An analysis on the deformation of the punch in sheet bending

AN ANALYSIS ON THE DEFORMATION OF THE PUNCH IN SHEET BENDING

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I. INTRODUCTION

In sheet bending practice where a straight contour is required, the bent part will usually have a different shape, not a straight but a slightly curved contour (see Fig.1), caused by a varying gap between punch and die due to

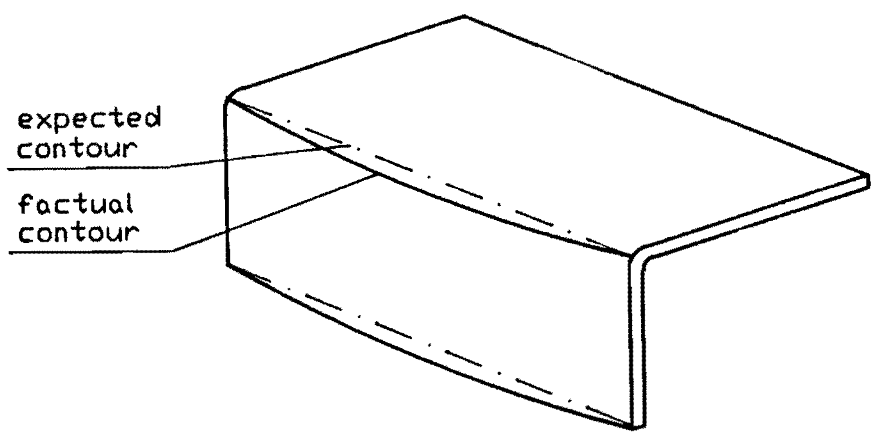

Fig.1 The Fault of the Bent Sheet

the forming force during the bending process. Consequently, it reduces the geometric accuracy of the part. It is necessary to find a practical method to indicate the deformation of the punch in order to conpensate it during forming. It is obvious that this deformation is depending on the stiffness of the components of punch, the conjunction between them, and also the stiffness of the supports of it.

This paper is going to analyse the deformation in the horizontal direction of the punch [1] during forming operation.

II. MODELLING THE STRUCTURE FOR ANALYSIS

The structure of the punch is briefly shown in Fig.2.1. The punch is horizontally supported by two blocks at both ends and three guides between them, each of which is connected with the foundation of the punch by four screws, it can freely move vertically with the motion of the punch up and down.

The load acting on the punch is caused by the workpiece being bent on the machine. The effective load to the deformation in horizontal direction is

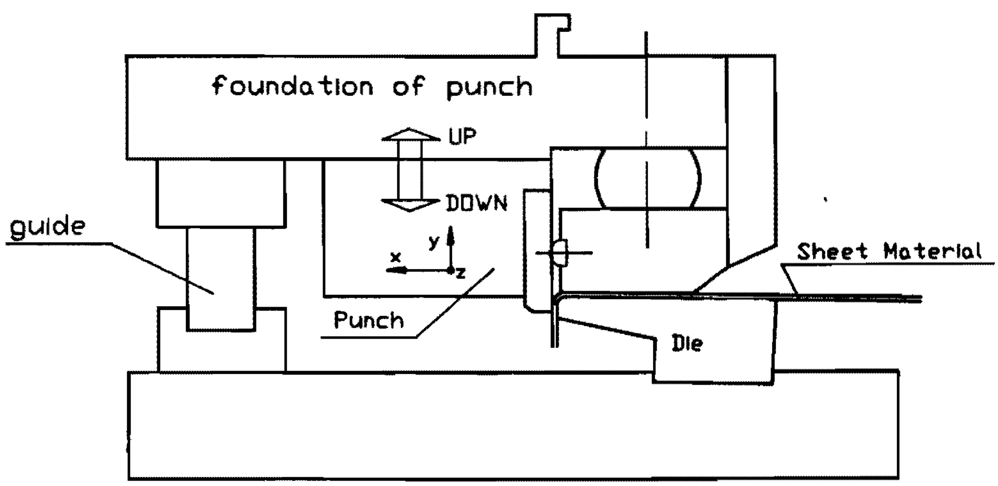

Fig.2.1 The Draft Drawing of Structure

the x–component F_x (See Fig.2.2), which is

$$F_x = F_n \cdot (\sin\varphi_a - \mu \cos\varphi_a) \qquad (2.1)$$

where μ is the friction coefficient constant.

According to [2], F_n becomes

$$F_n = \frac{M_b}{a_t + \mu \frac{s_0}{2}} \qquad (2.2)$$

where

$$M_b = Cbs_0^2 \cdot M_b^* \qquad (2.3)$$

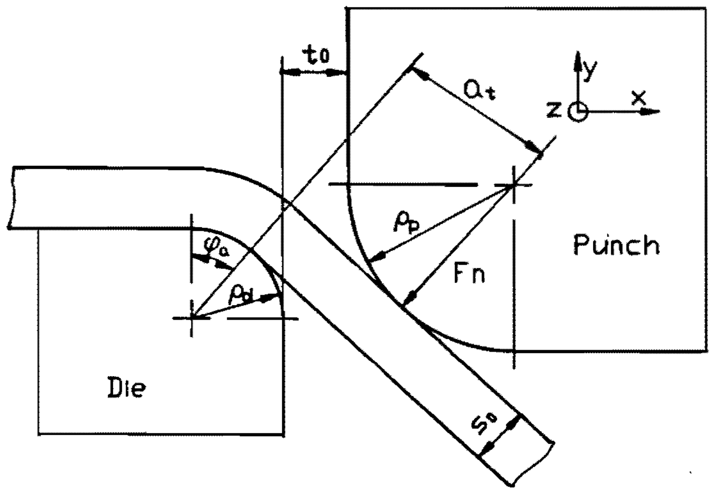

Fig.2.2 The Load Acting on the Punch

From [3], the dimensionless bending–moment M_b^* comes

$$M_b^* = \frac{E^* y_v^{*3}}{12(1-\nu^2)\rho^*} + \frac{\sqrt{3}\rho^{*2}}{(n+1)(n+2)} \cdot \left\{ \left(\frac{1-y_v^*}{\sqrt{3}\rho^*} + \bar{\epsilon}_0\right)^{n+1} \left[\frac{n+1+y_v^*}{\sqrt{3}\rho^*} - \bar{\epsilon}_0\right] - \right.$$

