Contribution to the mechanics of machining

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Contribution to the Mechanics of Machining

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SUMMARY. Based on the Merchant shear plane model and assuming global mechanical equilibrium between average values of stress in a state of plane stress, a shear angle relation is derived by identifying the direction of maximum strain rate with the direction of maximum principal stress.

It is shown that except for the von Mises plasticity condition no particular assumption as to minimum work has to be introduced. The shear angle solution is determined by the prevalent state of stress, which can be expressed in terms of the ratio between the average value of the maximum shear stress and the plasticity constant of the material being machined, which also holds when strain-hardening occurs.

A comparison is made with experimental results and a true strain–stress curve for the work-piece material, as obtained from the present theory, is given.

ZUSAMMENFASSUNG. Dieser Beitrag enthält Vergleiche zwischen Scherwinkeln, die nach der Merchant'schen Gleichung berechnet, und solchen, die gemessen wurden, ferner zwischen Fließkurven aus dem Zugversuch und dem Zerspanungsversuch.

Der Scherwinkel ist nicht so einfach versuchsmäßig zu bestimmen wie ein Reibwinkel, denn es bedarf der Messung der Spanstauchung. Es wird gezeigt, wie man eine ebensolche Genauigkeit wie beim Reibwinkel erzielt. Die rechnerischen Ergebnisse werden mit den versuchsmäßig ermittelten verglichen. Maßgebend ist das Verhältnis zwischen dem Mittelwert der größten Schubspannungen und der Fließspannung des Werkstoffs. Die Scherwinkelgleichung behält auch im Bereich der Kaltverfestigung ihre Gültigkeit.

In der Spannungs-Dehnungs-Beziehung findet der Verfasser, daß die Fließkurve im Bereich der hohen, beim Zerspanungsvorgang auftretenden Spannungen einem höheren Exponenten gehorcht als bei den kleineren Spannungen im Zugversuch.

RESUME. Partant du modèle de Merchant, l'auteur avance l'hypothèse que les tensions sont en équilibre dans le cas de l'état plan de tension.

Il obtient une relation contenant l'angle de cisaillement en identifiant la direction de la vitasse maximale de déformation avec la direction de la plus grande tension principale.

Si l'on excepte la condition de plasticité de Hencky-von Mises, aucune hypothèse, telle que par exemple celle du travail minimum, ne doit être faite. La valeur de l'angle de cisaillement est déterminée par l'état de tension. Celui-ci peut être exprimé en fonction du rapport entre la valeur moyenne de la tension maximale de cisaillement et la constante de plasticité du matériau usiné. Ceci reste valable pendant l'écrouissage.

L'auteur compare les résultats expérimentaux avec la courbe de tension-déformation vraie du matériau.

NOTATION

σ_1, σ_3	average principal stresses in shear zone Nm^{-2}	
σ_x, σ_y	average normal stresses in shear zone Nm^{-2}	
τ_s	average shear stress in shear plane Nm^{-2}	
τ_{max}	average maximum shear stress in shear zone Nm^{-2}	
φ	shear angle	
β	friction angle	
α	rake angle	
ψ	direction of maximum crystal elongation	with respect to the shear plane
Ω	direction of maximum principal stress	with respect to the shear plane
γ_s	shear strain, $\tan\gamma_s = \tan(\varphi-\alpha)+\cot\varphi$	
ε	engineering strain	
k	plasticity constant Nm^{-2}	
σ_ε	true tensile stress $= k\sqrt{3}$ Nm^{-2}	
g	$(2\sigma_y/\sigma_y-\sigma_x)$ stress parameter	
f	(τ_{max}/k) ratio factor	
t	feed m/rev	
d	depth of cut m	
r_c	chip thickness ratio	
v	cutting speed msec^{-1}	

1. INTRODUCTION

DURING the past decade or so a number of theories on the mechanics of machining have been published, in some of them the entire state of stress is investigated while in others equilibrium between average values of stress is assumed to be present in a geometric model of the cutting process. All theories are directed towards the formulation of a shear angle relation, which is an equation between measurable quantities predicting a unique steady-state configuration for tool rake and friction angle. A hypothesis of minimum work is generally introduced in order to secure the uniqueness of the shear angle solution.

It has been shown[1] that the search for uniqueness might well be considered as being fruitless, because a range of steady-state solutions of the Merchant shear-plane type[2] is to be expected within permissible regions of the characteristic angles describing the geometry and the mechanics of the cutting process.

Extant theories based on the assumption of global equilibrium between average values of stress have been reconsidered and it will be shown that when identifying the direction of maximum strain rate with the direction of maximum principal stress a shear angle relation can be formulated. In doing this it is not necessary to introduce any energy condition.

However, when aiming at a shear angle solution an additional assumption has to be made with regard to the prevalent state of stress, and this is equivalent to assuming a value of the maximum shear stress in the system in cases where materials being machined behave according to the von Mises condition of plasticity.

