

A note on the shape of the shear plane in orthogonal cutting

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In the classical model of orthogonal cutting, as developed by M.E. Merchant, the shear-plane is represented by a flat. ~~Microstructure~~-photographs have shown that this assumption can be considered a reasonable approximation of reality. However, it can be proved that this model is inconsistent with the fact that the chip curls during the cutting-process.

In this paper an effort has been made to produce an extension of the Merchant model, assuming a curved chip and, consequently, a curved shear-plane.

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This derivation can be used to find a stress distribution on the shear plane and the working point of the normal force on the chip plane.

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A note on the shape of the shear-plane in orthogonal cutting.

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Summary

In the classical model of orthogonal cutting, as developed by M.E. Merchant, the shear-plane is represented by a flat. Microstructure-photographs have shown that this assumption can be considered a reasonable approximation of reality.

However, it can be proved that this model is inconsistent with the fact that the chip curls during the cutting-process.

In this paper an effort has been made to produce an extension of the Merchant model, assuming a curved chip and, consequently, a curved shear-plane.

The classical model.

To describe this model the following quantities are defined:

f = feed per rev.

t_1 = thickness of chip

d = depth of cut

d_1 = width of chip

alpha) α = normal side rake angle

(phi) φ = shear angle

v = cutting speed

v_T = speed of chip along the rake

delta) Δ = tensile strain parallel to the cutting edge

(Fig. 1.)

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The tensile strain in the direction parallel to the cutting edge, Δ , follows from:

$$d_1 = d(1+\Delta) \quad (1)$$

Further, by geometrical consideration, it can be seen (Fig.1) that:

$$\frac{t}{b_1} = \frac{t \cos(\varphi-\alpha)}{\sin \varphi} \quad (2)$$

The relation between v and v_T follows from the continuity equation for the material which passes along the shear-plane:

$$t \cdot d \cdot v = \frac{t}{b_1} \cdot d_1 \cdot v_T \quad (3)$$

Using (1) and (2) we find from (3):

$$v_T = \frac{v \sin \varphi}{(1+\Delta) \cos(\varphi-\alpha)} \quad (4)$$

Thus we find v_T as a function of φ .

When v_T does not vary in direction X , the chip will come off straight. We know, however, that this is not correct: The chip actually comes off in a curved shape. This is only possible when a speed-distribution $v_T = v_T(x)$ exists, thus, using (4): $v_T = v_T(\varphi)$, leading to $\varphi = \varphi(x)$. This implies a curved shear-plane.

The curved shear-plane model (Fig. 2)

We will assume a chip contact-length of l_c . The velocity-field is chosen in such a way that all along any line ABC which is pseudo-parallel to the chip-boundary, the speed of the chip is constant. We shall also assume that the curved part of the chip is a rigid body. If the outside radius of the chip is R , we find the pole M according to Fig. 2. The relation between $v_T(0)$ and $v_T(x)$ is then given by

$$v_T(x) = v_T(0) \frac{R - x \cos(\varphi-\alpha)}{R} \quad (5)$$

The assumed speed-distribution implies that the part ODEF cannot be entirely rigid. Actually, from experiments it is known that secondary plastic deformation does take place in that region, especially near the ~~chip~~ rake-plane of the rake

tool. To describe the shape of the shear plane, we superimpose a variable angle $\psi = \psi(x)$ on the constant angle φ . Then, localisation of (4) gives:

(Fig. 2.)

$$v_T(x) = \frac{v \sin[\varphi + \psi(x)]}{(1+\Delta) \cos[\varphi - \alpha + \psi(x)]} \quad (6)$$

Substituting $x = 0$ in (6) we obtain:

$$v_T(0) = \frac{v \sin[\varphi + \psi(0)]}{(1+\Delta) \cos[\varphi - \alpha + \psi(0)]} \quad (7)$$

(6) and (7) in (5) gives:

$$\frac{\sin[\varphi + \psi(x)]}{\cos[\varphi - \alpha + \psi(x)]} = \frac{R - x \cos(\varphi - \alpha)}{R} \cdot \frac{\sin[\varphi + \psi(0)]}{\cos[\varphi - \alpha + \psi(0)]} \quad (8)$$

