

Two fluids model reexamined for a collisionless plasma in the stationary state

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TWO FLUIDS MODEL REEXAMINED

for a collisionless plasma in the stationary state

by

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ABSTRACT

The usual presentation of the Two Fluids model of a plasma is inadequate for the description of quasi-stationary state phenomena in fully ionized and magnetized plasmas. The mass rotation of the plasma is neglected incorrectly and the role of the electric field is overestimated. An alternative description is presented where the diamagnetic current is carried equally by electrons and ions, so that for each species the Lorentz force-density is in equilibrium with the pressure. The plasma possesses a non uniform mass rotation and the electric field is only of importance at the plasma boundary and in some cases at the plasma centre.

The "classical" diffusion theory yields erroneous values for the transport coefficients of a plasma as it neglects mechanisms (e.g. ion-ion collisions) which make the ions moving faster through the magnetic field than ion-electron collisions. Of the two alternative ways which were proposed respectively by BOHM and by SIMON in order to explain the following up of the electrons, only Bohm's "drain diffusion" of the electrons is acceptable. It seems that the drain mechanism is also responsible for the transfer of angular momentum from the wall to the plasma. At closer examination Simon's short circuit effect is undeserving of belief as the non ambipolar diffusion would be connected to a radial current leading to impossible big torques operating on the plasma.

A synthesis of these concepts - non uniform rotation, ion-ion collisions and drain diffusion - leads to a theoretical picture which is in good agreement with earlier experiments, performed with a fully ionized and magnetized plasma in a quasi-stationary state.

CONTENTS

1. INTRODUCTION

1.1. Nomenclature and Assumptions

2. SINGLE FLUID MODEL

2.1. Single Particle

2.2. Uniform Plasma

2.3. Plasma with ∇m

2.4. Plasma with E

2.5. Plasma with ∇m and collisions

3. TWO FLUID MODEL

3.1. Uniform Plasma

3.2. Plasma with ∇m

3.3. Plasma with ∇m and E

3.4. The collisionless model

3.5. A regression to the classical model

3.6. Plasma with collisions

3.6.1. Diffusion determined by ion-electron collisions

3.6.2. Diffusion determined by ion-ion collisions

3.7. Anomalous diffusion of the electrons

3.8. Relation to the MHD theory

3.9. Conservation of angular momentum

3.10. Experimental evidence

1. INTRODUCTION

Although numerous experimental and theoretical efforts have been made during the last decennia to understand the behaviour of a plasma in a magnetic field, many problems concerning transport phenomena are still unsolved. The question arises whether the relevant theories are wrong or the experiments do not satisfy sufficiently the theoretical assumptions. It seems that particularly the first suspicion may be justified, and we will try to show why.

In the theoretical sector the source of the difficulties lays in an abusive application of the usual two fluids theory to quasi-stationary states ($\omega \ll \omega_{ci}$). The pertinent equations are a simplification of the kinetic equations for an assembly of electrons and ions, equations which seem to be intractable for straightforward solutions. It is particularly the introduction of the concept of a pressure gradient in the simplified equations, which give rise to confusion. As VAN KAMPEN and FELDERHOF (1967) write in their book "Plasma Physics": "It is hard to see how the assumption of frequent collisions between particles of the same component can be reconciled with the assumption that collisions between different components are negligible".

As we will see the concept of a plasma pressure $p = nkT$ and its gradient ∇p must be handled carefully if dilute magnetized plasmas are under investigation. It are these plasmas which are dealt with in this article. No neutral atoms are present and only one kind of singly charged ions. We will start from single fluid models, first without, then with collisions. After that an electron and an ion fluid are brought together in an attempt to come to a satisfactory two fluids model. This can be done by using a modified form of an old concept of BOHM (1949) - the drift oscillations. These oscillations may turn out to be in our advantage as far as plasma containment is concerned.

From the beginning one has felt difficulties in reconciling the single particle model with the two fluids theory. Various "paradoxes" have been solved in order to prove that both theories are in agreement, if properly interpreted (see SCHLÜTER, 1959).

In a particular important paradox concerning the diamagnetism of a plasma it is pointed out that the presence of particles reflected from the wall is necessary to reconcile microscopic and macroscopic pictures. In this analysis it is shown that this is a misconception. There is no wall where the plasma particles are reflected. The plasma is diamagnetic and the diamagnetic current is carried equally by electrons and ions. Of great importance is also the fact that in a magnetic field all plasmas rotate.

The mathematics is kept as simple as possible, so that the physics is not camouflaged. Such an approach is stimulated by a look at the kinetic theory of gases where transport coefficients are derived in a lengthy way which can be found in two lines by a simple consideration, whereas experimentally it is often impossible to decide about a small difference in factors between the two.

In order to keep in touch with the physical reality, only cylindrical plasma columns are considered. As an infinite, uniform plasma is of no theoretical or experimental interest, it is assumed that the plasma radius is finite. Very interesting questions arise if one looks closer at the inevitable contact which the plasma has with the wall during its generation and during its extinction. This is of particular importance for the rotation of the plasma, as was already pointed out by HAINES (1965).

Some reference is made to experiments with a powerful hollow cathode discharge (BOESCHOTEN and DEMETER, 1968). Such a device has the advantage of providing a fully ionized plasma column, long and wide enough to satisfy at least the most modest requirements which we may ask from a plasma experiment ($L/\lambda_i \gg 1$ and $r_{ex}/r_{ci} \gg 1$). The phenomena which were observed in this experiment (plasma rotation and drift oscillations) have been observed in numerous experiments before and afterwards. Our results are, however, illustrative in so far as they show how well the plasma rotation can be explained with a single particle model in which ion-ion collisions are dominant. In the next sections a justification is given for the use of such a model, which we believe is applicable to all diluted, fully ionized and magnetized plasmas.

1.1. Nomenclature and Assumptions

A quasi-stationary state is considered ($\omega \lesssim \omega_{ce}$)

The cylindrical coordinates are denoted r, θ, z .

The c.g.s.-Gauss system is used throughout.

\underline{B} = magnetic field strength (in the z direction, pointing out of the paper: \odot)

\underline{E} = electric field strength

\underline{L} = total angular momentum

\underline{M} = total diamagnetic moment

n_e, n_i = surface density (cm^{-2}) of the guiding centres of the electrons, ions ($= n$ if $n_e = n_i$)

T_e, T_i = electron, ion temperature ($= T$ if $T_e = T_i$)

p = plasma "pressure"

\underline{v} = particle velocity

v_{et}, v_{it} = thermal velocity of electrons, ions ($= v_t$ in s.p.m.)

\underline{i} = particle flux ($\underline{i} = n \underline{v}$)

\underline{j} = electric current density ($\underline{j} = e \underline{i}$)

$\underline{\omega}$ = angular velocity

ω_{ce}, ω_{ci} = electron, ion cyclotron frequency (ω_c in s.p.m.)

Ω = frequency of drift waves ($\Omega \lesssim \omega_{ci}$)

λ_e, λ_i = mean free path length of electrons, ions

r_{ce}, r_{ci} = electron, ion cyclotron radius (r_c in s.p.m.)

τ_e, τ_i = electron, ion collision time (τ in s.p.m.)