The introduction of the von Mises condition implies accepting an energy condition. The latter however, regards exclusively the deformation of the workpiece material and does not refer to the cutting process as a whole.

A treatment of the problem along these lines proves able to account for the strain-hardening properties of the material.

2. THE DIRECTION OF MAXIMUM STRAIN AND A SHEAR ANGLE RELATION

In the present theory the Merchant shear plane geometric model according to Fig. 1 is accepted. The problem will be treated as a case of plane stress. From Fig. 1 follows the geometric condition:

$$\sigma_y = \tau_s \tan(\varphi+\beta-\alpha) \tag{1}$$

and hence from the Mohr equilibrium condition as represented in Fig. 2; it can be deduced that

$$\tan(\varphi+\beta-\alpha) = g \cot 2\Omega \text{ [4]} \tag{2}$$

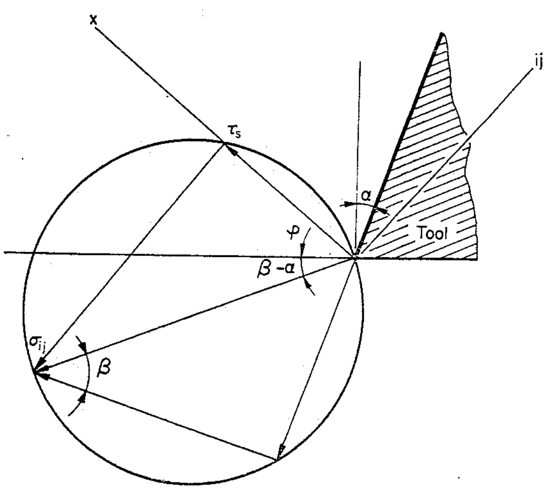

Fig. 1. The Merchant shear plane model and the geometric stress condition:

$\sigma_y = \tau_s \tan(\varphi+\beta-\alpha)$.

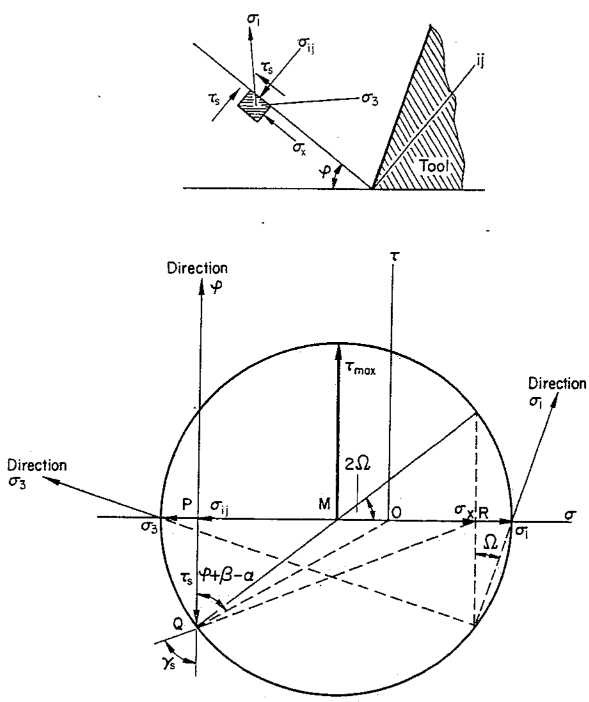

Fig. 2. The Mohr equilibrium condition and the stress parameter

$$g = \frac{OP}{MP} = \frac{2\sigma_y}{\sigma_y-\sigma_x},$$

from which is derived:

$$\tan(\varphi+\beta-\alpha) = g \cot 2\Omega.$$

If the direction Ω of the maximum principal stress is identified with the direction of maximum strain rate it can be shown that

$$\cot 2\Omega = \tfrac{1}{2}\tan\gamma_s$$

from which follows:

$$\sphericalangle PQR = \gamma_s$$

and hence the shear angle equation:

$$\tan(\varphi+\beta-\alpha) = \tfrac{1}{2} g \tan\gamma_s = \tfrac{1}{2} g[\tan(\varphi-\alpha)+\cot\varphi].$$

where the parameter g defines the state of stress. As is clear from Fig. 2 this parameter can be expressed in terms of the prevalent stresses:

$$g = \frac{OP}{MP} = \frac{2\sigma_y}{\sigma_y-\sigma_x}. \tag{3}$$

Thus $g = 1$ defines a state of pure shear.

Merchant introduces the angle ψ as the direction of the maximum value of the crystal elongation in the chip with respect to the shear plane, which can be interpreted as the direction of the maximum value of strain and strain rate and hence as the direction of the maximum (tensile) principal stress in the system. This is expressed by

$$\psi = \Omega. \tag{4}$$

Now, as shown in Fig. 3 an element AF of the workpiece material will be transformed by the cutting process into the state AF'.

