A notation convention in rigid robot modelling

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F.H.R. Lucassen
and
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Abstract

One of the major problems of modelling the dynamic behaviour of a rigid robot using only general theorems of dynamics and Newton-Euler equations, is finding a consistent notation for all the relevant variables.
A consistent notation can simplify the problem tremendously.
Apart from this, it facilitates a great deal of insight and surveyability.
In this report such a notation is proposed.

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1. Introduction

It is generally assumed that any mechanical manipulator can be considered to consist of n rigid bodies, called links of arms, connected in series by revolute or prismatic joints. One end of the open chain is attached to a supporting base, while the other end is free.
For advanced control and design of robot systems, knowledge of manipulator kinematics and dynamics is essentially important. Kinematics deals with robot arm position with respect to a fixed-reference coordinate system as a function of time and is often referred to as the "geometry of motion". Dynamics deals with the mathematical formulations of the equations of robot arm motion.

A robot is a complex mechanical system. Therefore the first step in the development of suitable control algorithms is the derivation of a dynamic model for the robot. Models of rigid robots are already well-known. The principles on which the description of mechanical manipulators are based, result from an energy consideration (Lagrange) or a forces/torques consideration (Newton-Euler). Although the energy considerations are from the point of view of physics the most elegant, their numeric substantial efficiency is less than the algorithms based on the principles of Newton-Euler (4,5). In this report we conform to the Newton-Euler consideration.

2. Notation convention

2.1 Motivation

Suppose each joint-link pair constitutes one rotational degree of freedom (d.o.f.). A joint allows a relative rotation around an axis determined by a unit vector ($\underline{e}$).
In every link a body fixed cartesian coordinate system is

introduced.
A fixed external cartesian coordinate system with a vertical z-axis is defined too (Fig. 1).
Every vector can be written in one of the (n+1) coordinate systems.
In Newton-Euler modelling it is important to compute the different representations of vectors in the different coordinate systems.

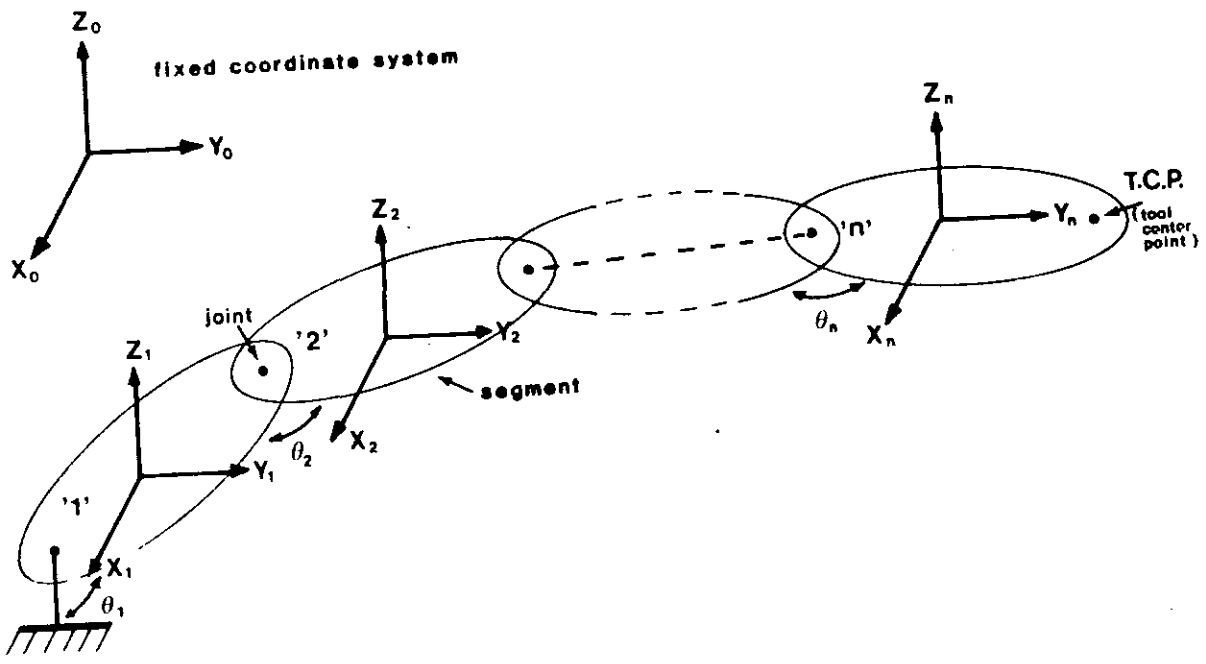

Fig. 1 Coordinate systems

One of the major problems of modelling the dynamic behaviour of a robot is finding a consistent notation for all the relevant variables. Not only the orientation of the joint axis and the position of the joints are important, but also the coordinate system in which these vectors are represented.

