Towards a better patientsupport

Citation for published version (APA):

Document status and date:
Published: 01/01/1995

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:
• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.
Towards a better patientsupport,
Modelling and control of
an electro-hydraulic positioning device

A.M. van Beek

TUE, Department of Mechanical Engineering
WFW rapport 95.142
Abstract

The goal of this survey is to analyze the horizontal positioning function of the patientsupport in order to make recommendations for future mechanical and control improvements.

A model for the horizontal movement is constructed and used for analysis of the system. With the use of a more refined experimental set-up a number of unmodelled effects was found. These unmodelled dynamics inhibit the use of the present model for setting a PID-type controller or developing more advanced control concepts.

A first step towards an improved model was made by describing the observed effects and offering possible explanations or directions for model improvement. Considerable extra effort will be needed however if an adequate simulation model is required.

Rigid friction compensation, used in the original controller, results in over- or undercompensation (depending on the load for which the compensation is set) causing a jerky motion and overshoot and/or an unacceptable steady state-error.

Without compensation integral control is necessary to achieve the desired position accuracy. Classic integral control however, induces limit-cycling for high gains, $K_i$, or overshoot for lower gains.

Therefore two modified integrators were considered, an anti-windup integrator and a switching and resetting integrator. The results with this last type of controller are the most promising.

Compared to the original controller the controller with switching integrator is seen to have a higher positioning accuracy (although the guaranteed accuracy is at present only slightly higher), and results in a much smoother movement (lower accelerations).
# Contents

1 Introduction  

2 The horizontal table model  
   2.1 The servo valve model  
   2.2 Cylinder and supply hoses  
   2.3 Transmission and load  
   2.4 Friction  
      2.4.1 Friction measurements  
      2.4.2 Friction models  
   2.5 Model analysis and simulation  
      2.5.1 Simplified pressure equations  
      2.5.2 System dynamics with closed valve  
      2.5.3 Friction analysis  
   2.6 Friction compensation  
      2.6.1 Compensation techniques  
      2.6.2 Simulations with PID-type control  

3 A flexible control environment  
   3.1 Experimental set-up  
      3.1.1 Measurement devices  
      3.1.2 Data Acquisition  
      3.1.3 Data manipulation software  
      3.1.4 Command signal  

4 Model evaluation  
   4.1 Offset pressure difference  
   4.2 Pressure dynamics  
   4.3 Friction  
   4.4 Pressure deviations  

5 Simple horizontal control  
   5.1 Original controller  
   5.2 Control configuration  
   5.3 Input trajectory  
   5.4 New control strategy  
   5.5 Modified PID-control  

- iii -
CONTENTS

5.5.1 PID with anti-windup integrator .................... 39
5.5.2 PID with switching and resetting integrator .......... 49
5.5.3 PI with switching and resetting integrator .......... 44
5.6 Summary ............................................ 46

6 Conclusions and recommendations 48
6.1 Conclusions ........................................ 48
  6.1.1 Model ........................................ 48
  6.1.2 Mechanical improvements .......................... 48
  6.1.3 Control improvements ............................. 49
6.2 Recommendations ..................................... 49
  6.2.1 Mechanical improvements .......................... 49
  6.2.2 Control improvements ............................. 50

Bibliography ............................................. 51
A The nominal model parameters ........................ 52
B The model equations .................................. 53
C Simulink model ....................................... 55
Acknowledgements ....................................... 59
Chapter 1

Introduction

Magnetic Resonance Imaging  Philips Medical Systems (PMS) produces a wide variety of medical equipment. At the plant in Best two different kinds of medical systems are being produced and researched:

- X-ray systems and
- Magnetic Resonance Imaging (MRI) systems.

MRI is a relative new, non-invasive, diagnostic method which can provide detailed images of nearly every part of the human body. Basically an image is acquired by measuring the response of hydrogen atoms to high frequent magnetic fields.

![Figure 1.1: A typical MRI-system: magnet and patientsupport (the optional trolley is used to transport patients lying on the tabletop).](image)
Patientsupport  In order to scan a part of a patients body it has to be positioned in or the near the magnets isocentrum. Naturally, this must be done as comfortable as possible and absolutely safe. The positioning device to perform this function is called the patientsupport (see Fig. 1.1). It has two sub-functions:

- vertical positioning, the table height can be lowered down to 500 (mm) to facilitate easy access for the patient,
- horizontal positioning, after the table is raised again to the operational height of 890 (mm) the scan-motion cycle can begin:

  1. Select the part of the body to be scanned by moving the patient under a light visor,
  2. travel a preset distance (the Travel To Scan (TTS) distance) to the magnet isocentrum,
  3. if necessary (when the part of the body is too big) move over smaller distances set by the operator,
  4. perform an inverted Travel To Scan, and
  5. if desired (e.g. after injecting contrast fluid) return to the last position in the magnet.