$D_{e\perp}, D_{i\perp}$ = electron, ion diffusion coefficient perpendicular to the magnetic field

L = length of plasma column

r_{pe} = radius of plasma column

$q = n/|\nabla n| = 1/|d \ln n/dr| = e$ -folding length (∇n is generally negative)

It is assumed that the plasma is fully ionized (collisions with neutral particles negligible) and magnetized ($\omega_{ce}\tau_e \gg \omega_{ci}\tau_i \gg 1$). That the plasma currents are small and the associated magnetic fields may be neglected compared to the externally applied uniform magnetic field \underline{B} . As a further external force only the electric field \underline{E} is considered. All particles of the same kind have the same absolute velocity, the ion temperature is equal to the electron temperature ($T_i = T_e$) and $\nabla T = 0$.

Only situations are considered where the radius of the plasma column accommodates many ion cyclotron radii ($r_{pe} \gg r_{ci}$). In practically all experiments the plasma radius r_{pe} is determined by some limiter and the plasma particles are confined by the magnetic field (Fig. 1). In the free space outside the limiting surface the particles may emerge up to a distance r_c , even if there are no collisions at all. Collisions which were neglected at first are introduced in the

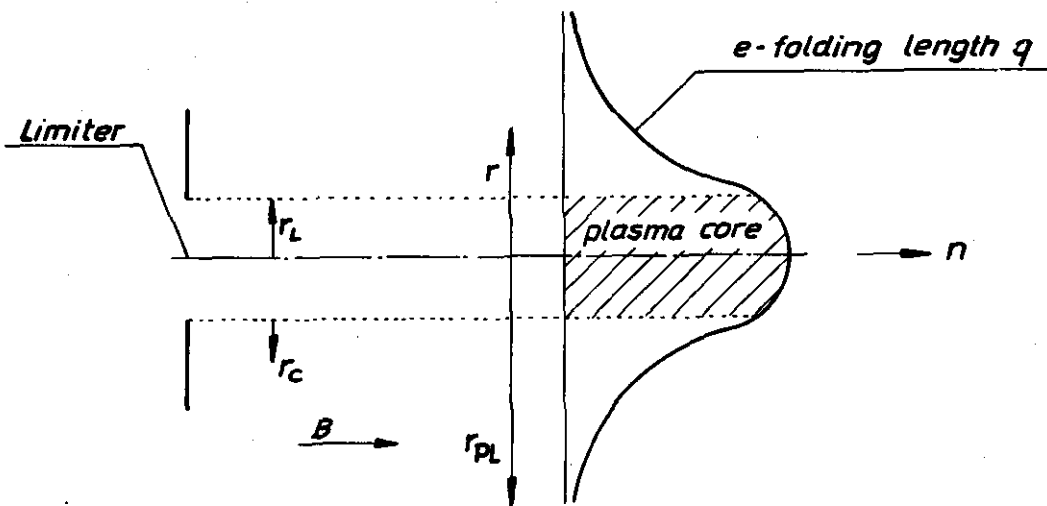


Fig. 1: Schematic of the density distribution

second approximation in order to explain the diffusion of plasma across the magnetic field. In section 3.6 it turns out that in a wide parameter range ion-ion collisions cause the ions to move faster through the magnetic field than the electrons. How the electrons manage to follow the ions in radial direction is explained in section 3.7.

The plasma density profile is determined by the geometry of the apparatus and by the motion of the plasma particles across the magnetic field. As pointed out in an earlier publication^{*)} (BOESCHOTEN 1964) plasma density profiles from the various experiments do not give us much information about the perpendicular diffusion. Except maybe for the fact that the e-folding length q is found to be about r_{ci} and not r_{ce} . q is an approximate quantity, always found to be $q/r_{ci} \geq 1$.

The plasma column is long ($L \gg \lambda$) but it has not to be infinite. The diffusion of particles along the magnetic field lines is determined by the ions as in the case without magnetic field. The behaviour of the plasma particles is analyzed in the r, θ plane. In this two dimensional model the plasma surface density \mathcal{M} is in units of cm^{-2} . At a first glance it is not clear at all why such an analysis should make sense - but it turns out in section 3.7 that it does.

2. SINGLE FLUID MODEL

We think about the plasma particles as gyrating in the magnetic field. This is a single particle model (section 2.1). If there are many particles of the same kind in a volume element, one has to assume that the Coulomb forces between them are cancelled by particles of opposite sign (Lorentz gas). In section 2.2 a uniform, but finite plasma is treated, in section 2.3 a density gradient is introduced and in section 2.4 an electric field. Finally collisions between the like particles are taken into consideration.

2.1. Single particle

The cyclotron radius r_c of a particle gyrating in a magnetic field is found by equating the centrifugal force acting on it: $m v_t^2 / r_c$ to the Lorentz force, $(j/c) \times B = (e/c) v_t B$. This yields $r_c = (eB/mc) v_t$ and calling $\omega_c = eB/mc$ the cyclotron frequency: $r_c = \omega_c v_t$. The electrons rotate in the direction of a corkscrew pointing in the direction of \underline{B} . We will call this direction the + direction. Thus the ions rotate in the - direction.

*) Although we would write many parts of this article differently now, the data about the experiments may still be used.

In the two dimensional analysis each particle circles around a point, called guiding centre. With no external forces present, this point stays on its place. In the presence of a static or quasi-static electric field \underline{E} , the guiding centres move with a drift velocity $\underline{v}_E = c \underline{E}/B$ which is the same for ions and electrons.

The angular momentum of the particle $|\underline{l}| = m v_t r_c = m v_t^2 / \omega_c$. It is in opposite direction for ions and electrons and much larger for the ions than for the electrons ($m_i/m_e \times$).

The magnetic moment $\underline{\mu}(\underline{l}) = (e/2mc) \underline{l}$ and $\mu = 1/2 m v_t^2 / B$. It has the same value for ions and electrons, and also the same direction. Both try to shield the plasma from the magnetic field, the plasma is diamagnetic.

2.2. Uniform plasma ($n = \text{constant for } r \leq r_{pe}; n = 0 \text{ for } r > r_{pe}$)

The particle flux at a point Q located at $r < r_{pe} - r_{ci}$ vanishes. This follows from the fact that all particles which gyrate through Q have their guiding centers on a circle with r_c as radius and Q as centre. The particle flux at a point P, located at $r_{pe} - r_{ci} < r < r_{pe} + r_{ci}$ does not vanish and can be calculated as follows:

Only particles with their guiding centre G on the arc ABC contribute to the particle flux at P with an amount $v_t \cos \alpha$. For small α :

$$i(r) \approx \frac{2}{\pi} n v_t \text{arc} \cos \frac{x}{r_c} \quad (1)$$

where x is the distance from P to the plasma surface $-r_c < x < r_c$. In this approximation it is assumed that $\cos \beta \approx \cos \alpha - (r/r_c) \sin \alpha \approx \cos \alpha$, so that it is valid only if $r_{pe}/r_c \gg 1$. In general we may write:

$$i(r) = \frac{2}{\pi} f(r/r_c) n v_t \text{arc} \cos \frac{x}{r_c} \quad (1a)$$

where $f(r/r_c) \approx 1$ for $r \leq r_{pe}$ and $f(r/r_c) = 0$ for $r < r_{pe} - r_c$ and $r > r_{pe} + r_c$. In Fig. 2 $i(r)$ is indicated for an ion gas. For an electron gas the rotation is in opposite direction.

