used in conjunction with a Fourier analyzer it is a tedious job. The random noise and swept sine methods also require an electromagnetic exciter and further require, by their low power spectral density, averaging over many measurements.
The swept sine method with time averaging is especially valuable because it is fast.

The only method not requiring an electromagnetic exciter is the pulse testing method. Without precautions, a lot of measurements have to be averaged for obtaining acceptable results. Because no time averaging can be applied, this is very time consuming. Application of the described k-function improves the results considerably. The most promising method, however, is the application of an exponential window, combined with the curve fitting program DYNPAR, which approximates the experimental curve by an analytical expression and further enables to remove the added exponential decay to obtain a smoothed original curve. The structural damping resulting from a shock excitation is larger than for the excitation methods using the electromagnetic exciter.

6. APPENDIX

Averaging in frequency-domain or in time-domain

Consider a linear, time-invariant single input-single output system. The response g(t) to an input signal f(t) is expressed by the convolution integral

$$g(t) = \int_{\infty}^{\infty} f(\tau).h(t-\tau)d\tau = f(t)\times h(t) \qquad (A1)$$

where h(t) is the impulse response of the system.
In the frequency domain, this corresponds to:

$$G(\omega) = H(\omega).F(\omega) \qquad (A2)$$

where G,H and F are the Fouriertransforms of resp. g, h and f. A convolution in time domain corresponds to a multiplication in frequency domain, and vice versa. The transfer function $H(\omega)$ is completely determined when $G(\omega)$ and $F(\omega)$ are known. Similarly, $H(\omega)$ is defined by:

$$P_{gf}(\omega) = H(\omega).P_{ff}(\omega) \qquad (A3)$$

where $P_{gf}(\omega) = G(\omega).F^*(\omega)$, the cross-power spectrum between input and output (* denotes complex conjugate)
$P_{ff}(\omega) = F(\omega).F^*(\omega)$, the auto-power spectrum of the input.

Theoretically, expressions (A2) and (A3) are equivalent for obtaining $H(\omega)$. However, when it comes to the practical determination of transfer functions in noisy environments, the use of power spectra is easier to apply. Indeed, to eliminate uncorrelated noise, averaging is generally applied. The frequency spectrum of a time domain signal depends upon the choice of the time origin.
A simple illustration of this is the frequency content of a sine and a cosine signal with same frequency. The single spectral line of the sine signal occurs at the same frequency as that of the cosine signal, but both lines are 90° shifted with respect to each other in the complex spectrum plane.
Consequently, for averaging frequency spectra, the time origin should be kept constant with respect to the signal. This requires special precautions for triggering the input signal.
These precautions are not necessary when working with power spectra, because there the relative phase between input and output is considered, which is independant of the starting phase of the input signal.

When it is possible to trigger the input at constant staring phase, it is far more favourable to average the time domain signals. Then a single calculation of Fourier spectra is sufficient and the total computing time is considerably reduced.
These considerations are especially valid if the Fourier transforms are calculated by software programs as it was the case for the experiments discussed in this paper. If the calculation is done in hardware, the difference between time and frequency averaging is less pronounced.

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