Both sub-functions are driven by hydraulic cylinders and controlled by electro-hydraulic servo valves (see Fig. 1.2 for a more detailed view). In this report the attention is focused on the horizontal positioning functioning.

![Figure 1.2: A detailed view of the patientsupport (stretcher without covers).](image)

Accurate and smooth positioning  Although in practice the present patientsupport performs reasonably, the original design specifications, see Dusseldorp [6], are not met. Especially the required accuracy (±1 (mm)) and maximum acceleration (100 (mm/s²))
CHAPTER 1. INTRODUCTION

are exceeded in a number of situations. Note that in the present scan-motion cycle
the emphasis is on the relative accuracy rather than the absolute accuracy, i.e. a small
deviation is acceptable if it can be reproduced during the same scan-cycle. In future
applications, e.g. interventional applications\(^1\), however the absolute accuracy is expected
to become more important.

**Objective: ‘better patientsupport’** The goal of this survey is to analyze the hori-
zontal positioning function of the patientsupport in order to make recommendations for
future improvements. The immediate objective is ‘accurate and smooth positioning’, i.e.
- absolute accuracy: \( \pm 1 \text{ (mm)} \)
- maximum acceleration: \( 100 \text{ (mm/s}^2 \text{)} \).

Although we focus on improvements of the patientsupport control, the impact of simple
mechanical modifications will be considered as well. The starting-point for every control
revision is the existent configuration, i.e. measuring only the tabletop position. Ideally
the objective is achieved with only a software update.

**Outline of the report** In Chapter 2 the horizontal movement is modelled, initially to
gain an understanding of the system, to predict the impact of small modifications, and to
test simple control concepts. If simple control concepts can not achieve the desired per-
formance this simulation model can be used in future to develop and test more advanced
control concepts.

The set-up of a 'flexible control environment', i.e. data acquisition hard- and software
is described in Chapter 3.

This experimental set-up is also used for further model evaluation (Chapter 4). In
this chapter model imperfections are discussed and possibilities for model adjustment are
given.

Subsequently, a simple controller for the horizontal positioning function, a PI(D)-
controller with a switching integrator, is implemented and evaluated (Chapter 5). Finally,
in Chapter 6 the reached conclusions are summarized and recommendations for a possible
follow-up are given.

\(^1\)Interventional applications involve injecting substances or inserting materials inside the patients body
to support diagnosis or even as a means of treatment, e.g.
- placing a catheter for the injection of contrast fluid,
- using a laser for removing sick tissue (laser ablation).
Chapter 2

The horizontal table model

The power unit for both the horizontal and the vertical movement consists of a fixed displacement pump, a pressure relief valve and a reservoir. As the pump delivers fluid at a constant rate, the supply pressure $p_0$ is determined by the setting of the pressure relief valve. The flow is controlled by throttling the fluid passing through the variable orifices of a proportional control valve, thus converting potential (pressure) energy into kinetic energy.

![Diagram of system to be modeled: four-way valve, symmetrical cylinder and cable-transmission.](image)

When the valve, shown in Fig. 2.1, is displaced by $q_v$, the piston with mass $M_p$ will move at a velocity $q_p$. As a result the payload $M_l$ moves at a velocity $q_i$. The transmission between both masses consists of a steel cable.

For future reference an equilibrium state is defined, i.e.

$$
\begin{align*}
\ddot{q}_v &= 0 \\
\dot{q}_p &= 0 \\
\dot{q}_l &= 0 \\
\dot{p}_A &= 0 \\
\dot{p}_B &= 0
\end{align*}
$$

(2.1)

when all masses are in their mid-positions, i.e.

$$
\begin{align*}
q_v &= 0 \\
q_p &= 0 \\
q_i &= 0
\end{align*}
$$

(2.2)
and the pump is left switched on for a certain amount of time, so

\[ p_A = p_B = \frac{p_0}{2}. \]  

(2.3)

Furthermore, there is an initial force \( F_c \) in the cable transmission due to the initial length of spring \( c \).