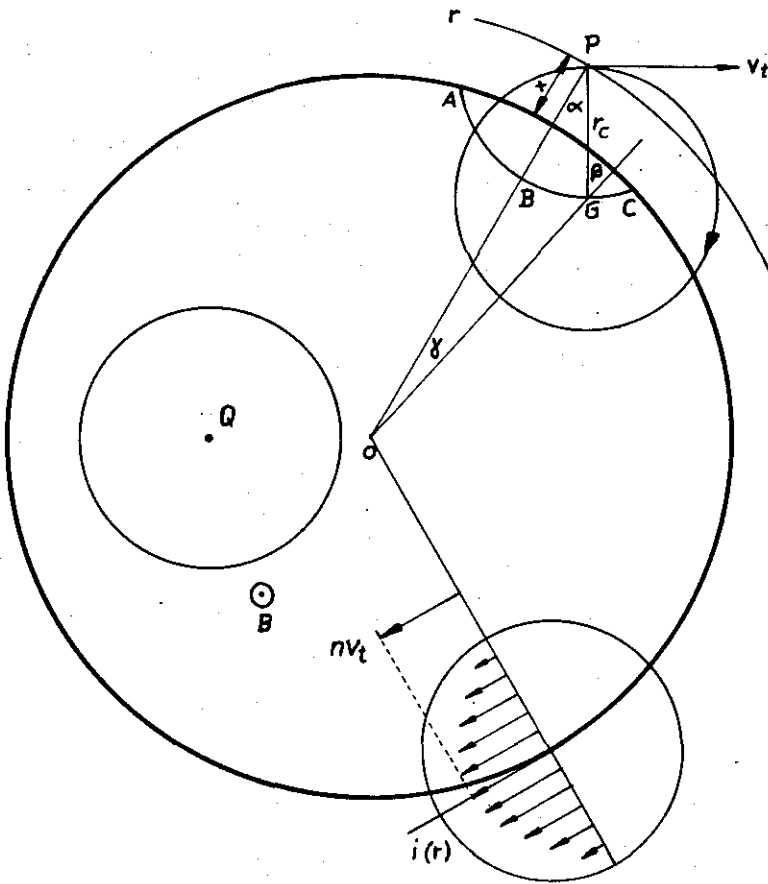


Fig. 2: Diamagnetic current of the ions. For electrons it is the same in opposite direction.

The total angular momentum of the fluid is

$$L = 2\pi \int_{r_c - r_e}^{r_c + r_e} m i(r) r^2 dr \quad \text{or}$$

$$L \approx \pi m m v_t r_c r_e^2 \quad (2)$$

for large r_e/r_c exactly equal to the sum of the angular momenta of the individual gyrating particles in the plasma "volume":

$$\underline{L} = \sum m \underline{r} \times \underline{v} = \sum m \underline{r}_g \times \underline{v} + \sum m \underline{r}_c \times \underline{v} = \sum m \underline{r}_c \times \underline{v}$$

(\underline{r}_g is the radius vector of the guiding centre).

The angular momentum is connected with a diamagnetism of the plasma which is $e/2mc$ times in value. If a reflecting wall is present at the plasma boundary, the diamagnetic current is cancelled by a counter current caused by reflections from the wall. This leads to an important theorem about the absence of diamagnetism in metals, but is of no importance for the situation considered here, where the plasma is well away from the wall. However, phenomena at the wall become of importance at the place where the plasma is generated. We come back to this question in section 3.9.

The current density at the plasma boundary is given by $\underline{j}(\alpha) = e \underline{i}(\alpha)$. This gives with \underline{B} a Lorentz force ($r_{pe}/r_c \gg 1$ approximation):

$$\frac{\underline{j}(\alpha)}{c} \times \underline{B} = \frac{2}{\pi} \frac{e}{c} m v_t B \text{ arc cos } \frac{\alpha}{r_c} \quad \text{or}$$

$$\frac{\underline{j}(\alpha)}{c} \times \underline{B} = \frac{2}{\pi} n m \frac{v_t^2}{r_c} \text{ arc cos } \frac{\alpha}{r_c} \quad (3)$$

Integration over α/r_c gives the total force (per cm^2): $n m v_t^2 / r_c$. If one likes, this may formally be written as ∇p , where $p = n m v_t^2 = n k T$ and $|\nabla n|/n = 1/r_c$:

$$\frac{\underline{j}}{c} \times \underline{B} = \nabla p \quad (4)$$

One should be careful with the interpretation of this formula. The particles gyrate around their guiding centres which stay on their place. The forces acting on each of them are the centrifugal force and the Lorentz force. By going to the macroscopic theory one could think of the magnetic field as a virtual containing wall and of the "plasma pressure" as a gas pressure. If the field were taken away and the plasma came into contact with a real physical wall, it would exert the same pressure $n k T$. What one forgets in this picture, is the rotational motion of the plasma. In the collisionless case the microscopic description seems preferable as it gives a better feeling for the real situation.

2.3. Plasma with ∇m

Quite analogously to the situation at the boundary of a uniform plasma, the particle flux at radius r within an inhomogeneous plasma may be calculated. One finds (in the $r/r_c \gg 1$ approximation):

$$i(r) \approx m \frac{r_c}{q} v_t \quad (5)$$

This is an approximate relation as all relations where q is used. The azimuthal velocity due to ∇m is:

$$v_{L\theta} = \frac{r_c}{q} v_t \quad (5a)$$

$q(r)$ is a physical quantity for which $q/r_c \geq 1$. Strictly speaking it only makes sense in the Two Fluids model, where it has to be explained why it is the same for ions and electrons (see sections 3.2 and 3.7).

The current density $\underline{j}(r) = e \underline{i}(r)$ gives with the magnetic field the Lorentz force $(\underline{j}/c) \times \underline{B} = m m v_t^2 / q = \nabla p$.

2.4. Plasma with \underline{E}

In this Single Fluid model it was assumed that there is no space charge, so that a radial electric field can only be introduced in the uniform cylindrical plasma if a cylindrical inner electrode (radius r_a) is present, concentric to the plasma surface. $\text{Div } \underline{E} = 0$ gives a field $\underline{E} = (r_a/r) \underline{E}_a$ (the direction of \underline{E} is supposed to be inward; $\nabla V > 0$). Independent on the temperature of the plasma, the guiding centres of the plasma particles will drift with a velocity $v_{E\theta} = c E/B = c (E_a/B) r_a/r$ in the + direction. $\omega_E = c (E_a/B) r_a/r^2$ - the radial electric field makes the plasma rotating like a vortex ($v_{E\theta} r = \text{const.}$).

The Lorentz force on an ion because of its guiding centre drift is $(\underline{j}/c) \underline{B} = (e/c) v_{E\theta} \underline{B} = e \underline{E}$ and is directed outward for a positive particle, thus compensating for the force of the electric field. (For electrons the direction of both forces is reversed.)

The situation in the region in the neighbourhood of the inner electrode (within a distance $\approx r_c$) is more complicated. Here magnetron like effects would occur. We come back to this question in sections 3.3 and 3.4.

