# Optimal age replacement versus condition based replacement : some theoretical and practical considerations

# Optimal Age Replacement versus Condition Based Replacement: Some Theoretical and Practical Considerations

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**When times to failure of a unit follow the Weibull distribution, achievement of optimal costs under age replacement is shown to be quite insensitive to the value of the shape parameter. Moreover, it is shown that under condition based replacement considerable improvement on even optimal age replacement costs is possible for any time to failure distribution with increasing failure rate, provided that the unit possesses a measurable prognostic characteristic which predicts the moment of failure within a reasonable margin of error and a sufficient lead time to undertake (preventive) replacement before a failure actually occurs.**

## Introduction

AGE replacement is still a very commonly employed preventive maintenance rule. It states that a unit is to be replaced by a new one if it has survived a certain age (preventive replacement) or has failed (corrective replacement), whichever occurs first. The new unit is subject to the same rule. "Age" or "time" is to be interpreted as the variable upon which the risk of failure of the unit is dependent. This could be calendar time, running hours, number of products produced, number of operations performed, etc. For age replacement to be a sensible rule, the risk of failure must be an increasing function of the age of the unit and a corrective replacement must cost more than a preventive replacement. Its application requires knowledge of the two replacement costs, the current age of the unit and—in order to determine the "optimal" replacement age—the time to failure probability distribution of the unit. Glasser (1969) provides an excellent discussion of age replacement.

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KEY WORDS: Age Replacement, Condition Based Replacement, Prognostic Characteristic.

Condition based replacement is gradually becoming a feasible alternative to age replacement, especially with the advent of more varied and more sophisticated condition monitoring equipment. Condition based replacement requires a unit to be inspected at intervals or monitored continuously and replaced just before its failure, the moment of failure being predicted from measurements on some "prognostic characteristic" which is the object of the monitoring process. The new unit, again, will be subject to the same rule.

The essence of the difference between age replacement (AR) and condition based replacement (CR) is illustrated by the following example. Consider the wear-out (*not* the accidental puncturing) of an automobile tire under average driving conditions. AR would consider the tire as a "black box" and set its replacement at a certain predetermined mileage, regardless of its actual condition, thus preventing some (not all) wear-out failures, throwing away part of the useful life of some tires, and executing replacements more often than necessary. CR, however, would regularly measure the remaining tread depth and from the pattern of its decrease predict the remaining mileage to failure of the tire. Just before the moment of this failure the tire would be replaced, thus in principle preventing all wear-out failures, making use of the entire useful life of the tire, and yet incurring only the cost of preventive replacement at each replacement. CR may have considerable advantages over AR if the inspection or monitoring costs are not prohibitive.

Effective application of CR does not require detailed knowledge of the probability distribution of time to failure (TTF), though complete knowledge of it would make the determination of (near) optimal inspection intervals possible, as demonstrated by Sherwin (1979). For a general and theoretically deeper treatment of CR models, see Gertsbakh (1977). Geraerds (1972) describes a more engineering oriented approach. CR has been applied for some time in aircraft maintenance, where it is known under a variety of names (see Nowlan et al. (1978)). The more or less systematic checking of temperatures and vibrations of bearings may be considered as an early and somewhat informal industrial application.

This article has a dual purpose. In the first place, we shall discuss the optimal cost of AR and its relative insensitivity to exact knowledge of the probability distribution of time to failure. In the second place, we shall indicate the improvement on this "optimal" AR that is theoretically obtainable by application of CR. First, however, we must introduce the most important assumptions underlying the AR and CR models, and discuss their relations to practical situations. We shall follow as much as possible Glasser's (1969) notation.

## Assumptions

### Failure distribution

We shall assume that the distribution of times to failure $X$ is Weibull:

$$F(x) \begin{cases} = 1 - \exp(-(\alpha x)^{\beta}) & x \geq 0, \alpha > 0, \beta > 0 \\ = 0 & x < 0 \end{cases}$$

$$E(X) = \alpha^{-1}\Gamma(1 + \beta^{-1})$$

$$\mathrm{Var}(X) = \alpha^{-2}\{\Gamma(1 + 2\beta^{-1}) - \Gamma^2(1 + \beta^{-1})\}$$

where

$$\Gamma(z) = \int_0^{\infty} \tau^{z-1}e^{-\tau}\, d\tau$$

is the complete gamma function, which is described and tabulated by, for example, ~~in~~ Abramowitz and Stegun (1965).

The Weibull distribution has the advantage that it is extremely flexible and often describes available failure data reasonably well. Moreover, limitation to the Weibull seems more serious in theory than it is in practice: in order to distinguish among the exact forms of distribution one needs more data than usually can be found in maintenance records. Also, Barlow and Marshall (1965) show that distributions with increasing failure rate do not differ all that much, as long as they have equal means and equal variances and $F(x) = 0$ for $x < 0$.

