On the critical radius in sheet bending

ON THE CRITICAL RADIUS IN SHEET BENDING

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ON THE CRITICAL RADIUS IN SHEET BENDING

by J.A.G. Kals and P.C. Veenstra

INTRODUCTION

Actually it is very difficult to find a suitable experimental deforming limit for sheet bending operations. In many cases literature only provides data for particular sheet materials and in general the criterion connected with the data is either not very well defined or not mentioned at all. Different values might be due to different criteria on which the experiments are based.

The deforming limit could be defined to be the amount of strain which starts a remarkable granulation in the outer surface of the bending zone. In another case a visible orientation of this granulation or the start of surface cracks might be more relevant. In general, however, it is very difficult to carry out experiments based on direct visual observation.

In this paper a simple theoretical model is proposed, which proved to be rather useful in a number of practical applications. It is based on the consideration that local strain effects like localized granulation, necking, cracks etc. in a more or less uniformly stressed area arise from strain peaks which must have been preceded by a local instability. Since plastic instability can be calculated from tensile test data, it is possible to derive a safe upper limit for the bending radius which can serve as a guideline for tool and product design.

LOCAL SURFACE INSTABILITY

Starting from the consideration that a peak strain will occur at the outer surface of the bending zone for homogeneous materials, we have to deal with a plane stress situation in a thin surface layer :

(1) $\sigma_2 = i\,\sigma_1 \qquad (\sigma_3 = 0)$,

where σ_1 is the bending stress. Then the stress-strain relations are

$$(2)\quad \begin{cases} d\varepsilon_1 = \frac{d\lambda}{2}\sigma_1\ (2-i) \\ d\varepsilon_2 = \frac{d\lambda}{2}\sigma_1\ (2i-1) \\ d\varepsilon_3 = -\frac{d\lambda}{2}\sigma_1\ (1+i) \end{cases}$$

Now, by substituting the strain increments given by Equations (2) into the general expression for the effective strain increment

$$d\bar{\varepsilon} = \sqrt{\frac{2}{3}\left(d\varepsilon_1{}^2 + d\varepsilon_2{}^2 + d\varepsilon_3{}^2\right)}\ ,$$

it is easily shown that

$$(3)\qquad d\bar{\varepsilon} = -\frac{2\ d\varepsilon_3}{1+i}\ I \qquad\qquad (d\varepsilon_3 \leqslant 0)$$

where

$$(4)\qquad I = \sqrt{i^2 - i + 1}$$

Consider a small (uniformly strained) surface element

$$A = x_1 x_2\ ,\quad \text{then}$$

$$dA = x_1 dx_2 + x_2 dx_1,\quad \text{and}$$

$$(5)\qquad \frac{dA}{A} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} = d\varepsilon_1 + d\varepsilon_2 = -\ d\varepsilon_3$$

when constancy of volume is assumed.

With Equation (5), Equation (3) can be written as follows :

$$(6) \qquad d\bar{\varepsilon} = \frac{2I}{1+i} \frac{dA}{A}$$

For a constant value of i we have a straight strain path and Equation (6) can be integrated to

$$(7) \qquad \bar{\varepsilon} = \frac{2I}{1+i} \ln \frac{A}{A_o}$$

where A_o is the initial value of A. Using the strain hardening equation according to Ludwik :

$$(8) \qquad \bar{\sigma} = C\,(\bar{\varepsilon}+\varepsilon_o)^n$$

where C denotes the characteristic flow stress and n is the strain hardening exponent, the work done per unit volume will be given by the equation

$$(9) \qquad dW_s = \bar{\sigma}\, d\bar{\varepsilon} = C(\bar{\varepsilon}+\varepsilon_o)^n\, d\bar{\varepsilon}$$

Considering Equations (6) and (7), this will give the result :

$$(10) \qquad \frac{dW_s}{dA} = \frac{C}{A} \left(\frac{2I}{1+i}\right)^{n+1} \left(\ln\frac{A}{A_o} + \frac{1+i}{2I}\,\varepsilon_o\right)^n$$

If after a certain amount of strain, the dissipation of work per unit surface increase shows a maximum value the relevant surface element becomes a weakened spot, whilst the surrounding surface elements are strengthening further. Thus we see, that the stability limit is found by

$$(11) \qquad \frac{d}{dA}\left(\frac{dW_s}{dA}\right) = 0$$

In other words

$$(12)\qquad \ln \frac{A_c}{A_o} + \frac{1+i}{2I}\,\varepsilon_o = n \quad ,$$

where c indicates the critical value of the surface element.
For i = constant we find, considering Equations (2), (3) and (6), after integration

$$(13)\qquad \begin{cases} \varepsilon_{1c} = \dfrac{2-i}{1+i}\left(n - \dfrac{1+i}{2I}\,\varepsilon_o\right) \\[2ex] \varepsilon_{2c} = \dfrac{2i-1}{1+i}\left(n - \dfrac{1+i}{2I}\,\varepsilon_o\right) \end{cases}$$

Curves representing these equations are shown in Fig. 0.
Actually Equations (13) are defining the forming limit diagram for the sheet material on the base of local instability, which is the first condition for necking.