Its original position is fixed by an angle p relative to the co-ordinate system, the position after deformation being defined by the angle q.

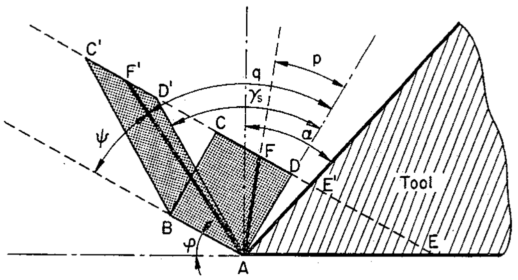

Fig. 3.

Determination of the direction $q = \frac{\pi}{2} - \psi = \frac{\pi}{2} - \Omega$ *of maximum strain and strain rate.*

An element of material AF is deformed by the cutting process into the state AF′ and is thus strained to the amount

$$\varepsilon = \frac{AF'}{AF} - 1 = \frac{\cos p}{\cos q} - 1.$$

From the condition

$$\frac{d\varepsilon}{dq} = 0,$$

can be derived the direction of maximum strain:

$$\cot 2\Omega = \tfrac{1}{2}\tan\gamma_s = \cot 2\psi.$$

The strain resulting from the deformation amounts to

$$\varepsilon = \frac{AF' - AF}{AF} = \frac{\cos p}{\cos q} - 1. \tag{5}$$

Furthermore, it follows from Fig. 3 that

$$\tan q = \tan\gamma_s + \tan p \tag{6}$$

and hence

$$\cos p = \left[\frac{1}{1+(\tan q - \tan\gamma_s)^2}\right]^{1/2} \tag{7}$$

Combining equations (5) and (7) gives

$$\varepsilon = \frac{1}{\cos q}\left[\frac{1}{1+(\tan q - \tan\gamma_s)^2}\right]^{1/2} - 1. \tag{8}$$

The direction of the maximum strain in terms of the angle q now follows from

$$\frac{d\varepsilon}{dq} = 0$$

which gives

$$\tan q_{\varepsilon_{\max}} = \cot\psi$$

$$= \tfrac{1}{2}\tan\gamma_s + [\tfrac{1}{4}\tan^2\gamma_s + 1]^{1/2} \tag{9}$$

and from this the relation between ψ and γ_s is given by

$$\cot 2\psi = \tfrac{1}{2}\tan\gamma_s. \tag{10}$$

Using equations (4) and (2)

$$\tan(\varphi+\beta-\alpha) = \tfrac{1}{2}g\tan\gamma_s. \tag{11}$$

Substituting for the shear strain in terms of φ and α according to Merchant, results in:

$$\tan(\varphi+\beta-\alpha) = \tfrac{1}{2}g[\tan(\varphi-\alpha)+\cot\varphi] \tag{12}$$

which is a shear angle relation valid in a state of stress defined by the parameter g.

The value of the maximum strain follows from equations (9) and (8):

$$\varepsilon_{\max} = [1+\tan\gamma_s \{\tfrac{1}{2}\tan\gamma_s + (1+\tfrac{1}{4}\tan^2\gamma_s)^{1/2}\}]^{1/2} - 1 \tag{13}$$

By now it is possible to derive the shear angle relation equation (11) in a direct way from the Mohr equilibrium diagram Fig. 2.

According to equations (4) and (10) the equality holds:

$$\frac{MP}{PQ} = \cot 2\Omega = \tfrac{1}{2}\tan\gamma_s$$

and as

$$MP = \tfrac{1}{2}PR$$

it follows that

$$\sphericalangle\, PQR = \gamma_s$$

by which a graphical interpretation is obtained of the relation between the shear strain and the characteristic angle $(\varphi+\beta-\alpha)$.

From this follows the shear angle relation:

$$\tan(\varphi+\beta-\alpha) = \frac{OP}{MP}\tfrac{1}{2}\tan\gamma_s$$

and hence:

$$g = \frac{OP}{MP}$$

as already defined in equation (3).

Finally the shear strain, which originally has been defined merely as a geometric quantity, can now be expressed in terms of stress, and it can be concluded from Fig. 2 that

$$\tan\gamma_s = \frac{\sigma_x - \sigma_y}{\tau_s}.$$

3. STRESS PARAMETER g

In the case where the value of the stress parameter g is known, the shear angle relation equation (12) allows for a shear angle solution, i.e. the determination of the shear angle in its dependence on the friction angle, with the rake angle as a parameter.

As equation (13) incidates that the strain in the material can be expressed merely in terms of the shear strain, and thus in terms of the shear angle φ, this means that an analytical formulation will be obtained accounting for the interaction between the friction on the rake of the tool—whatever the physical background of this particular process might be—and the deformation of the workpiece material in the shear zone. Thus it is important to investigate the physical meaning of the stress parameter g, apart from its definition in equation (3).