Proceeding on the assumption of the Weibull distribution we shall need numerical values for $\alpha$ and $\beta$ if we are to set an optimal replacement age for AR. This means that $\alpha$ and $\beta$ must be estimated from the data. Since in practice a unit is rarely purposely allowed to run to failure, our maintenance records often show censored data; that is, running times that were not completed by a failure, but by preventive replacement. Even under these circumstances, estimates of $\alpha$ and $\beta$ can be obtained in various relatively simple ways, as discussed by Nelson (1982).

Given the usual paucity and incompleteness of data, one cannot expect the resulting estimates of $\alpha$ and $\beta$ to be very accurate. However, we shall see that this is not of extreme importance when setting an optimal replacement age for the AR rule, and irrelevant when applying CR.

### Costs

The economic parameters influencing preventive replacement rules are the costs incurred by a preventive replacement before failure of the unit, $c_b$, and those incurred when a corrective replacement is effected, after failure of the unit, $c_a$. Actually, it is the ratio $k = c_a/c_b$ that is important. The cost $c_b$ usually includes labor and part costs. The cost $c_a$ is comprised of $c_b$ and, in addition, costs due to possible additional damage as a consequence of the actual breakdown of the unit and costs due to disruption of the production schedule. When consequence and disruption costs are not negligible, as is often true in practice, $k > 1$, but its exact value becomes difficult to establish and is not likely to be very accurate. Of course, preventive replacement, whether AR or CR, can be efficient only when $k > 1$ (and $\beta > 1$).

The costs to acquire estimates of $\alpha$ and $\beta$ in the AR case and the costs to own and operate the monitoring equipment and interpret its results in the CR are not explicitly included in the models. They are difficult to determine, yet they may not be negligible. Where applicable, we shall comment on these costs.

## Prognostic characteristic

A prognostic characteristic (PC) of a unit that is subject to failure is a property, or a composite of properties, that changes gradually from an initial value to a "fatal limit". The fatal limit is the value of the property at which the unit enters the failed state (Geraerds (1972)).

The applicability of CR crucially depends on the existence of a prognostic characteristic, for example, tread depth, and its measurability and predictability. The cost effectiveness of CR relative to AR depends moreover on the accuracy with which measurements on a PC predict the remaining TTF of a unit, had it been allowed to run to failure, and on the cost of obtaining such measurements. However, certain simple measures such as drop of oil pressure, play in bearings, or measurements of quality characteristics of products can presumably be fairly accurately extrapolated over time. These extrapolations can provide an estimate of the point where the unit in question becomes unserviceable; that is, fails. Also, the lead time with which a sufficiently accurate prediction of remaining TTF can be obtained plays a role since it determines whether a preventive replacement can still be undertaken.

We shall not discuss the existence and measurability of a PC. They are matters of physics and engineering that are dealt with in detail in, for example, the "Guide to the Condition Monitoring of Machinery" (1979) and Collacott (1977). These works discuss a wealth of condition checking and monitoring techniques and their areas of applicability. However, we shall discuss briefly and in a qualitative way the practical problems of measurement and prediction accuracy, and prediction lead time, and their interactions with monitoring costs.

In the ideal situation, as described by Geraerds (1972), the measurements of the PC predict the moment of failure without error and with sufficient lead time to undertake preventive replacement. Figure 1 illustrates this situation, where the shape of the relationship between a PC measure and the remaining TTF of the unit is supposed to be known, that is, parabolic in case a and linear in b. Early errorless measurements, ~~two~~ three in case a and ~~three~~ two in case b, are sufficient to determine the values of the parameters of the relationship and predict without error the moment of failure of the unit.

In practice, however, there are various sources of uncertainty. The actual shape of the relationship may not be known and may have to be estimated,

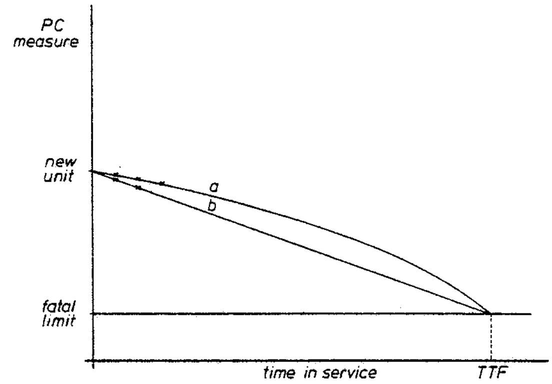


**FIGURE 1. Relation Between Time in Service and Prognostic Characteristic (PC), Ideal Situation; a: Parabola, b: Straight Line**

together with its parameters, from the measurements as they become available. The measurements themselves may not be without error or, due to internal or external circumstances, the actual value of the PC may show random fluctuations. Also, the value of the PC measure at which the unit fails or is considered unserviceable, that is, the fatal limit, may not be very well established. The overall result will be that, in practice, the predicted remaining TTF will be an uncertain quantity, at best described by a probability density as depicted in Figure 2. This also makes the available replacement lead time a stochastic quantity.