In the case of simple sheet bending (i.e. bending in one plane) the surrounding surface material is not preventing the instable spot from strongly increasing its strain.

BENDABILITY CRITERION

In order to make a first step towards a solution of the practical problem the displacement of the neutral surface of bending is neglected. The difference in the result is not very important for materials with a low degree of bendability. Further it is assumed that the resulting tension in the bending zone is zero and that bending takes place under plane-strain conditions (the average transverse strain being zero during the bending).
So

$$(14)\qquad \varepsilon_2 = 0$$

Using the last of the Equations (13) we find that $i = \frac{1}{2}$. Substitution of this value into the first of Equations (13) leads to

$$(15)\qquad \varepsilon_{1c} = n - \varepsilon_o \frac{\sqrt{3}}{2}$$

As usual it is assumed that plane sections remain plane during the bending. The distribution of elongation (or engineering strain) of fibres across the sheet is therefore linear. The bending strain in any fibre is

$$\ln\left(1 + \frac{y}{\rho}\right)$$

where ρ is the radius to the central surface and y is the distance of the fibre from the central surface.
So the bending strain in the outer surface of the bending zone is

$$(16) \qquad \varepsilon_1 = \ln\left(1 + \frac{s}{2\rho}\right)$$

as a first approximation. Thus, using Equation (15) and Equation (16) provides the bending geometry connected with an instable surface layer :

$$(17) \qquad \frac{\rho_c}{s} = \frac{1}{2\{\exp\left(n - \varepsilon_o \frac{\sqrt{3}}{2}\right) - 1\}}$$

An easy practical approximation is achieved for

$\varepsilon_o = 0$, $s \approx s_o$ and $e^x \approx 1+x$:

$$(18) \qquad \frac{\rho_c}{s_o} \approx \frac{1}{2n}$$

The curves representing Equations (17) and (18) are shown in Fig. 1. The left part of the diagram is of practical importance and shows a neglectable difference between the curves.

BENDING IN SUCCESSIVE STEPS

For the sake of simplicity the following calculations are based on Equation (18). From Fig. 1 it becomes clear that for sheet materials with a small n-value difficulties can arise when a relatively sharp bending edge is wanted. Apart from choosing a more ductile sheet material, a sharp edge may be achieved by bending in steps applying an annealing operation in between the steps.

The development of a practical guideline for the planning of the steps may start from the assumption, that the ductility of the material can be restored completely by annealing. So the increase in bending strain in the sheet surface

$$(19) \qquad \Delta\varepsilon_1 = \ln\left(1+\frac{s}{2\rho_j}\right) - \ln\left(1+\frac{s}{2\rho_{j-1}}\right)$$

during any bending step j is restricted to the value given by Equation (15). So for $\varepsilon_o = o$, $s = s_o$ and $e^x \approx 1+x$ we obtain the simple law

$$(20) \qquad \left(\frac{s_o}{\rho_j}\right)_c \approx 2n + (n+1)\,\frac{s_o}{\rho_{j-1}}$$

which enables a successive calculation of the critical bending radius by each step using the value of the initial radius. From this formula Equation (21) can be derived, if powers of n are neglected.

$$(21) \qquad \left(\frac{s_o}{\rho_j}\right)_c \approx 2n\left(j + n\sum_{a=o}^{j-1} a\right)$$

This approximate formula gives the critical values of the bending radius directly for each step.
The curves representing Equation (20) are shown in Fig. 2.

DISCUSSION

Startingfrom an analysis of local plastic instability under biaxial stress Equation (20) is obtained.
This is a very important result, since it allows us to determine a safe upper limit for the first bending operation of sheet and useful indication for planning the following steps. Since instability cannot be observed, a direct experimental verification was impossible. However, from a number of practical applications the criterion proved to be a very useful one.
In addition to this the criterion developed is in agreement with a number

of recommandations in literature. For example : Oehler {1} mentions a value $\rho_c \simeq 2.5\ s_o$ for steel. Since the average n-value for steel is 0.2 Equation (18) provides the same value. However, the strain-hardening exponent is not often mentioned in literature, therefore a comparison is difficult.

Moreover a first investigation has been carried out in order to check the theoretical bending criterion in a mediate way. For that purpose three different steel sheets with a thickness of 12.0 mm and a polished surface have been subjected to bending operations in a V-tool.
By watching changes of the surface appearance by eye very carefully the values of the bending radius have been measured for beginning mattness, for the start of perceptible directionality in the surface appearance and finally for visible crack initation. The results of these tests are represented in Fig. 3. The observations come up to expectations : mattness is not connected with instability, but directionality of the surface structure and crack formation are obviously preceded by local instability. With this the practical significance of the theoretically developed criterion is provisionally established, although the relation between this and different practical criteria has to be investigated in a more systematical way. Particularly, the effect of sheet thickness is of practical importance.