The angle ψ is small. Thus we approximate:

$$\cos \psi \approx 1$$

$$\sin \psi \approx \psi \approx \tan \psi = \frac{dy}{dx} = y' \quad (9)$$

From (8) we can now easily derive:

$$\frac{\sin \varphi + y' \cos \varphi}{\cos(\varphi - \alpha) - y' \sin(\varphi - \alpha)} = \frac{R - x \cos(\varphi - \alpha)}{R} \cdot \frac{\sin \varphi + y'(0) \cos \varphi}{\cos(\varphi - \alpha) - y'(0) \sin(\varphi - \alpha)} \quad (10)$$

or:

$$y' = \frac{Ax + B}{Cx + D} \quad (11)$$

in which

$$\begin{aligned}
 A &= -\cos^2(\varphi - \alpha) \{ \sin \varphi + y'(0) \cos \varphi \} \\
 B &= y'(0) R \cos \alpha \\
 C &= -\cos(\varphi - \alpha) \sin(\varphi - \alpha) \{ \sin \varphi + y'(0) \cos \varphi \} \\
 D &= R \cos \alpha
 \end{aligned}
 \tag{12}$$

Integration of (11) gives:

$$y = \frac{A}{C} \cdot x + \frac{BC - AD}{C^2} \ln(Cx + D) + C_1 \tag{13}$$

with boundary conditions:

$$y(0) = 0 \quad \text{and} \quad y\left(\frac{l}{\sin \varphi}\right) = 0 \tag{13a}$$

By substitution of (13a) in (13) we find 2 equations, which enable us to solve the unknown quantities C_1 and $y'(0)$. Then the shape of the shear-plane is known.

We will illustrate this consideration with a numerical example:

$$\begin{aligned}
 \varphi &= 30^\circ \\
 \alpha &= 10^\circ \\
 R &= 5 \text{ mm} \\
 \frac{l}{\sin \varphi} &= 0.6 \text{ mm/rev.}
 \end{aligned}
 \tag{14}$$

From these assumed data, which may be measured from experiments, we calculate according to (12):

$$\begin{aligned}
 A &= -0.4415 - 0.7647 y'(0) \\
 B &= 4.9240 y'(0) \\
 C &= -0.1607 - 0.2783 y'(0) \\
 D &= 4.9240
 \end{aligned}
 \tag{15}$$

Substitution of (15) in (13) gives:

$$\begin{aligned}
 y &= 2.7475 x - \frac{4.9240 y'(0) - 13.5287}{0.2783 y'(0) + 0.1609} \cdot \\
 &\quad \cdot \ln[\{ -0.1607 - 0.2783 y'(0) \} x + 4.9240] + C_1
 \end{aligned}
 \tag{16}$$

The boundary conditions

$$y(0) = 0 ; \quad y(1.200) = 0 \quad (16a)$$

can now be substituted in (16). Then, we find;

$$0 = \frac{4.9240 y'(0) - 13.5287}{0.2783 y'(0) + 0.1607} \ln 4.9240 + C_1 \quad (17a)$$

$$0 = 3.297 - \frac{4.9240 y'(0) - 13.5287}{0.2783 y'(0) + 0.1607} \ln \{4.7312 - 0.3340 y'(0)\} + C_1 \quad (17b)$$

Elimination of C_1 from (17a) and (17b) gives:

$$\ln\{4.731 - 0.3340 y'(0)\} - \frac{8.7669 y'(0) - 21.0363}{4.9240 y'(0) - 13.5287} = 0 \quad (18)$$

(18) can be solved by successive approximation according to Newton.

Then (17a) gives C_1 . We find:

$$\begin{aligned} y'(0) &= 0.0576 \\ C_1 &= -119.4901 \end{aligned} \quad (19)$$

For the equation of the shear-plane, we find in this case, by substitution of (19) in (16):

$$y = 2.7475 x - 119.4901 + 74.9581 \ln(-0.1767 x + 4.9240) \quad (20)$$

From (11) we find

$$\psi = y' = \frac{-0.4856 x + 0.2836}{-0.1767 x + 4.9240} \quad (21)$$

Table I gives some local values of the actual shear angle:

(Table I)

Remark: It seems only correct to mention a limitation of this model; we have, indeed, neglected the fact that a temperature-distribution $T=T(x)$ exists, which will influence the value of R as long as the chip is in touch with the tool. After the chip has come off, we may measure a different outside radius, since the temperature-distribution then has disappeared.