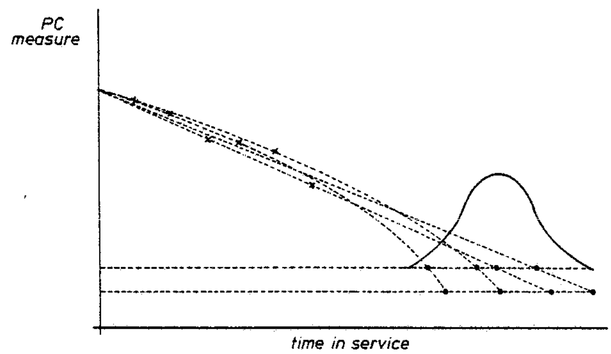


**FIGURE 2. Various Sources of Uncertainty Which Make the Prediction of TTF a Stochastic Variable**

Some of the uncertainties may be reduced by engineering information or experimentation. Remaining uncertainty may be reduced by taking more measurements on the PC and by continuing to do so beyond the first few, or by using more accurate measuring equipment. Either way the cost of monitoring the condition of the unit would increase. Eventually the lower confidence limit of the estimated available replacement lead time will come close to the required lead time and a decision will have to be made about the moment of preventive replacement of the unit. A conservative choice of lower confidence limit on the predicted moment of failure will assure that an actual failure will only rarely occur. It will also involve more frequent replacements at (low) cost $c_b$ than in the ideal case. A liberal choice may save on unnecessarily early replacements, but would also run an increased risk of actual failure with the associated (high) cost $c_a$. We shall return to this question later.

The unit cost of obtaining PC measurements depends on whether the condition measuring equipment and its use are expensive and dedicated to one unit, as in some nuclear reactors, or relatively cheap and universal, as is the case with shock pulse meters.

Despite the number and complexity of these practical problems, and the expenses involved, impressive net savings can be realized in practice by CR relative to AR. This is probably due to the large potential savings discussed in the next section. Baldin (1979) and Loynes (1981) describe practical cases in point.

## Age Replacement

Glasser (1969) gives a leisurely and easy to follow development of the formula for the cost per unit of time $R(t)$ as a function of the preventive replacement age $t$ under the AR rule. We quote his result for the case where the time to failure $X$ of a unit follows a Weibull distribution $F(x)$ as defined in the Assumptions section.

$$R(t) = [c_b \exp\{-(\alpha t)^\beta\} + c_a\{1 - \exp(-(\alpha t)^\beta)\}] \Big/ \int_0^t \exp\{-(\alpha x)^\beta\}\, dx$$

In the right hand side of this expression the numerator represents the expected cost per replacement; that is, the cost of a preventive replacement plus the cost of a corrective replacement, each weighted by the probability of its occurrence. The denominator represents the expected time until a replacement takes place; that is, the time until a preventive replacement (at time $t$) plus the time until a corrective replacement (when failure occurs at any time $< t$), all weighted by their respective probabilities of occurrence:

$$t\{1 - F(t)\} + \int_0^t x\, dF(x) = \int_0^t \{1 - F(x)\}\, dx.$$

Glasser (1967, 1969) gives charts from which the optimal value $t^*$ of $t$ can be read for values of $\beta > 1$. The charts also provide the ratio of the cost of preventive replacement at the optimal age $t^*$ to the cost of preventive replacement at infinite age, $\rho(t^*) = R(t^*)/R(\infty) = R(t^*)/\{c_a/E(X)\}$. Preventive replacement at infinite age is equivalent to no preventive replacement since failure has already occurred.

However, for our purposes we need the entire cost ratio curve $\rho(t)$, not just the optimal value $t^*$ and the corresponding ratio $\rho(t^*)$. In order to have a basis of comparison for all our curves we shall construct them for distributions that all have the same mean time to failure (MTTF), which we arbitrarily set equal to one. Thus, the MTTF $\equiv E(X)$ becomes the unit of time measurement and $t$ in our graphs must be interpreted as "dimensionless time", that is, a fraction or multiple of the MTTF. Also, when expressed as a function of dimensionless time, $\rho(t^*) = R(t^*)/c_a$.

Since the MTTF of a Weibull distribution is a function of both $\alpha$ and $\beta$, we have to adjust $\alpha$ as $\beta$ varies, in order to keep MTTF $\equiv E(X) = 1$. This means we have to choose $\alpha$ such that $\alpha = \Gamma(1 + \beta^{-1})$. The resulting combination of $\alpha$ and $\beta$ then uniquely determines the standard deviation of the distribution. Figure 3 summarizes specifications of some of the distributions used in the construction of Figures 4, 5, and 6, including the values of $v$, the MTTF measured in standard deviation units. Note that $v^{-1}$ = variation coefficient = standard deviation/mean.

Figures 4, 5, and 6 give $\rho(t)$ as a function of the dimensionless time described above, for values of $k = 2$, 4, and 16, respectively, and several values of $\beta$. Also indicated are values of $F(t^*) = P\{X \leq t^* E(X)\}$, the remaining corrective (unpredictable) replacements under AR as a fraction of all replacements when preventive replacements take place at the optimum dimensionless age $t^*$.