Actually, practical values {2} of the bending radius exceed the minimum values given by the criterion very often. But the demands made on products differ considerably and generally are not defined very well.

Especially in the case of dynamic loads or chemical treatment of the product after bending, the instability criterion seems to meet the needs in a proper way. Also for thick sheet materials, where the strain gradient in normal direction has a smaller value in connection with surface instability, it might be wise to restrict the bending radius according to the limit of surface instability.

It is neither possible nor desirable to discuss all the practical bending processes here. It is only suggested that some bending problems can be analysed in a more systematical way.
Especially in the case of additional drawing stresses in the plane of the sheet, it is necessary to take the shift of the neutral plane of bending into account. It can be calculated by equating the total tension to the difference between the bending tension and compression.

1. OEHLER/KAISER : *Schnitt-, Stanz- und Ziehwerkzeuge.* Springer Verlag, 1957, p. 189,537.

2. W.P. ROMANOWSKI : *Handboek voor de moderne Stanstechniek.* Kluwer, p. 77 (translated from Handbuch der Stanzereitechnik, VEB Verlag Technik, Berlin).

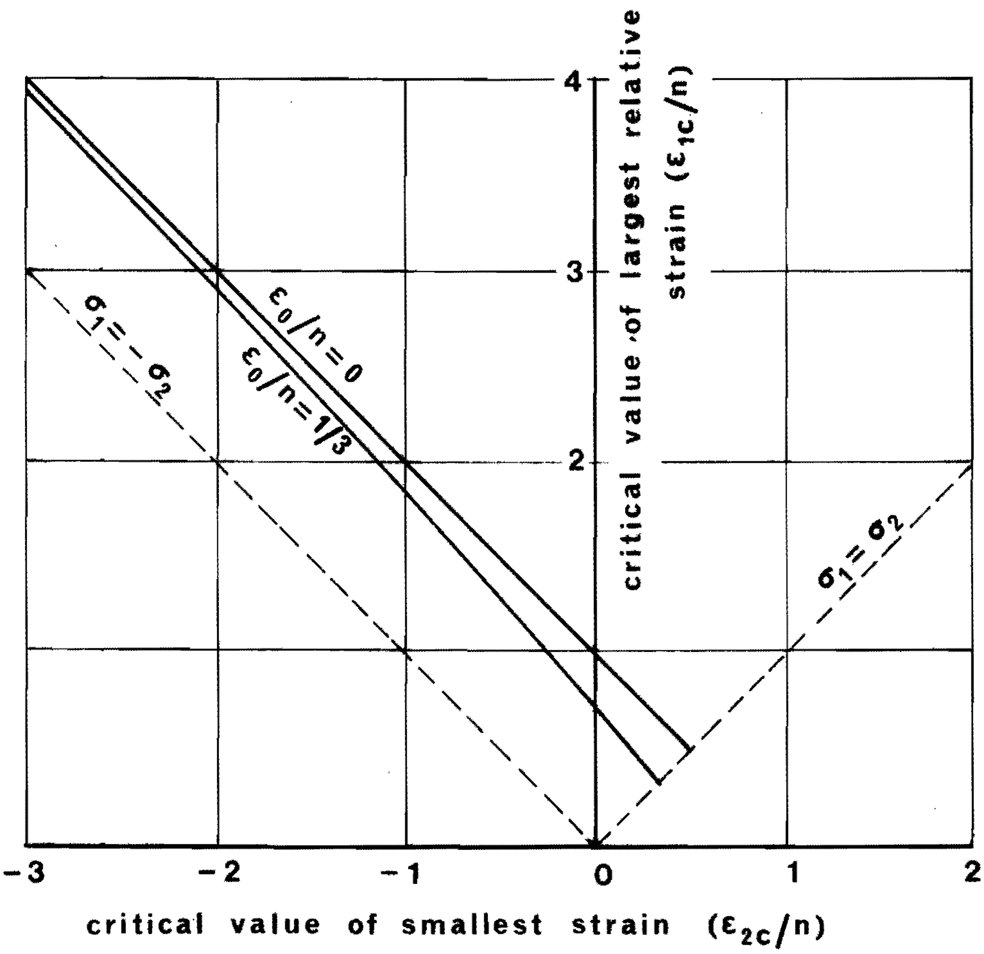

fig.0 Local instability limit for stretching of sheet material

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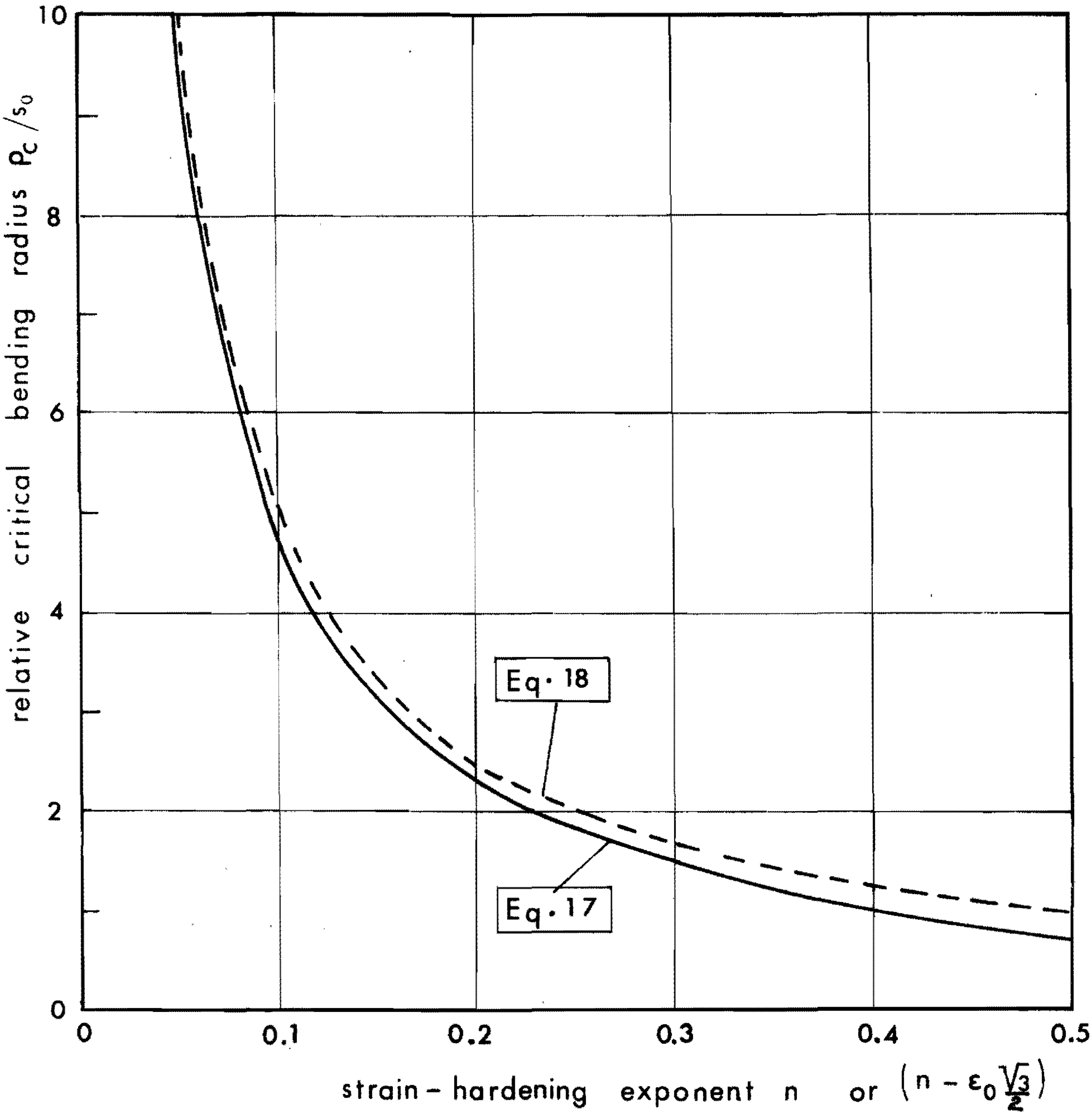