The general plasticity condition of von Mises reduces to

$$\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_s^2 = 3k^2 \qquad (14)$$

for the state of plane stress.

The plasticity constant k is considered to be a function of the strain ε, and hence equation (14) remains valid when strain-hardening occurs.

This means that the plasticity ellipse, when transferred to the co-ordinate system of principal stresses:

$$\sigma_1^2+\sigma_3^2-\sigma_1\sigma_3 = 3k^2 \qquad (15)$$

shows semi-axes of variable magnitude depending of the state of strain at a given strain rate.

The equilibrium condition according to Fig. 2 requires:

$$\sigma_x = \sigma_y-2\tau_s \cot 2\Omega. \qquad (16)$$

The geometric condition as to the stresses has been formulated in equation (1).

Now the solution of equations (1), (14) and (16) refers to a state of stress satisfying simultaneously the geometric condition prescribed by the shear plane model, the condition of global equilibrium and finally the condition of plasticity at the given state of strain and strain rate.

The solution is:

$$-\tau_s = \frac{k\sqrt{3}}{[\tan^2(\varphi+\beta-\alpha)-2\tan(\varphi+\beta-\alpha)\cot 2\Omega+4\cot^2 2\Omega+3]^{1/2}}$$

$$-\sigma_y = \frac{k\sqrt{3}\tan(\varphi+\beta-\alpha)}{[\tan^2(\varphi+\beta-\alpha)-2\tan(\varphi+\beta-\alpha)\cot 2\Omega+4\cot^2 2\Omega+3]^{1/2}} \qquad (17)$$

while σ_x can be solved from equation (14).

The counterpart of equation (16) also comes from Fig. 2

$$\tau_s = -\tau_{\max}\sin 2\Omega \qquad (18)$$

and

$$2\tau_{\max} = \sigma_1-\sigma_3.$$

Now two different extreme situations of stress may occur:

(*a*) *A state of linear stress*

$$\sigma_1 \neq 0 \quad \text{or} \quad \sigma_1 = 0$$

$$\sigma_3 = 0 \quad \text{or} \quad \sigma_3 \neq 0$$

In this case it follows from equations (15) and (18) that

$$\tau_{\max} = \tfrac{1}{2}k\sqrt{3}.$$

(*b*) *A state of pure shear*

$$\sigma_3 = -\sigma_1$$

where it follows from the same equations that

$$\tau_{\max} = k.$$

In general this can be written as

$$\tau_{\max} = f.k$$

and

$$\tau_s = -fk\sin 2\Omega \qquad (19)$$

where

$$\tfrac{1}{2}\sqrt{3} \leqq f \leqq 1.$$

From this it is clear that any *a priori* assumption with regard to the value of the maximum shear stress in terms of the plasticity constant defines a state of stress. In particular the condition $\tau_{\max} = k$, which is quite common in extant theories, defines a state of pure shear.

Substitution of equation (19) into (17) and using equations (4) and (10) again leads to a shear angle relation:

$$\tan(\varphi+\beta-\alpha) = \left[1 \pm \left\{3\left(1+\frac{4}{\tan^2\gamma_s}\right)\left(\frac{1}{f^2}-1\right)\right\}^{1/2}\right]\tfrac{1}{2}\tan\gamma_s. \qquad (20)$$

Comparison with equation (11) shows that

$$g = 1 \pm \left[3\left(1+\frac{4}{\tan^2\gamma_s}\right)\left(\frac{1}{f^2}-1\right)\right]^{1/2} \qquad (21)$$

from which it is obvious that the state of stress defined in terms of the parameter g at a given state of strain has its physical origin in the ratio f between the average value of the maximum shear stress and the plasticity constant of the material.

The positive sign in equation (21) implies:

$$|\sigma_y| > \sigma_x$$

the negative sign means:

$$|\sigma_y| < \sigma_x$$

The condition $f = 1$ is compatible with $g = 1$, and defines a state of pure shear, as shown earlier.

4. SHEAR ANGLE SOLUTIONS

(*a*) *The case of pure shear*

From the foregoing it will be clear that the shear angle relation equation (20) or (12) reduces to:

$$\tan(\varphi+\beta-\alpha) = \tfrac{1}{2}[\tan(\varphi-\alpha)+\cot\varphi] \qquad (22)$$

from which φ can be expressed as a function of β

for given values of the rake angle α. The solution has been plotted in Fig. 4, where the shear angle φ appears as a function of the angle $\beta-\alpha$. A remarkable fact is that in the present theory the rake angle operates as a parameter which definitely influences the solution obtained. This is shown for the values

