Selection of actuators and sensors for an actively suspended tractor-semitrailer

Selection of Actuators and Sensors for an Actively Suspended Tractor-Semitrailer
(Trainee Project)

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Abstract

Prior to controller design, appropriate actuators and sensors must be chosen in order to achieve an acceptable closed loop performance. A recently developed method, where input-output sets are subjected to 8 viability tests, is used to decide on suitable actuators and sensors for an active suspension system of a model with ten degrees of freedom of a tractor-semitrailer. These 8 viability conditions are necessary and, together, sufficient conditions for the existence of a controller achieving a required $\mathcal{H}_\infty$ norm bound. Due to the large number of input-output sets, an efficient search method, where not all sets have to be submitted to the viability tests, is used.

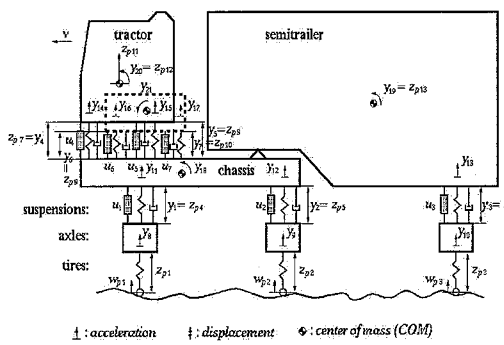


Figure 1: 10 DOF (Degrees Of Freedom) truck model with actuators u, sensors y, road irregularities w_p and regulated variables z_p.

1. Introduction

One step in the design of a control system is the choice of an appropriate number, place, and type of actuators (inputs) and sensors (outputs). This process is called Input Output (IO) selection. One way to start for a vehicle suspension is by placing actuators where the suspension elements are situated and sensors in every place where deflections and accelerations can be measured, which gives a total of 7 actuators and 21 sensors, see Fig. 1. Using all these components would lead to a good quality of control. This would also lead to a very complex and expensive suspension system due to the price of the actuators and to a lesser extent the displacement and acceleration sensors, respectively. The following question arises when a cheaper solution is needed: Can a required performance level be achieved with less actuators and sensors, and if that is possible, what is the minimum number of actuators and sensors that should be used? The answer to the first question is simple, but answering the second question is very complex. Due to the complexity of the model and the huge number of combinations of sensors and actuators, simply using physical insight can't solve such a problem. Here a systematic and automated method for IO selection will be used, as proposed in [9]. The idea is to identify all combinations of inputs and outputs (IO sets) for which there exist a controller that achieves a desired level of performance. The performance specifications are in terms of a bound on the $\mathcal{H}_\infty$ norm of the closed loop system to guarantee robustness. Due to the large number of input-output sets, an efficient search method, where not all sets have to be submitted to the viability tests, is used. In this paper IO selection will be performed based on Nominal Performance (guarantee performance and stability in the absence of uncertainties) and Robust Performance (guarantee performance and stability in the presence of uncertainties).

2. System description and control objectives

The considered system is a 10 DOF model of a tractor-semitrailer freight vehicle, which is given in Fig.1. In [1, Appendix] the linear state-space description and the numerical values of the parameters are given.

Three conflicting performance objectives have to be dealt with when designing a suspension for a tractor-semitrailer: 1) the suspension deflections should be kept within bounds, 2) the vehicle should have good handling properties and road surface damage should be avoided as much as possible, and 3) the human driver comfort should be good and cargo damage should be avoided. In [1] this is discussed in more detail. The three objectives should also be achieved in the face of disruptive influences and model uncertainties. The above gives rise to the following RP *control goal: guarantee stability and satisfactory achievement of the performance objectives 1), 2) and 3), in the face of road surface disturbances and model uncertainties.*

The regulated variables (the signals to be kept small) are collected in z_p (see Appendix). Apart from variables representing the three performance objectives mentioned above, the inputs u are included in z_p in order to keep them small. The exogenous variables consist of the road surface and sensor noise. Road surfaces could be classified as stochastic or deterministic. Both cases are considered here: a stochastic road surface is used to formulate the shaping filter for the road surface irregularities $w_{p1,2,3}$, while deterministic road surfaces are used in the simulations.

To control the active suspension, 7 candidate inputs u (actuators) and 21 candidate outputs y (sensors) are proposed, see Fig.1. Two of the candidate force actuators ($u_{1,2}$) are placed between the axles and the chassis, one (u_3) between the axle and the trailer, two ($u_{4,5}$) between the chassis and the cabin and two ($u_{6,7}$) between the chassis and the engine. The actuators are connected parallel with the springs and dampers of the original, passive, suspension. With the 21 candidate sensors the suspension deflections and accelerations of the cabin, engine, chassis, axles and trailer are measured.

3. Performance specification

In this section, shaping filters W_p for the exogenous variables and weighting filters Z_p for the controlled variables are proposed.

Stochastic road input signals are often modelled as low-pass filtered, white noise, see, e.g., [1]. In this paper, w_{p2} and w_{p3} are considered a delayed version of w_{p1} and hence the tree shaping filters are chosen the same:

$$W_{P1,2,3} = \alpha_1 \frac{s}{s/\omega_1 + 1}$$

It is emphasized that the delay between w_{p1}, w_{p2} and w_{p3} is not explicitly modelled here. For this purpose Padé approximations could be employed, but this will increase the order of the generalized plant G.