Inspection of the figures leads to the following observations:

1. For $\beta \leq 1$, $t^* = \infty$, regardless of the value of $k$; a trivial observation, since when $\beta \leq 1$ the failure rate is not increasing and nothing can be gained by preventive replacements.
2. For $\beta > 1$, $\rho(t)$ is rather flat on both sides near its minimum $\rho(t^*)$ (with the exception of curves for extremely high $\beta$).
3. For fixed $k$ and $\beta > 1$ the variations in $t^*$ that are accompanied by moderate variations in $\beta$ do not cause large differences in $\rho(t^*)$.

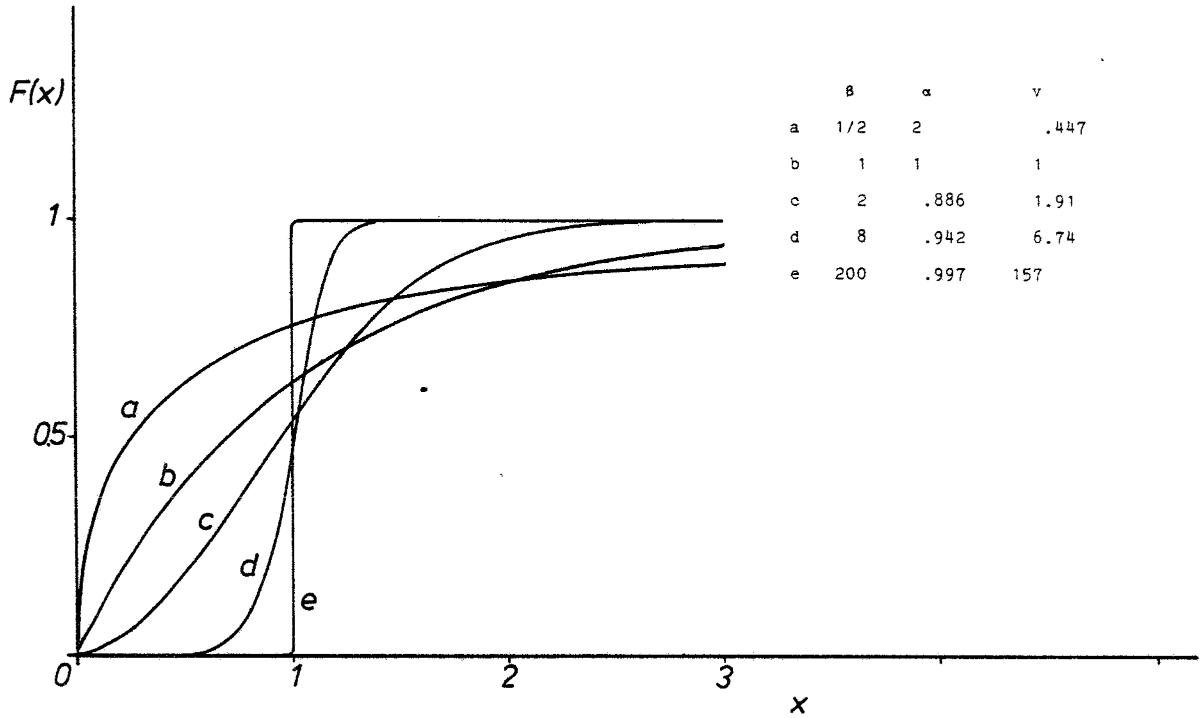


FIGURE 3. Weibull Distribution Functions $F(x) = 1 - \exp[-(\alpha x)^{\beta}]$ for Various Values of $\beta$; $E(x)$ = MTTF = 1; $v \equiv$ MTTF in Standard Deviation Units