fig. 1 Surface instability boundary in sheet bending

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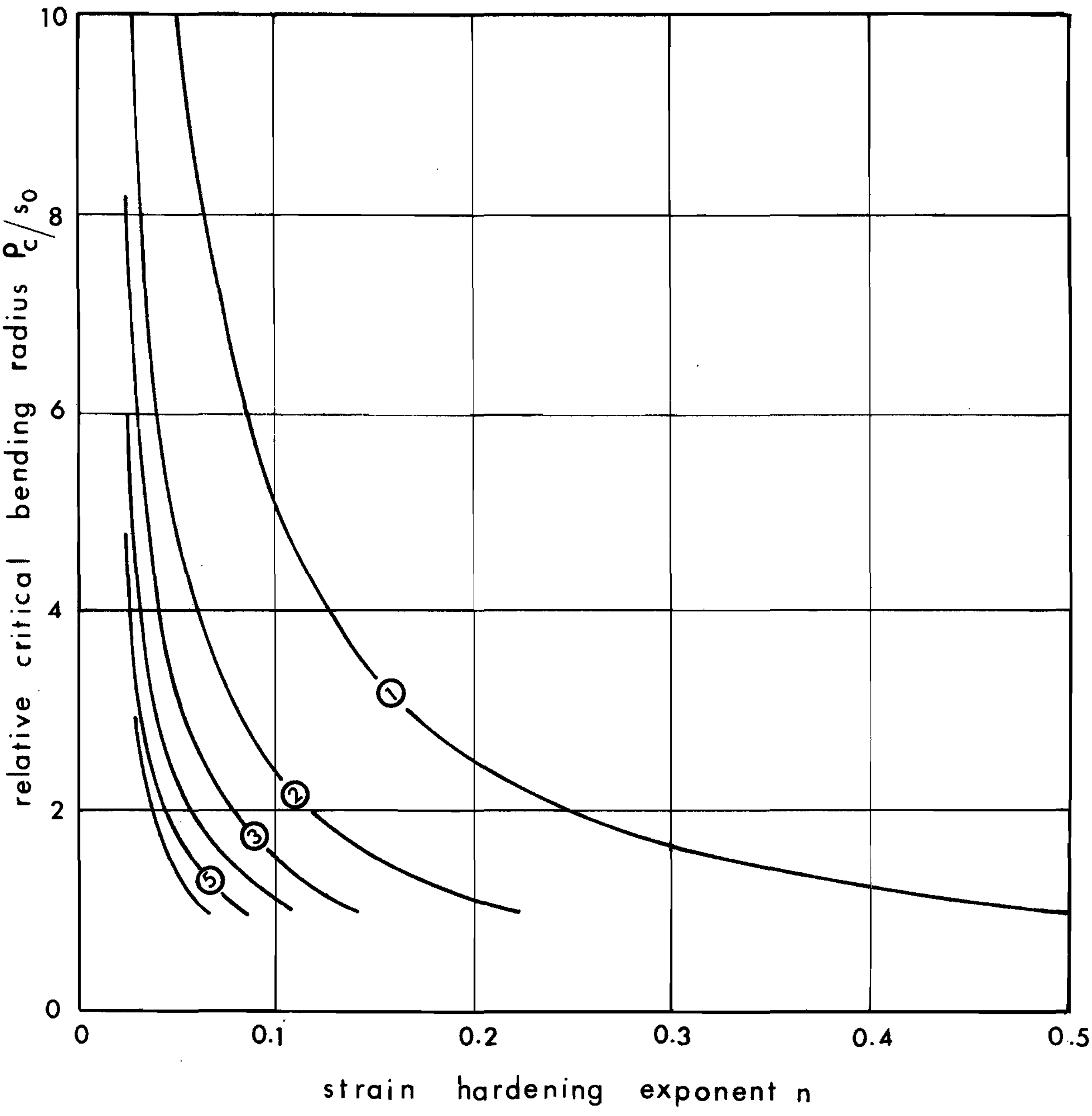

fig. 2 Surface instability limit for successive bending steps ($\varepsilon_0 = 0$)

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material (s_0 = 12 mm)		ρ/s_0		
ind.	n	matt-ness	direct	cracks
a	0.049	8.75	7.75	7.25
b	0.057	8.50	6.83	4.50
c	0.139	6.65	3.50	2.08