The measurements are assumed to be disturbed with zero mean noises. The suspension deflections $y_{1,..,7}$ can be measured with an accuracy of 10^{-3} [m]. The noise of the acceleration sensors $y_{8,...,21}$ is assumed to stem from two sources. First, the measurement are disturbed with noise with an RMS value of $2.5 \cdot 10^{-2}$ [m/s^2]. Second, the acceleration sensors $y_{8,..,21}$ are sensitive to accelerations in the transverse direction, i.e., 1.3% of the transverse accelerations is passed through to these sensors as if these were vertical accelerations. This assumption is only valid when the sensors are placed levelled. In case the truck is tilted the error may be larger and should be corrected. An RMS level of $2.5 \cdot 10^{-2}$ [m/s^2] was used as a suitable representation of a worst-case situation. For the measurements constant weights are taken as proposed in [9, section 6.4.1]:

$$W_{p_4} = ... = W_{p_{10}} = 1.0 \cdot 10^{-3}$$
$$W_{p_{11}} = ... = W_{p_{24}} = 2.5 \cdot 10^{-2} \cdot \sqrt{2}$$

Next weighting filters Z_p for the controlled variables must be designed. In order to adapt the filters, for the regulated variables, to the desired performance, all filters are multiplied by a parameter ρ_n, where the index n is equivalent to the number of the filter. The initial values of these parameters are chosen equal to 1. For the suspension and tire deflections constant weights are used, in order to avoid damage to the chassis, cabin and tires, and these equal the inverse of the minimum value of the absolute deflection limits, multiplied with a scaling factor to be determined later:

$$Z_{p_1} = \rho_1/0.034 \qquad Z_{p_2} = \rho_2/0.024$$
$$Z_{p_3} = \rho_3/0.02 \qquad Z_{p_4} = \rho_4/0.09$$
$$Z_{p_5} = \rho_5/0.09 \qquad Z_{p_6} = \rho_6/0.09$$
$$Z_{p_7} = \rho_7/0.05 \qquad Z_{p_8} = \rho_8/0.05$$
$$Z_{p_9} = \rho_9/0.02 \qquad Z_{p_{10}} = \rho_{10}/0.02$$

To guarantee good comfort, avoid cargo damage and obtain a long lifetime the vertical and rotational accelerations of the tractor should be limited. The selected filters for the regulated variables z_{p11} and z_{p12} are based on data of the human sensitivity in vertical and horizontal direction, as provided by [1]. First, the sensitivity data for the vertical acceleration zp11 is approximated by the magnitude of [2]:

$$Z_{p_{11}} = \rho_{11} \frac{s/\omega_3 + 1}{s/\omega_2^2 + 2\zeta s/\omega_2 + 1}$$

With $\omega_2 = 2\pi\sqrt{(5 \cdot 16)}$ [rad/s] and $\omega_3 = 2\pi \cdot 2$ [rad/s] representing the 'corners' of the sensitivity data. Parameter ζ=0.8834 is computed so that it minimizes the mean, squared error of the relative difference in magnitude between the actual sensitivity data and the weighting filter. The numerical value of ρ_{11} will be determined later in this section. Since the driver's horizontal acceleration can be approximated by a constant multiplied by the rotational acceleration z_{p12} the weight for z_{p12} is based on the sensitivity data for the horizontal acceleration:

$$Z_{p_{12}} = \rho_{12} \frac{1}{s/\omega_4 + 1}$$

With $\omega_4 = 2\pi \cdot 2$ [rad/s] and ρ_{12} a constant to be determined. The appropriate frequency dependency of the weighting filter Z_{p13} for the semitrailer's rotational acceleration z_{p13} strongly depends on the cargo in question. Here it is taken as a constant to be determined:

$$Z_{p_{13}} = \rho_{13}$$

The inputs u, represented by $z_{p14,..,20}$ should be bounded to avoid saturation. To avoid a higher-order generalized plant, the first three corresponding weighting filters are taken constant:

$$Z_{p_{14,15,16}} = \rho_{14,15,16} \cdot 10^{-5}$$

In fact, 10^{-5} is the reciprocal value of the peak force limitation (100[kN]) suggested in [1] for the tractor rear axle if fully loaded. For the system considered here, this peak force is approximately equal to the maximum, absolute force that the *passive* suspension generates for the particular types of road surfaces discussed later in this section. The peak forces acting on the cabin and the engine are much smaller. The exact values of these forces, which can be used to determine weighting filters for the variables $z_{p17,..,20}$ are not known and these won't be determined due to the fact that these weights will be adapted any way, later on in this section. Hence, for the peak forces of the cabin and engine actuators an approximate value of 20% of the peak forces of the chassis and the trailer is chosen. The weights are taken as follows:

$$Z_{p_{17,18,19,20}} = \rho_{17,18,19,20} 5 \cdot 10^{-5}$$

The "ρ parameters" $\rho_{1,..,20}$ in the weighting filters are chosen such that the three control objectives are met. Here, the ρ parameters are determined by iterative $\mathcal{H}_\infty$ optimizations and closed loop simulations for the nominal model and the full IO set.

For the input signals of the road surface, rounded pulses are used, where the road height q_r is defined as a function of the horizontal position r by

$$q_r(r) = q_{\max} \left(\frac{e}{2}\right)^2 \left(2\pi \frac{r}{l_d}\right)^2 e^{(-2\pi r / l_d)}$$

with r =0 at the start of the rounded pulse. The parameters $q_{\max}$ (maximum height) and l_d (typical length) can be used to let $q_r(r)$ represent road irregularities. The road profile as function of time t depends on the two parameters and the vehicle's longitudinal speed v. For constant speed it holds that $r = vt$, with t=0 at the beginning of the rounded pulse and the road profile $q_r(t)$ is parameterized by $q_{\max}$ and l_d/v only. The velocity v =25 m/s and the distance (see [1]) between the tires is used to determine the delay time t_d between the tree input signals w_{p1}, w_{p2} and w_{p3}.

For five typical rounded pulses, the parameters $q_{\max}$ and l_d/v are set such that the passively suspended system hits one of the tire or suspension limits, see [1, section 3.1]. These five rounded pulses are called Huge, Large, Medium, Small and Tiny with l_d/v decreasing in this order.