$$\left. - \bar{\epsilon}_0^{\,n+1} \cdot \left[\frac{(n+2)y_v^*}{\sqrt{3}\rho^*} - \bar{\epsilon}_0\right] \right\} \qquad (2.4)$$

where

$$\rho^* = \rho / s_0$$

$$E^* = E / C$$

$$y_v^* = \frac{2(1-\nu^2) \cdot \bar{\epsilon}_0^n \rho^*}{E^* \sqrt{\nu^2 - \nu + 1}}$$

In the three expressions above, E, C, and ν are the material constants, $\bar{\epsilon}_0$ the prestrain of the bent sheet, ρ the radius of bending, or $\rho = \rho_d + s_0/2$, and s_0 the thickness of the sheet.

However, the maximum value of F_x occurs when the value of the angle φ_a tends to $\pi/2$. In this case (see Fig.2.3),

$$a_t = \sqrt{(\rho_d+\rho_p+t_0+U_x)^2 - (\rho_d+\rho_p+s_0)^2} \qquad (2.5)$$

where ρ_d and ρ_p are the radii of the die and punch, t_0 the gap between them, whose value may be $1.0s_0$ to $1.1s_0$, and U_x the displacement in the x–direction of the punch from the initial position.

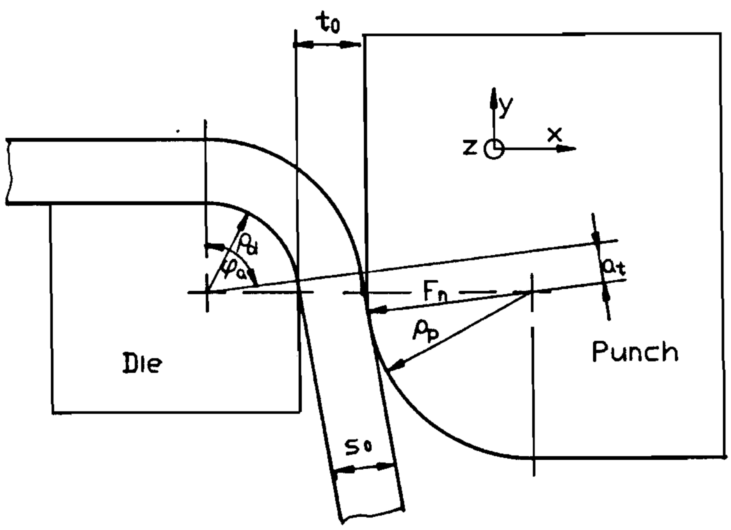

Fig.2.3 The Maximum Load on the Punch

From Eqs.(2.1), (2.2), and (2.5), the load contributing to the deformation is a function with respect to the deformation U_x, so it is difficult to yield a simple expression like $U_x = U_x(F_x)$. In this case, it is convenient to use numerical method by means of superposition.

According to the structure, the applied load, and our purpose mentioned above, we can consider the punch as a beam simply supported at two ends and supported by three springs between the two ends. In practice, the bent part is usually positioned in the middle of the punch, so the load is applied as a symmtrically distributed one.

In the following chapters, we will offer an analysis based on elastic deformation of a beam in a situation mentioned above.

III. DETERMINATION OF THE REACTIONS ON THE REDUNDANT SUPPORTS

As known, a one–degree beam that is supported by five points is indeterminate in a third–degree (see Fig.3.1). Fortunately, the problem in our project is a symmtric one due to the symmtric structure and the symmtric load.

In Fig.3.1, L is the span of the punch between the two blocks, A,B,C the three spring–supported points, b the distance between the left spring support and the left block, K the spring constant of supports, and l is the length of the loaded range, or the width of the bent sheet. In a first approximation, the applied load is assumed as an uniform q, dividing the range of load into $2n$ equal sub–parts that have the same length

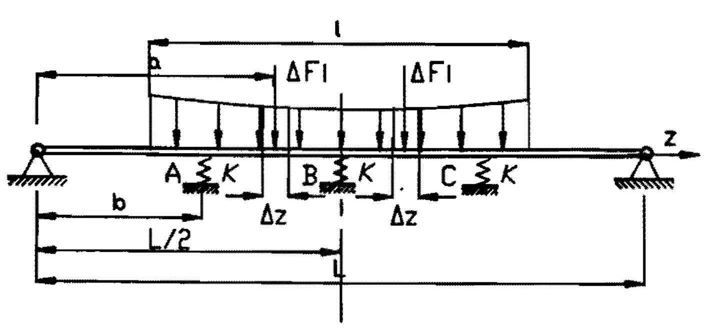

Fig.3.1 The Model for Anaysis

$$\Delta z = l/2n$$

the sub–load on every part is

$$\Delta F_i = q \cdot \Delta z \qquad (3.1)$$

Moreover, a is the distance between the sub–part and the block, which is

$$a = (L-l)/2 + (i-1)\cdot\Delta z \qquad (3.2)$$

where i is the order number of the sub–part.

In order to obtain the displacement of the beam, we may superpose the sub–displacements caused by the sub–loads. But, before this we have to know the reactions on the redundant supports because the beam in our problem is indeterminate. So, first of all we will determine the reactions of redundant supports by means of elastic displacement superposition method.