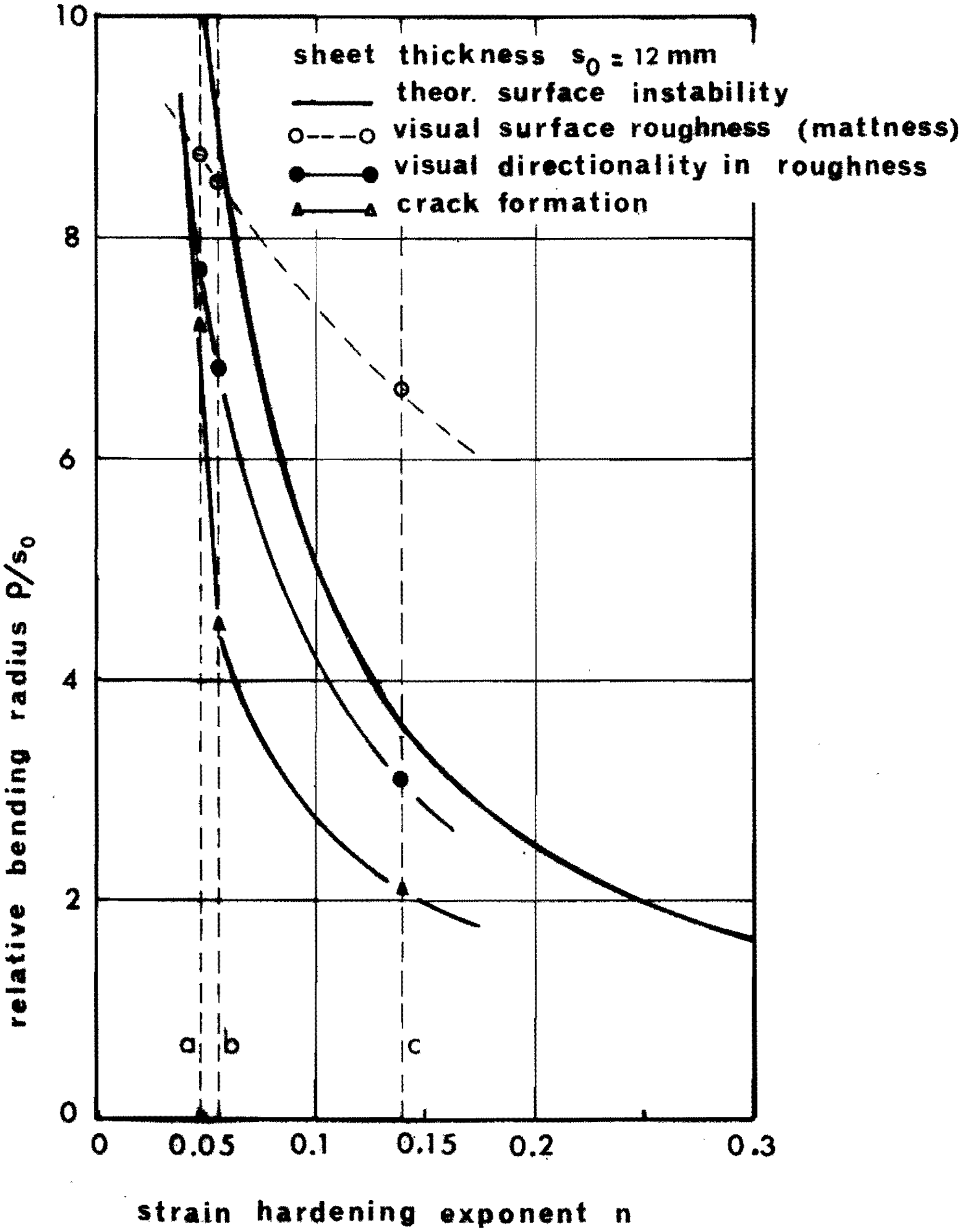

fig.3 Experimental formability criteria in sheet bending

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On the Critical Radius in Sheet Bending

J. A. G. Kals (2), and P. C. Veenstra (1)

Introduction

It is very difficult to find for sheet bending operations a suitable experimental deforming limit. In many cases literature provides data only for particular sheet materials and, in general, the criterion connected with the data is either not very well defined or not mentioned at all. Different values might be due to different criteria on which the experiments are based.

The deforming limit could be defined as the amount of strain which starts a noticeable granulation in the outer surface of the bending zone. In another case a visible orientation of this granulation or the start of surface cracks might be more relevant. In general, however, it is very difficult to carry out experiments based on direct visual observation.

In this paper a simple theoretical model is proposed, which has proved rather useful in a number of practical applications. It is based on the consideration that local strain effects such as localized granulation, necking, cracks, etc. in a more or less uniformly stressed area arise from strain peaks which must have been preceded by a local instability. Since plastic instability can be calculated from tensile test data, it is possible to derive a safe upper limit for the bending radius which can serve as a guideline for tool and product design.

Local Surface Instability

Starting from the consideration that a peak strain will occur at the outer surface of the bending zone for homogeneous materials, we have to deal with a plain stress situation in a thin surface layer:

$$\sigma_2 = i\,\sigma_1 \qquad (\sigma_3 = 0) \tag{1}$$

where σ_1 is the bending stress. Then the stress-strain relations are

$$\left.\begin{aligned} d\varepsilon_1 &= \frac{d\lambda}{2}\,\sigma_1\,(2-i)\\ d\varepsilon_2 &= \frac{d\lambda}{2}\,\sigma_1\,(2i-1)\\ d\varepsilon_3 &= -\frac{d\lambda}{2}\,\sigma_1\,(1+i)\end{aligned}\right\} \tag{2}$$

Now, by substituting the strain increments given by Equations (2) into the general expression for the effective strain increment

$$d\bar{\varepsilon} = \sqrt{\frac{2}{3}\,(d\varepsilon_1^2 + d\varepsilon_2^2 + d\varepsilon_3^2)}$$

it is easily shown that

$$d\bar{\varepsilon} = -\frac{2\,d\varepsilon_3}{1+i}\cdot I \qquad (d\varepsilon_3 \leq 0) \tag{3}$$

where

$$I = \sqrt{i^2 - i + 1} \tag{4}$$

Consider a small (uniformly strained) surface element $A = x_1 x_2$, then

$$A = x_1 x_2 \;,$$

$$\text{then} \quad dA = x_1 dx_2 + x_2 dx_1 \;,$$

$$\text{and} \quad \frac{dA}{A} = \frac{dx_1}{x_1} + \frac{dx_2}{x_2} = d\varepsilon_1 + d\varepsilon_2 = -\,d\varepsilon_3 \tag{5}$$

when constancy of volume is assumed.