In the following first three sections the model equations will be derived for each of the components: the servo valve, the cylinder and the combination of transmission and load. In section 2.4 the modelling of friction on load and cylinder piston will be addressed. Finally the influence of several parameters on the resulting model will be considered and simplifications will be made (section 2.5).

2.1 The servo valve model

The electro-hydraulic servo valve consists of a piston which is positioned by an electromagnet and opposing spring, i.e. when an electric current is fed to the spool the valve-piston is displaced over a distance \( q_u \), until the resulting Lorenz force equals the spring force.

Modelling the valve the following assumptions are made:

- friction in the valve can be neglected, and
- the density of the oil, \( \rho \), is independent of both temperature and pressure variations across the valve.

Valve piston equation A model of the valve sub-system can be obtained using experimental data supplied by the manufacturer. For now a second order model is assumed to be sufficiently accurate, i.e.

\[ \ddot{q}_v + \omega_v^2 q_v + 2\beta_v \omega_v \dot{q}_v = \omega_u^2 K_u u \]  

(2.4)

where

- \( \omega_v \) is the natural frequency of the valve-piston,
- \( \beta_v \) the damping factor of the valve-piston,
- \( u \) the input variable, i.e. the electrical current to the servo-valve, and
- \( K_u \) the gain factor.

Depending on the dynamics of the cylinder and the combination of transmission and load the valve model can be simplified, resulting in a first order or even a proportional model.

Flow equations

A fairly general relationship between valve opening, pressure drop and controlled flow \( Q \) in a throttle flow valve is, for turbulent flow, given by the equation

\[ Q = C_d \sqrt{\frac{2}{\rho} a \text{ sign}(p_2 - p_1) \sqrt{|p_2 - p_1|}}. \]  

(2.5)

where

\[ p_1 \xrightarrow{a} \frac{p_1 + p_2}{2} \xrightarrow{p_2} Q \]
2.1 The servo valve model

$C_d \quad \text{is the coefficient of discharge (determined by valve dimensions),}$

$a \quad \text{the orifice area, and}$

$p_2 - p_1 \quad \text{the pressure drop across the orifice.}$

In a servo valve the orifice area for both in- and outlet flows can be adjusted by displacing the valve piston, as shown in Fig. 2.2.

![An overlapped servo valve.](image)

Figure 2.2: An overlapped servo valve.

In the flow equations for the considered system new ‘valve factor’ is introduced, given by

$$C_v = C_d \sqrt{\frac{2}{\rho} a(q_v) \text{sign}(q_v)}. \quad (2.6)$$

For an ideal valve the orifice area $a$ and therefore $C_v$ is a function of the valve displacement $q_v$. If the valve has an overlap $q_{vo}$ (see Fig's 2.2 and 2.3) a dead zone is introduced and $C_v$ is a function of $q_v - q_{vo}$.

![The valve factor $C_v$ for an overlapped servo valve.](image)

Figure 2.3: The valve factor $C_v$ for an overlapped servo valve.

As can be seen in Fig. 2.3 there are 5 different ranges for the valve displacement $q_v$ and thus the valve factor $C_v$.

Now the relations for the in- and outlet flows $Q_A$ and $Q_B$ to and from the cylinder

---

- 6 -
2.2 Cylinder and supply hoses

can be given for each of the 5 different ranges of the valve displacement $q_v$, i.e.

$$
\begin{align*}
-(q_{vo} + w_v) & \leq q_v \leq -(q_{vo} + w_v); & C_v = -C_{v_{max}}; & Q_A = C_v\sqrt{p_A - p_r} \quad \text{and} \\
-q_{vo} & \leq q_v \leq q_{vo}; & C_v = 0; & Q_A = 0 \quad \text{and} \quad Q_B = 0; \\
q_{vo} & \leq q_v \leq q_{vo} + w_v; & C_v = \frac{q_{vo} + w_v}{w_v} C_{v_{max}}; & Q_A = C_v\sqrt{p_0 - p_A} \quad \text{and} \\
q_{vo} + w_v & \leq q_v; & C_v = C_{v_{max}}; & Q_B = C_v\sqrt{p_B - p_r}.
\end{align*}
\tag{2.7}
$$

2.2 Cylinder and supply hoses

With respect to the cylinder and its supply hoses (see Fig. 2.4) the following assumptions are made:

- The influence of temperature variations on the density, $\rho$, of the oil can be neglected and, $\rho$ only depends on the pressure $p$, i.e. $\rho = f(p)$. This implies that