The closed loop response depends on the class of rounded pulses that are used. Therefore the ρ parameters have to be chosen in such a way that suspension and tire deflection bounds are not exceeded for several kinds of deterministic road surfaces. One way to chose the ρ parameters would be to approximate the behaviour of the responses of the passive system with respect to the largest allowable rounded pulse magnitude q_{max} as a function of the parameter l_d/v. Another way to do this, which is used here, would be to determine the ρ parameters for one rounded pulse, in this case the huge one, and than verify if the desired closed-loop response is also achieved for the other deterministic pulses. It will appear that for the huge, large and medium pulses, the closed-loop responses are in approximation suitable. For the tiny and the small rounded pulses almost all responses of the actively suspended system will exceed that of the passively suspended system by a large amount. One can still consider these filters as suitable, due to the fact that the responses for the actively suspended and passive system acquired with MATLAB simulations for small pulses are not totally correct due to the finite tire radius. For small pulses the displacement of the tire axles won't be the same as the displacement caused by the pulse.

Taking all these assumptions into account one can still say that the filters are suitable for the overall system and its different inputs. When the filters are tuned for the large pulse, one can start by taking $\rho = 1$ and making Bode plots of the closed loop in order to have an indication of how much the value of the ρ parameters must increase/decrease, followed from the difference between the maximum amplitude of each individual transfer function and the $\mathcal{H}_\infty$ norm. After having multiplied or divided the parameters with suitable values one can begin with a fine tuning by adapting these parameters with an iterative process where the maximum value of actively and passively suspended systems are compared: if the maximum values of the responses of the actively suspended system are smaller/larger than for the passively system, then the corresponding ρ's are decreased/increased accordingly. This iterative process is continued until the closed loop responses are acceptable, in the sense that the maximum values of the responses of the actively suspended system are below that of the passively suspended system. For some responses the actively suspended system will slightly exceed that of the passive system. No attempt is made to further improve the response, since this is beyond the scope of this research which does not aim at designing a controller for the system.

To determine the values for the weights such that a good performance is achieved for all inputs is a hard and sometimes almost impossible job. First, the springs and dampers used were chosen such that there is an optimal response when the trailer is fully loaded, which is the case now. Secondly, the employed controller used minimises the maximum singular value in a certain frequency range but in some cases this causes an increase in the singular values for other frequencies. Due to this, the responses of the actively suspended system could exceed that of the passive system for inputs with frequencies which are not in the range of the frequency of the maximum singular value. By using non-constant weights this could be avoided but this would also increase the order of the plant G.

The following values finally result:

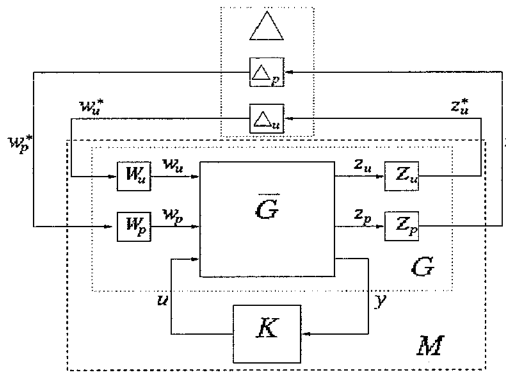
$$\rho_1 = 1.9 \quad \rho_2 = 1.6 \quad \rho_3 = 1.8 \quad \rho_4 = 3.2 \quad \rho_5 = 1$$
$$\rho_6 = 2.8 \quad \rho_7 = 1.2 \quad \rho_8 = 0.4 \quad \rho_9 = 0.7 \quad \rho_{10} = 1.1$$
$$\rho_{11} = 0.2 \quad \rho_{12} = 0.6 \quad \rho_{13} = 0.5$$

The resulting values are still close to 1, which means that the initial choice of 1 was a reasonable one.

The values for parameters ρ_{14}, ρ_{15} and ρ_{16} are kept at one, because the other responses deteriorate when these increase. To improve the other responses the weights for $z_{p17,18,19,20}$ are decreased. This can be done due to the fact that the force acting on these actuators is very small and far from reaching their limits. After doing this the forces still remain small compared to the limits. This can also be concluded when the Bode plots are studied. The following values are chosen:

$$\rho_{14} = 1 \quad \rho_{15} = 1 \quad \rho_{16} = 1$$
$$\rho_{17} = 0.4 \quad \rho_{18} = 0.4 \quad \rho_{19} = 0.4 \quad \rho_{20} = 0.4$$

4. Standard control system set-up and standard assumptions in $\mathcal{H}_\infty$ controller design



$$w = \begin{pmatrix} w_u \\ w_p \end{pmatrix} \qquad z = \begin{pmatrix} z_u \\ z_p \end{pmatrix}$$

Figure 2: Standard control system set-up

In Figure 2 a "standard" set-up of a finite-dimensional, linear and time-invariant control system is given. The following signals play a role: the manipulated variables u; the measured variables y; the controlled variables z, which have to be kept small; the exogenous variables w, such as reference signals and disturbances (in w_p). The generalized plant G consists of the nominal plant $\bar{G}$ augmented with the filters W=diag(W_u, W_p) and Z=diag(Zu, Zp) which characterize performance specifications and uncertainty. The block Δ=diag(Δ_u, Δ_p), which is assumed to be stable, serves to extract model uncertainties (Δ_u) from the nominal plant and it transforms performance specifications (Δ_p) into stability specifications. The closed loop of the controller K and the generalized plant G, is called the generalized closed loop plant M.

The goal of *suboptimal* $\mathcal{H}_\infty$ controller design is to find a controller K which 1) stabilises the nominal closed loop M and 2) guarantees a specified closed-loop ∞-norm $\|M\|_\infty < \gamma$, where γ is larger than the smallest achievable value γ_0. The *optimal* $\mathcal{H}_\infty$ design aims at finding a stabilizing K which minimises $\|M\|_\infty$.