We consider a third–degree indeterminate beam is loaded by a pair of concentrated forces ΔF_i (see Fig.3.2a) which are symmtric with the middle point, and is supported by three springs with spring–constant K at the points A, B, and C. After releasing the three redundant supports by three unkown reactions R_{ai}, R_{bi},and R_{ci}, we can obtain the deformations at the points A, B,

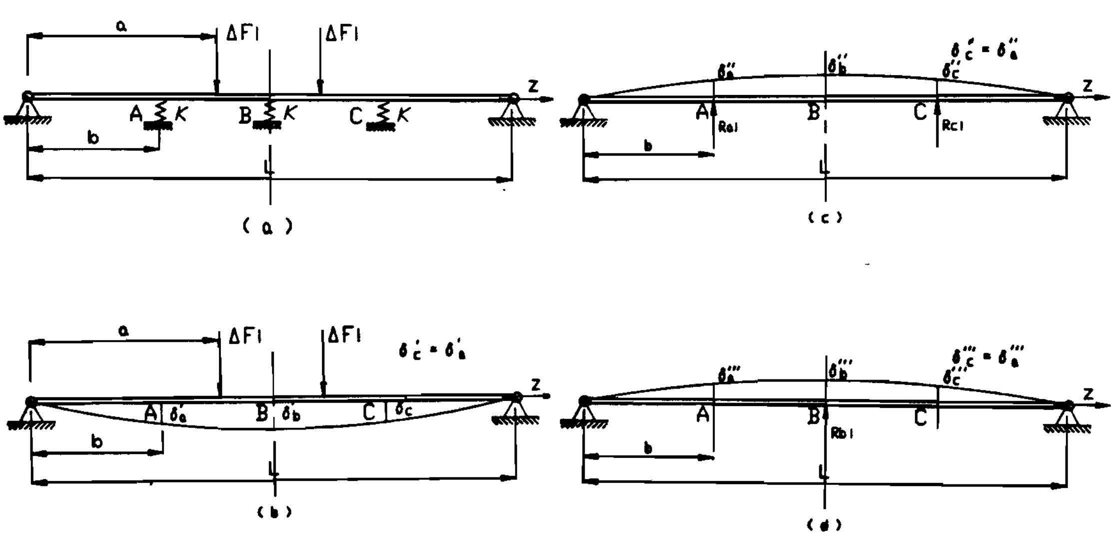

Fig.3.2 The Reactions Caused by Applied Load

and C caused by the applied load ΔF_i itself (see Fig.3.2b)

$$\delta_a' = \frac{\Delta F_i b}{6EI}(3La - 3a^2 - b^2) \qquad (a \geq b) \qquad (3.3a_1)$$

or

$$\delta_a' = \frac{\Delta F_i a}{6EI}(3Lb - 3b^2 - a^2) \qquad (a<b) \qquad (3.3a_2)$$

and

$$\delta_b' = \delta_{max} = \frac{\Delta F_i a}{24EI}(3L^2 - 4a^2) \qquad (3.3b)$$

$$\delta_c' = \delta_a' \qquad (3.3c)$$

where I is the moment of inertia of the beam, which will be obtained in Chapter V.

The deformations at points A, B, and C caused by the reactions R_{ai} and R_{ci} (see Fig.3.2c), which are considered equal and symmtric, are

$$\delta_a'' = \frac{R_{ai} b}{6EI} \cdot (3bL - 4b^2) \qquad (3.4a)$$

$$\delta_b'' = \frac{R_{ai} b}{24EI} \cdot (3L^2 - 4b^2) \qquad (3.4b)$$

$$\delta_c'' = \delta_a'' \qquad (3.4c)$$

Again, the deformations at points A, B, and C caused by the reactions R_{bi} (see Fig.3.2d) are

$$\delta_a''' = \frac{R_{bi} b}{48EI} \cdot (3L^2 - 4b^2) \qquad (3.5a)$$

$$\delta_b''' = \frac{R_{ai} L^3}{48EI} \qquad (3.5b)$$

$$\delta_c''' = \delta_a''' \qquad (3.5c)$$

Superpositions of Exps.(3.3), (3.4), and (3.5) respectively must satisfy the deformation conditions

$$\delta_a = \delta_a' - \delta_a'' - \delta_a'''$$

$$= R_{ai}/K \qquad (3.6a)$$

$$\delta_b = \delta_b' - \delta_b'' - \delta_b'''$$

$$= R_{bi}/K \qquad (3.6b)$$

$$\delta_c = \delta_a \qquad (3.6c)$$

Substitution of Exps.(3.3), (3.4), and (3.5) into Exp.(3.6) gives the equations

$$\frac{\Delta F_i b}{6EI}(3La-3a^2-b^2)-\frac{R_{ai}b}{6EI}(3bL-4b^2)-\frac{R_{bi}b}{48EI}(3L^2-4b^2)=R_{ai}/K$$

$$\frac{\Delta F_i a}{24EI}(3L^2-4a^2)-\frac{R_{ai}b}{24EI}(3L^2-4b^2)-\frac{R_{ai}L^3}{48EI}=R_{bi}/K$$

$$(\text{ for } a\geq b) \qquad (3.7)$$

or

$$\frac{\Delta F_i a}{6EI}(3Lb-3b^2-a^2)-\frac{R_{ai}b}{6EI}(3bL-4b^2)-\frac{R_{bi}b}{48EI}(3L^2-4b^2)=R_{ai}/K$$

$$\frac{\Delta F_i a}{24EI}(3L^2-4a^2)-\frac{R_{ai}b}{24EI}(3L^2-4b^2)-\frac{R_{ai}L^3}{48EI}=R_{bi}/K$$

$$(\text{ for } a<b) \qquad (3.8)$$

Solving these equations and noting that the reaction at point C is the same as at point A, we obtain the reactions at point A, B, and C

