The inverted slide-crank used for the design of an approximated straight-line mechanism

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Evert A. Dijksman*)

The design of a centric inverted slider-crank producing a symmetrical curve touching a common tangent at three pairs (PP-PP-PP) of instantaneously near coupler-point positions, requires, apart from a scale-factor only one design-parameter ($\phi_0 = \sphericalangle K_1 B_0 K_3$). The mechanism may be seen as a degenerate λ-mechanism, the λ-mechanism being a 4-bar linkage (A_0A-K-BB_0) for which $\overline{AB} = \overline{B_0B} = \overline{KB}$. Then, if B goes to the infinite, the λ-mechanism turns into an inverted slider-crank.

Approximations of the straight-line by the coupler curve of these mechanisms are usually better than those obtained by the corresponding λ-mechanism they are derived from. A graph showing the double relative deviation ($\Delta h/L$) of the straight-line set out against the dimensionless length ($L/h_{\min}$) of the straight-line demonstrate this result. In practice, the graph may be used to determine the singular design-parameter (ϕ_0) from the desired length of the straight-line. In the special case for which $\phi_0 = 0$, the well-known result is recovered where the tracing-point happens to be Ball's point (Bl_2) with excess 2 (the curve then touches the common tangent at 6 infinitesimally near coupler point positions).

1. Introduction

Symmetrical curves produced by inverted slider-cranks may be approximated by a straight-line, see Fig. 1. This can be done by catching a stretch of the curve within a rectangular box of length L and width Δh. The middle of L then joins the symmetry-axis intersecting the rectangle and the curve at two infinitesimally near positions ($K_3 = K_4$) of the lower tangent. This tangent coincides with a common tangent for two other pairs of infinitesimally near positions of the tracing point. Thus, the 6 tracing- or coupler point positions $K_1 = K_2$, $K_3 = K_4$ and $K_5 = K_6$ all join the lower (common) tangent of the curve for which $K_1K_3 = K_4K_6$. The upper tangent of our rectangle touches the curve at two symmetrical but separate positions of K and intersects the curve at two other locations, leading to the length of the straight-line (note that both the lower- and the upper tangent intersect the curve at 6 intersections being the maximum number possible for the 6th order curve). We conclude that the design of the

*) Dr. *E.A. Dijksman*, Faculty of Mechanical Engineering, Eindhoven University of Technology Eindhoven, The Netherlands.