Like the generalized plant G, the *(sub)optimal* controller to be designed is restricted to be real-rational, proper, stabilizable, and detectable. In order to find the parameterisation of all admissible, suboptimal controllers $\mathcal{K}_A$, based on conditions on the solutions to two Algebraic Riccati Equations (ARE) (see [10, section 4.1.3]), four assumptions must then be met for the state space description of the generalized plant:

$$G: \begin{cases} \dot{x} = Ax + B_1 w + B_2 u \\ z = C_1 x + D_{11} w + D_{12} u \\ y = C_2 + D_{21} w + D_{22} u \end{cases},$$

1. D_{12} has full column rank n_u

2. D_{21} has full row rank n_y

3. $rank\left(\Sigma_1(\omega) = \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}\right) = n_x + n_u$

4. $rank\left(\Sigma_2(\omega) = \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}\right) = n_x + n_y$

where n_x, n_u and n_y represent the number of states, inputs and outputs, respectively. If these conditions are satisfied, a stabilizing $\mathcal{H}_\infty$ controller can be designed based on Riccati techniques, provided the conditions in the next section are met.

5. The IO selection method

During input output selection, candidate IO sets are subjected to the conditions for existence of a controller achieving a desired γ_t, which are called the viability conditions. These conditions are as follow (see [9]):

1. (A, B_2) *is stabilizable*,
2. (C_2, A) *is* detectable,
3. $\max(\sigma(D_{11_{11}}, D_{11_{12}}), \sigma(D_{11_{11}} D_{11_{11}}^T)) < \gamma_t$,
4. $H_U \in dom(Ric)$,
5. $U_\infty = Ric(H_U) \geq 0$,
6. $H_Y \in dom(Ric)$,
7. $Y_\infty = Ric(H_Y) \geq 0$,
8. $\rho(X_\infty Y_\infty) < \gamma_t^2$,

with D_{1111} and D_{1112}, the first and the second component, respectively, of the first row of the matrix D_{11} obtained after performing non-singular transformation on u and y, together with unitary transformations on w and z. U_∞ and Y_∞ are solutions of the two ARE, which are uniquely determined by the Hamiltonians H_u and H_y ,respectively, and hence are denoted as a function '*Ric*" of these Hamiltonians. The domain of *Ric*, denoted by *dom* (*Ric*) consists of these two Hamiltonians, respectively.

Condition 1 and 2 check that the generalized plant *G* is stabilizable (i.e., that the unstable modes are controllable by u) and detectable (i.e., that the unstable modes are observable from y) while condition 3 checks if the open-loop system's direct feedthrough from w to z restricts the achievable $\mathcal{H}_\infty$ norm. For condition 3 it has to be noticed that normally an overlined σ is used as symbol for the maximum singular value, while in this paper σ, which is not overlined, is used.

Condition 4 on H_U and 5 on U_∞ check if a controller employing full information on x and w solves the suboptimal $\mathcal{H}_\infty$ control problem while condition 6 on H_Y and 7 on Y_∞ check if a controller having full access to x and z also solves this problem. Finally, condition 8 checks if a suitable combination of state-estimation and state-feedback can solve the suboptimal $\mathcal{H}_\infty$ control problem.

Only if all these conditions are fulfilled at the same time for a certain IO combination, than this IO combination is termed viable. Otherwise it is termed nonviable. In this case there are a total of 7 sensors and 21 actuators, which means that there are a total of 2^{28}=268 · 10^6 candidate IO sets, which is a worst-case situation because combinations without actuators or sensor are also present in this set.

IO sets where there are no actuators or sensors are all considered nonviable without submitting them to the viability conditions test because the open loop (passive system) is not viable. Submitting all the rest of the sets to the viability test will still become a great burden, their number being $(2^7\text{-}1)(2^{21}\text{-}1)$ =266 · 10^6, which is still of the same order, and it will also take a great amount of time. Due to this, the selection has to be made more efficient. The key idea in deriving an efficient selection method, which is only valid under some conditions, is as follows: if a certain combination of sensors and/or actuators achieves a value beneath γ_t, adding other sensors and/or actuators will not cause performance degradation (in this case increase the achievable γ_t) and, vice versa, if a combination is not acceptable, removing sensors and/or actuators from this combination will not improve performance. Due to this, it won't be necessary to perform viability checks for all combinations where sensors and/or actuators are added or removed, because the result is already known. This reduces the number of IO sets submitted to the viability conditions. An algorithm for this has been implemented in a C program (see [4]).

6. IO selection procedure based on nominal performance

In order to gain insight and obtain a suitable value for γ_t, the optimal γ_{opt} for some typical IO sets are computed with $\mathcal{H}_\infty$ optimization. These typical sets are for: (0) the open loop system *G*, (1) the cabin with its actuators and sensors, (2) the engine with its actuators and sensors, (3) trailer, chassis and axles with the appropriate actuators and sensors, combinations of these 3 IO sets and the full IO set case. The results are given in Table 1.

Table 1: Optimal γ's for typical IO sets

IO set	Inputs	Outputs	γ
0	–	–	4.1890
1	$u_{4,5}$	$y_{4,5,14,15,20}$	0.1971
2	$u_{6,7}$	$y_{6,7,16,17,21}$	0.2087
3	$u_{1,2,3}$	$y_{1,2,3,8,9,10,11,12,13,18,19}$	0.1031
1+2	$u_{4,5,6,7}$	$y_{(1)+(2)}$	0.1971
1+3	$u_{1,2,3,4,5}$	$y_{(1)+(3)}$	0.0989
1+2+3	$u_{1,..,7}$	$y_{1,..,21}$	0.0798

One observation is that control of only the cabin, which is IO set (1), gives better performance than control of only the engine, which is IO set (2), in terms of γ. The smaller the value of gamma the better the performance level. Another observation is that with control of the chassis and trailer a suitable performance is achieved compared to (1) and (2). The performance is improved (γ is smaller) when the cabin (1) is also controlled. One can now try to estimate a suitable value for γ_t. Initially γ_t is chosen to be 0.1. This doesn't mean yet that this is a suitable value for γ_t. After IO selection and after verifying if the results make sense one can conclude if γ_t was chosen correctly. If the minimum number of actuators/sensors is not satisfactory, in the sense that it is too large, γ_t has to be increased.