$$R_{ai}=\Delta_a/\Delta \qquad (3.9a)$$

$$R_{bi}=\Delta_b/\Delta \qquad (3.9b)$$

where determinants Δ_a, Δ_b, and Δ for $a\geq b$ are

$$\Delta_a=\begin{vmatrix} \frac{\Delta F_i b}{6EI}(3La-3a^2-b^2) & \frac{b}{48EI}(3L^2-4b^2) \\ \frac{\Delta F_i a}{24EI}(3L^2-4a^2) & \frac{L^3}{48EI}+\frac{1}{K} \end{vmatrix} \qquad (3.10a)$$

$$\Delta_b=\begin{vmatrix} \frac{b}{6EI}(3Lb-4b^2)+\frac{1}{K} & \frac{\Delta F_i b}{6EI}(3La-3a^2-b^2) \\ \frac{b}{24EI}(3L^2-4b^2) & \frac{\Delta F_i a}{24EI}(3L^2-4a^2) \end{vmatrix} \qquad (3.10b)$$

$$\Delta=\begin{vmatrix} \frac{b}{6EI}(3Lb-4b^2)+\frac{1}{K} & \frac{b}{48EI}(3L^2-4b^2) \\ \frac{b}{24EI}(3L^2-4b^2) & \frac{L^3}{48EI}+\frac{1}{K} \end{vmatrix} \qquad (3.10c)$$

$$(\text{ for } a\geq b) \qquad (3.10)$$

and for $a<b$ are

$$\Delta a = \begin{vmatrix} \frac{\Delta F_i a}{6EI}(3Lb-3b^2-a^2) & \frac{b}{48EI}(3L^2-4b^2) \\ \frac{\Delta F_i a}{24EI}(3L^2-4a^2) & \frac{L^3}{48EI}+\frac{1}{K} \end{vmatrix} \tag{3.11a}$$

$$\Delta_b = \begin{vmatrix} \frac{b}{6EI}(3Lb-4b^2)+\frac{1}{K} & \frac{\Delta F_i a}{6EI}(3La-3a^2-b^2) \\ \frac{b}{24EI}(3L^2-4b^2) & \frac{\Delta F_i a}{24EI}(3L^2-4a^2) \end{vmatrix} \tag{3.11b}$$

$$\Delta = \begin{vmatrix} \frac{b}{6EI}(3Lb-4b^2)+\frac{1}{K} & \frac{b}{48EI}(3L^2-4b^2) \\ \frac{b}{24EI}(3L^2-4b^2) & \frac{L^3}{48EI}+\frac{1}{K} \end{vmatrix} \tag{3.11c}$$

(for $a<b$) (3.11)

The determinations of Exps.(3.9a) and (3.9b) are the reactions at the redundant support points A and B, including the point C (or $R_{ci}=R_{ai}$), caused by the sub–load ΔF_i. The total reactions at points A and B are

$$R_a = \sum_1^n R_{ai} \tag{3.12a}$$

$$R_b = \sum_1^n R_{bi} \tag{3.12b}$$

where n is the number of sub–parts in the half length of the beam.

VI. DETERMINATION OF THE DEFORMATION

Having obtained the reactions on the redundant supports, we can easily determine the deformations by means of superposition. The deformation at any point can be considered as the combination of the deformations caused by applied load and by the reactions (see Fig.4.1). The displacement at the point in a distance z from the left end of the beam caused by applied load ΔF_i (see Fig.4.1a) is

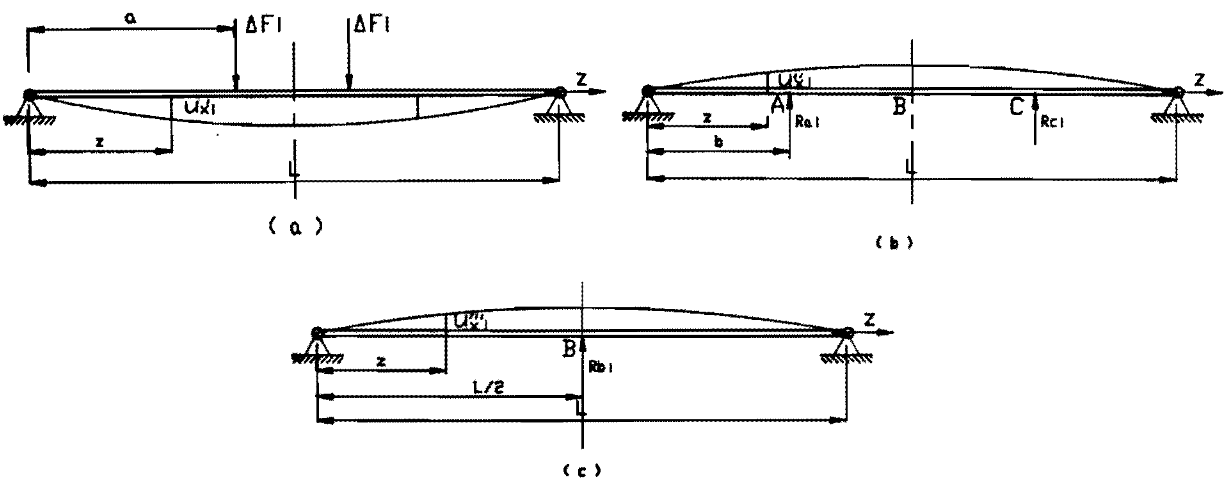

Fig.4.1 The Deformation of Beam

$$u_{xi}' = \frac{\Delta F_i z}{6EI} \cdot (3La - 3a^2 - z^2) \qquad (0 \le z \le a) \qquad (4.1a)$$

or

$$u_{xi}' = \frac{\Delta F_i a}{6EI} \cdot (3Lz - 3z^2 - a^2) \qquad (a \le z \le L/2) \qquad (4.1b)$$

The displacement at the point z caused by the reactions R_{ai} and R_{ci} (see Fig.4.1b, and note $R_{ai}=R_{ci}$) is

$$u_{xi}'' = \frac{R_{ai} z}{6EI} \cdot (3Lb - 3b^2 - z^2) \qquad (0 \le z \le b) \qquad (4.2a)$$

or

$$u_{xi}'' = \frac{R_{ai} b}{6EI} \cdot (3Lz - 3z^2 - b^2) \qquad (b \le z \le L/2) \qquad (4.2b)$$