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{p}}{E_{oil}}; \quad \frac{1}{E_{oil}} = \frac{1}{\frac{df}{dp}}$$
\tag{2.8}

where $E_{oil}$ can depend on $p$. According to Backé [2] the relation is

$$E_{oil}(p) = 9 \times 10^8 \ln \left( \frac{9}{28 \times 10^5} p + 3 \right),$$
\tag{2.9}

where $E_{oil}$ and $p^1$ are both in SI-units ($N/m^2$),

- There is no side leakage (out of the cylinder),

- The pressure drop over the supply hoses can be neglected, and

- The volume $V_h$ of the supply hoses changes linear with the pressure $p$, i.e.

$$V_h(p) = V_h(p_{atm}) + C_h l_h (p - p_{atm}),$$
\tag{2.10}

where

- $p_{atm}$ is atmospheric pressure,

- $C_h$ the hydraulic capacity (in $m^3/Pa_m$), and

- $l_h$ the length of the hose (the hoses have equal lengths).

Cylinder piston equation From Fig. (2.5) it is readily seen that the motion of the piston $M_p$ can be described by

$$M_p \ddot{q}_p = A_c (p_A - p_B) + F_{ex} - F_{fp}; \quad |q_p| \leq \frac{\text{cylinder stroke}}{2},$$
\tag{2.11}

\(^1\)In section 2.5 will be shown that the pressure dependency of $E_{oil}$ can be neglected as well. Hence the constant value for $E_{oil}$ in Appendix A.
2.2 Cylinder and supply hoses

where

- \( M_p \) is the mass of the cylinder-piston,
- \( A_c \) the effective cylinder-piston area,
- \( F_{ex} \) forces, exerted by the cables, and
- \( F_{fp} \) the friction force.

**Pressure equations** The pressure equations describing \( p_A \) and \( p_B \) can be derived using the mass balances on the cylinder chambers and the supply hoses from valve to cylinder and back (see Fig. 2.4).

![Figure 2.4: The cylinder, simplified.](image)

The mass balance for side A results in

\[
V_A(0)\rho_A(0) + \int_0^t \rho_A(t)Q_A(t)\,dt - \int_0^t \rho_A(t)Q_i(t)\,dt = V_A(t)\rho_A(t).
\] (2.12)

The total volume under compression \( V_A(t) \) is given by \( V_0 + A_cq_p + C_hl_hp_A \) where \( V_0 \) is the total volume when \( q_p = 0 \) and \( p_A = 0 \) (including the volume of the supply-hose from valve to cylinder). Differentiation of Eq. (2.12) with respect to time and substitution of Eq. (2.8) yields, after some trivial elaboration

\[
\dot{p}_A = \frac{E_{oil}}{V_0 + A_cq_p + C_hl_h(E_{oil} + p_A)}(-A_c\dot{q}_p + Q_A - Q_i).
\] (2.13)

Similarly for \( \dot{p}_B \) can be written

\[
\dot{p}_B = \frac{E_{oil}}{V_0 - A_cq_p + C_hl_h(E_{oil} + p_B)}(A_c\dot{q}_p - Q_B + Q_i).
\] (2.14)
2.3 Transmission and load

The following assumptions are made:

- no friction in the bearings of the pullies,
- the moment of inertia of the wheels is negligible, and
- Coulomb friction between piston and cylinder is small compared to the initial force $F_{cr}$.

![Diagram of forces and displacements.]

Figure 2.5: Forces and displacements.

The external force $F_{ex}$ in Eq. (2.11) is given by $F_{ex} = 4(F_B - F_A)$ where $F_A$ and $F_B$ are the cable forces left and right of the load $M_l$.

From Fig. (2.5) it is readily seen that the motion of the load $M_l$ can be described by

$$M_l q_t = F_A - F_B - F_{ft} = -\frac{F_{ex}}{4},$$

(2.15)

where $F_{ft}$ is the friction force between the load and the support.

The cable forces For the moment it is assumed that in all relevant situations both cable parts, $l_A$ and $l_B$, remain stressed, i.e. $F_A > 0$ and $F_B > 0$. When the transmission is rigid (no spring and a rigid cable) the motion of piston and load are directly related: $q_l = 4 q_p$. When the flexibility of the transmission is taken into account then

$$q_l - 4 q_p = -\Delta l_A = \Delta l_B + \Delta l_c,$$

(2.16)

where $\Delta l_A$, $\Delta l_B$ and $\Delta l_c$ are elongations of, respectively, the cable parts and the spring $c$ with respect to reference lengths determined by the initial force $F_{cr}$ in the spring, i.e.