2.2 Notation formalism

A notation formalism with the ability to cope with:
- representations in different coordinate systems,
- same sort of vectors in different links,

- different sort of vectors,

would be most attractive.

The modelling of multi-body systems is simplified tremendously and it gains a lot of insight if a good, comprehensive and non-trivial notation is used.
The following notation makes a self-correcting modelling-algorithm possible.

1st position and rotation vectors

$$ {}^{c}\underline{a}_{e}^{d} $$

where

$\underline{a}$ represents any of the following vectors:

- $\underline{p}$: an arbitrary point
- $\underline{z}$: the center of gravity (c.o.g.)
- $\underline{j}$: a joint (vector going from the c.o.g. to the rotation-axis of the joint)
- $\underline{e}$: a unit vector of rotation

and the indices mean:

- c the coordinate system in which the vector $\underline{a}$ is represented.
  (default = 0, inertial coordinate system)
- d vectors referring to joints only ($\underline{e}^{\ell}, \underline{e}^{u}, \underline{j}^{\ell}, \underline{j}^{u}$)
  - ℓ: referring to the joint with the preceding segment
  - u: referring to the joint with the following segment
- e the segment in which the vector $\underline{a}$ is situated.

Examples:

- the following vectors describe the geometric of the i^{th} link:

  ${}^{i}\underline{e}_{i}^{\ell}$, ${}^{i}\underline{e}_{i}^{u}$, ${}^{i}\underline{j}_{i}^{\ell}$, ${}^{i}\underline{j}_{i}^{u}$

- ${}^{p}\underline{j}_{q}^{\ell}$ is the vector connecting the c.o.g. of the q^{th} segment to the joint with the $(q-1)^{th}$ segment presented in the p^{th} coordinate system.

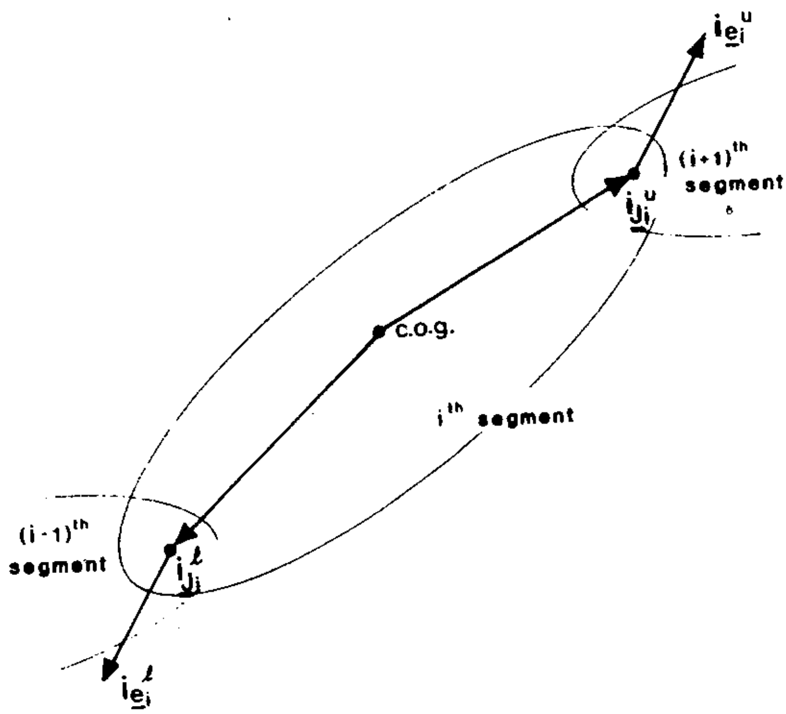

Fig. 2 Position and rotation vectors of the i^{th} segment

2nd Forces and moments

$${}^{c}_{b}\underline{a}^{d}_{e}$$

where

$\underline{a}$ represents any of the following vectors:

$\underline{F}$: force

$\underline{M}$: moment or torque

and the indices mean:

b variables referring to the c.o.g. or joints with other segements, namely:

a: component parallel to the rotation axis (joints only),

r: component perpendicular to the rotation axis (joints only),

t: total vector (= default),

i: resultant of inertial forces and/or moments.

c the coordinate system in which the vector $\underline{a}$ is represented
(default = 0, inertial coordinate system)

d the point of attachment of the vectors ($\underline{F}^u$, $\underline{F}^\ell$, $\underline{M}^u$, $\underline{M}^\ell$):

- ℓ: referring to the joint with the preceding segment
- u: referring to the joint with the following segment

e the segment in which the vector $\underline{a}$ is situated.