With Equation (5), Equation (3) can be written as follows:

$$d\bar{\varepsilon} = \frac{2I}{1+i}\,\frac{dA}{A} \tag{6}$$

For a constant value of i we have a straight strain path and Equation (6) can be integrated to

$$\bar{\varepsilon} = \frac{2I}{1+i}\,\ln\frac{A}{A_o} \tag{7}$$

where A_o is the initial value of A. Using the strain hardening equation according to Ludwik:

$$\bar{\sigma} = C\,(\bar{\varepsilon} + \varepsilon_o)^n \tag{8}$$

where C denotes the characteristic flow stress and n is the strain hardening exponent, the work done per unit volume will be given by the equation

$$dW_s = \bar{\sigma}\,d\bar{\varepsilon} = C\,(\bar{\varepsilon} + \varepsilon_o)^n\,d\bar{\varepsilon} \tag{9}$$

Considering Equations (6) and (7), this will give the result:

$$\frac{dW_s}{dA} = \frac{C}{A}\left(\frac{2I}{1+i}\right)^{n+1}\left(\ln\frac{A}{A_o} + \frac{1+i}{2I}\,\varepsilon_o\right)^n \tag{10}$$

If after a certain amount of strain, the dissipation of work per unit surface increase shows a maximum value the relevant surface element becomes a weakened spot, whilst the surrounding surface elements are strengthened further. Thus the stability limit is found by

$$\frac{d}{dA}\left(\frac{dW_s}{dA}\right) = 0 \tag{11}$$

In other words

$$\ln\frac{A_c}{A_o} + \frac{1+i}{2I}\,\varepsilon_o = n \tag{12}$$

where c indicates the critical value of the surface element. For i = constant we find, considering Equations (2), (3) and (6), after integration

$$\left.\begin{aligned} \varepsilon_{1c} &= \frac{2-i}{1+i}\left(n - \frac{1+i}{2I}\,\varepsilon_o\right)\\ \varepsilon_{2c} &= \frac{2i-1}{1+i}\left(n - \frac{1+i}{2I}\,\varepsilon_o\right)\end{aligned}\right\} \tag{13}$$

Actually Equations (13) define the forming limit diagram for the sheet material on the base of local instability, which is the first condition for necking.

In the case of simple sheet bending (i.e. bending in one plane) the surrounding surface material does not prevent the instable spot from strongly increasing its strain.

Bendability Criterion

In order to make a first step towards a solution of the practical problem the displacement of the neutral surface of bending is neglected. The difference in the result is not very important for materials with a low degree of bendability. Further it is assumed that the resulting tension in the bending zone is zero and that bending takes place under plane-strain conditions (the average transverse strain being zero during the bending).

So

$$\varepsilon_2 = 0 \tag{14}$$

Using the last of the Equations (13) we find that $i = 1/2$. Substitution of this value into the first of Equations (13) leads to

$$\varepsilon_{1c} = n - \varepsilon_o\,\frac{\sqrt{3}}{2} \tag{15}$$

As usual it is assumed that plane sections remain plane during the bending. The distribution of elongation (or engineering strain) of fibres across the sheet is therefore linear. The bending strain in any fibre is

$$\ln\left(1 + \frac{y}{\rho}\right)$$

where ϱ is the radius to the central surface and y is the distance of the fibre from the central surface.

So the bending strain in the outer surface of the bending zone is

$$\varepsilon_i = \ln\left(1 + \frac{s}{2\rho}\right) \tag{16}$$

as a first approximation. Thus, using Equation (15) and Equation (16) provides the bending geometry connected with an instable surface layer:

$$\frac{\rho_c}{s} = \frac{1}{2\left\{\exp\left(n - \varepsilon_o \frac{\sqrt{3}}{2}\right) - 1\right\}} \tag{17}$$

An easy practical approximation is achieved for

$$\varepsilon_o = 0\,, \quad s \simeq s_o \quad \text{and} \quad e^x \simeq 1 + x \quad :$$

$$\frac{\rho_c}{s_o} \simeq \frac{1}{2n} \tag{18}$$

The curves representing Equations (17) and (18) are shown in Fig. 1. The left part of the diagram is of practical importance and shows a neglectable difference between the curves.

Bending in Successive Steps

For the sake of simplicity the following calculations are based on Equation (18). From Fig. 1 it becomes clear that for sheet materials with a small n-value difficulties can arise when a relatively sharp bending edge is wanted. Apart from choosing a more ductile sheet material, a sharp edge may be achieved by bending in steps, applying an annealing operation in between the steps.

The development of a practical guideline for the planning of the steps may start from the assumption that the ductility of the material can be restored completely by annealing. So the increase in bending strain in the sheet surface

$$\Delta\varepsilon_1 = \ln\left(1 + \frac{s}{2\rho_j}\right) - \ln\left(1 + \frac{s}{2\rho_{j-1}}\right) \tag{19}$$

during any bending step j is restricted to the value given by Equation (15). So for $\varepsilon_o = o$, $s = s_o$ and $e^x \simeq 1 + x$ we obtain the simple law

$$\left(\frac{s_o}{\rho_j}\right)_c \simeq 2n + (n+1)\,\frac{s_o}{\rho_{j-1}} \tag{20}$$

which enables a successive calculation of the critical bending radius by each step using the value of the initial radius. From this formula Equation (21) can be derived, if powers of n are neglected.

$$\left(\frac{s_o}{\rho_j}\right)_c \simeq 2n\cdot\left(j + n \sum_{a=o}^{j-1} a\right) \tag{21}$$

This approximate formula gives the critical values of the bending radius directly for each step.