2.5. Plasma with ∇n and collisions

In this simple Single Fluid model only collisions between like particles (collision time τ) are to be considered. This case was treated by SIMON (1955 b), who came to the conclusion: "The diffusion velocity due to the like particle collisions is clearly proportional to B^{-4} and does not obey Fick's law. Instead the diffusion rate depends on the second and third space derivatives of the particle density:

$$v_{\perp} = \frac{3}{8} \frac{r_c^4}{\tau} \frac{\partial}{\partial r} \left(\frac{1}{n} \frac{\partial^2 n}{\partial r^2} \right) \quad (6)$$

The outward flux vanishes if n varies linearly or exponentially with r . Often the density profile in a region outside the core of the plasma is approximately:

$$n \approx n_0 e^{-r^2/q^2} \quad (\text{see section 3.4}). \text{ This yields:}$$

$$v_{\perp} \approx 3 \frac{r}{\tau} \left(\frac{r_c}{q} \right)^4 \quad (6a)$$

τ varies $\propto n^{1/2}$, so that this flux is much larger for a fluid of ions than for a fluid of electrons. Generally q is a function of r and (6a) is only valid in a region of uniform rotation at some distance of the core.

Without volume production and recombination, the particle conservation equation requires for the stationary state: $\nabla \cdot n \underline{v} = 0$. With ambipolar diffusion in axial direction and a negligible radial electric field (see section 3.4) this relation reduces to $\partial(r n v_{\perp}) / \partial r = 0$. Equation (6a) fulfils this requirement rather well in the first approximation. In higher approximation it should be remembered that ω is not exactly constant and ∇T not exactly zero.

3. TWO FLUIDS MODEL

A fluid of ions and a fluid of electrons are taken together. The mixing provides for the bulk space charge compensation, which is assumed in the Single Fluid model. Differences in behaviour must be reconciled, for which the plasma has a solution: drift waves and drain diffusion.

3.1. Uniform plasma

In the inner region where $\nabla M = 0$, the mass velocity \underline{V} and the current density \underline{j} vanish. Like in the Single Fluid model the phenomena of interest occur at the boundary in a skin of thickness $\sim r_{ci}$. As $r_{ci} \gg r_{ce}$ the ions tend to come out further than the electrons. In order to overcome the presence of large polarization fields, the radius of the guiding centres of the electron fluid, r_{pe} , has to be somewhat larger than the radius of the ion fluid.

$$r_{pe} < r_{pe} \leq r_{pe} + r_{ci} \quad (7)$$

The angular momentum of the ion fluid is much larger than that of the electron fluid. From equation (2):

$$L = L_i - L_e = L_i = \pi m \frac{kT}{\omega_{ci}} r_{pe}^2 \quad (8)$$

The plasma rotates in the - direction

The magnetic moments add together:

$$M = M_i + M_e = \pi m \frac{kT}{B} r_{pe}^2 \quad (9)$$

points in the direction of \underline{L} , their relation being:

$$\underline{M} = \frac{e}{mc} \underline{L} \quad (10)$$

a factor two different from the single particle case. Relation (10) indicates that a plasma should show a kind of Einstein - de Haas effect.

3.2. Plasma with ∇M

The density gradients of the electron fluid and the ion fluid must be the same, so that according to equation (5a) the azimuthal velocity of the electron fluid is equal and in opposite direction to the azimuthal velocity of the ions

$$v_{L0e} = -v_{L0i}$$

$$v_{L0i} = \frac{r_{ci}}{q} v_{it} = \frac{r_{ce}}{q} v_{et} = \frac{ckT}{eBq} \quad (11)$$

q is not a constant all over the radius and there is a shear in the angular frequency $\omega_{Li} = v_{L0i}/r$.

3.3. Plasma with ∇M and E

E fields are generally forced into the plasma from the outside and are closely related to its generation. The plasma can shield itself easily from E fields, but will except them to a certain extent if necessary for its existence.*) Electric equipotential planes are determined by the potential distribution at the chamber walls and by sheaths between the plasma and the walls. Most fully ionized stationary plasmas in cylindrical geometry are generated by an electron beam in at least some part of the core region. In experiments made under the conditions $r_{ex}/r_{ci} \gg 1$ and $\omega_{ci}\tau_i \gg 1$, the radial electric fields have the largest magnitude in the core region (radius r_c) and are negligible between the core and the plasma boundary. Apparently a space charge is generated in the centre of the plasma by a slight excess of electrons ($\delta m^+ \approx kT/e^2 r_c$). The core appears as a singular region where the centrifugal forces, the Coriolis forces, the electric forces and the Lorentz forces all have the same order of magnitude. Theoretical investigation of this region is very difficult and experimentally it is not very accessible

*) In gas discharge physics it is quite common to think about sheaths with potential drops where electrons are accelerated which are necessary for ionisation. In such qualitative descriptions, which are given by lack of quantitative explanations, a kind of vitality is attributed to the plasma.

for diagnostic measurements (small dimension, small Doppler shifts, melting of probes).

At larger radii, outside the centre region, the situation is much simpler. The electric field is practically zero and the plasma behaves as described in the previous section ($\nabla M \neq 0$). One might call this the true plasma region. There is not much reason why inside a collisionless plasma static E fields should be present. The dielectric constant $\epsilon \approx c^2/v_A^2 \gg 1$ and it is also easy to demonstrate experimentally that it is impossible to introduce a transverse electric field into a magnetized plasma.

3.4. The collisionless model

In the true plasma region ($r_a \lesssim r \lesssim r_{pe} - r_{ci}$) the electric field is negligible small ($E \approx 0$). A density gradient is present ($\nabla M \neq 0$) with an e-folding length $q \gtrsim r_{ci}$. The situation is simply sketched in Fig. 3

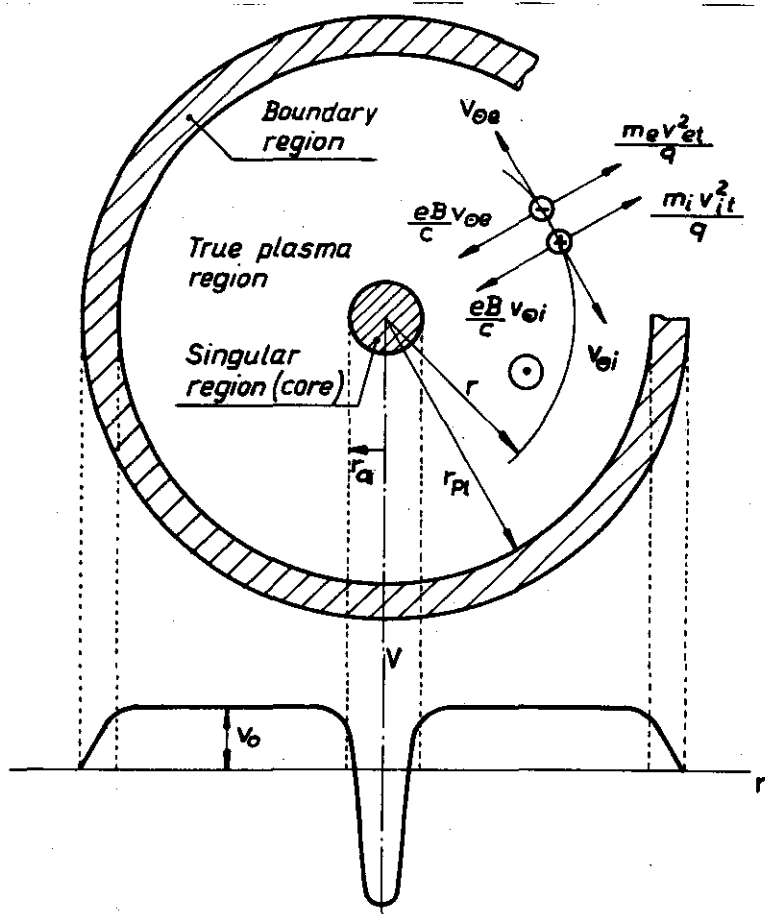


Fig. 3: The collisionless model with the three regions of plasma. A singular region is found in many plasma experiments (particularly in cylindrical geometry) but it has not to be necessarily present.

