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Improved μ -Synthesis Control Design for an XY-table

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Abstract

The paper discusses the tracking control of uncertain nonlinear systems. The specific system studied is an XY-table. The control system consists of an "exact" linearizing state feedback and a linear robustifying controller in an outer loop. For the outer loop a controller is synthesized using the μ -methodology. Iterative improvements in the robustness of the controlled system are obtained by stepwise refinements in the structure of the μ -synthesis problem and in the choice of performance weights and uncertainty weights, at the cost of decreased performance. This uncovers the need for preciseness in the choice of design weights.

1. Introduction

The application of robust control theory in the field of (nonlinear) systems is, although progress has been made in the past, still a difficult problem. The difficulty mainly stems from the problems associated with devising accurate descriptions of the model errors or uncertainties. On one side, the use of general, but conservative, error descriptions is advantageous due to easily available theory and design software, making a rapid control design and evaluation cycle possible. On the other side, such general and nonspecific descriptions may result in controllers that are too conservative, and therefore lack performance characteristics that could be obtained by careful tuning of controller parameters.

In general, there is not a strict distinction between general and specific error descriptions, but a whole range of descriptions can be used. A case study is used to get insight in the effects of choosing a specific point in this range. The insight may lead to guidelines for the selection of the appropriate point in this range.

The work reported in this paper is a continuation and improvement of the work reported in [1].

2. Model and Control

The systems considered are nonlinear mechanical systems, modeled as multi-body systems with the number n of control inputs f equal to the number of DOF (degrees-of-freedom) q . The model used is

$$M(q, \theta)\ddot{q} + g(q, \dot{q}, \theta) = f$$

with M the mass matrix, g the vector of Coriolis, centrifugal, gravity, and friction forces, f the motor torques, and θ the model parameters. The control system proposed consists of two levels. A lower level exact linearizing state feedback

$$f = \hat{M}(\dot{q}_d + u) + \hat{g}$$

where $\hat{M} = M(q, \hat{\theta})$, $\hat{g} = g(q, \dot{q}, \hat{\theta})$, with $\hat{\theta}$ an estimate of the parameters, q_d the desired trajectory, and u the output of the second level linear robustifying controller. After applying this law and assuming $\theta = \hat{\theta}$ there follows for the outer loop model $\ddot{q} = \dot{q}_d + u$. We consider μ -synthesis controllers C for the plant model $P = q(s)/u(s) = 1/s^2$, neglecting the influence of the feedforward \dot{q}_d . Information available to this controller is the tracking error $e = q_d - q$, so $u(s) = C(s)e(s)$. Model errors to be conquered by the controllers stem from flexibility in the system (real number of DOF is larger than n), uncertainty in the model parameters ($\theta \neq \hat{\theta}$), and discretization errors. Because the

nominal plant P for the μ -synthesis design is a double integrator, problems may arise with $j\omega$ -axis poles and zeros.

The study tries to assess the influence of the type of μ -synthesis problem, of the choice of performance weights, and of the choice of model error characteristics on the control system performance and robustness. The following three cases are studied.

1. The weighted mixed-sensitivity problem with a sensitivity weight W_1 based on performance and a complementary sensitivity weight W_3 based on model errors (unmodeled dynamics). No consideration is given to the system poles and zeros that are on the $j\omega$ -axis.
2. The same mixed-sensitivity problem, but with shifted poles and adapted weights to accommodate the $j\omega$ -axis poles and zeros.
3. The robust performance problem that also accounts for the parameter errors, and not only for unmodeled dynamics. This leads to a more involved uncertainty model with a real parametric uncertainty pulled out of the plant.

3. The System

The system used for the simulations is a three degrees-of-freedom XY-table, moving in the horizontal plane, with three prismatic joints, two of which are parallel and coupled by a spindle with adjustable flexibility k_s . For a picture of the XY-table see Fig. 1.

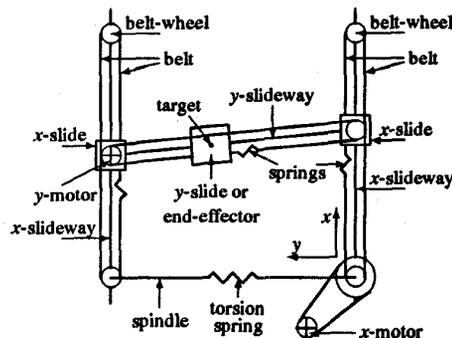


Figure 1: Schematic drawing of XY-table

The system is nonlinear (mainly Coulomb friction) and some of the joints are flexible. For more details of the system see [2].

4. μ -synthesis controller design

Assuming sufficient knowledge of μ -synthesis design, see, e.g., [3], a fresh up is not offered.

The nominal model is a 2 DOF one, assuming the spindle to be very stiff. By changing the spindle stiffness k_s , the unmodeled dynamics in x -direction changes. Because the unmodeled dynamics in x -direction can be made significant, we concentrate on the tracking controller in this direction.

In the first case two scalar weights are used. The transfer function W_1 weights the sensitivity function from the reference x_d to the tracking error e_x for performance. It is selected based on steady state error and

bandwidth considerations, and chosen as

$$W_1(s) = \frac{\alpha \left(s^2 + 2\xi_a \frac{\omega_c}{\sqrt{\alpha}} s + \frac{\omega_c^2}{\alpha} \right)}{s^2 + 2\xi_b \frac{\omega_c}{\sqrt{\beta}} s + \frac{\omega_c^2}{\beta}}$$

with $\alpha = .09$, $\beta = 10^4$, $\xi_a = \xi_b = 1$, and ω_c a tuning factor.