The displacement at the point z caused by the reaction R_{bi} (see Fig.4.1c) is

$$u_{xi}''' = \frac{R_{bi} z}{48EI} \cdot (3L^2 - 4z^2) \tag{4.3}$$

Combination of Exps.(4.1), (4.2) and (4.3) gives the real deformation at the point z caused by the sub–load ΔF_i

$$u_{xi} = u_{xi}' - u_{xi}'' - u_{xi}''' \tag{4.4}$$

The summation of them, considering i from 1 to n, gives the total deformation at point z, that is,

$$U_{xi} = \sum_1^n u_{xi} \tag{4.5}$$

V. DETERMINATION OF THE PARAMETERS NEEDED

In previous chapters, we mentioned several parameters such as I, the moment of inertia of the beam, and K, the spring–constant of the three mediate supports, which are depending on the structure of the machine tool. In this chapter, they will be determined according to the Drawing No.10 75 18 008 9. It should be mentioned that some of the dimensions which are missing in the drawing are evaluated by the ruler and the scale.

1) Determination of the Moment of Inertia I (see Fig.5.1)

The total area of the section is

$$A = \Sigma A_i$$

$$= 7.2 \cdot 9.5 + 32.0 \cdot 4.6$$

$$= 215.6 \text{ cm}^2$$

The centre of gravity is at the point $C(x_c, y_c)$,

$$x_c = \frac{\Sigma A_i x_i}{A}$$

$$= \frac{7.2 \cdot 9.5 \cdot (24.2 - 9.5/2) + 32.0 \cdot 4.6 \cdot 16.0}{215.6}$$

$$= 17.08 \text{ cm}$$

$$y_c = \frac{\Sigma A_i y_i}{A}$$

$$= \frac{7.2 \cdot 9.5 \cdot (4.6+3.6) + 32.0 \cdot 4.6 \cdot 2.3}{215.6}$$

$$= 4.17 \text{ cm}$$

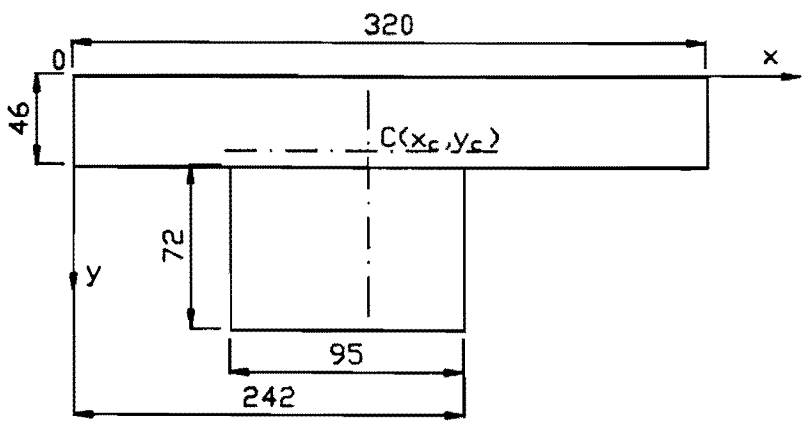

Fig.5.1 The Dimension of Section of Punch (*mm*)

therefore, the moment of inertia of the section is

$$I_y = \Sigma I_{yi} + \Sigma A_i \cdot (x_i - x_c)^2$$

$$= \frac{7.2 \cdot 9.5^3}{12} + 7.2 \cdot 9.5 \cdot (24.2 - 9.5/2 - 17.08)^2 + \frac{4.6 \cdot 32^3}{12} + 4.6 \cdot 32 \cdot (17.08 - 16)^2$$

$$= 13596 \text{ cm}^4$$

We only need the value of the moment of inertia I_y as I for our project.

2) Determination of the Spring Coefficient of Intermediate Supports

According to the drawing, the three guides (see Fig.5.2), as the mediate supports in our project, are connected to the foundation of the punch with four screws whose diameters are about 14 *mm*. Considered as the elastical parts, the spring coefficient of the guide is depending on the stiffness of guide and screw. The spring constant means the force which occurs one unit of displacement at the end of the guide, which is caused by the deflection of guide itself and the extension of screw. The deflection of the guide caused

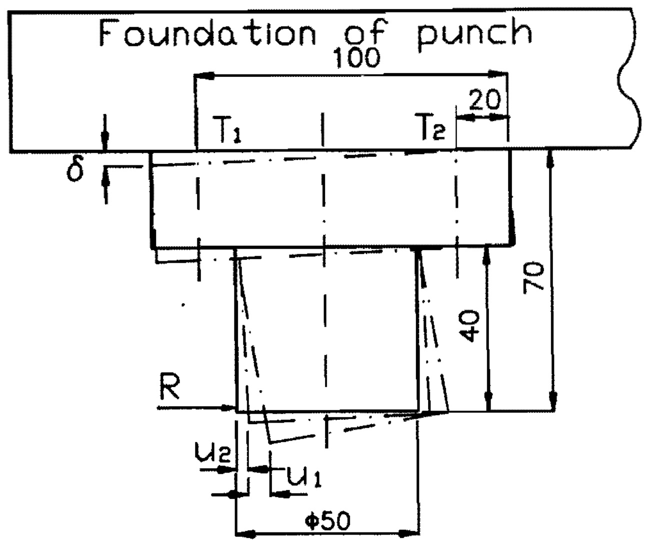

Fig.5.2 Connection of the Guide

by a centain force R, considered the guide as a cantilever, is

$$u_1 = \frac{R\,l_g^3}{3EI} \tag{5.1}$$

where l_g is the length of guide, which is about 40 mm, I the moment of inertia of guide. The extension of screw caused by force R also gives the displcement u_2 at the end of guide, which is