$$V_{L0i} = V_{L0e} = \frac{cKT}{eBq}$$

q is generally a function of radius and so are V_{L0i} and $\omega_{Li} = V_{L0i}/r$ (both pointing in the - direction). The direction of the angular momentum is determined by the ions which represent the mass of the plasma. In the presence of an electric field which points to the centre, the ions drift in the + direction and the total angular (mass) frequency $\omega = \omega_{Li} - \omega_{Ei}$. In the true plasma region ω_{Ei} is negligible and $\omega \approx \omega_{Li}$. So we find from $m v_{it}^2 / q = (e/c) B V_{L0i}$ with $1/q = -|d \ln n / dr|$ and $V_{L0i} = \omega r$ ($n_0 =$ particle density in the centre):

$$n = n_0 e^{-\frac{\omega_{ci}}{v_{it}^2} \int_0^r r \omega dr} \quad (12)$$

Experimentally it is found that in a small region ($\Delta r \approx r_{ci}$) directly near the core, the rotation is vortex-like, whereas at some ion gyro radii away from the centre the rotation is more nearly uniform over a large part of the plasma radius. From (12) one finds for the region of uniform rotation ($\omega \approx \text{const.}$):

$$n \approx e^{-\frac{1}{2} \frac{\omega_{ci} \omega}{v_{it}^2} r^2} \quad (13)$$

or

$$n \approx e^{-r^2/q^2} \quad \text{with } q = (2/\omega \omega_{ci})^{1/2} v_{it} \quad (13a)$$

At a larger distance from the centre $\omega \rightarrow 0$. For $\omega \approx r^{-1}$ the density is found to vary as $n \approx \exp -\alpha r$.

The density profiles derived from equation (12) are in good agreement with the experiments. The equation is not valid in the direct neighbourhood of the core, where large electric fields may be present.

A density profile like given by equation (13) is also found from a stationary solution of the Vlasov-equation (see e.g. SCHINDLER (1962)). Basically this direct solution is similar to the collisionless model presented here in case of uniform rotation. The electron fluid and the ion fluid rotate as solid bodies in opposite directions. Generally, however, the rotation is non uniform (q varies with radius) and then this special solution of the Vlasov-equation should be replaced by a more general one.

In plasmas where τ_e/τ_i is not $\gg 1$ and $\omega_{ci}\tau_i$ is not $\gg 1$ (low B fields), the external electric fields may penetrate also the plasma outside the core, which cannot be regarded any more as a true plasma. The plasma may rotate even opposite to $v_{L\theta i}$ ($\omega_{Ei} > \omega_{Li}$). In these (ion) magnetron-like situations the Coriolis force may not be neglected and may become even larger than the centrifugal force. We will not go into further detail in the behaviour of these kind of plasmas. They may be regarded as a transition to the case where $B = 0$.

3.5. A regression to the "classical" model

The plasma does not rotate $m_i v_{\theta i} = m_e v_{\theta e}$, so that $v_{\theta e} \gg v_{\theta i}$. The force equation for the electrons is:

$$-\frac{e}{c} B v_{\theta e} = -e E_r - \frac{\nabla p_e}{m}$$

For the ions:

$$0 = e E_r - \frac{\nabla p_i}{m}$$

The electrons carry alone the diamagnetic current $j = m_e v_{\theta e}$. The Lorentz force $(1/c) \underline{j} \times \underline{B}$ does not compensate only for the

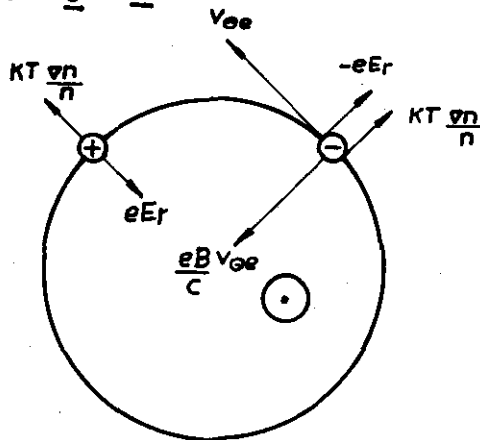


Fig. 4: The "classical" model

"pressure" of the electron gas but also for the force of a radial electric field. This field which confines the ions is called "ambipolar" field. (As a matter of fact a bad name, as it relates to ambipolar diffusion. Diffusion is a collisional phenomena, but this field must also be present in the collisionless case).

Addition of the equations yield $(1/c) \underline{j} \times \underline{B} = \nabla (p_i + p_e)$
 and subtraction gives $(1/c) \underline{v}_{\theta e} B = 2 E_A$ with
 $e E_A = \nabla p_e / m = \nabla p_i / m$ and $v_{\theta e} = 2 r_{ce} / q v_{et} = 2 c kT / e B q$
 Compare this expression and $v_{\theta i} = 0$ with equation (11).

If this model is used - as is often done - to describe the behaviour of the plasma in the laboratory system, it leads to wrong conclusions, e.g. the angular momentum of the plasma is missing. It describes the behaviour of the plasma in a coordinate system moving with the mass velocity $\underline{v}_{L\theta i}$. This does not make much sense from the theoretical side, because $\underline{v}_{L\theta i}$ is generally a function of radius and matters get unnecessary complicated, and neither from the experimental side because the measurements are made in the laboratory system.

The use of a moving coordinate system leads to an overestimate of the role played by electric fields in stationary state plasmas. As mentioned before the electric field is negligible in the laboratory system and there is no reason to expect static electric fields to play an important role inside a collisionless plasma with a very high dielectric constant. The so called "generalized Ohm's law" is not of much use either. Infrequent collisions ($\omega_c \tau_i \gg 1$) lead to resistivity and diffusion of plasma particles across the magnetic field. But the collision terms generally have no simple meaning (see section 3.8). All of this makes the usual presentation of the Two Fluids model of a plasma inadequate for the description of quasi-stationary state phenomena in fully ionized and magnetized plasmas.

3.6. Plasma with collisions

Collisions make the particles moving radially through the magnetic field. This particle transport is called diffusion through the magnetic field. The collisions represent a friction force $m \underline{v}_{\theta} / \tau$ oppositely to the azimuthal velocity of the particles. The equations

of motion in the \ominus direction are:

$$\frac{eB}{c} v_{zi} = \frac{m_i v_{\theta i}}{\tau_i}, \text{ or } v_{zi} = \frac{v_{\theta i}}{\omega_{ci} \tau_i} \quad \text{for ions} \quad (14)$$

$$\frac{eB}{c} v_{ze} = \frac{m_e v_{\theta e}}{\tau_e}, \text{ or } v_{ze} = \frac{v_{\theta e}}{\omega_{ce} \tau_e} \quad \text{for electrons} \quad (14a)$$

Depending on what value is taken for τ_i we can distinguish different cases.