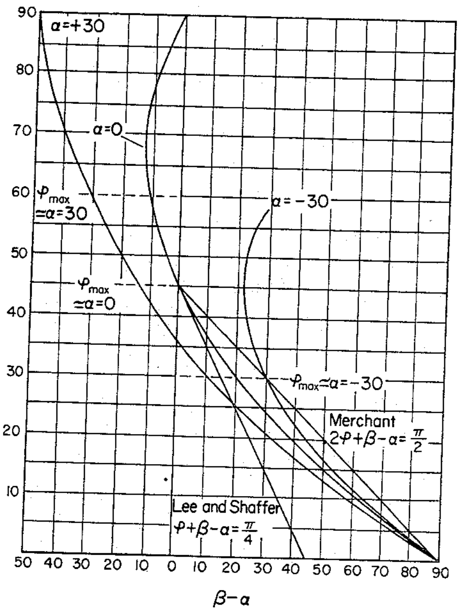

Fig. 4. The shear angle solution equation (20) in the state of stress of pure shear, as defined by the condition $\sigma_3 = -\sigma_1$ and hence by $\tau_{max} = k$ or $f = g = 1$.

The effect of the rake angle as a parameter is shown. A comparison is made with both the Merchant and the Lee and Schaffer solutions.

$\alpha = \pm 30°$ and $\alpha = 0°$. For a comparison the Merchant *et al.*[5] solutions have been plotted. The present theory arrives at values intermediate between those predicted by the other theories, as it should do whenever it has a chance to cover reality. It is observed that for $0 \leq \varphi \leq \frac{1}{2}\pi$, the theory apparently does not allow for unique solutions, the shear strain passes through a minimum value as a function of the shear angle φ

$$\frac{d \tan \gamma_s}{d\varphi} = \frac{1}{\cos^2(\varphi-\alpha)} - \frac{1}{\sin^2\varphi} = 0.$$

Hence the minimum value of the shear strain is reached at

$$\varphi = \tfrac{1}{4}\pi + \tfrac{1}{2}\alpha \tag{23}$$

where the friction angle β has the value zero—shown by substitution of equation (23) into (22).

In this state the cutting process dissipates energy only by deformation of the workpiece material in the absence of friction on the rake of the tool. It seems obvious that this never can be a physical reality and thus the uniqueness of the solution of equation (22) is secured by:

$$\beta \geqq 0$$
$$\varphi \leqq \tfrac{1}{4}\pi + \tfrac{1}{2}\alpha. \tag{24}$$

In Fig. 4 the region of physical significance is restricted to:

$\varphi = 30°$ for $\alpha = -30°$, to $\varphi = 45°$ for $\alpha = 0°$ and to $\varphi = 60°$ for $\alpha = +30°$.

The solutions again prove to be unique.

(b) The case of a general state of stress

As discussed, the general state of stress prevails when:

$$f = \frac{\tau_{max}}{k} < 1.$$

The system is governed by the shear angle relation, equation (20), from which after expressing the shear strain in terms of shear angle and rake angle, shear angle solutions can be obtained with both f and α as parameters.

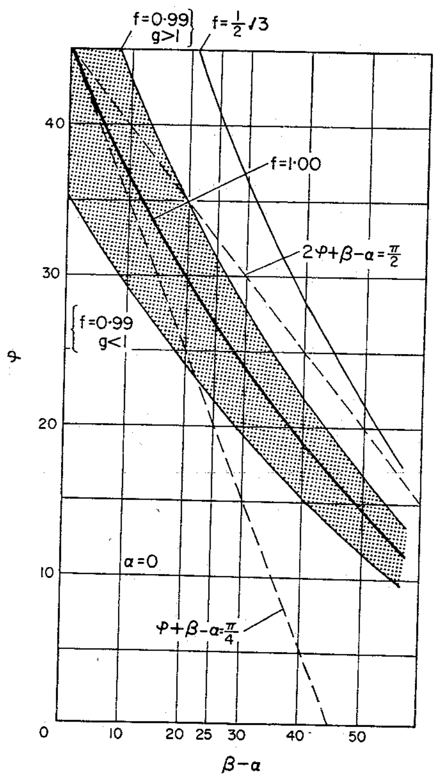

Fig. 5. The shear angle relation equation (20) for the rake angle $\alpha = 0$ and different values of the ratio

$$f = \frac{\tau_{max}}{k}.$$

The sensitiveness of the solution with respect to minor changes in f in the region close to $f = 1$ is shown. Both the possible solutions have been plotted according to the value $f = 0{\cdot}99$, corresponding with the two possible different states of average stress.

It is shown in Fig. 5 that the ratio f has a very strong influence on the course of the shear angle solution, and it does so particularly in the region close to $f = 1$. When considering Fig. 5 it should be kept in mind that every value of the parameter

f gives two different shear angle solutions, these correspond to the choice of sign in equations (20) and (21), and hence are dependent on the modulus of the ratio between the principal stresses, which can be expressed in terms of $g \gtreqless 1$, as shown earlier.