$$\Delta l_A = \frac{1}{k_A}(F_A - F_{cr}); \quad k_A = \frac{EA_c}{l_{A_0}},$$

(2.17)

$$\Delta l_B + \Delta l_c = \left(\frac{1}{k_B} + \frac{1}{c}\right)(F_B - F_{cr}); \quad k_B = \frac{EA_c}{l_{B_0}}.$$
2.3 Transmission and load

Combination of Eq's (2.16), (2.17) and (2.18) and use of \( l_{A_0} = l_{B_0} \) yields

\[
F_A = -k_A(q_l - 4q_p) + F_{c_s},
\]

\[
F_B = \frac{k_Bc}{k_B + c}(q_l - 4q_p) + F_{c_s}.
\]

It is emphasized that these relations hold only if \( F_A > 0 \) and \( F_B > 0 \).

Most of the model parameters, introduced in this and preceding subsections, can be determined from information supplied by the manufacturers of the various components (see Appendix A for the numerical values).

**Transmission stiffness measurements** For accurate positioning the transmission stiffness is an important and possibly limiting parameter. Therefore a few simple experiments were performed to verify the transmission model. Because both cable parts are practically equal in length \( k_A = k_B = k \) the model transmission (output) stiffness can be defined by

\[
c_t = \frac{F_B - F_A}{q_l - 4q_p} = \frac{k^2 + 2kc}{k + c}.
\]

With \( c = 2.94 \times 10^4 \) and \( k = 3.36 \times 10^5 \) from Appendix A the expected transmission stiffness \( c_t = 3.63 \times 10^5 \).

**Experimental set-up:** With a hand-held force transducer (accuracy ±0.1 (N)) a force is exerted on the tabletop in different positions. The resulting displacement is measured (accuracy ±0.1 (mm)). Measurements:

1. force on the table against the position when the piston is fixed against the left- or right cylinder wall, as a result the transmission stiffness is measured,
   a. stiffness of a transmission with a standard cable,
   b. stiffness of a transmission with a cable with double diameter,

2. force on a single cable part against the position in a set-up outside the patientsupport, as a result only the cable stiffness is measured.

In Fig. 2.6 the result is displayed of a typical stiffness measurement.
2.3 Transmission and load

![Figure 2.6: Typical result for the stiffness measurements (here experiment 1.b).](image)

Table 2.1: Stiffness measurements.

<table>
<thead>
<tr>
<th>Stiffness (N/m)</th>
<th>transmission measurements 1.a and b</th>
<th>cable measurement 2</th>
<th>transmission $c_t$ (Eq. 2.21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard cable</td>
<td>$3.7 \times 10^4$</td>
<td>$3.1 \times 10^5$</td>
<td>$3.63 \times 10^5$</td>
</tr>
<tr>
<td>doubled cable diameter</td>
<td>$9.4 \times 10^4$</td>
<td></td>
<td>$1.46 \times 10^6$</td>
</tr>
</tbody>
</table>

In Table 2.1 the (average) results of experiments are summarized. In both directions, so independent from the reference spring $c$, the transmission stiffness is much lower then expected. Apparently other components (e.g. frame, pulleys), individually stiffer than the cable, together result in a $10 \times$ lower stiffness. Therefore using a stiffer cable (e.g. increasing the diameter) has only a limited effect.

In future simulations the stiffness will be set to $c_t = 3.7 \times 10^4$. 
2.4 Friction

Another important factor expected to determine the system's behaviour is friction. Several simple experiments were performed in order to determine and identify an appropriate friction model. Subsequently, a simulation model is chosen.

2.4.1 Friction measurements

Experimental set-up:
With a hand-held force transducer (accuracy ±0.1 (N)) a force is exerted on the shuttle or the tabletop\(^2\) (with different loads). The resulting displacement is measured with a cable potentiometer (accuracy ±0.6 (mm)). The cylinder bypass, directly connecting the cylinder chambers, is opened.

Measurements:

1. force on the uncoupled table against table position,
2. force on the coupled table against table position,
3. force when the shuttle starts moving,
4. force when the uncoupled table starts moving.

Note that in this set-up the influence of dither can not be measured since the oil flows do not pass through the valve. Measurement 2 but especially measurement 3 therefore represent worst-case situations, i.e. the actual friction could be smaller.

![Figure 2.7: Experiment 1 and 2: force against table position for the uncoupled (left) and the coupled table (right) and for three different total (mass tabletop included) loads.](image)

In Fig. 2.7 some results of experiment 1 and 2 are plotted. In both experiments peaks in the force occur at about 500 and 850 (mm). These peaks arise when a new pair of wheels has crossed the gap between patientsupport and magnet.