3.6.1. Diffusion determined by ion-electron collision ($\tau_i = \tau_{i,e}; \tau_e = \tau_{e,i}$)

This is the simplest case, which occurs if we take for τ_i the collision time of the ions with the electrons, $\tau_i = \tau_{i,e}$ and for τ_e the collision time of the electrons with the ions $\tau_e = \tau_{e,i}$. As

$\tau_{i,e} = m_i/m_e \tau_{e,i}$ we find with the aid of equation (11):
an ambipolar diffusion:

$$v_{zi} = v_{ze} = \frac{\tau_{ci}^2}{\tau_{i,e} q} = \frac{\tau_{ce}^2}{\tau_{e,i} q} \quad (15)$$

Describing the motion with Fick's law: $n v_z = -D_{\perp} \nabla n$ we find for the "perpendicular diffusion coefficient":

$$D_{i\perp} = D_{e\perp} = \frac{\tau_{ci}^2}{\tau_{i,e}} = \frac{\tau_{ce}^2}{\tau_{e,i}} \quad (15a)$$

This is the same value as in the "classical" case, but note that now the diffusion takes place without an "ambipolar" electric field.

3.6.2. Diffusion determined by ion-ion collisions ($\tau_i = \tau_{i,eff}$)

In section 2.5 we saw that ions may move through the magnetic field by ion-ion collisions with a velocity: $v_{zi} = 3 \left(\tau / \tau_{i,i} \right) \left(\tau_{ci} / q \right)^4$
In order to make an estimate of q , we use equation (13a):

$$\left(q / \tau_{ci} \right)^2 = 2 \left(\omega_{ci} / \omega \right) \quad \text{and find:}$$

$$v_{zi} \approx \frac{3}{4} \frac{\tau}{\tau_{i,i}} \left(\frac{\omega}{\omega_{ci}} \right)^2 \quad (16)$$

or

$$v_{zi} = \frac{3}{4} \frac{v_{\theta i}}{\omega_{ci} \tau_{i,i}} \frac{\omega}{\omega_{ci}} \quad (16a)$$

Relation (14) may be maintained if formally an "effective ion collision time" is introduced:

$$\tau_{i,eff} = \left(\frac{4}{3} \frac{\omega_{ci}}{\omega} \right) \tau_{i,i} \quad (16b)$$

As $\tau_{i,i} = (m_e/m_i)^{1/2} \tau_{i,e}$, the transport because of the ion-ion collisions is about $(\omega/\omega_{ci})(m_i/m_e)^{1/2}$ times faster than the diffusion determined by ion-electron collisions.

In conclusion it has to be stated that the value taken for in equation (16b) is an approximate value, to be used only in a certain region of the plasma, where the density profile is given by equation (13a). By no means equation (16) should be considered as the only possible transport velocity caused by ion-ion collisions. Also it is not impossible that other mechanisms as ion-ion collisions or ion-electron collisions make the ions move through the magnetic field. In these cases another value for $\tau_{i,eff}$ as given in equation (16b) will be found. But in any case the inequality (21) must be satisfied (see section 3.7).

3.7. Anomalous diffusion of the electrons

In case the radial transport of the ions is larger than caused by ion-electrons collisions the question arises how the electrons may follow the ions. For this two mechanisms have been proposed: "Drain diffusion" of the electrons by BOHM (1949) and "Short circuiting" of the electrons by SIMON (1955a). In Fig.5 both mechanisms are sketched schematically.

Suppose an ion moves from place 1 to place 2. In the case of Bohm diffusion one electron follows by drifting in an azimuthal a.c. electric field $E_{\theta} \sim$ (frequency $\Omega \leq \omega_{ci}$). In the case of Simon diffusion one electron follows by going to the endplate, shortcircuiting and going back in the plasma to reach the ion at 2. We consider the latter case first. A simple consideration (BOESCHOTEN, 1967) shows rigorously that for every ion reaching the wall, one electron flows to the end plate. This must lead to a radial ion current: $j_r = ne v_{A_i}$ which is impossibly high, together with a huge torque K_{θ} which would continuously accelerate

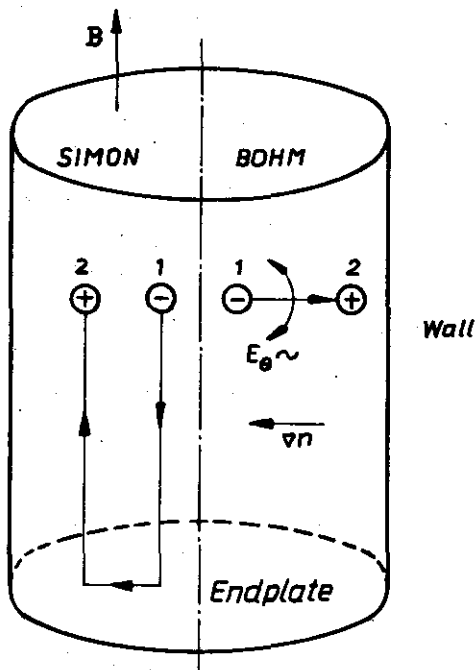


Fig. 5: Anomalous diffusion of electrons through the magnetic field

the plasma. Apart from this theoretical objection, the short circuit mechanism is also highly improbable from a physical point of view, particularly if the end plates are non conducting. These arguments makes us to disregard the "short circuit" model as a possible mechanism for anomalous diffusion of electrons through the magnetic field.

The "drain" mechanism on the other hand gives a quite natural explanation for the following up of the electrons. The outward motion of the ions cause the excitation of driftwaves in the plasma, with an oscillating azimuthal electric field $E_{\theta} \sim$. The frequency of the drift oscillations is approximately given by $\Omega = (k r_{ci})(r_{ci}/q)\omega_{ci}$ where the wave number $k \geq 1/q$. Such oscillations are found in numerous plasma experiments and as in all these experiments $q \geq r_{ci}$ $\Omega \lesssim \omega_{ci}$. (Note that Ω is not proportional to B). As the frequency Ω is close to ω_{ci} , inertia effects of the ions make that practically only the electrons move in the oscillating drift field. The density gradient makes that on the average more electrons are moving outward than inward.

The drain diffusion coefficient is easily derived if one simplifies the calculation by assuming a one way drain motion of the electrons. Electrons that move to a region of lower density do not return, particularly not because the collisional moved ions prevent them in doing so. (In fact the rigorous calculation has to be made on a statistical basis). The drift velocity of the electrons is given by:

$$V_{D\sim} = c \frac{E_{\sim}}{B} = c \frac{E_{\theta\theta}}{B} \sin \Omega t$$

The distance over which they move during one oscillation (assuming that they do not return because of ∇n):

$$s_0 = \int_0^{\pi/\Omega} V_{D\sim} dt = \frac{2cE_{\theta\theta}}{\Omega B} \quad (17)$$

$E_{\theta\theta}$ may be estimated by assuming that a fraction γ of the kinetic energy of the electrons is invested in these waves:

$$\gamma kT = \int_0^{s_0} e E_{\sim} ds = \frac{\pi e c}{\Omega B} (E_{\theta\theta})^2 = \frac{\pi}{2} e E_{\theta\theta} s_0$$

$$(E_{\theta\theta})^2 = \gamma \frac{\Omega B kT}{\pi e c} \quad (18)$$

For the "diffusion" coefficient we find with $\tau_e \approx \pi/\Omega$

$$D_{e1} = \frac{1}{3} \frac{s_0^2}{\tau_e} = \frac{1}{3} s_0^2 \frac{\Omega}{\pi} = \frac{4}{3\pi} c^2 \frac{(E_{\theta\theta})^2}{B^2} \quad \text{so that}$$

$$D_{e1}(\text{drain}) = \frac{4}{3\pi^2} \gamma \frac{c kT}{e B} \quad (19)$$

independent on Ω ! γ is not a fixed quantity - it adjusts itself so that the electrons can move with the same speed through the magnetic field as the ions. It is a process of adjustment of the electrons to the radial movement of the ions. Without such an "adjustment" a Two Fluid system could not exist. Bohm took $\gamma = 1/2$ (half of the thermal energy of the electrons available to the drift waves) and stated: "The exact value of D_{e1} is uncertain within a

factor 2 or 3". This gave the impression that γ has a fixed value and could be higher than 1/2, a cause of much confusion with later authors, who always conscientiously quoted the factor 1/16. The misunderstanding was even worse where $D_{e\perp}$ was taken as the diffusion coefficient of the ions. It is just the other way around - the electron drain depends on the diffusion rate of the ions. The ions are the "first movers".