Before submitting candidate IO sets to the viability conditions, some modifications have to be made in the MATLAB program VIABCHK (viability check) made for IO selection (see [10]). For some IO sets the generalized plant will give some numerical problems, which is probably caused by ill conditioned system matrices. This problem can be solved by using the MATLAB command **norminf** instead of **hinfnorm**. Another way is to increase the tolerance of **hinfnorm**, which also gives the same results. The results of IO selection are listed in Table 2 and 3. The minimum number of actuators (m_u) and minimum number of sensors (m_y) needed are determined for three different values of γ_t. The cardinality m_t of the IO sets with m_u and m_y is also given in Table 2. These sets with minimum number of actuators and sensors are a subset of the set of all minimal dependent sets J. A minimal dependent set is a combination of actuators and sensors that is not viable when any of the elements of the set is excluded. The cardinality of J is also listed in Table 2, obtained with the efficient method described in [4].

Table 2: Minimum number of inputs and outputs for three different γ_t, and their total number of combinations and cardinality of J

γ_t	m_u	m_y	$\|m_t\|$	$\|J\|$
0.10	4	3	1221	4962
0.11	3	2	12	2262
0.12`	2	2	21	683

As can be expected, the number of actuators and sensors needed to achieve a certain performance increases as the value of γ_t decreases. This means that more actuators and sensors are needed, as the requirements become more severe.

In Table 3 the actuators, which are always required to achieve $\gamma < \gamma_t$, are listed. Also the sensors, which are not used in any of the combinations of IO sets m_t are listed in Table 3. The remaining actuators and sensors are used in the viable minimal IO sets mt to form the number m_u and my given in Table 2. It has to be emphasized that not all IO sets which can be made with these actuators and sensors are viable, only those combinations in the minimal IO set m_t.

Table 3: Actuators and sensors used to achieve $\gamma < \gamma_t$

γ_t	u_i	y_i
0.10	1, 3, 6/7	(-) 1,2, 4, 5, 6, 7, 9
0.11	1, 3	(-) 1,2,4,5,7,8,9,17,18,19,20,21
0.12	1, 3	(-) 5, 6, 7, 18, 19, 20, 21

(-) not used in any IO set m_t

For all IO sets in J and for all three γ_t 's, the actuators u_1 and u_3 are always needed

For some of these IO sets, an $\mathcal{H}_\infty$ controller is computed to check if the responses of the closed-loop really fulfil the preceding specifications with the chosen weights for the full system. If this is not the case then the filters have to be adapted to that specific IO set. Goal of this is to verify if the assumption holds that using the weights Z_p and W_p based on the full IO set is good to asses performance. For all the chosen IO sets the obtained responses fulfilled the requirements, in the sense that the maximum values of the responses of the actively suspended system do not or do only slightly exceed that of the passive system. This means that the weights for the full IO set also gives suitable performance for these chosen IO sets.

7. Model uncertainties and the structured singular value

Until now model uncertainties were not taken into account. The values of the parameters used in the preceding sections were only for a fully loaded truck. Due to possibly large variations in the cargo, the main parametric uncertainties are assumed to be the semitrailer mass M_t and the semitrailer inertia J_t. Actually, variation in the cargo also causes variation in the stiffness k_{sr} of the rear tractor suspension. This is due to the fact that this air-spring is self-leveling, making the rear suspension deflection at steady state independent of the load.

Other sources of uncertainties are for example due to the fact that the dampers- and springs characterized by b_i and k_{sr}, are considered as linear, whereas they are far from linear. However, this non-linear behavior will not be considered to avoid that the uncertainty model becomes too extensive. The following relations are used to model the uncertain semitrailer mass, inertia and spring constant:

$$\widetilde{M}_t = M_t(1+\tau\delta),\quad \widetilde{J}_t = J_t(1+\tau\delta),\quad \widetilde{k}_{sr} = k_{sr}(1+\tau\delta)$$

With $\delta \in$ R, $|\delta|\leq 1$. It is assumed that the location of the semitrailer's center of mass is invariant under cargo variations and that J_t and k_{sr} are proportionally related to M_t. The nominal values corresponding to M_t ((30 $\cdot 10^3$-1.6 $\cdot 10^3$)/2), Jt and k_{sr} are taken as the means of the values of the fully and empty loaded truck. On this basis, the parameter expressing the magnitude of the uncertainties is given by an approximated value of $\tau = 0.90$. As demonstrated in [9, Appendix D], pulling δ out of the uncertain plant gives rise to a repeated, real block $\Delta_u = \delta I_4$ (see Fig.2). The corresponding scaling filters, forming the upper left blocks in W and Z, are chosen as:

$$W_u = Z_u = \sqrt{\tau} I_4$$

In order to derive a condition to guarantee RP, the small gain theorem is used. This condition is as follows:

Consider Figure 2 with M and Δ internally stable and let γ be a positive real scalar. Then the feedback loop $M\Delta$ is internally stable for all $\Delta \in \mathcal{M}(\underline{\Delta}^{1/\gamma})$, if and only if $\|M\|_\infty \leq \gamma$.