3rd Transition matrices:

$$^{c}A_{e}$$

A is the transition matrix from the e^{th} coordinate system to the c^{th}coordinate system (default = 0)

4th Inertial matrices:

$$^{c}J_{e}$$

J is the inertial matrix of the e^{th} segment expressed in the c^{th} coordinate system.

5th Velocities and accelerations

$$^{c}\underline{a}_{e}$$

$\underline{a}$ represents any of the following vectors;

- v: linear velocity of the c.o.g.
- $\underline{\dot{v}}$: linear acceleration of the c.o.g.
- $\underline{\omega}$: angular velocity of the c.o.g.
- $\underline{\dot{\omega}}$: angular acceleration of the c.o.g.

c means the coordinate system in which the vector is represented

e means the segment in which the vector is situated.

Some trivial formulas are e.g.

$$\underline{e}_i^\ell = \underline{e}_{i-1}^u$$

$$A_i = \prod_{\ell=0}^{i-1} {}^{\ell}A_{\ell+1}$$

$$\underline{j}_i^{\ell} = A_i \ {}^{i}\underline{j}_i^{\ell}$$

Note: The right sub index of a transition matrix should always match to the left super index of the following matrix or vector; writing down this sort of formulas has become very easy.

At first sight this convention may look intricate, but later its compactness will be appreciated. The charm of this formalism will be clarified in the continuation of this paper.

3. Coordinate systems

There are three possible origins for the body fixed coordinate system:

1° In the center of gravity (c.o.g.) of the segment.

2° In one of the two joints of the segment $\underline{j}_i^u$ or $\underline{j}_i^{\ell}$.

3° Arbitrarily.

The origin will be in the center of gravity for mechanical simplicity.
There are three possible orientations for the coordinate system:

1° Parallel to the principal axis of the segment

2° The Denavit-Hartenberg convention [3]

3° Arbitrarily.

It can be proved that for real-time computations the Denavit Hartenberg convention is the most attractive one.
So:

z-axis $\rightarrow$ parallel to $\underline{e}_i^u$

x-axis $\rightarrow$ parallel to $\underline{e}_i^{\ell} x \underline{e}_i^u$

y-axis $\rightarrow$ parallel to $\underline{e}_i^u x \ (\underline{e}_i^{\ell} \ x \ \underline{e}_i^u) = \underline{e}_i^{\ell} - (\underline{e}_i^{\ell} . \underline{e}_i^u)\underline{e}_i^u$

where x denotes the cross product.

The transformation matrix from the $(i+1)^{th}$ to the i^{th} coordinate system is:

$$
{}^{i}A_{i+1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix}
$$

with $\cos\alpha_i = ({}^{i}\underline{e}_i^u . {}^{i}\underline{e}_i^\ell)$ and θ_i is the rotation of link i.

4. Kinematic relations

In order to avoid complex expressions and derivations we shall use recurrent expressions for segment velocities $(\underline{\omega}_i, \underline{v}_i)$ and accelerations $\dot{\underline{\omega}}_i, \dot{\underline{v}}_i)$ with $i=1, \ldots, n$.
The following recurrent expressions can be stated [1].

- for the angular velocity of the i^{th} segment

$$\underline{\omega}_i = \underline{\omega}_{i-1} + \dot{\theta}_i \underline{e}_i^\ell$$

- for the linear velocity of the i^{th} segment

$$\underline{v}_i = \underline{v}_{i-1} + \underline{\omega}_{i-1} \times \underline{j}_{i-1}^u + \underline{\omega}_i \times (-\underline{j}_i^\ell)$$

- for the angular acceleration of the i^{th} segment

$$\dot{\underline{\omega}}_i = \dot{\underline{\omega}}_{i-1} + \underline{\omega}_i \times \underline{e}_i^\ell \dot{\theta}_i + \ddot{\theta}_i \underline{e}_i^\ell$$

- for the linear acceleration of the i^{th} segment

$$\dot{\underline{v}}_i = \dot{\underline{v}}_{i-1} + \dot{\underline{\omega}}_{i-1} \times \underline{j}_{i-1}^u + \dot{\underline{\omega}}_i \times (-\underline{j}_i^\ell)$$

$$+ \underline{\omega}_{i-1} \times (\underline{\omega}_{i-1} \times \underline{j}_{i-1}^u) + \underline{\omega}_i \times (\underline{\omega}_i \times (-\underline{j}_i^\ell)$$

Starting with $\underline{\omega}_0 = \underline{v}_0 = \dot{\underline{\omega}}_0 = \dot{\underline{v}}_0 = \underline{0}$ all other velocities and accelerations can be calculated.