Conclusion and prospects

We have shown that from experimental data the local value of the shear angle can be computed. Its variation may be in the order of $\pm 10\%$. In the existing literature on orthogonal cutting, based on the Merchant model, in many cases average stresses are calculated, dependent on the value of the shear angle.

With the help of this theory, these calculated values may be localised, thus giving a stress distribution. In practice this means, that the measurement of the radius R in addition to the usual quantities measured, enables us to find a stress distribution on the shear plane from a number of experiments which each individually provide us with values of average stresses. The solution will, however, be only kinematically admissible. If $\sigma_y(x)$ is the normal stress distribution on the shear-plane and P_T the normal pressure on the tool, we may find the working point of P_T using the equilibrium of moment of the chip round O :

$$\int_0^{a/\sin \phi} d \cdot \sigma_y(x) \cdot x \, dx = P_T \cdot b \tag{22}$$

where b is the distance between O and the working point of P_T . In (22) we may substitute either $d = \text{constant}$ or $d = d(x)$.

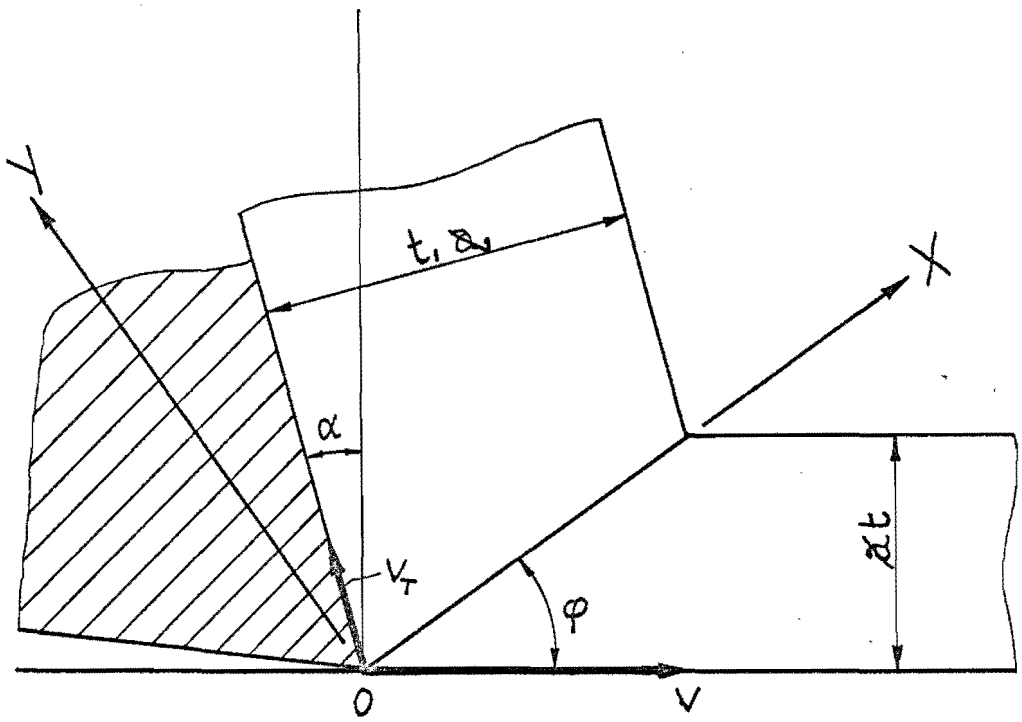


Fig. 1. The classical model of orthogonal cutting.