The set $\mathcal{M}(\underline{\Delta})$ defined as: $\mathcal{M}(\underline{\Delta})$:={ $\Delta \in \mathcal{RH}_\infty$: $\Delta(j\omega) \in \mathbb{C}^{n_w \times n_z}$ $\forall$ $\omega \in \mathbb{R}$), while the norm bounded subset of $\mathcal{M}(\underline{\Delta})$ is defined as $\mathcal{M}(\underline{\Delta}^\beta)$:={$\Delta \in \mathcal{M}(\underline{\Delta})$: $\|\Delta\|_\infty \leq \beta$} and $\underline{\Delta}$ is a set of block-diagonal matrices.

Based on the small gain theorem a sufficient condition to guarantee RP is now formulated as follows: *Consider figure 2. Given that K stabilizes M, stability of the closed-loop system under all "uncertainties"* Δ =diag(Δ_u , Δ_p) *with* $\|\Delta_u\|_\infty \leq 1$ *and* $\|\Delta_p\|_\infty \leq 1$ *is achieved if* $\|M\|_\infty < 1$.

Note that the RP condition is sufficient, but not necessary. This means that, even if $\|M\|_\infty < 1$, the closed loop may perform robustly for all full Δ_u's with $\|\Delta_u\|_\infty \leq 1$ and for all full Δ_p's with $\|\Delta_p\|_\infty \leq 1$. This is due to the inability of the small gain theorem to account for possible structure in the uncertainty block Δ. More specific, it does not account for off-diagonal zero-blocks in Δ as in Δ =diag(Δ_u , Δ_p) with Δ_u and Δ_p full blocks.

A way to avoid this conservatism, by incorporating the knowledge on the off-diagonal zeros, is offered by the introduction of the structured singular value: *Given a matrix M* $\in \mathbb{C}^{n_z \times n_w}$ *and a compatible block-diagonal structure* $\underline{\Delta}$, *the structured singular value* $\mu_{\underline{\Delta}}(M)$ *of M with respect to* Δ *is defined as:*

$$\mu_{\underline{\Delta}}(M) = \frac{1}{\min_{\Delta \in \underline{\Delta}}\{\sigma(\Delta : \det(I - M\Delta) = 0\}},$$

unless no $\Delta \in \underline{\Delta}$ *makes I-M*Δ *singular, in which case* $\mu_{\underline{\Delta}}(M)$=0.

The following provides a necessary and sufficient condition for RP: *Consider the uncertain TFM between* w_p *and* z_p, *which is given by the following upper LFT:*

$$\mathcal{F}_u(M,\Delta_u) = M_{22} + M_{21}\Delta_u(I - M_{11}\Delta_u)^{-1}M_{12}$$

let $\gamma_\mu > 0$ *and let* Δ_u *and M be stable. For all* $\Delta_u \in \mathcal{M}(\underline{\Delta_u}^{1/\gamma})$, *with* $\mathcal{M}(\underline{\Delta}^\beta)$: ={$\Delta \in \mathcal{M}(\underline{\Delta})$: $\|\Delta\|_\infty \leq \beta$}, *the uncertain system* $\mathcal{F}_u$ *(M,* Δ_u*) is stable and* $\|\mathcal{F}_u(M, \Delta_u)\|_\infty < \gamma_\mu$, *i.e., robust performance is achieved, if and only if:*

$$||M||_\mu = \sup_\omega \mu_{\underline{\Delta}}(M(j\omega)) < \gamma_\mu ,$$

which is the supremum of the structured singular value of M with respect to Δ across the frequency ω. For the computation of the $\mu_{\underline{\Delta}}(M)$: the following upper bound can be used (see [4]):

$$\mu_{\underline{\Delta}}(M) \le \sigma(M),$$

By pre- and post multiplication of M (see Fig. 2) with appropriate matrices D_Z and D^{-1}_W, the upper bound of the singular value can be pushed downwards in order to obtain a practical estimation of $\mu_{\underline{\Delta}}(M)$:

$$\mu_\Delta(M) \le \inf_{D\in \underline{D}} \sigma(D_z M D_w^{-1}) = \mu_u(M)$$

The *optimal* control problem of minimizing M with respect to K is now replaced by minimizing its upper bound with respect to K and D.

$$\min_K \inf_{D\in \underline{D}} \| D_z M D_w^{-1} \|_\infty$$

This is tried to be solved by a procedure called '$\mathcal{D}$-K-iteration", which could proceed as follows:

0 *Initial controller design*: An initial controller K is computed by $\mathcal{H}_\infty$ optimization for the generalized plant G.

1. *μ -Analysis:* For the closed loop system $M = \mathcal{F}_l(G, K)$, the convex optimization problem $\inf_{D\in\underline{D}} \sigma (D_Z(j\omega)\ M(j\omega)\ D_W^{-1}(j\omega))$ is solved point-wise in the relevant frequency grid Ω_ω. The supremum is $||M||_{\mu u}$.
2. *D-scale approximation:* Across the frequency grid Ω_ω, a minimum-phase, $\mathcal{RH}_\infty$ TFM $\check{D}(s)$ is fit to the data $D(j\omega)$ from the previous step. This fit needs only be performed in magnitude, since adding phase to $D(j\omega)$ does not effect $\mu_u(M(j\omega))$.
3. *$\mathcal{H}_\infty$ Optimization for the scaled plant:* The generalized plant is scaled with $\check{D}$, $\|\check{D}_Z\ \mathcal{F}_l(G,K)\ \check{D}_W^{-1}\|_\infty$ is minimized with respect to all admissible K. For the newly obtained and unscaled $M = \mathcal{F}_l(G,K)$ the μ-analysis in Step 1 is repeated.

8. Input output selection based on robust performance

In this paper two methods, called DSE (D-Scales Estimates) and DKI method in [10] will be used to perform IO selection for the model with uncertainties. For the first method the D-scales are determined by D-K-iteration for the generalized plant G corresponding to the full IO set. For this system G, augmented with its fixed D-scales, IO selection is done by using the same procedure as in section 6, where the candidate IO sets are submitted to the viability conditions. For the second method, again the same procedure as described in section 6, is used, but now the candidate IO sets are subjected to a D-K-iteration test as a viability test. If the value of γ_μ of an IO set stays above a certain $\gamma_{\mu t}$ after a given number of iteration steps than this set is called nonviable. Otherwise it is called viable.