\(^2\)the tabletop can be uncoupled by lifting it from the shuttle which is directly connected to the cable.
2.4 Friction

In Fig. 2.8 the average force needed to move the tabletop (coupled and uncoupled) is plotted against the total load. Fitting lines through the data, it can be seen that the total friction of the system in motion can be modelled by Eq. (2.22).

\[ F_I = \text{sign}(\dot{q}_p) F_{I_p} + \text{sign}(\dot{q}_l) F_{I_l} = \text{sign}(\dot{q}_p) 45 + \text{sign}(\dot{q}_l) 0.04 M_I g. \]  

(2.22)

**Velocity dependency** When the shuttle starts moving (experiment 3) a fall of approximately 5 (N) in the force occurs. Stick-slip friction measurements on the decoupled table (experiment 4) result in range of values for the fall in friction when the tabletop starts moving: for loads varying from 17 to 107 (kg) the difference between stick and slip friction increases from approximately 2 to 10 (N).

In neither cases a rise in force is observed when the velocity is increased.

2.4.2 Friction models

Based on the measurements now a theoretical model and an appropriate simulation model are chosen. These models can be used to study the friction mechanism by means of simulation.

In literature (e.g. Armstrong-Hélouvry et al. [1]), a wide variation of friction models can be found. When time dependency is left out of consideration there are three basic models (see Fig. 2.9):

1. coulomb friction,
2. coulomb and static friction, and
3. coulomb and stribeck friction.
2.4 Friction

Modelling the horizontal movement it was assumed that friction forces occur at only two locations, namely acting on the cylinder-piston and on the load. The friction measurements suggest that

- stick-slip occurs at both locations, and
- during slip there is no significant rise in friction force when the velocity is increased.

Based on the simple experiments model 2 coulomb and static friction is chosen to model friction.

Simulation model  By adding this friction model to the model for the horizontal movement discontinuities, which generally occur inside integration subintervals, are introduced at zero velocity. To solve this discontinuous differential equations an integration method with a small or variable step size is required. Consequently the simulation time rises, especially at near zero velocity where a continuous switching of the friction force can occur.

One possible method to overcome these simulation problems is the use of Karnopp's friction-velocity model [7], given by:

\[
F_f(\dot{x}, F_{\text{external}}) = \begin{cases} 
\text{sign}(\dot{x}) F_{\text{slip}} & |\dot{x}| > D_v \\
\text{sign}(F_{\text{external}}) \max(F_{\text{external}}, F_{\text{stick}}) & |\dot{x}| \leq D_v,
\end{cases}
\]

where

\[
F_{\text{external}} \rightarrow \dot{x} \quad F_f \quad F_{\text{slip}} < F_{\text{stick}}
\]

A small neighbourhood of zero velocity is defined by \(D_v\). Outside this neighbourhood, friction is a function of velocity. Inside velocity is considered to be zero and friction is
force dependent, i.e. the external force can increase until the stick force is reached. In Fig. 2.10 the block diagram of this friction model is given.

![Friction Model Diagram](image)

Figure 2.10: Karnopp’s friction-velocity model.

In a coulomb and static friction model there are three parameters: the slip friction force, \( |F_{\text{slip}}| \), the stick friction force, \( |F_{\text{stick}}| \), and the band of zero velocity \( D_v \). Note that this simulation model could also be used for coulomb friction (\( F_{\text{stick}} = F_{\text{slip}} \)) or coulomb and stribeck friction (\( F_{\text{slip}} = F_{\text{slip}}(v) \)).

Assuming there are two locations where friction occurs, namely on the cylinder piston and on the load, the following parameter values are chosen:

- friction between cylinder and piston,
  \[
  |F_{\text{slip}}| = 4 \times 40 \ (N) \\
  |F_{\text{stick}}| = 4 \times 40 + 4 \times 5 \ (N)
  \]
  (the force was measured at the output but occurs in the cylinder, hence the factor 4).

- friction between load (tabletop) and patientsupport,
  \[
  |F_{\text{slip}}| = 0.04 M_I \ g \ (N) \\
  |F_{\text{stick}}| = 0.04 M_I \ g + (2 + 0.009 M_I g) \ (N)
  \]

- band of zero velocity, \( D_v = 1.0 \times 10^{-5} \ (m/s) \).