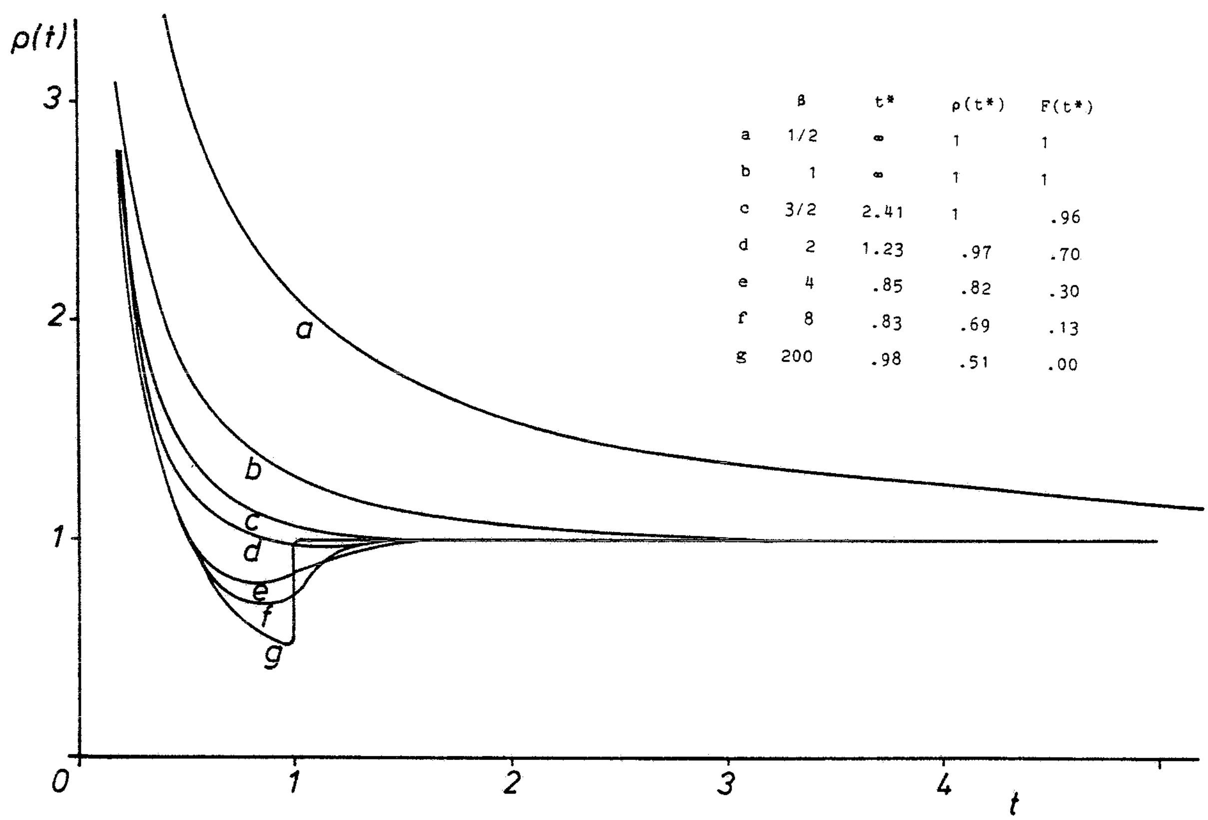


FIGURE 4. Relative Cost $\rho(t)$ as a Function of $t \equiv$ Dimensionless Time of Planned Replacement for Various Weibull Distributions $F(x)$ of TTF; $k = 2$; $t^* \equiv$ Optimal $t$

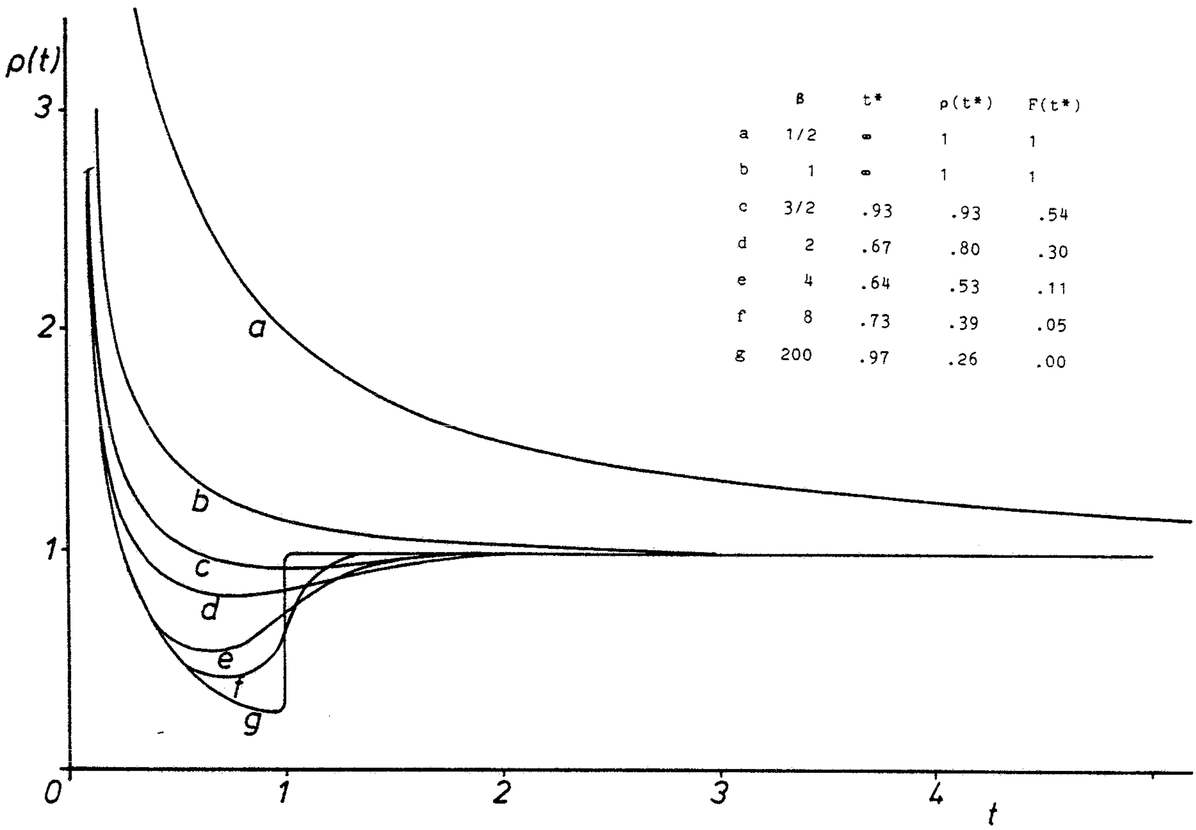


**FIGURE 5. Relative Cost $\rho(t)$ as a Function of $t$ ≡ Dimensionless Time of Planned Replacement for Various Weibull Distributions $F(x)$ of TTF; $k = 4$; $t^*$ ≡ Optimal $t$**