The DKI method is more effective then the DSE method, because the DSE method incorrectly rejects some IO sets due to the fact that

$$\| M_{IOset} \|_\mu < \| D_{z\,fixed}\ M_{IOset}\ D^{-1}_{w\,fixed} \|_\infty .$$

Due to this some IO sets will be considered as nonviable in terms of the $\mathcal{H}_\infty$ norm, while in terms of the μ bound they are viable.

The DKI method is less efficient than the DSE method, because for all candidate IO sets that are subjected to a viability test a D-K-iteration is done, which is time assuming.

To guarantee RP, which in this case is to guarantee a good closed loop response and stability in the presence of uncertainties, the weighting filters have to be adapted before a D-K-iteration is done in order to achieve a γ_μ smaller than or equal to one. By adapting the filters for two extreme situations, in this case for a fully loaded and an empty semitrailer, a good closed loop response is guaranteed. All the weighting filters are divided by a scaling factor sf and some ρ parameters are also changed. These parameters are determined for the huge pulse and as in section 3, they are not determined to achieve the best possible closed loop response. Finally the following values are obtained for the ρ parameters and the scaling factor sf.

$\rho_2 = 1.9 \quad \rho_2 = 1.6 \quad \rho_3 = 1.8 \quad \rho_4 = 3.2\cdot 10^3 \quad \rho_5 = 3.2\cdot 10^3$

$\rho_6 = 4.2\cdot 10^3 \quad \rho_7 = 1.3\cdot 10^2 \quad \rho_8 = 2.8\cdot 10^2 \quad \rho_9 = 3.9\cdot 10^2$

$\rho_{10} = 9\cdot 10^2 \quad \rho_{11} = 3.2\cdot 10^1 \quad \rho_{12} = 1.2\cdot 10^2 \quad \rho_{13} = 0.5$

$sf = 130$

The values for the ρ parameters of the weighting filters for the actuators are the same as in section 3.

The results of IO selection with the two methods are listed in Table 4 and 5. For method 2 (*DKI*) only one value of $\gamma_{\mu t}$ is used because of the large amount of computing time needed when this method is used. Table 4 shows the minimum number of actuators (m_u) and sensors (m_y) needed for five values of $\gamma_{\mu t}$. The cardinality of the IO sets (m_t) with these minimum number of actuators and sensors is also listed in Table 4:

Table 4: Minimum number of inputs and outputs for three different $\gamma_{\mu t}$

$\gamma_{\mu t}$	m_u	m_y	$\lvert m_t\rvert$	$\lvert J\rvert$
		DSE method		
1.1	4	5	263	3832
1..2	3	5	58	2010
1.3	3	4	31	1847
1.4	3	3	6	769
1.5	2	3	2	503
1.6	2	3	2	2104
		DKI method		
1.6	1	1	3	58

From Table 4 it can seen that the DSE method truly rejects IO sets incorrectly. The difference between the two methods is quite large. In Table 5 the actuators that are used to form the number m_u given in Table 4 are listed. The actuators which are used in all the viable minimal IO sets for the selected $\gamma_{\mu t}$ are denoted with a superscript *. For $\gamma_{\mu t}$ =1.1, 1.2 and 1.3 the sensors that are not used in any of the viable IO sets for the tree different γ_t's are listed. Also the sensor y_2 is listed, which is used in all these three IO sets. For $\gamma_{\mu t}$ =1.4, 1.5, and 1.6 all the sensors used in the viable IO sets are listed. Again, as in section 6, it has to be emphasized that not all the combinations which can be made with these actuators and sensors are feasable, only the combinations in m_t.

One observation that can be made from Table 5 is, that the actuators u_2 and u_3 are used in all the IO sets. This can be explained by the fact that it is much easier for the controller to deal with uncertainties at or near places where these are located. In this case u_2 is situated at the same place as the uncertain spring k_{sr} and u_3 is situated at the semitrailer with uncertain mass and inertia.

Table 5:Actuators and sensors required to achieve $\gamma_\mu < \gamma_{\mu t}$

$\gamma_{\mu t}$	u_i	y_i	
	DSE method		
1.1	2*,3*,4,5,6,7	(-) 1,4,5,6,7,21	; (+) 2
1.2	2*,3*,4*	(-) 4,5,6,7,21	; (+) 2
1.3	2*,3*,4*,5	(-) 6,7,21	; (+) 2
1.4	2*,3*,4,5,6	(+) 2,(3/13),18	
1.5	2*,3*	(+) 2,(3/13),18	
1.6	2*,3*	(+) 2,(3/13),18	
	DKI method		
1.6	3*	(+) 3,10,13	

* = always used (-) not used (+)used

For some the accepted IO sets with the DSE method a $\mathcal{H}_\infty$ controller is computed via *D-K*-iteration process. The closed loop responses are than determined in order to check if the filters used for the full IO set case also give good closed loop responses for these sets. For the IO sets, accepted for small values of $\gamma_{\mu t}$ ($\gamma_{\mu t} \leq 1.3$), the required performances is not achieved, because the response of the actively suspended system exceeds that of the passive system. The weights for these IO sets must be adapted in order to obtain a good closed-loop performance. When $\gamma_{\mu t} > 1.3$, most of the responses of the actively suspended system are suitable.

9. Conclusions

A systematic and automated method was used to perform IO selection for a large-scale problem with 28 input/output devices to choose from. This method can be called "efficient" when the number of tested IO sets are compared with the number of all possible IO sets ($266 \cdot 10^6$) with 7 inputs and 21 outputs. For all the desired level of performances tested this number was relative small compared to the total number of IO sets possible. Although this method can be called "efficient" in terms of the total number of IO sets tested, it still requires a great amount of computing time to perform IO selection.