$$u_2 = \frac{70}{100}\delta$$

$$= 0.7\delta \tag{5.2}$$

where δ is the extension of screws, which is determined by the tension T_1 in left screws and T_2 in right screws. However, according the deformation condition, T_1 and T_2 have the relation

$$T_1 : T_2 = 100:20 = 5:1$$

The equilibrium of moment on the guide gives

$$100\,T_1 + 20\,T_2 = 70R \longrightarrow T_1 \cong 0.7R \tag{5.3}$$

and the extension of the left screw is

$$\delta = \frac{T_1 \cdot l_s}{2EA} \qquad (5.4)$$

where there is a 2 under the fraction line because there are two screws in the left side, and l_s is the effective length of screw, which is about 30_{mm}. Therefore, combination of Exps.(5.1) to (5.4) gives the displacement at the end of the guide

$$\begin{aligned} u &= u_1 + u_2 \\ &= \frac{R\, l_g^3}{3EI} + 0.7\frac{T_1 \cdot l_s}{2EA} \\ &= \frac{40^3 \cdot R}{3 \cdot 210000 \cdot \pi \cdot 50^4/64} + 0.7\frac{0.7R \cdot 30}{2 \cdot 210000 \cdot \pi \cdot 7^2} \\ &= 0.55848 \cdot 10^{-6} R \end{aligned} \qquad (5.5)$$

To obtain the spring coefficient K, let u=1, therefore

$$\begin{aligned} K &= R|_{u=1} \\ &= 1.79057 \cdot 10^6 \ N/_{mm} \end{aligned} \qquad (5.6)$$

VI. COMPUTATION OF THE DEFORMATION

The computation of the deformation of the punch is carried out by a numerical approach method by the follwing steps:

(1). Assuming the initial deformation of the punch to be zero, or U_{xi}=0, it follows the parameter a_t by Exp.(2.5);

(2). Dividing the half width of the sheet $l/2$ into n equal sub–parts, to calculate the approximate applied load ΔF_i on each sub–part by Exp.(3.1), where the value of q, at first, is assumed as an uniform, then it will change with the change of the value of a_t, which is

$$q = \frac{C s_0^2}{a_t + \frac{s_0}{2}} M_b^* \qquad (6.2)$$

(3). Calculating the reactions on intermediate suports caused by the applied load ΔF_i by Exps.(3.9*a*) and (3.9*b*);

(4). Calculating the deformation U_{xi} of each sub–part by Exp.(4.5). If the computational difference between the approximations of two following calculations is satisfactory regarding the accuracy (numerical stability of the solution), or,

$$\text{abs}(\ U_{xi} - U_{xi\text{-}1}\ ) \leq \epsilon \qquad (6.3)$$

where ϵ is a small positive number (eg. ϵ=10^{-8}), the value of U_{xi} is regarded as the appropriate result, or if not, the computation goes back to carry out the steps (2), (3), and (4) again and again until the approximation of deformation satisfies Exp.(6.3).

In practice the convergence speed of the computation is quite fast. Usually after less than five times of approaches, the appropriate result is obtained (see Fig.6.1).

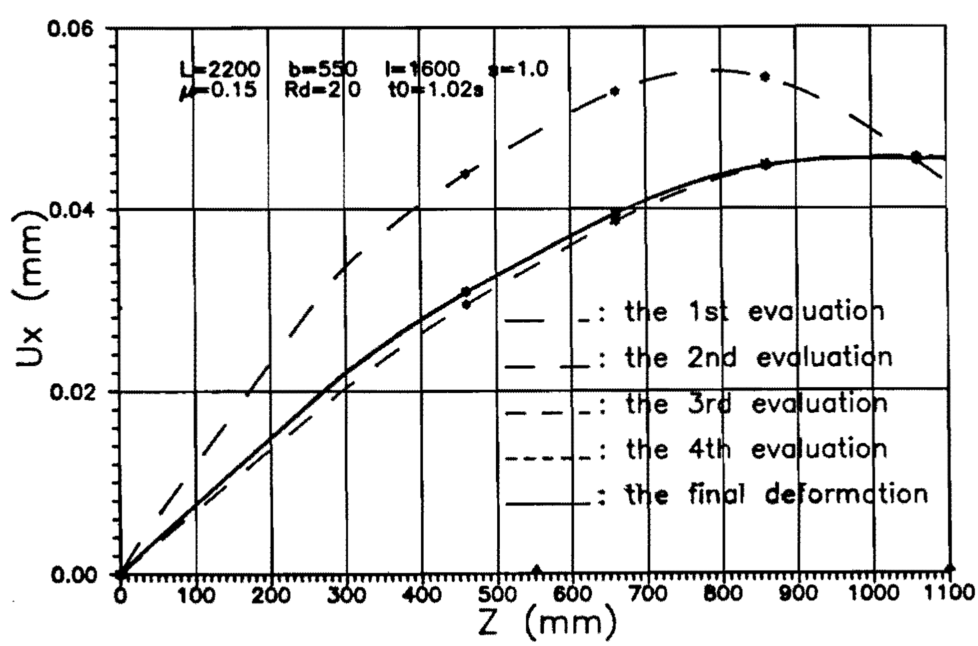

Fig.6.1 The Approaches of Computation

In following pages, several figures show us the results obtained by the varying parameters such as the width of sheet (see Fig.6.2), the gap between punch and die (see Fig.6.3), the coefficient of friction (see Fig.6.4), and the thickness of the sheet (see Fig.6.5). Moreover, in order to know how the deformation is affected by the stiffness of intermediate supports, Fig.6.6 shows us the influence of the possible stiffness.

In these figures, the following basic parameters of sheet material and punch material are applied,

$E = 210000\ N/_{mm^2}$	(for punch and sheet)
$C = 580\ N/_{mm^2}$	(for sheet)
$\nu = 0.45$	(for sheet)
$\bar{\epsilon}_0 = 0.005$	(for sheet)

and the rest of parameters needed are shown in each figure.