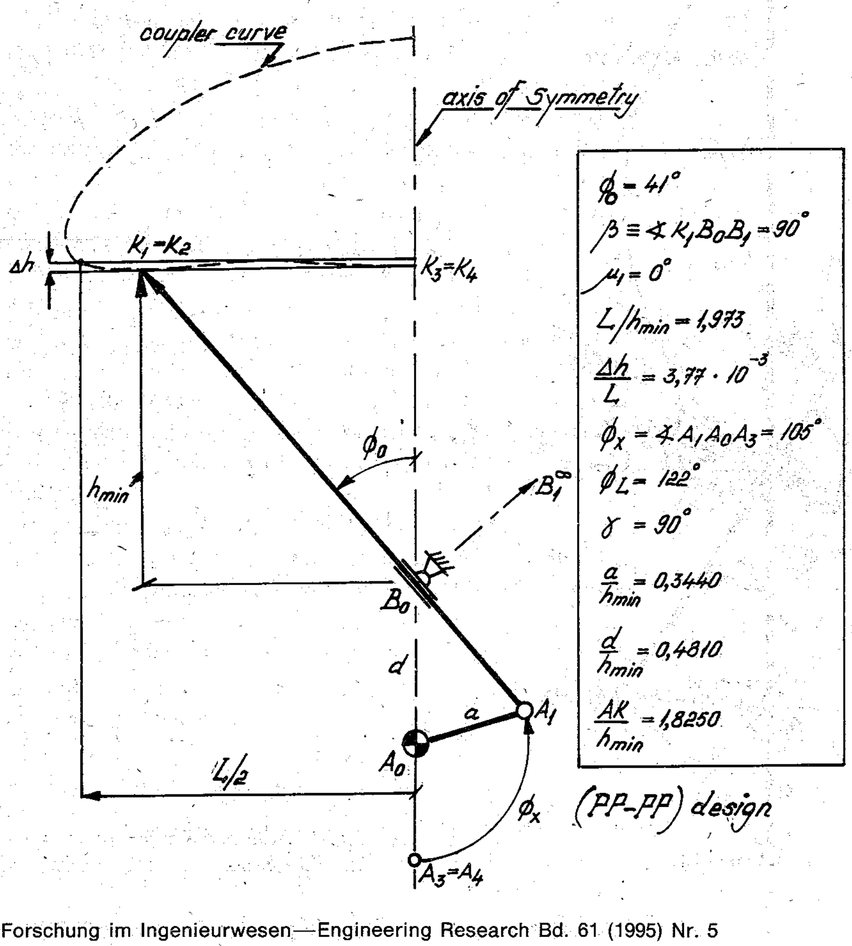

Fig. 1. Inverted slider-crank used as a straight-line mechanism (bordercase for λ-type crank-and-rocker mechanisms)

inverted slider-crank may be based on 3 pairs of infinitesimally near accuracy points K_i on the lower tangent, representing an application of multiply separated position (MSP)-theory, here being a mixture of separate and instantaneously near positions.

The symmetry of the curve is obtained by considering only centric inverted slider-cranks, that is to say by obliterating the two possible eccentricities in the mechanism. That leaves only 3 lengths, to wit $a = \overline{A_0A}$, $d = \overline{A_0B_0}$ and $l = \overline{AK}$. Of these, one is needed to have a common lower tangent. Thus, apart from a scaling-factor, only one design degree of freedom is left. For this, one takes the angle $\phi_0 = \sphericalangle K_1 B_0 K_3$.

2. Geometrical design

λ-type 4-bar linkages for which $\overline{KB} = \overline{B_0B} = \overline{AB}$, producing symmetrical 4-bar coupler curves, turn into centric inverted slider-crank mechanisms when joint B goes to infinity [5]. Thus, if $(A_0A\text{-}K\text{-}BB_0)$ represents the 4-bar and $B \Rightarrow B^{\infty}$, then the circle about B joining the points A, B_0 and K, merges into the slider KB_0 containing The crank-joint A, Fig. 2. Indeed, by turning B to infinity, the inverted slider-crank while being centric, will have a coupler point joining the slider. Thus, the derivation from the λ-mechanism guarantees an absence of eccentricities. This causes the symmetry-axis of the curve to joint the centres (A_0 and B_0) of the frame.

Because of the symmetry of the curve and of the mechanism not three but only two pairs of coincident positions are needed for its design. They correspond with the pairs $(K_1 = K_2)$ and $(K_3 = K_4)$.

The two times two positions define the 6 centres of rotation or rotation poles $P_{12}, P_{32} = P_{24} = P_{41} = P_{13}$ and P_{34}. From them only P_{12} and P_{34} represent (instantaneous) velocity poles. The remaining four are just (virtual) rotation poles all coinciding at one and the same location. The pole P_{12} lies at the intersection of the normals to the slider at B_0 and to the curve at $K_1 = K_2$. Similarly, the velocity pole P_{34} coincides with the frame center B_0.

The condition for the common (lower) tangent to touch the curve at the same height, being h_{min}, needs one design degree of freedom. Thus, if point S is defined as the intersection of the path-normal $P_{12}K_1$ with the diameter-circle about $\overline{B_0K_1}$, the condition is represented by the equation $\overline{SK_1} = (h_{min}) = \overline{B_0K_3}$. Hence, the mechanism is fully determined by the (scaling) length of h_{min} and by the independent parameter $\phi_0 = \sphericalangle K_1 B_0 K_3$.