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Appendix

Table 1: Variables in the linear plant model *G*.

Symbol	Description
	State variables x: see [1, Appendix]
	Exogenous variables w_p
$w_{p1,2,3}$	front, rear and trailer road displacement
$w_{p4,..,24}$	sensor noise
	Regulated variables z_p
$z_{p1,2,3}$	front, rear and trailer tire deflections
$z_{p4,5,6}$	front, rear and trailer suspension deflections
$z_{p7,..,10}$	front and rear cabin/engine deflection
$z_{p11,12}$	vertical and rotational acceleration around COM cabin
z_{p13}	rotational acceleration around COM trailer
$z_{p14,..,20}$	actuator forces
	Uncertainty-related variables w_u, z_u
$w_{u1,..,4}$	output from uncertainty block
$z_{u1,..,4}$	input to uncertainty block
	Manipulated variables (inputs) u
$u_{1,..,7}$	vertical actuator forces
	Measured variables (outputs) y
$y_{1,2,3}$	front, rear and trailer suspension deflections
$y_{4,5,6,7}$	front and rear cabin /engine deflections
$y_{8,9,10}$	front, rear and trailer axle acceleration
$y_{11,12,13}$	front and rear chassis and trailer acceleration
$y_{14,..,17}$	front and rear cabin/engine acceleration
$y_{18,..,21}$	acceleration around COM chassis, trailer, cabin, engine

In [9, Appendix D.1] a state space model is derived for the 6 DOF model with uncertainties. The same procedure is used to derive a state space model with uncertainties for the 10 DOF model. The state space model for the system without uncertainties derived in [1] is than augmented with the following uncertainty matrices:

$$\Delta M_{22} = \begin{bmatrix} 1 & c & d \\ c & c^2+g^2 & cd+fg \\ d & cd+fg & J_t/M_t+d^2+f^2 \end{bmatrix} M_t, \; \Delta M = \begin{bmatrix} 0_{3\times3} \\ \Delta M_{22} \\ 0_{4\times3} \end{bmatrix}$$

$$\Delta K_{5:} = \begin{bmatrix} 0 & -1 & 0 & 1 & b & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T k_{sr}$$

$$\overline{B}_{1_u} = \begin{bmatrix} 0_{10\times4} \\ -M^{-1}[\Delta M \quad \Delta K_{5:}] \end{bmatrix} \qquad \overline{C}_{1_u} = \begin{bmatrix} \{A\}_{14-16:} \\ \{C_{1_p}\}_{5:} \end{bmatrix}$$

$$\overline{D}_{11_{uu}} = \begin{bmatrix} \{B_{1_u}\}_{14-16:} \\ 0_{1\times4} \end{bmatrix} \qquad \overline{D}_{11_{up}} = \begin{bmatrix} \{B_{1_p}\}_{14-16:} \\ 0_{1\times3} \end{bmatrix}$$

$$\overline{D}_{11_{pu}} = \begin{bmatrix} 1_{3\times4} \\ 0_{7\times4} \\ \{B_{1_u}\}_{17} \\ \{B_{1_u}\}_{18} \\ \{B_{1_u}\}_{16} \end{bmatrix} \qquad \overline{D}_{12_u} = \begin{bmatrix} \{B_2\}_{14-16:} \\ 0_{1\times7} \end{bmatrix}$$

$$\overline{D}_{21_u} = \begin{bmatrix} 0_{7\times4} \\ \{B_{1_u}\}_{11-13:} \\ \{B_{1_u}\}_{14:} - a\{B_{1_u}\}_{15:} \\ \{B_{1_u}\}_{14:} - b\{B_{1_u}\}_{15:} \\ \{B_{1_u}\}_{14:} - c\{B_{1_u}\}_{15:} + e\{B_{1_u}\}_{16:} \\ \{B_{1_u}\}_{17:} - l\{B_{1_u}\}_{18:} \\ \{B_{1_u}\}_{17:} + m\{B_{1_u}\}_{18:} \\ \{B_{1_u}\}_{19:} - n\{B_{1_u}\}_{20:} \\ \{B_{1_u}\}_{19:} - o\{B_{1_u}\}_{20:} \\ \{B_{1_u}\}_{15-16:} \\ \{B_{1_u}\}_{18:} \\ \{B_{1_u}\}_{20:} \end{bmatrix}$$

The state-space description is:

$$\overline{G} : \begin{cases} \dot{\overline{x}} = \overline{A}\overline{x} + [\overline{B}_{1_u} \ \overline{B}_{1_p}] \begin{bmatrix} \overline{w}_u \\ \overline{w}_p \end{bmatrix} + \overline{B}_2 u \\ \begin{bmatrix} \overline{z}_u \\ \overline{z}_p \end{bmatrix} = \begin{bmatrix} \overline{C}_{1_u} \\ \overline{C}_{1_p} \end{bmatrix} \overline{x} + \begin{bmatrix} \overline{D}_{11_{uu}} & \overline{D}_{11_{up}} \\ \overline{D}_{11_{pu}} & \overline{D}_{11_{pp}} \end{bmatrix} \begin{bmatrix} \overline{w}_u \\ \overline{w}_p \end{bmatrix} + \begin{bmatrix} \overline{D}_{12_u} \\ \overline{D}_{12p} \end{bmatrix} u \\ y = \overline{C}_2 \overline{x} + [\overline{D}_{21_u} \ \overline{D}_{21_p}] \overline{w}_p + D_{22} u \end{cases} ,$$

with the subscripts $\{\cdot\}_u$ referring to the uncertainty–related variables and $\{\cdot\}_p$ referring to the performance-related variables.