When it is accepted that the average value of the maximum shear stress as a resultant of a hypothetic stress distribution might differ by as much as 2 per cent from the plasticity constant of the material being machined, quite a number of the observations published in current literature are covered by the present theory. It might even be that the extreme sensitiveness of the shear angle solution with respect to the state of stress suggests the lack of a unique solution to the problem.

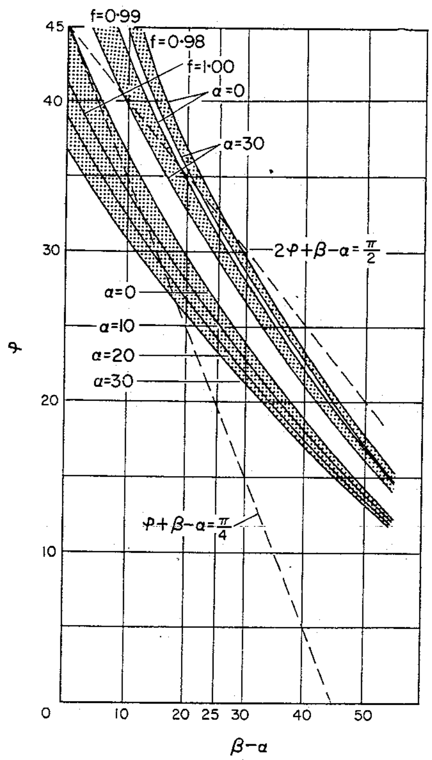

Fig. 6. The shear angle relation equation (20) *for different values of both the rake angle α and the ratio f.*

Only the solutions corresponding to the positive sign in the equations (20) *and* (21) *have been plotted, as this refers to the majority of the practical cases.*

The decreasing importance of the rake angle is noted as the ratio f decreases due to the moving of the system out of a state of pure shear.

A more complete picture is given in Fig. 6 where the effect of both of the two parameters is shown simultaneously under the condition $g \geq 1$, which appears to be usual in the majority of the practical cases investigated. It is observed that the influence of the rake angle decreases rapidly as the value of f decreases, i.e. when the average behaviour of the system moves out of the state of pure shear.

In conclusion experimental data given by Oxley[6] are compared as an example with the present theory in Fig. 7.

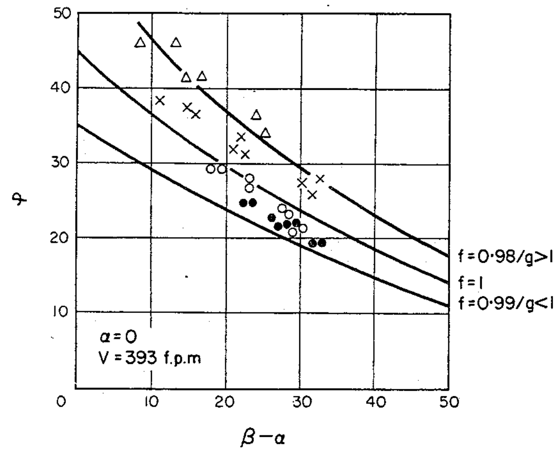

Fig. 7. Comparison of a number of shear angle values as used by Oxley[6] *with the predictions of the present theory according to equation* (20).

△ *SAE* 4135 *RC*—35
× *SAE* 4135 *RC*—26
○ *SAE* 4135—*as received*
● *SAE* 4135—*annealed.*

5. EXPERIMENTAL RESULTS

A major difficulty in verifying shear-angle solutions arises from measuring the shear-angle φ with an accuracy comparable with that which can be obtained when measuring the friction angle β by means of dynamometry. As a rule cutting forces will be recorded during a considerable length of time and hence an average value of the friction angle can be determined with fair precision. Determination of the shear-angle depends on measuring the chip-ratio from samples of the chip. A large number of samples must be taken in order to arrive at an accuracy comparable with that obtained by dynamometry. Now, in a programme of investigation of cutting temperatures, an extensive study has been made of the behaviour of the chip contact length in relation to the cutting conditions[7]. When machining obliquely an annealed steel C45 with a carbide tool of grade S2 (ISO−P20) a definite relation between feed, speed and chip-ratio is found to exist:

$$r_c = \frac{0{\cdot}615t}{0{\cdot}205\times 10^{-3}+0{\cdot}850t-0{\cdot}029\times 10^{-3}v} \tag{25}$$

in the speed range $1 \leq v \leq 5\ \mathrm{msec}^{-1}$, in the feed range $0{\cdot}2\times 10^{-3} \leq t \leq 1{\cdot}0\times 10^{-3}$ m/rev and at the depth of cut of $d = 3\times 10^{-3}$ m.