4. For $\beta > 1$ and fixed, large variations of the value of $k$ are accompanied by appreciable relative variations in the values of both $t^*$ and the corresponding minimum $\rho(t^*)$. This effect persists up to high values of $\beta$.
5. As $\beta$ approaches infinity, that is, as $F(x)$ approaches a step function, $t^*$ approaches 1 and $\rho(t^*)$ approaches its lowest possible value, $1/k$. This is as it should be: if the unit fails with certainty at exactly its expected lifetime, the corresponding cost per unit of dimensionless time then is $c_b/1$ and the ratio $\rho(t^*) = 1/k$. (It can actually be shown that for any finite $\beta > 1$, $\rho(t^*) > 1/k$, and $\lim_{\beta \to \infty} \rho(t^*) = 1/k$).

Observations 2 and 3 together lead to the conclusion that, (at least for Weibull distributed times to failure) for a given value of $k$, achievement of near-optimal costs does not require a very accurate estimate of $\beta$ nor a very precise setting of $t$, so long as the distribution has a value of $\beta > 1$, which is commonly encountered in practice. It also implies that planning of the execution of preventive replacements can be done with some flexibility when the planned replacement age is near optimal. Of course, the fact remains that while a relative difference from optimal costs may be small, the absolute difference may still be considerable. However, it is doubtful that it would be possible in practice to achieve exactly optimal costs anyway, given the uncertainties in $k$ and $\beta$.

Observation 4 causes more concern, especially in view of the likely difficulties in estimating $c_a$. It leads to the conclusion that the estimate of $k$ has to be fairly accurate, or a near optimal value of $t$ may not be set and, consequently, a near optimal value of $\rho(t)$ may not be achieved. For fixed $\beta > 1$, a grossly overestimated value of $k$ and the resulting erroneous setting of $t$, say $t^+$, may even result in $\rho(t^+) > 1$. For a gross underestimate of $k$ the worst that may happen is that $\rho(t) \approx 1$. Thus, when there is extreme uncertainty about the correct value of $k$, it would seem preferable to err on the low side.

To illustrate the conclusions reached so far, assume that a unit has a time to failure distribution that is Weibull with true value $\beta = 4$, and assume that the true value of $k = 4$. Then $t^* = 0.64$ and $\rho(t^*) = 0.53$. Errors in the estimates $\hat{\beta}$ and $\hat{k}$ lead