Formula (19) gives a radial transport velocity for the electrons:

$V_e = (2\gamma/15) c kT / eBq$. Comparing with equation (14a) we can introduce formally an "effective electron collision" time:

$$\tau_{e\text{eff}} = \frac{15}{2\gamma} \frac{1}{\omega_{ce}} \quad (19b)$$

From this section and the previous one it follows that the plasma diffusion is in any case ambipolar $V_{ie} = V_{ii}$, with

V_{ii} indicative and generally determined by ion-ion collisions. Both transport velocities are limited. V_{ii} by the fact that $\omega < \omega_{ci}$, thus:

$$V_{ii}(\text{max}) \lesssim \frac{3}{4} \frac{v_{oi}}{\omega_{ci} \tau_{i,i}} = \frac{3}{4} \frac{1}{\omega_{ci} \tau_{i,i}} \frac{c kT_i}{eBq} \quad (20)$$

$V_{ie} = (\omega / \omega_{ci}) V_{ii}(\text{max})$ and as generally $(\omega / \omega_{ci}) \ll 1$ the limiting value of equation (20) will be seldom reached. The value of γ cannot surpass the factor 1/2 so that $V_{ie}(\text{max})$ is given by the Bohm value:

$$V_{ie}(\text{max}) = \frac{D_{e\perp}(\text{drain})}{q} = \frac{1}{16} \frac{c kT_e}{eBq} \quad (20a)$$

In any case the inequality:

$$V_{ii} \lesssim V_{ie}(\text{max}) \quad (21)$$

should hold, as the electrons cannot follow faster the ions than with the transport velocity $V_{ie}(\text{max})$. Comparing V_{ii} with $V_{ie}(\text{max})$ we find:

$$\frac{V_{ii}}{V_{ie}(\text{max})} = \frac{6}{\gamma} \frac{kT_i}{kT_e} \left(\frac{\omega}{\omega_{ci}} \right) \frac{1}{\omega_{ci} \tau_{i,i}} \quad \gamma = 1/2 \quad (22)$$

*) TAYLOR (1961) obtained from stochastic considerations a similar upper limit for the ion-diffusion rate.

Looking at equation (16) one could think about the possibility of bringing down the radial transport of the ions (and thus also of the electrons) by making ω small. In case $\omega = 0$, only the diffusion determined by ion-electron collisions (equation 15) should remain. It seems difficult to prevent a mass rotation of the plasma and moreover in the limit $\omega = 0$ the formula (16) is not valid any more, as it is based on a density profile as given in equation (13a). Nevertheless it does not seem impossible that something could be achieved by a proper generation of the plasma (see section 5.9).

As $V_{\perp i}$ varies as $(\omega / \omega_{ci})^2$, another way of bringing $V_{\perp i}$ down is by increasing the magnetic field. It is confirmed experimentally that (ω / ω_{ci}) varies proportional to B^{-1} , provided that $\omega_{ci} \tau_{ii} \gg 1$ (see section 3.10). This condition, which was assumed to be fulfilled throughout the preceding theoretical calculations, is not so easy to fulfill experimentally. There are clear indications that $V_{\perp i}$ decreases strongly with B if it is. More efforts along this line should be done.

Before concluding this section it should be stressed that the presence of the drain diffusion mechanism simplifies the description of the plasma considerably. In cases where $L / \lambda_i \gg 1$ one may neglect end effects and consider the plasma as two dimensional. This is the justification for the considerations in the previous sections.

3.8. Relation to the MHD theory

The complete equation of motion for the ions is:

$$m_i \frac{d\mathbf{v}_i}{dt} = e \mathbf{E} + \frac{e}{c} \mathbf{v}_i \times \mathbf{B} - \frac{\nabla P_i}{n} - \frac{m_i \mathbf{v}_i}{\tau_i} \quad (23)$$

For the electrons a similar equation can be written. In the preceding sections it is shown, however, that no simple meaning can be ascribed to the collision times τ_i and τ_e and that the radial plasma transport may take place in a more complicated and faster way than would simply follow from $\tau_i = \tau_{i,e}$ and $\tau_e = \tau_{e,i}$. The best way to describe the behaviour of a magnetized plasma in the quasi-stationary state ($\omega < \omega_{ci}$) is to use the equation of motion for the ions alone. The electrons are not able to effect the ions motion, neither in

parallel nor in perpendicular direction to the magnetic field. They follow the ions and the mass velocity of the plasma is exclusively determined by the velocity of the ions, $\underline{v} = \underline{v}_i$ *)

In the collisionless case, equation (23) reduces in the true plasma region (E negligible) to:

$$\rho \frac{d\underline{v}}{dt} = \frac{\underline{j}}{c} \times \underline{B} - \nabla p_i \quad (23b)$$

Addition of the equality $0 = \frac{\underline{j}_e}{c} \times \underline{B} - \nabla p_e$ found in section 3.4 yields the usual MHD equation of motion.

There is no such an equation as the "generalized Ohm's law". The second MHD equation $\underline{j} = \sigma (\underline{E} + \frac{\underline{v}}{c} \times \underline{B})$ is only valid for the case $\omega_{ce} \tau_e \ll 1$, a fact that is very often overlooked, e.g. in the confidence on the validity of Ferraro's law. The MHD model should not be applied to dilute fully magnetized plasmas.

For the study of rotational phenomena it is useful to take the curl of equation (23). With $\underline{\omega} = \nabla \times \underline{v}$:

$$\frac{\partial \underline{\omega}}{\partial t} + \nabla \times \left(\frac{\underline{v}}{\tau_i} \right) = \nabla \times \left\{ \underline{v} \times \left(\frac{e \underline{B}}{m_i c} + \underline{\omega} \right) \right\} \quad (24)$$

Equilibrium configurations of a collisionless plasma may be found by equating the right hand side of (24) to zero. As the plasma obtains at its generation inevitably a rotational velocity, this seems a better starting point for finding equilibria than the corresponding equation $\underline{j} \times \underline{B} = \nabla p$.