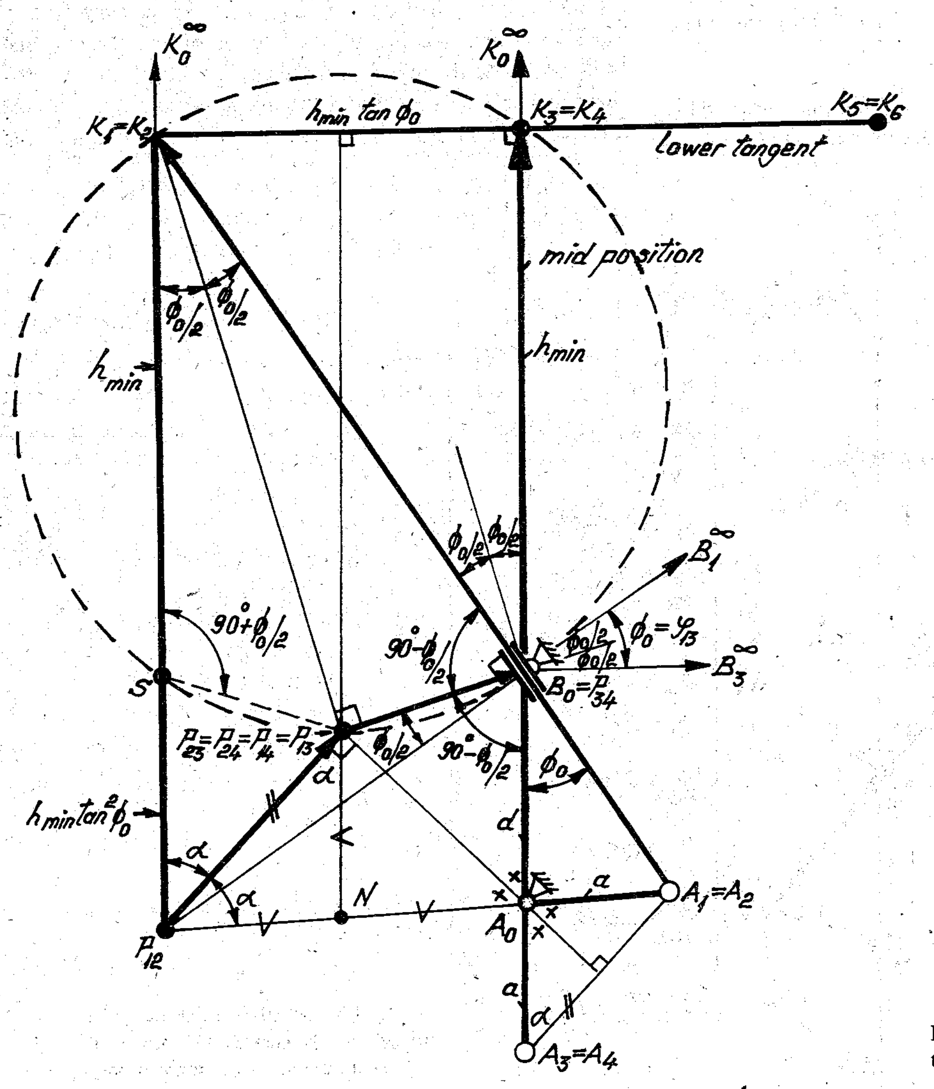

Fig. 2. Geometrical design of an inverted slider-crank

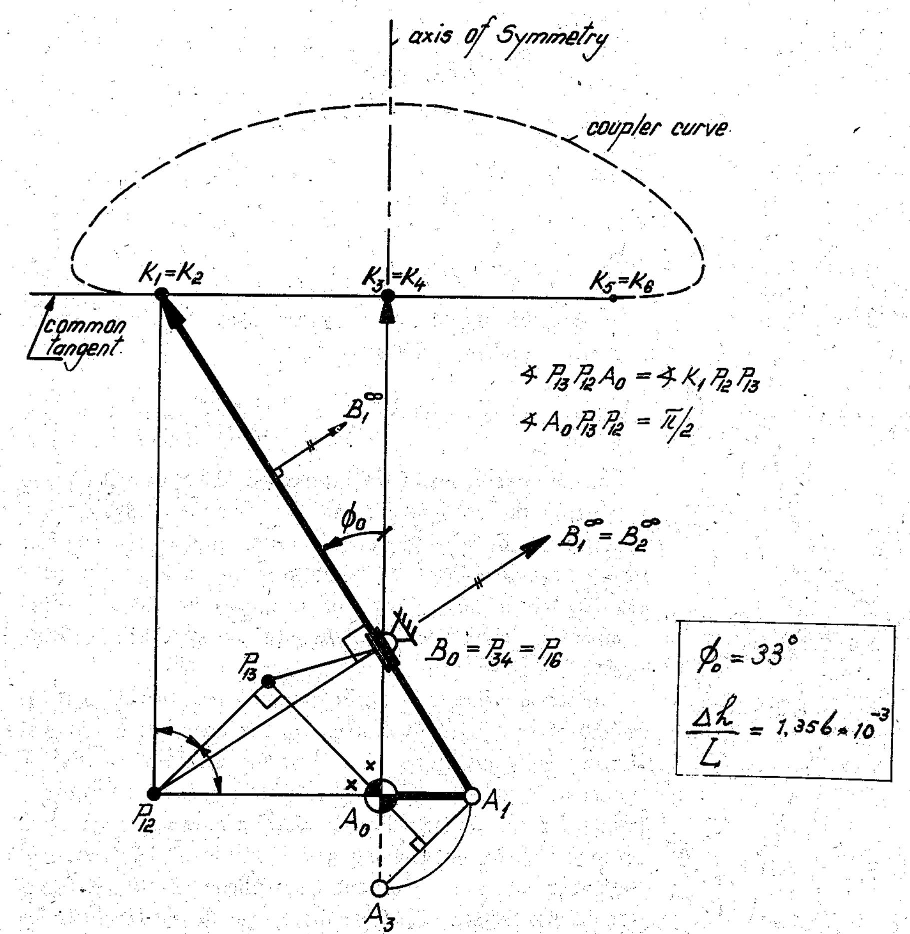

Fig. 3. Simplified design for an inverted slider-crank very neatly approximating a straight-line.

The rotation pole P_{13}, being the coincident pole of the remaining poles, joins the intersection of the midnormal of $\overline{K_1K_3}$ with the angle bisector of the $\sphericalangle B_1^\infty B_1 B_3^\infty$. Hence, $\sphericalangle P_{13}B_0K_1 = 180° - (\sphericalangle K_1B_0B_1^\infty + \frac{1}{2}\phi_0) = 90° - \frac{1}{2}\phi_0$, while the equality $\overline{SP_{13}} = \overline{P_{13}B_0}$ leads to the fact that $\sphericalangle B_0K_1P_{13} = \sphericalangle P_{13}K_1S = \frac{1}{2}\phi_0$, finally yielding to $\sphericalangle K_1P_{13}B_0 = 90°$. Thus, the rotation pole P_{13} joins the diameter-circle about $\overline{K_1B_0}$ containing the points K_1, K_3, B_0, P_{13} and S.

As further the points P_{12}, A_0 and A_1 all join the path-normal of the crankjoint A_1, while P_{13} meets the midnormal of $\overline{A_1A_3}$, the rotation-pole P_{13} will join the angle-bisectors through the vectices K_1, B_0, and A_0 of the quadrilateral $K_1P_{12}A_0B_0$. Whence P_{13} represents the center of a circle being inscribed in the quadrilateral $K_1P_{12}A_0B_0$, whereas the line $P_{12}P_{13}$ coincides with the 4th angle-bisector of that quadrilateral.

Hence, $\sphericalangle K_1P_{12}P_{13} = (\alpha) = \sphericalangle P_{13}P_{12}A_0$, Fig. 3.

The midnormal of $\overline{K_1K_3}$ intersects the path-normal $P_{12}A_0$ of A_1 at a point N for which $\overline{P_{13}N} = \overline{P_{12}N} = \overline{NA_0}$ giving $\sphericalangle A_0P_{13}P_{12} = 90°$.

This determines the exact location of the crank-center A_0 at the axis of symmetry from the already known locations of the poles P_{12} and P_{13}.