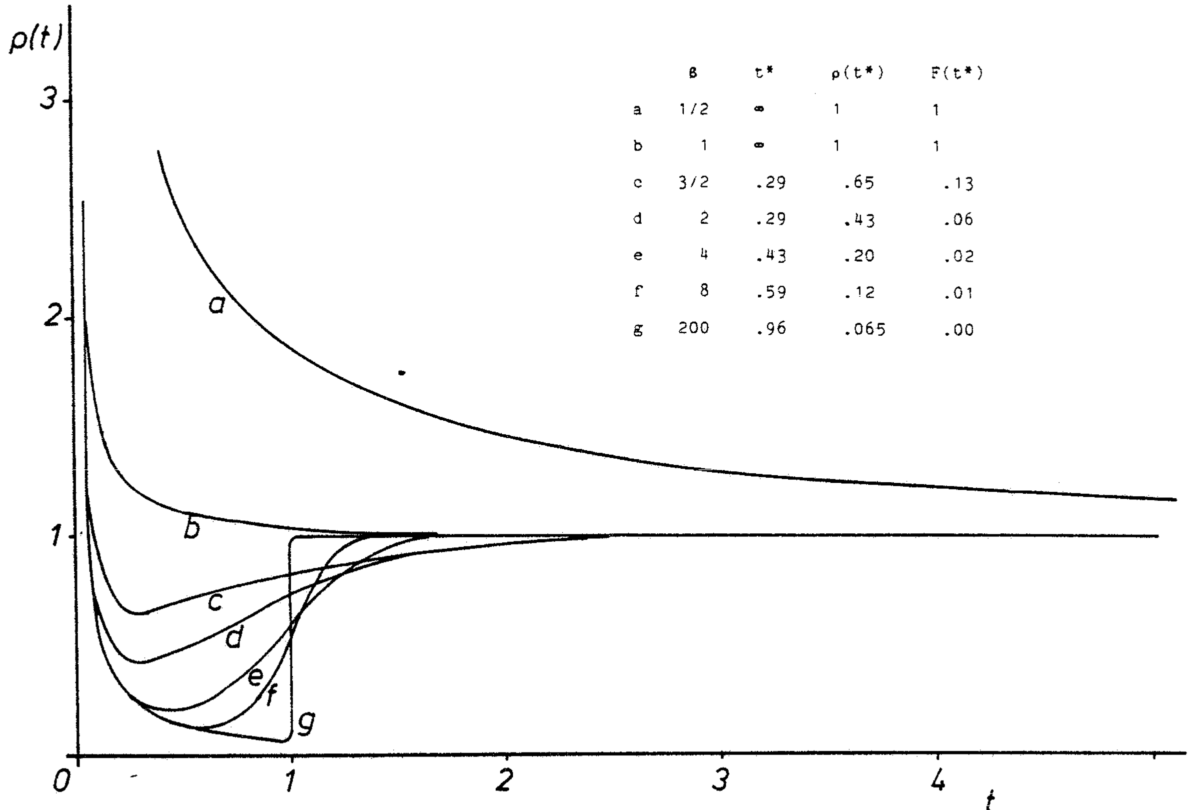


**FIGURE 6. Relative Cost $\rho(t)$ as a Function of $t \equiv$ Dimensionless Time of Planned Replacement for Various Weibull Distributions $F(x)$ of TTF; $k = 16$; $t^* \equiv$ Optimal $t$**

to an incorrect "optimal" choice $\hat{t}$ of the replacement age. The effect of such an incorrect choice can be calculated by evaluating $\rho(t)$ at $t = \hat{t}$ for the true values of $\beta = 4$ and $k = 4$. Table 1 gives some results for the percentage increase in cost over the optimal $\rho(t^*) = 0.53$.

**TABLE 1. Percentage Increase (lower entry) Over Optimal Cost Due to Incorrect Choice of $t$ (upper entry) Resulting From Erroneous Estimates $\hat{\beta}$ and $\hat{k}$; True Values are $\beta = 4$ and $k = 4$**

| $\hat{k}$ | $\hat{\beta}$ = 2 | 4 | 8 |
|---|---|---|---|
| 2 | 1.23<br>64 | .85<br>13 | .83<br>10 |
| 4 | .67<br>0 | .64<br>0 | .73<br>3 |
| 8 | .43<br>18 | .52<br>5 | .65<br>0 |

## Condition Based Replacement

Observation 5, from the previous section, implies that under the AR rule, for any given $k$, the lowest possible value of $\rho(t^*)$ is $1/k$. It is achieved (when $\beta = \infty$) as each replacement takes place just before the age of failure of the unit. The expected replacement age then equals the MTTF and no failures occur. The results of applying the CR rule are identical: the expected replacement age is the MTTF and no failures occur. At least in theory. How close the CR rule can approach this theoretical ideal depends on the accuracy with which the point in time of the occurrence of a failure can be predicted over a lead time sufficiently long to undertake preventive replacement.

Now we may interpret the $\rho(t)$ curves for $\beta = \infty$ (which are virtually indistinguishable from the curves for $\beta = 200$) as applicable to the CR rule, regardless of the actual value of $\beta > 1$ of the distribution of the unit's times to failure. In fact, the curves for $\beta = \infty$ are theoretically applicable to the CR rule for any distribution with increasing failure rate, Weibull or not. They are described by

$$\rho_C(t) = \begin{cases} (kt)^{-1} & \text{for } 0 \le t < E(X) = 1 \\ 1 & \text{for } t \ge E(X) = 1. \end{cases}$$

One notices an agreeable property of the CR rule here. Even if one employs conservative predictions of the remaining time to failure, appreciable relative savings can be obtained by CR over AR for most values of $\beta$ that are commonly encountered in practice. We shall now illustrate the practical importance of this property.

In the ideal situation, the PC predicts without error the TTF of the unit in service. That unit presumably was drawn at random from the population of units with TTF distribution $F(x)$. In practice, however, there is some uncertainty associated with the prediction of each TTF. It may be unbiased but, as we have seen, its variance probably is not negligible. In trying to avoid the actual occurrence of a failure we therefore employ a conservative lower confidence limit of the predicted TTF each time we make a decision about the moment of (preventive) replacement of a unit in service.

Let us assume, for the purpose of illustration, that this lower limit is such that each time-replacement takes place at 75 percent of the mean of the predicted TTF of the unit then in service. We also assume that the choice of this lower limit is indeed effective in avoiding the occurrence of any and all failures. If the predictions are unbiased, we thus replace the units on average at 75 percent of their true TTF. Therefore, the expectation of the fraction of the MTTF at which we replace will equal 0.75. For example, when $k = 4$ this means that instead of achieving $\rho_C(t = 1 = t^*) = 1/k = 0.25$ under the CR rule, we expect to achieve approximately $\rho_C(t = 0.75) = 1/(kt) = 0.33$. However, this is still a reduction of 38 percent from the optimal value $\rho(t^* = 0.64) = 0.53$ under the AR rule for $\beta = 4$. Thus, roughly speaking, owning and operating equipment to monitor the unit might cost per time unit as much as 38 percent of the optimal cost per time unit under AR, and still CR would be cost effective.