The function W_3 weights the complementary sensitivity function from x_d to the output x to account for multiplicative model errors (parasitic dynamics). A detailed analysis of models for a system with and without flexible spindle, and for a range of spindle stiffnesses, leads to the following choice for W_3 , slightly modified from [1],

$$W_3(s) = \frac{0.67s^2 + 0.14s + 0.02}{0.67s^2 + 20s + 418}$$

This case results in a two block uncertainty Δ in the D step of the DK iteration used to solve the μ -synthesis problem. One block is for the model error and the other for the performance. A solution can be computed if it is slightly suboptimal.

In the second case two problems are solved. First, the poles of the system on the $j\omega$ -axis are warped to the left half plane by a bilinear transformation and the resulting controller is given the inverse treatment. This may result in a suboptimal controller. Second, the zeros at infinity on the $j\omega$ -axis are removed by choosing an improper weight function, namely $\left(\frac{s}{25}\right)^2$, so the generalized plant (system plus weight functions) is proper and has no $j\omega$ -axis zeros at infinity anymore. The improper weight is taken care of by absorbing the double differentiator in the plant and weighting the plant input by the factor

$$W_3 = \left(\frac{1}{25}\right)^2$$

This weight represents an additive model error. The structure of Δ does not change. For this case $\omega_c = 9$ was used. This resulted after one DK iteration in a controller with 6 states, that was reduced to 5 states giving $\mu = .985$, compared with $\mu \approx 1.15$ for an \mathcal{H}_∞ controller. Further iterations were diverging.

In the third case uncertainty in the mass matrix \hat{M} due to varying load is accounted for. This is approximately equivalent with uncertainty in the plant gain, modeled by an inverse multiplicative error at the input of the plant. A constant W_2 weights this error. The uncertainty in the parameter is real, but in the controller design it is considered complex to make the design computations simpler. This leads to a diagonal Δ , with three entries. For the setup of this case see Fig. 2. The previous two cases are simple modifications of this setup.

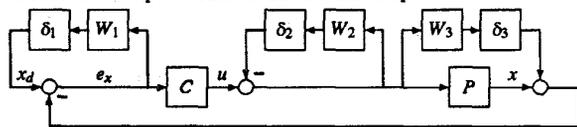


Figure 2: Setup of design problem, case 3

This case with $\omega_c = 6$ and $W_2 = .1$ resulted, after one iteration, in a controller with 12 states, that was reduced to 6 states giving $\mu = 1.0008$, compared with $\mu \approx 5$ for an \mathcal{H}_∞ controller. Further iterations were diverging. A plot of μ for the μ -synthesis and \mathcal{H}_∞ controller is in Fig. 3.

5. Simulation Results

The three controllers are used to control a simulation model of the XY-table, see [2] for details of the model. Because the simulation should mimic reality as closely as possible, the controllers are implemented in discrete time and with low order. So, the controllers were first reduced in order, as discussed before, and then discretized (Tustin). Results are given in Fig. 4. The task was to track a circle for x . Several values for the spindle stiffness k_s , all unrealistically small, were

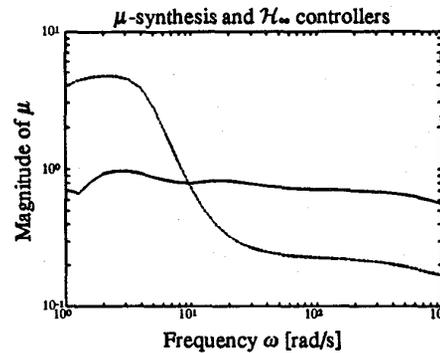


Figure 3: Values for μ , case 3. —: μ -synthesis, - -: \mathcal{H}_∞

used to access robustness for unmodeled dynamics. The performance measure is the RMS of the tracking error in x -direction e_x .

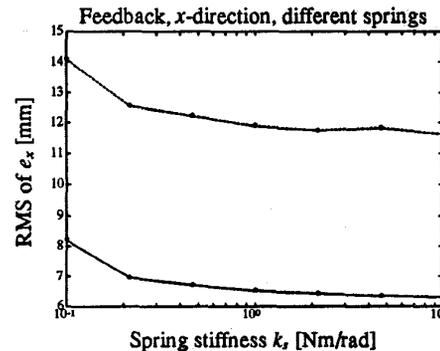


Figure 4: Simulation results. —: case 2, - -: case 3

The results for case 1 and 2 did not differ much, so only those for case 2 are included. Because the controlled system is stable for a large range of k_s , the design goals are achieved, although the multiplicative uncertainty weight W_3 did not completely cover the set of plants parameterized by k_s . The controller's stability margin is larger than required by W_3 , resulting in diminished performance. The presence of zeros and poles on the $j\omega$ -axis did not seem to be a problem.

The performance for case 3 is worse, because the model error description is more strict, and therefore more conservative. Adaptation of W_3 , that in the previous cases also catered for parameter uncertainty, is needed to overcome this disadvantage. But remember, this case should be more robust for variations in \hat{M} .

6. Further Research

Current and future research will try to address

- the acquirement by experimental identification of a nominal model and an error description
- the use of a parameterized error model, with the parameter k_s pulled out and made explicit in the error description
- the use of a 3 DOF nominal model, including the flexible spindle, with k_s pulled out as a real parametric uncertainty
- a careful and detailed analysis of the uncertainties in the Coulomb friction, removing the assumption $\hat{g} = g$.

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