5. Mechanism dynamics

Let us consider the i^{th} segment.

Further let ${}_i\underline{F}_i$ and ${}_i\underline{M}_i$ be the total resultant force and moment relation to the segment's c.o.g..
Now according to Newton-Euler:

$$ {}_i\underline{F}_i = m_i \, \dot{\underline{v}}_i $$

$$ {}_i\underline{M}_i = \underline{\omega}_i \times J_i \underline{\omega}_i + J_i \dot{\underline{\omega}}_i $$

with: m_i = mass of the i^{th} segment

$J_i = A_i \, {}^iJ_i \, {}^iA$

= inertia tensor of the i^{th} segment with respect to the inertial coordinate system.

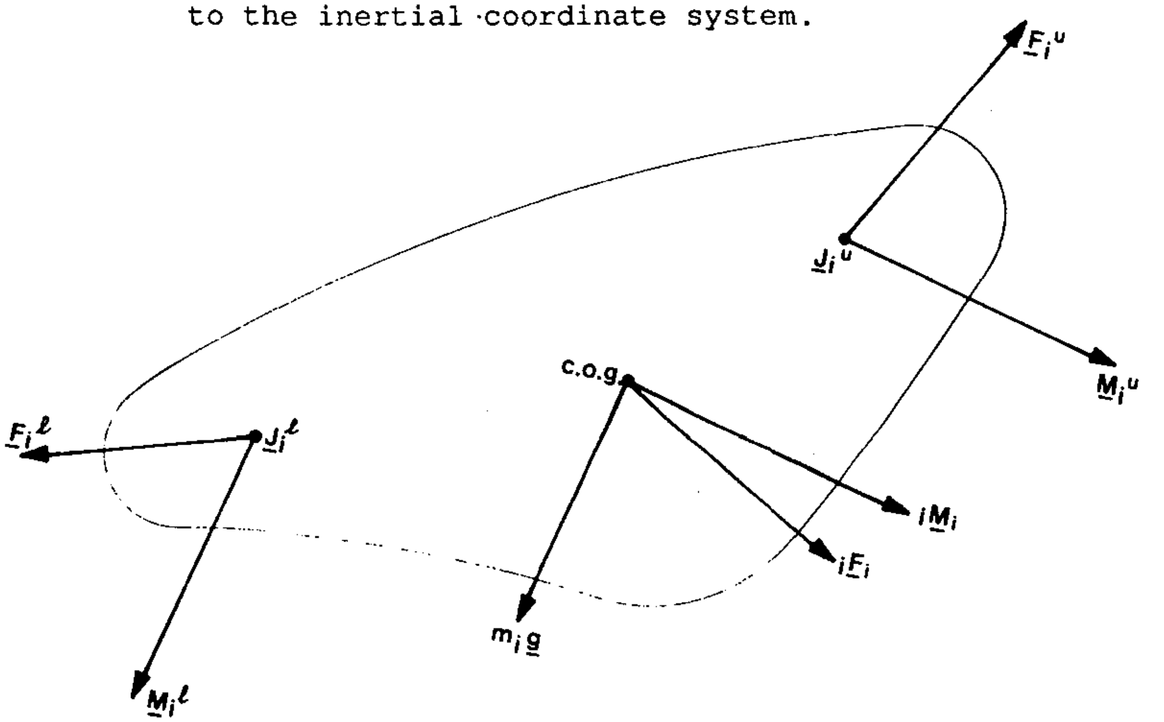

Fig. 3 Forces and moments acting on the i^{th} segment

Finally, the relation between the resultant forces/moments and the forces/moments in the joints are given by (see Fig. 3):

$$ {}_i\underline{F}_i + \underline{F}_i^{\ell} + \underline{F}_i^{u} + m_i\,\underline{g} = \underline{0} $$

$$ {}_i\underline{M}_i + \underline{M}_i^{\ell} + \underline{M}_i^{u} + \underline{j}_i^{\ell} \times \underline{F}_i^{\ell} + \underline{j}_i^{u} \times \underline{F}_i^{u} = \underline{0} $$

where $\underline{g}$ is the gravitational acceleration vector,
or:

$$ \underline{F}_i^{\ell} = -\,(m_i\underline{g} + \underline{F}_i^{u} + {}_i\underline{F}_i) $$

$$ \underline{M}_i^{\ell} = -\,(\underline{M}_i^{u} + {}_i\underline{M}_i + \underline{j}_i^{u} \times \underline{F}_i^{u} + \underline{j}_i^{\ell} \times \underline{F}_i^{\ell}) $$

Starting with $\underline{M}_n^u$ and $\underline{F}_n^u$ all other forces and moments can be calculated.