### 2.5 Model analysis and simulation

With the use of the derived model now the influence of several model parameters on the system behaviour can be studied in detail. Important aspects of the system behaviour are:

- the lowest eigenfrequency (subsection 2.5.2), because of its limiting effect on the bandwidth of the controlled system, and

- stick-slip friction (subsection 2.5.3), which can cause large steady-state errors, large break-away accelerations, and limit-cycling, i.e. a self-sustained vibration with a constant amplitude.

Where possible, model simplifications will be made. Unless otherwise mentioned, the default parameters from Appendix A are used.
2.5 Model analysis and simulation

2.5.1 Simplified pressure equations

Before further analysis, a simplification is made. In the denominators of the pressure equations the pressures $p_A$ and $p_B$ are significantly smaller than the oil modulus, $E_{oil}$, i.e.

\[
\max(p_A, p_B) = p_0, \quad \frac{p_0}{E_{oil}} \leq 0.0067,
\]

so

\[
\dot{p}_A = \frac{E_{oil}}{V_0 + A_c q_p + C_h l_h E_{oil}} \left( -A_c \dot{q}_p + Q_A - Q_l \right),
\]

\[
\dot{p}_B = \frac{E_{oil}}{V_0 - A_c q_p + C_h l_h E_{oil}} \left( A_c q_p - Q_B + Q_l \right).
\]

(2.24)

2.5.2 System dynamics with closed valve

When the valve is closed ($Q_A = Q_B = 0$) a further simplification can be made, resulting in a linear sub-system. The stiffness and eigenfrequencies of this system can be expressed in the various model parameters in order to illustrate their influence on the system behaviour. Defining the 'load' pressure $p_L = p_A - p_B$ and

- with closed valves, $Q_A = Q_B = 0$,
- without leakage, $Q_l = 0$, and
- without friction, $F_{jp} = F_{j1} = 0$,

the load and system dynamics (see Appendix B) reduce to

\[
M_p \ddot{q}_p - A_c p_L(q_p, \dot{q}_p) - 4 c_1(q_l - 4q_p) = 0,
\]

\[
M_l \ddot{q}_l + c_1(q_l - 4q_p) = 0,
\]

(2.25)

where

\[
\dot{p}_L(q_p, \dot{q}_p) = \frac{-2A_c}{V_0 + C_h l_h} \left( 1 - \frac{A_c q_p}{V_0 + C_h l_h E_{oil}} \right)^2 \dot{q}_p = -\xi(q_p) \dot{q}_p.
\]

(2.26)

Because the influence of the piston position $q_p$ on $\xi$ is small, \(\frac{A^2 \dot{q}_p^2}{(V_0 + C_h l_h E_{oil})^2} \leq 0.07\), $\xi$ can be approximated by a constant, i.e.

\[
\xi = \frac{2A_c}{V_0 + C_h l_h}.
\]

(2.27)

Initially the system is in the equilibrium state, i.e. $p_L(0) = q_p(0) = 0$, so $p_l = \xi q_p$, thus resulting in the following linear sub-system

\[
M_p \ddot{q}_p + A_c \xi q_p - 4 c_1(q_l - 4q_p) = 0,
\]

\[
M_l \ddot{q}_l + c_1(q_l - 4q_p) = 0.
\]

(2.28)

The eigenfrequencies of this system are

\[
\omega = \frac{1}{\sqrt{2}} \sqrt{-\frac{c_1}{M_l} - \frac{16c_1}{M_p} - \frac{A_c \xi}{M_p}} \pm h,
\]

(2.29)
2.5 Model analysis and simulation

where

\[ h = \frac{\sqrt{256 c_t^2 M_i^2 + 32 A_c c_t \xi M_i^2 + A c^2 \xi^2 M_i^2 + 32 c_t^2 M_i M_p - 2 A_c c_t \xi M_i M_p + c_t^2 M_p^2}}{M_i M_p}, \]

so

\[ f_t = \frac{\text{min}(\omega)}{2\pi} \quad \text{and} \quad f_h = \frac{\text{max}(\omega)}{2\pi}. \]

Only for qualitative insight, two simple ‘textbook approximations’ can be made

\[ f_{t_a} = \frac{1}{2\pi} \sqrt{\frac{c_t}{M_i}} \quad \text{and} \quad f_{h_a} = \frac{1}{2\pi} \sqrt{\frac{A c \xi}{M_p}}. \]

In Fig. 2.11 the influence of 4 model parameters on \( f_t \) and \( f_h \) is considered:

- Table-load \( M_i \) (plot A and B),
  if the load is increased from 17 to 167 (kg) \( f_t \) (A) decreases from 6.25 to 1.8 (Hz). Load variation has no influence on the hydraulics, so \( f_h \) (B) remains constant.