3. Computational design

Application of the Rule of Sine at $\Delta P_{12}P_{13}K_1$ results into a relation between the angle α and the design-parameter ϕ_0:

$$\frac{\sin\alpha}{\sin(\alpha + \frac{1}{2}\phi_0)} = \cos\phi_0 \cdot \cos(\tfrac{1}{2}\phi_0),$$

or

$$\cot\alpha = \frac{4}{\sin 2\phi_0} - \cot\tfrac{1}{2}\phi_0 = 2\tan\phi_0 - \tan\tfrac{1}{2}\phi_0 \qquad (1)$$

The Rule of Sine applied at $\Delta A_0A_1B_0$ leads to the formula

$$\frac{d}{a} = \frac{\sin(\phi_0 + 2\alpha)}{\sin\phi_0} \qquad (2)$$

in which the angle α may be established through Eq. (1)

Further,

$$\overline{A_0B_0} + \overline{B_0K_3} = \overline{P_{12}K_1} - \overline{K_1K_3} \cdot \tan(90° - 2\alpha)$$

or

$$d + h_{\min} = (h_{\min}/\cos^2\phi_0) - \tan\phi_0 \cdot \cot 2\alpha$$

leading to the expression

$$d/h_{\min} = \tfrac{1}{2} \cdot \tan\phi_0 \cdot \{\tan(\tfrac{1}{2}\phi_0) + \tan\alpha\} \qquad (3)$$

Division of the Eqs. (2) and (3) leads to

$$a/h_{\min} = \tfrac{1}{2} + \tfrac{1}{2} \cdot \tan\phi_0\{\tan(\tfrac{1}{2}\phi_0) - \tan\alpha\} \qquad (4)$$

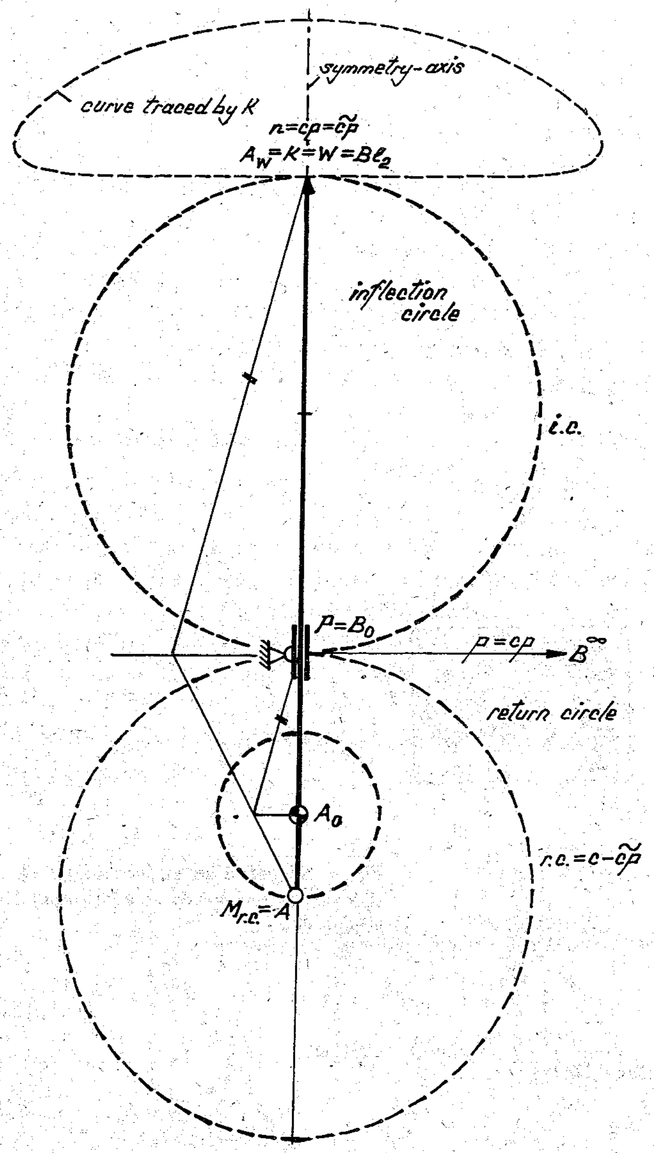

Fig. 4. Inverted slider-crank with a Ballpoint with excess 2

Clearly,

$$\overline{A_3A_0} + \overline{A_0B_0} + \overline{B_0K_3} = \overline{A_1K_1} = l$$

whence,

$$a + d + h_{\min} = l$$

and

$$l/h_{\min} = 1 + (a/h_{\min}) + (d/h_{\min}) \tag{5}$$

Substituting the obtained expressions (3) and (4) into the expression for $l/h_{\min}$ gives rise to

$$\frac{l}{h_{\min}} = \frac{3}{2} + \tan\phi_0 \cdot \tan\tfrac{1}{2}\phi_0 = \frac{3}{2} - \frac{2}{1 - \operatorname{cotan}^2(\frac{1}{2}(\phi_0))} \tag{6}$$