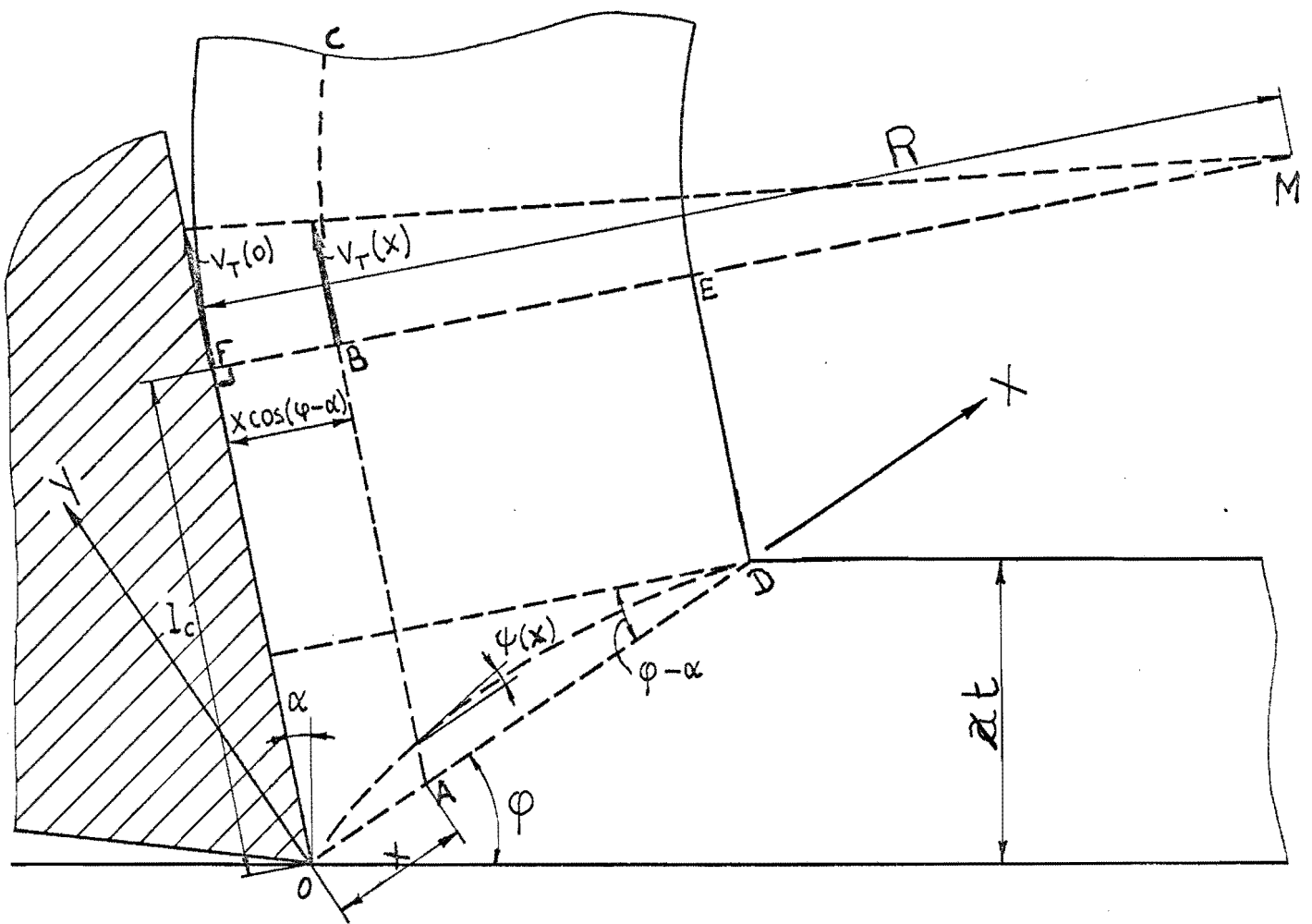


Fig. 2. The curved shear-plane model.

x (mm)	ϕ (rad)	ϕ degrees	shear angle
0	0.0576	3° 19'	33° 19'
0.2	0.0385	2° 13'	32° 13'
0.4	0.0184	1° 4'	31° 4'
0.6	-0.0014	-0° 5'	29° 55'
0.8	-0.0215	-1° 14'	28° 46'
1.0	-0.0415	-2° 22'	27° 38'
1.2	-0.0622	-3° 34'	26° 26'

Table I. Local values of the shear angle.