Note 1: $\underline{F}_n^u$ and $\underline{M}_n^u$ are the forces/moments corresponding with the load in the T.C.P. (tool center point).

Note 2: Remember: $\underline{F}_i^u = -\,\underline{F}_{i+1}^{\ell}$; $\underline{M}_i^u = -\,\underline{M}_{i+1}^{\ell}$.

The required torque in the i^{th} joint is:

$$ {}_a M_i = (\underline{M}_i^{\ell} \cdot \underline{e}_i^{\ell}) $$

6. The algorithm

In setting up the algorithm we have to start with two different tupes of input data

- data describing the robot configuration; these are parameters such as ${}^i\underline{j}_i^u$, ${}^i\underline{j}_i^{\ell}$, ${}^i\underline{e}_i^u$, ${}^i\underline{e}_i^{\ell}$, ${}^iJ_i$, m_i

- data describing a given trajectory; principally this is a sequence of $\theta(k)$, $\dot{\underline{\theta}}(k)$, $\ddot{\theta}(k)$.

With the presented formalism we can draw the following block scheme (Fig. 4) and using it writing down the algorithm is a minor task.

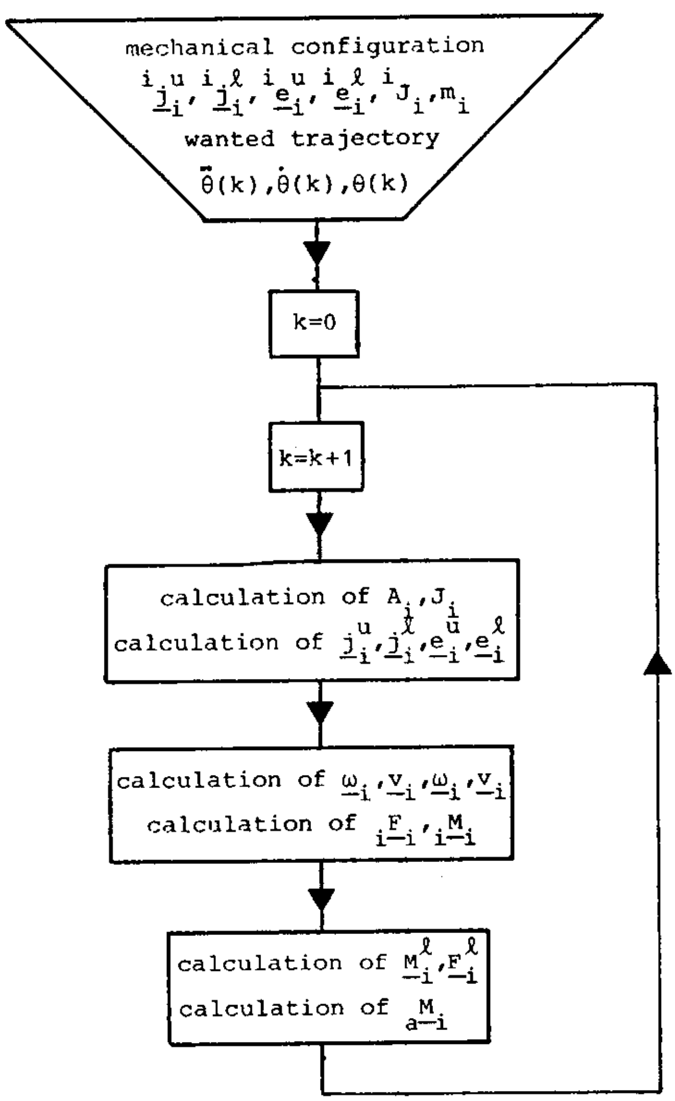

Fig. 4 Block scheme of the algorithm

7. Conclusion

By using the proposed consistent notation the modelling of the dynamic behaviour of rigid bodies is simplified tremendously and increases the insight and surveyability. It is easy to find the required algorithm.

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