As τ_i varies inverse proportional to the density, the second term on the left hand side is found to be: $\nabla \times \left(\frac{\underline{v}}{\tau_i} \right) = \left\{ \underline{\omega} - \left(\frac{\underline{e}_z}{q} \right) \times \underline{v} \right\} / \tau_i$ where \underline{e}_z is the unit vector in the z direction. $\underline{\omega}$ and $\left(\frac{\underline{e}_z}{q} \right) \times \underline{v}$ are of the same magnitude and oppositely directed, so that they cancel

*) For the description of non stationary phenomena with $\Omega \gg \omega_{ce}$ the predominant role is played by the electrons and the equation of motion of the electrons is the most important one. This is the reason that wave phenomena in plasmas are described by the usual Two Fluid model much more successfully than quasi-stationary phenomena.

each other partly. These terms determine the collisional damping of the rotation and are related to the diffusion of the ions.

3.9. Conservation of angular momentum

If we think about a plasma in a magnetic field as an assembly of gyrating particles with a total angular momentum $L = \pi n r_{pe}^2 kT / \omega_{ci}$ (8), we still have to answer the question: "where is this momentum coming from?" It has to be created by some reaction - in laboratory plasmas apparently in reaction with the wall of the apparatus.

Generally the plasma is created by electrons which leave an electrode or the wall and which are then accelerated by an externally applied electric field. During the time that an electron leaves the wall, the magnetic field exerts a force on it in the - direction. The wall (or electrode) receives a reaction in the + direction. If the electron leaves the wall with its thermal velocity ^{*}, the impulse transfer is $\Delta p = (v_{et} / \lambda \pi) m_e$ and the change in angular momentum is $\Delta p r_{pe}$. The number of cooperative particles at the wall is $\lambda \pi r_{pe}^2 n_{ce} m_e$, so that the total change in angular momentum is given by $\Delta L = m r_{pe}^2 n_{ce} m_e v_{et} = m r_{pe}^2 kT / \omega_{ce}$. The momentum transfer takes place during a time $\approx 1 / \omega_{ce}$ so that the torque on the plasma is $K_{\theta} \approx m r_{pe}^2 kT$. As shown by HAINES (1965) this torque works effectively during half an ion cyclotron period on the plasma, so that $L = K_{\theta} \pi / \omega_{ci} = \pi m r_{pe}^2 kT / \omega_{ci}$. The total angular momentum (8) may indeed be obtained in this way. The torque $K_{\theta} \approx m r_{pe}^2 kT$ may be compared with the torque of the same magnitude due to the Bohm electron diffusion "current" (BOESCHOTEN, 1967). This raises the surmise that the drain diffusion mechanism is connected to the transfer of angular momentum from the wall to the plasma. It seems that the drain mechanism is at the same time operative at the plasma generation - providing its angular momentum - and at the plasma loss - diffusion through the magnetic field.

Granted this all is very speculative, but one should not forget that the whole problem of plasma contact with the wall is a very mysterious one, where still much has to be learned.

*) Generally the velocity with which the electron leaves the wall, will be lower and it is accelerated in an electric field which prevails at the plasma boundary. This does not change the qualitative picture, however.

3.10. Experimental evidence

It is not so easy (or better inexpensive) to create a fully ionized and magnetized ($\omega_{ci}\tau_i \gg 1$) plasma with $L/\lambda_i \gg 1$ and $\tau_{pe}/\tau_{ci} \gg 1$. One of the best ways to come to it was found by LUCE (1958) in a powerful Hollow Cathode Discharge. With such a device we made some measurements (BOESCHOTEN and DEMETER, 1968) to which we will refer for illustration of the foregoing theory.

The arc had a length of about 1 m ($L/\lambda_i \approx 50$); the magnetic field could be varied from 500 - 8000 G. At 3000 G, $\tau_{ci} \approx 1$ cm. The core of the arc was about 2 cm in diameter and $\tau_{pe} \approx 8 \tau_{ci}$ with $q \approx \tau_{ci}$. The arc was fed with different gases, but most experiments were done with Argon gas, for which was found $n \approx 1.2 \times 10^{14} \text{ cm}^{-3}$, $\kappa T_i \approx 10 \text{ eV}$ and $\kappa T_e \approx 3 \text{ eV}$.

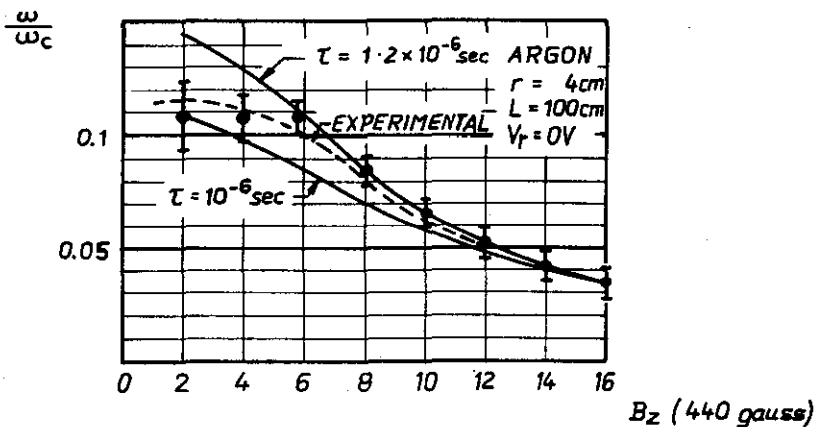


Fig. 6: Comparison between experimental (--) and theoretical (-) values for Argon arc

The angular frequency of the mass rotation was measured as a function of B - the results are reproduced in Fig. 6. A good theoretical explanation of the experimental results was obtained with a Single Fluid model for the ions. This picture refers to Argon, but the same good agreement was found with other gases. It may be seen how important it is to fulfill the condition $\omega_{ci}\tau_i \gg 1$.

The ratio ω/ω_{ci} does not decrease with B until $\omega_{ci}\tau_{ei} \geq 1$ *)
 The particle density increases strongly if this value is surpassed.

Oscillations of a frequency $\Omega \leq \omega_{ci}$ were found in agreement with what is expected for drift waves in a plasma with $q \approx r_{ci}$. These oscillations were coherent along the plasma column and the amplitude of the oscillating electric field indicated that the energy in the oscillations was of the same order of magnitude as the electron temperature. Thus $\gamma \approx 1/2$ (18) and the transport of the electrons takes place at maximum velocity (20b). This was found also from the particle balance. The agreement with (22) is also good. The low value of $V_A \leq 2 \times 10^3$ cm/sec was not understood at that time, as from equation (14) with $\omega_{ci}\tau_{ei} \approx 1$ it is expected that $V_{A1} \approx V_{e1} \approx 1,5 \times 10^5$ cm/sec. The larger part (about a factor 25) of the missing factor is now explained by equation (16b). The remaining part of the factor may be due to the fact that the factor 3/8 in equation (6) is somewhat too low (LONGMIRE and ROSENBLUTH (1956) found theoretically a 1.4 times larger like particle diffusion than Simon.)

Finally we refer to a rotating plasma experiment (ALDRIDGE and KEEN, 1970) with a hollow cathode discharge where the ion temperature is relatively low ($kT_i \leq 0.5$ eV) and the electric field high (E several V/cm), so that the plasma rotation is in the + direction. Such a plasma is of interest for the study of the singular plasma region (see section 3.4.2).

Acknowledgement

I want to thank Dr. M. Weenink for critical reading of the manuscript and for the many clarifying discussions I had with him.

*) At that time we did not realize the importance of like particle collisions and we used for τ_{ei} the collision time of singly and doubly charged Argon ions, which are both found spectroscopically in the arc.

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