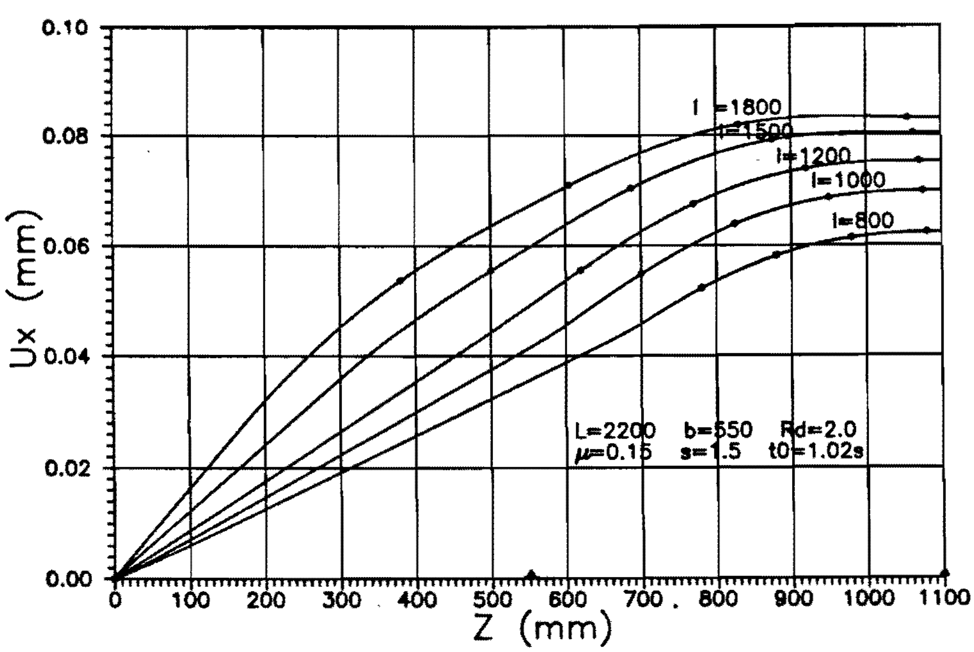

Fig.6.2 The Deformations of Different Width of Sheet

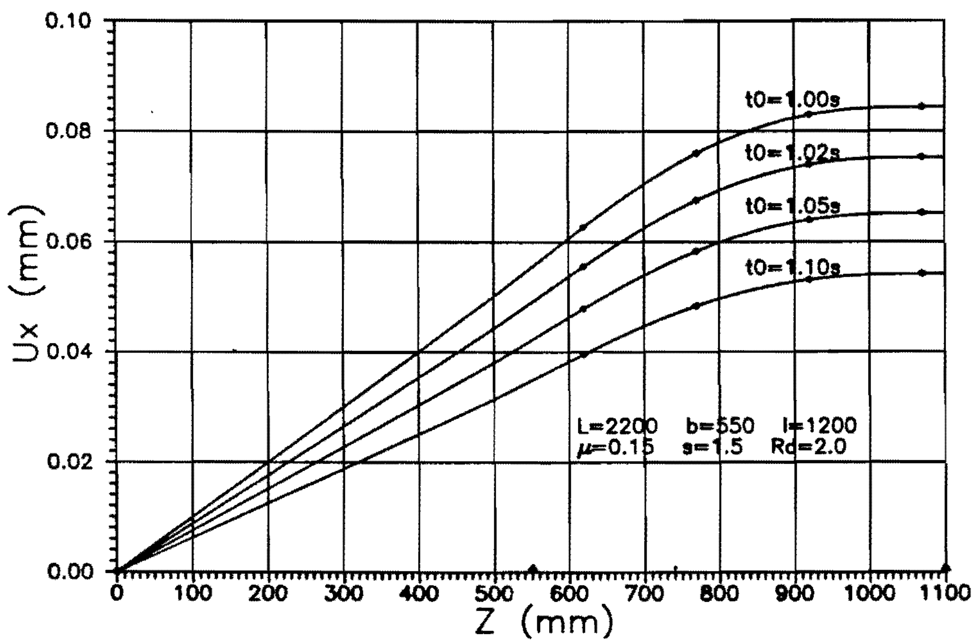

Fig.6.3 The Deformations of Different Gap

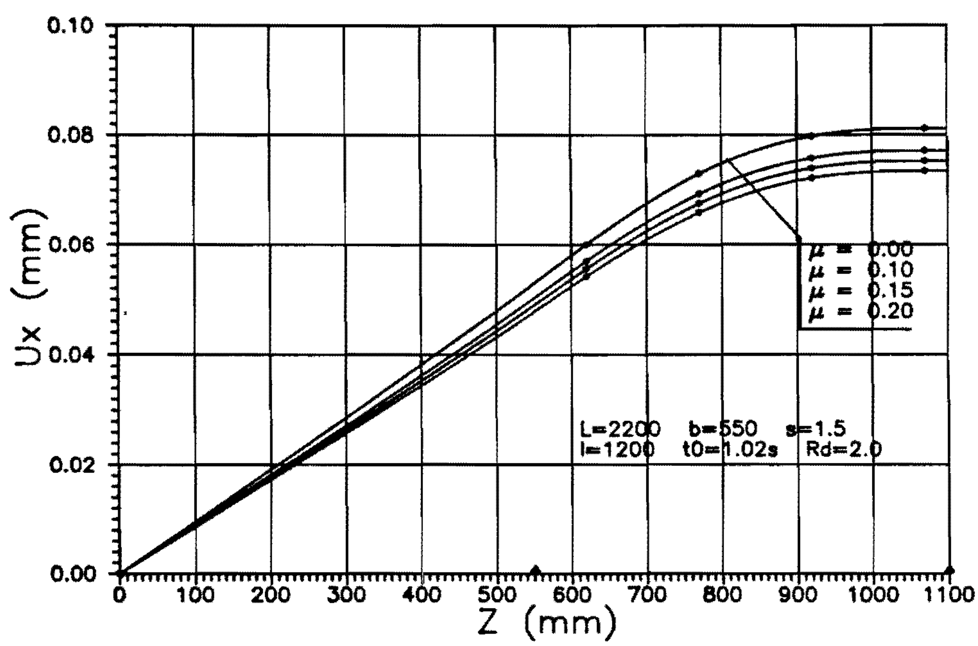

Fig.6.4 The Deformations of Different
Coefficient of Friction

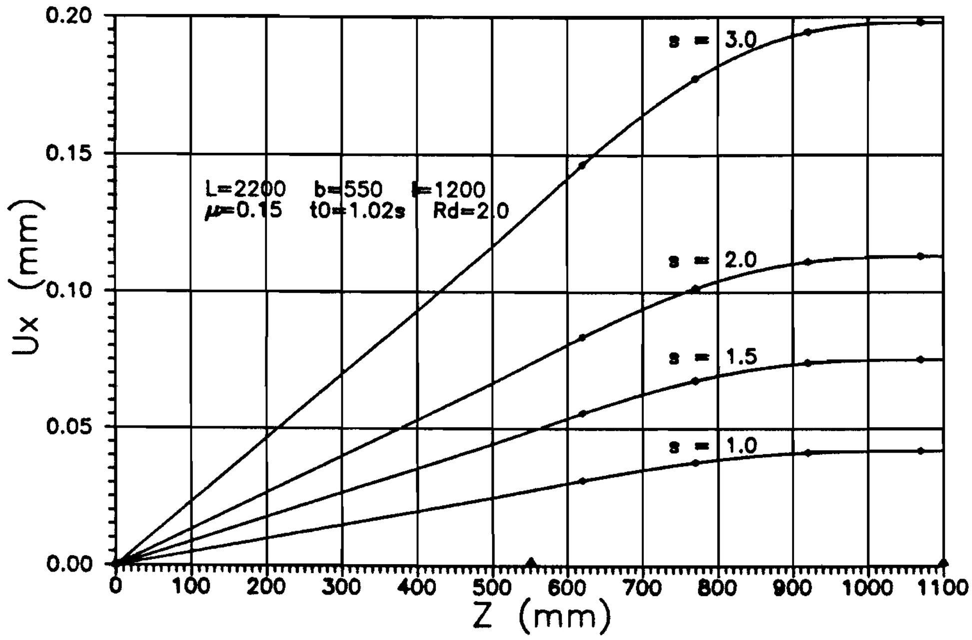

Fig.6.5 The Deformations of Different Thickness of Sheet

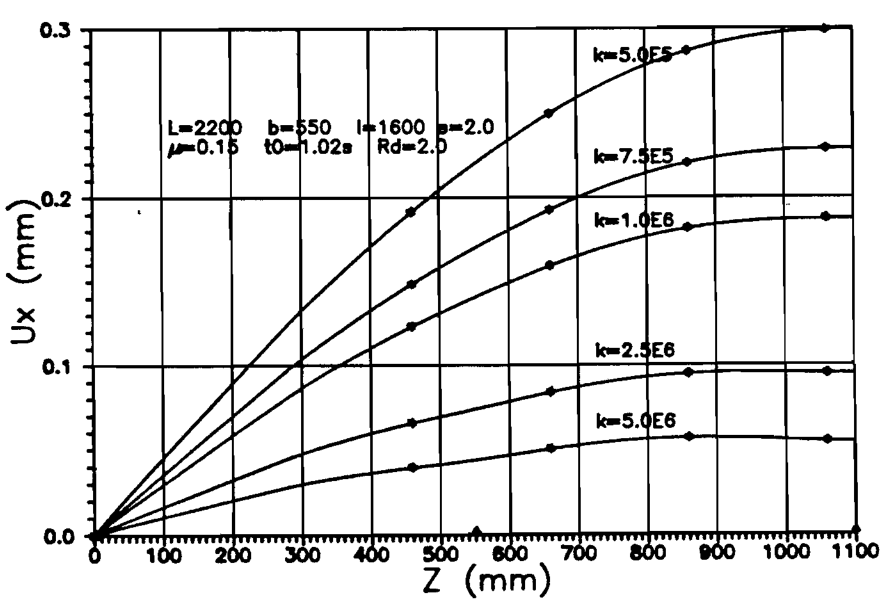

Fig.6.6 The Deformation Changes with the Stiffness

VII. CONCLUSIONS AND RECOMMENDATIONS

— As well the width of the sheet as the gap between punch and die (t_0–s_0) have less proportional influence on the deformation of the punch (see Fig.6.2 and 6.3). So a very accurate adjustment of the gap seems not to be of great importance. In addition it is noted that the influence of the friction is almost neglectible (see Fig.6.4).

— Both the thickness of the sheet and the stiffness factor K have great influence on the punch deformation (see Fig.6.5 and 6.6). Experiments are necessary to determine the validity of modelling and analysis. In this, especially a correct estimation of the value of K is of major importance.

— Further research has to be done to establish the relation between the deformation of the punch and the geometry of the bent sheet after spring back. Nevertheless a substantial improvement of the sheet geometry may be expected by raising the value of K.

REFFERENCE

[1] The copy of Drawing of Punch and Die, whose Drawing Number is 10 75 18 008 9

[2] ir. S.M.Hoogenboom, ir. A.C.E.C.Melis, & ir. A.B.Perduijn, Analysis Strijkbuigen, TUE – WPA report 0767, Aug. 1989

[3] ir. Guo Minghai & ir. A.B.Perduijn, Elastic Springback and Residual Stresses in Sheet Bending Where $E=E(\bar{\epsilon})$, TUE – WPA report 0926, Aug. 1990