As we have seen, when applying CR it is important to use the failure time predictions obtained from the measurements on the prognostic characteristic in a conservative way. The equivalence of the $\rho(t)$ curves under AR for $\beta = \infty$ with the $\rho(t)$ curves under CR does not hold if, despite CR, wear-out failures do occur. It is implicitly assumed that CR, when applied, does lead to the prevention of (virtually) all failures due to wear-out. Thus, the conservative choice of 75 percent of the mean of the predicted TTF, as used in the illustration, is supposed to result in the prevention of all impending failures. This assumption is realistic only if the accuracy of the time to failure predictions is such that the occurrence of an *actual* failure at less than 75 percent of the *predicted* time to failure has negligible probability. Clearly, there is a possibility here to trade off the degree of conservatism required (and the corresponding decrease of potential savings) against predictive accuracy (and the corresponding monitoring expense). If, under CR, in practice one would allow a small fraction $p$ of failures to occur, then $\rho_C(t)$ may be approximated by $\rho_C'(t) = 2(kt)^{-1}\{1 + p(k-1)\}/(2-p)$. As shown in the Appendix, this approximation has a relative error of less than $p/(2-p)$ for $0 < t < 1$.

The (theoretical) elimination of actual failures by the CR rule not only allows undisrupted production, it also allows all replacements to be planned. Thus, not only the production department saves expenses (as reflected by the difference between $c_a$ and $c_b$), but the maintenance department also will have a larger fraction of planned jobs to do. This may be an additional bonus to the extent that it has not been accounted for in the difference between $c_a$ and $c_b$. It is implicitly assumed here that inspection of the unit, if not possible while the machine is running, can be performed at times when it is not needed for production. If that is not possible there would be additional costs associated with production losses due to the required inspection time.

## Summary

When times to failure of a unit follow the Weibull distribution, achievement of optimal costs under age replacement is rather insensitive to the exact value for $\beta \gtrsim 1$. The accuracy of the value of $k = c_a/c_b > 1$ is more important. The *value* of the optimal costs depends strongly on the values of $\beta$ and $k$. For any given $k$ the smallest possible value of the optimum costs equals $1/k$. It can be achieved when $\beta$ approaches infinity. Under CR, however, this smallest possible value may be closely approximated for any value of $\beta > 1$, indeed for any distribution with increasing failure rate. The potential cost savings of CR relative to even optimal AR are generally considerable. Achievement of these savings requires the existence of a prognostic char-

acteristic which gives good predictions of the impending moment of failure. The effort and accuracy with which these predictions can be obtained determines the overall cost effectiveness of CR versus AR.

## Appendix

Under the CR rule we preventively replace a unit at a fraction $t < 1$ of its predicted TTF. If, nevertheless, failures may occur with probability $p$, we do not achieve the intended lifetime fraction $t$ for units that fail. Therefore, the expected value of the time until replacement will be less than $tE(X)$. If the predictor of the TTF is unbiased, the expected value of the time until replacement is $(1-p)tE(X) + p\tau E(X)$, where $0 \le \tau < t < 1$. The expected cost for a replacement is then $(1-p)c_b + pc_a$, so that the expected cost per expected time until replacement is

$$R_C(t) = \{E(X)\}^{-1}\{(1-p)c_b + pc_a\} / \{(1-p)t + p\tau\}.$$

The expected cost per replacement with no preventive replacements at all, that is, with all replacements being corrective, is $c_a$ and the expected cost per expected time until replacement is $R_C(\infty) = c_a/E(X)$. Thus

$$\rho_C(t) = R_C(t)/R_C(\infty) = \{[(1-p)/k] + p\}/\{(1-p)t + p\tau\}.$$

For $p = 0$ this reduces to $\rho_C(t) = (kt)^{-1}$. For $t > p > 0$ we have

$$(1-p)t < (1-p)t + p\tau < t.$$

Therefore

$$\{[(1-p)/k] + p\}/\{(1-p)t\} > \rho_C(t) > \{[(1-p)/k] + p\}/t$$

We take the harmonic average of the left hand side (LHS) and the right hand side (RHS) of this inequality as an approximation of $\rho_C(t)$, that is,

$$\rho'_C(t) = 2(kt)^{-1}\{1 + p(k-1)\}/(2-p).$$

For the relative error $\varepsilon = \{\rho'_C(t) - \rho_C(t)\}/\rho_C(t)$ we have

$$\frac{[\rho'_C(t) - \text{LHS}]}{\text{LHS}} < \varepsilon < \frac{[\rho'_C(t) - \text{RHS}]}{\text{RHS}}$$

$$-p/(2-p) < \varepsilon < p/(2-p).$$

## Acknowledgments

The author would like to thank his colleague C. W. Gits and an anonymous referee for very helpful comments and suggestions on an earlier draft of this paper. Thanks are also due to P. J. A. Doorakkers for production of the figures and to A. C. J. Kirkels for typing numerous revisions of the manuscript.

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