As equation (25) has been obtained from the study of the average behaviour of the chip contact length as recorded in a natural way in the wear pattern on the rake of the tool, the accuracy in determining the shear-angle from it proves to be about the same as in determining the friction-angle from recordings obtained with a sensitive strain-gauge dynamometer[8]. Statistical evaluation

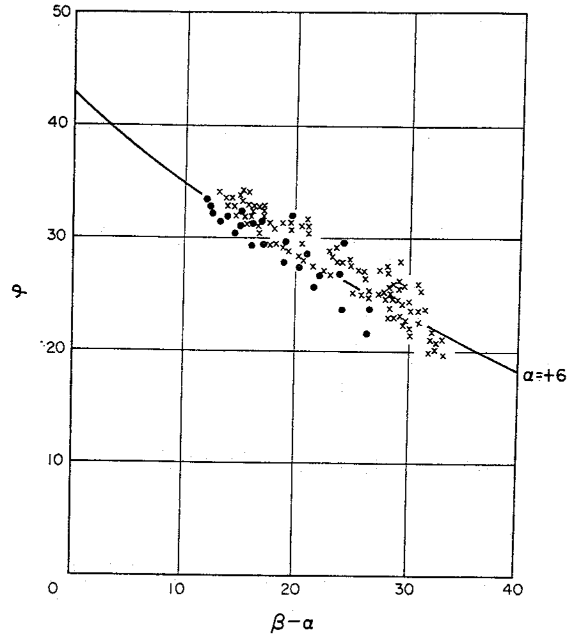

Fig. 8.

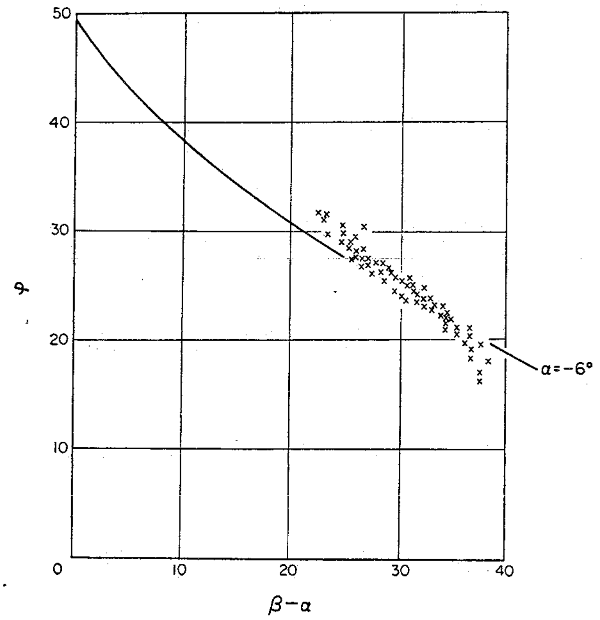

Fig. 9.

Figs. 8 *and* 9. *Comparison of experimental results with the predictions of the present theory, equation* (22), *when machining an annealed steel C45.*
Speed range 1–5 $msec^{-1}$, *feed range* 0·2–1·0 *mm/rev, depth of cut* 3 *mm.*
● = *determined indirectly from chip ratio relation equation* (25).
× = *measured directly from chip ratio by sampling.*

shows a relative error of 2 per cent in the shear-angle and a relative error of 2·5 per cent in the angle $\beta-\alpha$. The experimental results have been plotted in Fig. 8, where both values of the shear-angle obtained by use of equation (25) and those obtained by direct measurement of the chip ratio have been used. The presence of a systematic error is evident. The agreement with the present shear-angle relation equation (22) is pretty good, from which it might be concluded that the material is machined in an average state of pure shear, and probably behaves according to the von Mises condition of plasticity.

A second series of experiments has been performed with a negative value of the rake-angle. The results are shown in Fig. 9 from which the conclusion drawn might well be the same.

In conclusion it is remarked that equation (17) when used in connection with dynamometric experiments allows for investigation into the plastic

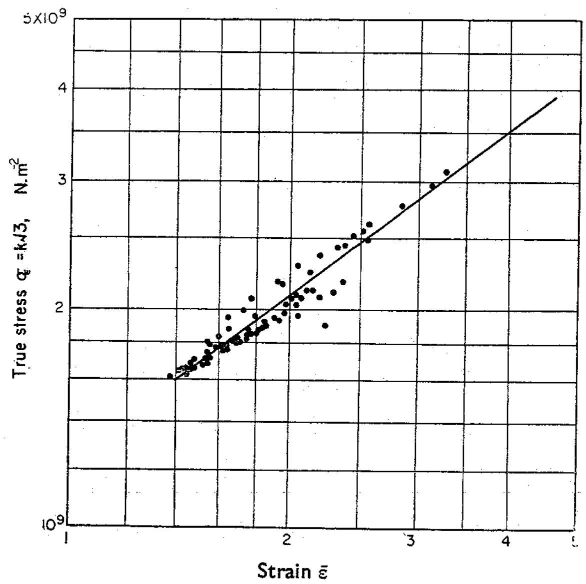

Fig. 10. *The stress–strain relation of an annealed steel C 45 in metal cutting, in accordance with equations* (13) *and* (26) *and based on the measurements of Fig.* 8.