So, the dimensions of the inverted slider-crank mechanism producing the straight-line, are all to be established as a function of the scale-value $h_{\min} = B_0K_3$ and of the singular design-parameter ϕ_0, to be taken from the graph in which the relative width $\Delta h/h_{\min}$ of our rectangle has been plotted against the relative length $L/h_{\min}$ of the straight-line being approximated.

The straight-line to be approximated runs horizontal at the height of $(h_{\min} + \frac{1}{2}\Delta h)$ intersecting the curve at 6 separate coupler point positions (since first the locations of these 6 points were unknown, they have not been used as accuracy points for the design, though older programs based on a probing Tchebychev-spacing have done so, [1; 3]). The method developed here, represents a so-called (*PP-PP*) approach of a (over a distance of $\frac{1}{2}\Delta h$) shifted straight-line produced by the inverted slider-crank. Whence the maximum deviation from the straight-line equals $\frac{1}{2}\Delta h$ within the rectangular box the curve was captured in.

The crank-angle needed to reach position 1 from the mid-position 3 of the mechanism, meets the equation

$$\sphericalangle A_1A_0A_3 = 180° - 2\alpha \tag{7}$$

in which the angle α is to be calculated through Eq. (1)

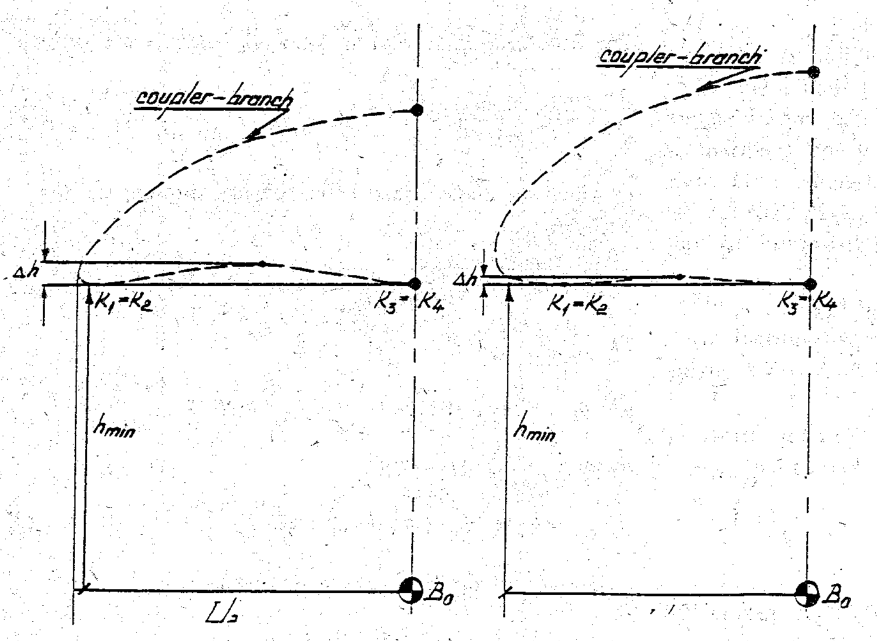

Fig. 5. Determination of the length *L* of the straight-line

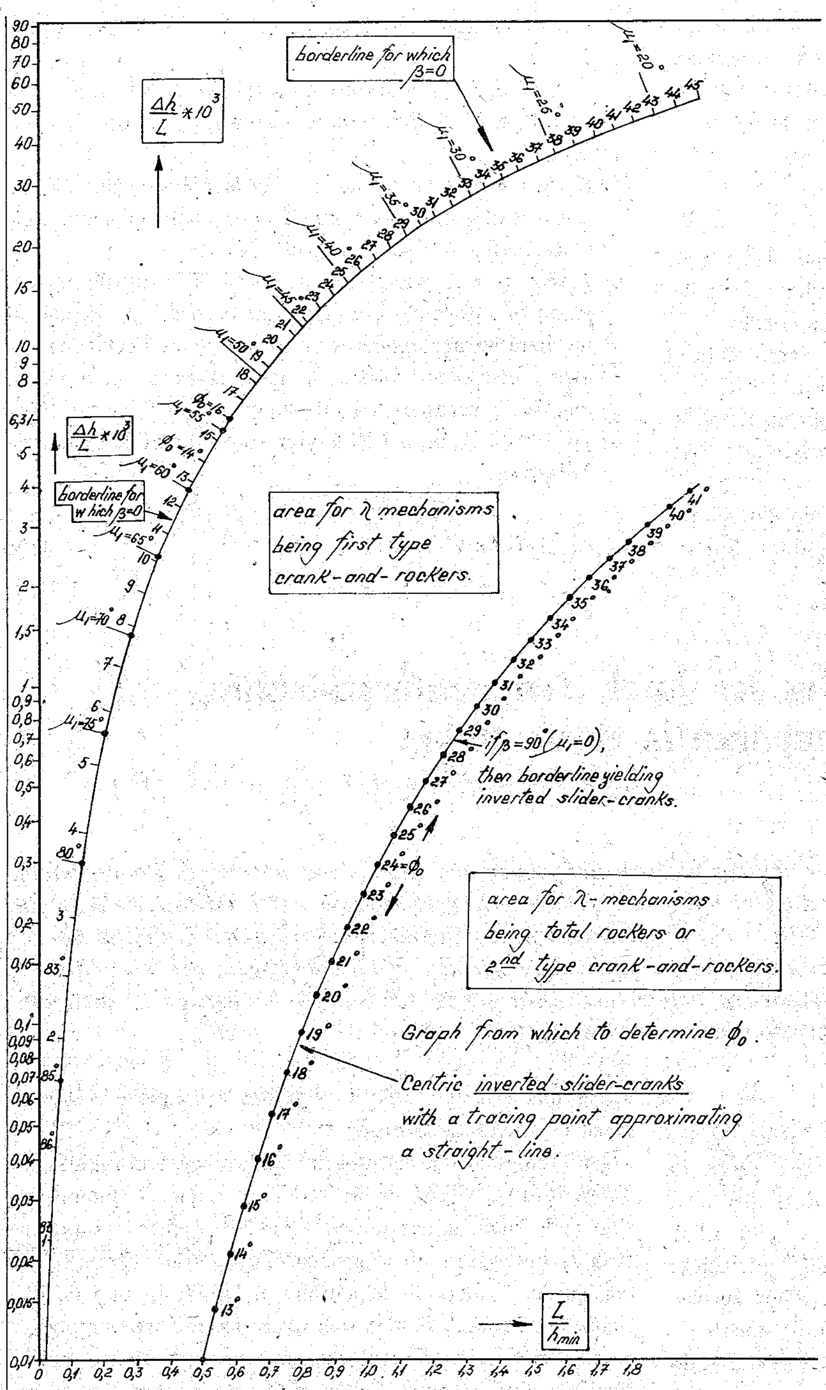

Fig. 6. Centric inverted slider-cranks with a tracing point approximating a straight-line

In the particular case for which $\phi_0 = 0$, the two design-positions merge into a singular position being the inverse of the elliptic position, sometimes named the cardioidal position [4]. Then, a border-mechanism results meeting the dimensions

$$d/h_{\min} = \tfrac{1}{3};$$

$$a/h_{\min} = \tfrac{1}{6}$$

and

$$l/h_{\min} = \tfrac{3}{2} \qquad (8)$$

Whence, $d/a = 2$ and $l/a = 9$, Fig. 4

Thus, when $\phi_0 = 0$, all 6 accuracy points coincide (i.e. $K_1 = K_2 = K_3 = K_4 = K_5 = K_6$), yielding a 6-point contact between lower tangent and coupler curve. The coupler point then represents Ball's point with excess 2.

Larger values for ϕ_0 however, always lead to longer stretches L of the straight-line that has to be approached. Of course, larger deviations of $\tfrac{1}{2}\Delta h$ have then to be taken into account. The designer may pick his choice and finding the corresponding value for ϕ_0 by observing the graph showing values of $10^3\ \Delta h/L$ as a function of $L/h_{\min}$, Fig. 6.