The curves representing Equation (20) are shown in Fig. 2.

Discussion

Starting from an analysis of local plastic instability under biaxial stress Equation (20) is obtained.

This is a very important result, since it allows us to determine a safe upper limit for the first bending operation of sheet and useful indication for planning the following steps. Since instability cannot be observed, a direct experimental verification is impossible. However, from a number of practical applications the criterion proves to be a very useful one. In addition to this the criterion developed is in agreement with a number of recommendations in literature. For example: Oehler [1] mentions a value $\varrho_c \simeq 2.5\, s_o$ for steel. Since the average n-value for steel is 0.2, Equation (18) provides the same value. However, the strain-hardening exponent is not often mentioned in literature, therefore a comparison is difficult.

Actually, practical values [2] of the bending radius very often exceed the minimum values given by the criterion. But the demands made on products differ considerably and, generally, are not defined very well.

Especially in the case of dynamic loads or chemical treatment of the product after bending, the instability criterion seems to meet the needs in a proper way. Also for thick sheet materials, where the strain gradient in normal direction has a smaller value in connection with surface instability, it might be wise to restrict the bending radius according to the limit of surface instability.

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1. Oehler/Kaiser, Schnitt-, Stanz- und Ziehwerkzeuge. Springer Verlag, 1957, p. 189, 537.
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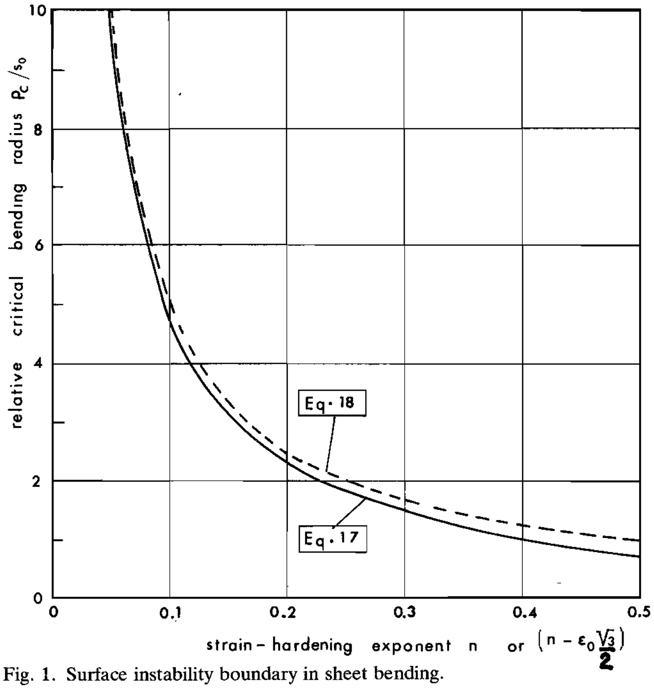

Fig. 1. Surface instability boundary in sheet bending.

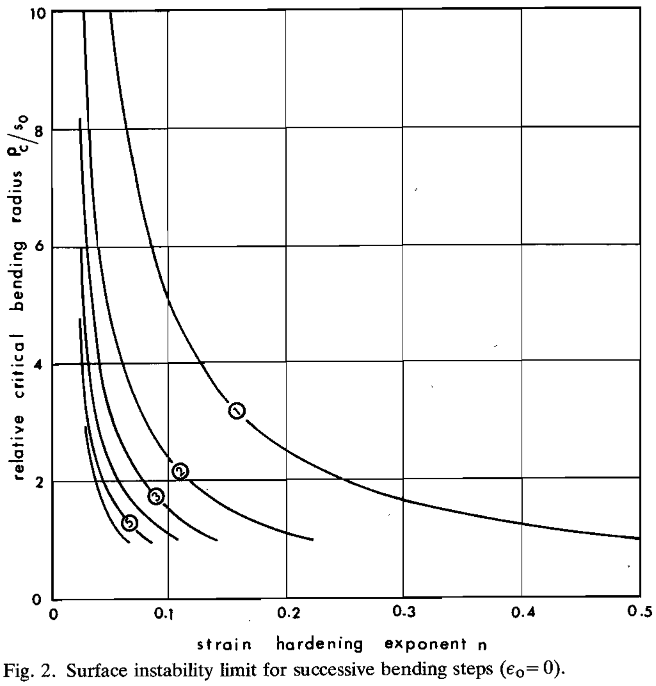

Fig. 2. Surface instability limit for successive bending steps ($\varepsilon_o = 0$).