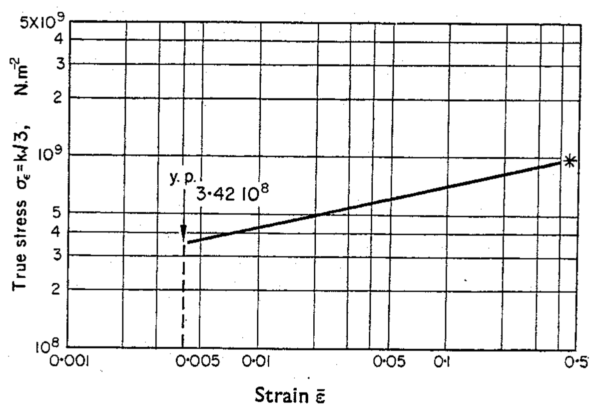

Fig. 11. *The stress–strain relation of an annealed steel C 45 as obtained from a step by step interrupted tensile test. Yield point* $3{\cdot}42\times10^{8}\ Nm^{-2}$ (34·2 kg/mm^2), *fracture* $10^9\ Nm^{-2}$ (100 kg/mm^2).

behaviour of the workpiece material under machining conditions when assuming validity of the von Mises condition. When using

$$\cot 2\Omega = \tfrac{1}{2}\tan \gamma_s,$$

as derived earlier, equation (17) can be written in the form

$$\sigma_\varepsilon = k\sqrt{3} = \tau_s[\tan^2(\varphi+\beta-\alpha)-\tan(\varphi+\beta-\alpha)\tan\gamma_s+\tan^2\gamma_s+3]^{1/2} \quad (26)$$

by which the true stress σ_ε is expressed in dynamometric quantities and hence can be calculated from numerical experimental values. The amount of computing work is considerably reduced when the material is machined in a state of pure shear, as in this case equation (17) reduces to $\tau_s = k \sin 2\Omega$. From equation (13) the natural effective strain can be calculated from the prevalent value of the shear strain and thus a stress–strain relation in the range of machining conditions can be plotted. This is shown in Fig. 10 as based on the measurements of Fig. 8 when machining an annealed steel C45. The mechanical properties of the material are illustrated in Fig. 11, which represents the true stress–strain relation as obtained from a step by step interrupted tensile test. The yield point of the material is reached at a true stress of $3{\cdot}42\times10^8$ Nm^{-2} (34·2 kg/mm^2), and fracture occurs at a value of the stress close to 10^9 Nm^{-2} (100 kg/mm^2). For the region in between, the stress–strain curve behaves almost perfectly in accordance with the power function:

$$\sigma_\varepsilon = 1{\cdot}18\times10^9\varepsilon^{0{\cdot}22}.$$

From Fig. 10 it may be concluded that in the region of high strain and strain rate, as typical for conditions of machining,

$$\sigma_\varepsilon = 1{\cdot}2\times10^9\varepsilon^{0{\cdot}7} \quad \text{approx.}$$

A simultaneous representation of both the relations is given in Fig. 12.

This preliminary investigation into the plastic behaviour of the material under conditions of machining does not conclude to a significant influence of the cutting speed and hence of the strain rate on the strain hardening. By far the most important factor in strain hardening, referring to the particular material studied, appears to be the value of the strain. So far, however, no experiments have been performed in the region of strain which links the ultimate values of the quasi-static tensile test with the minimum values achievable in machining, in order to investigate whether some continuous transition from the one region to the other might exist.

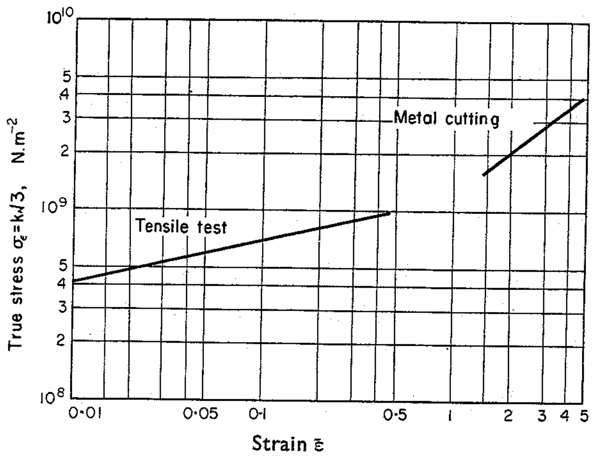

Fig. 12. Comparison of strain hardening in a tensile test and in the process of metal cutting.

$$\bar{\varepsilon} = \frac{2}{\sqrt{3}}\, ln\,(\varepsilon+1)$$

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