The length L of the straight-line has been calculated through the coupler curve equation and depends on if the curve has, or not has, a vertical tangent before leaving the area lying under the upper tangent, Fig. 5. In case the curve shows to have a vertical tangent within that area, the length L will be the horizontal distance between the two vertical tangents. In the other case, the length L represents the distance between the outermost intersections of the curve with the upper tangent.

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Näherungsweise Berechnung der durch Temperaturschichtung hervorgerufenen Axialspannungen in einem Rohr

Gerhard Sauer*)

Eine einfache Formel zur näherungsweisen Berechnung der Axialspannungen eines Rohres infolge der von einer Temperaturschichtung im Rohrinnern hervorgerufenen Temperaturfelder wird vorgestellt. Die Formel basiert auf der konventionellen Balkentheorie. Durch Vergleiche der von der Näherungsformel gelieferten Axialspannungen mit den Axialspannungen, die mit Hilfe von Finite-Elemente-Rechnungen erhalten werden, wird gezeigt, daß die Näherungsformel die maximalen Axialspannungen ausreichend genau wiedergibt.

1. Einleitung

In waagerecht verlegten Rohren füllt die Strömung bei einem schwachen Mediumdurchsatz nicht den gesamten Rohrquerschnitt aus: Nur ein Teil der Rohrinnenoberfläche wird benetzt. Wird durch das Rohr ein Fluid in einen Behälter gefördert, der bis oberhalb der Einmündung des Rohres gefüllt ist, kann vom Behälter Fluid in das Rohr zurückfließen. Das vom Behälter eindringende Fluid okkupiert den durch das geförderte Medium nicht beanspruchten Rohrquerschnitt. Unterscheiden sich die Temperaturen des geförderten und des im Behälter vorhandenen Mediums, bilden sich im Rohr waagerecht verlaufende Strömungsbänder unterschiedlicher Temperaturen. Dieses Phänomen wird als Temperaturschichtung bezeichnet. Ist die Temperatur des Behälterfluids höher als die des geförderten Fluids und wird aus dem Behälter die gleiche Fluidmenge abgeführt, die durch das Rohr in den Behälter transportiert wird, sind die Temperaturschichten örtlich und zeitlich ziemlich konstant. Nur an der Grenzfläche zwischen den Temperaturschichten vermischen sich beide Fluidströme. Solange sich der Förderstrom durch das Rohr nicht ändert, bleibt die Temperaturschichtung erhalten. Bild 1 zeigt das prinzipielle Aussehen einer Temperaturschichtung.

Die Temperaturschichtung im Rohrinneren beeinflußt die Temperaturverteilung in der Rohrwand. Die Temperaturen sind nicht mehr axialsymmetrisch wie bei einer Füllung des Rohres mit einem Fluid gleicher Temperatur. Sie variieren infolge der Temperaturschichtung in Umfangs- und Radialrichtung. Dadurch ändert sich auch der Beanspruchungszustand des Rohres. Die Rohraxialspannungen nehmen zu und überwiegen die anderen Spannungskomponenten. Der stationäre Beanspruchungszustand eines Rohres infolge Temperaturschichtung kann durch die Axialspannungen charakterisiert werden.

Der durch die Temperaturschichtung hervorgerufene Spannungszustand kann exakt nur numerisch, z.B. mit Hilfe einer Finite-Elemente-Rechnung, bestimmt werden. Die numerische Rechnung gliedert sich in zwei Stufen. Zuerst wird das stationäre Temperaturfeld in der Rohrwand berechnet. Danach werden die von diesem Temperaturfeld verursachten Spannungen ermittelt.

Näherungsweise können die Rohraxialspannungen jedoch auch analytisch berechnet werden. Voraussetzung für die analytische Berechnung ist lediglich die Vorgabe eines genügend einfachen Temperaturfeldes. Ein einfaches Temperaturfeld kann man sich aus der tatsächlichen Temperaturver-

*) Dipl.-Ing. *G. Sauer*, TÜV Bayern Sachsen, München.