- Transmission stiffness \( c_t \) (plot C and D),
  if the stiffness is varied between \( 0.5 \times 3.7 \times 10^4 \) and \( 2 \times 3.7 \times 10^4 \) (N/m) a small increase in \( f_t \) (C) is seen (1.4 to 2.2 (Hz)). The highest eigenfrequency, \( f_h \) (D), rises from 125 to 165 (Hz).

- Hydraulic capacity \( C_h \) (plot E and F),
  the hydraulic capacity is varied between 0 (rigid hoses) and \( 2 \times 2.18 \times 10^{-13} \) (m\(^4\)/N). The lowest eigenfrequency, \( f_t \) (E) drops from 2.15 to 1.6 (Hz), \( f_h \) (F) drops from 220 to 120 (Hz).

- Pressure \( p \) (plot G and H),
  The pressure influence arises from the pressure dependent \( E_{cil} \) in Eq. (2.8). The effect is small, \( f_t \) (G) varies between 1.78 and 1.87 (Hz), \( f_h \) (H) between 136 and 147 (Hz).

**Figure 2.11:** The eigenfrequencies, \( f_t \) and \( f_h \), and their approximations, \( f_{t_a} \) and \( f_{h_a} \) for variations in load \( M_i \) (A and B), transmission stiffness \( c_t \) (C and D), hydraulic capacity \( C_h \) (E and F) and the pressure \( p \) (G and H). The dotted vertical lines represent the default parameters.
2.5 Model analysis and simulation

To show the value of the simple approximations they are also plotted in Fig. 2.11. Some parameter influences are obviously not described by the approximation (plot B, D, E and G). Note especially the deviation in lowest eigenfrequency vs. $c_t$ (C).

Conclusions

- Besides the practical difficulties to increase the transmission stiffness its influence on $f_i$ is small (especially when compared with the textbook approximation).
- When the valve is directly placed on the horizontal valve, and thus the flexibility of the supply hoses has no influence on the dynamics ($C_h = 0$), a small increase (1.8 to 2.2 (Hz)) in $f_i$ is expected.
- The influence of pressure variations on $E_{coil}$ and thus on the eigenfrequencies is very small. Therefore $E_{coil}$ is set to a constant value (see Appendix A).

2.5.3 Friction analysis

At low velocities stick-slip is known to cause a phenomenon called limit-cycling, i.e. a self-sustained vibration with a constant amplitude. This phenomena will be examined by means of simulation. The objective is to predict for which conditions limit-cycling will occur and to determine what can be done to prevent or reduce this effect.

All simulations presented in this subsection are without control. The input current $I_c$ is chosen to achieve, in steady state and without feedback, the following load velocity trajectory:

$$\dot{q}_{\text{nom}}(t) = \begin{cases} 
0.1t & 0.1 t < \dot{q}_{\text{max}} \\
\dot{q}_{\text{max}} & 0.1 t \geq \dot{q}_{\text{max}} 
\end{cases} \quad (2.33)$$

![Diagram showing stick-slip simulation](image)

*Figure 2.12: Stick-slip simulation for $\dot{q}_{\text{max}} = 10 \times 10^{-3} \text{ m/s}$. In the top plot (from left to right): $q_{\text{nom}}$, $4 \times q_p$ and $q_l$, in the middle: $\dot{q}_l$ and at the bottom: $\ddot{q}_l$. *
In Fig. 2.12 it can be seen that for a load $M_l = 167$ (kg) and moving at a nominal velocity of approximately 10 (mm/s) limit-cycling occurs. The load moves in 'stick-slip steps', $\Delta q_l$, of approximately 5 (mm) and with a frequency, $f_{ss}$ ($= \frac{1}{T_{stick}} + \frac{1}{T_{slip}}$) of 1.8 (Hz).

Note furthermore the high break-away accelerations when the load starts moving and the relative small effect of piston friction on the load trajectory (Fig. 2.12, bottom plot). Neglecting coulomb and/or static friction between piston and cylinder has no significant influence on the occurring stick-slip.

The accompanying forces on the piston and the tabletop are plotted in Fig. 2.13.

Various simulations were carried out to examine the influence of several conditions and model parameters on $\Delta q_l$ and $f_{ss}$. In Fig. 2.14 the results are summarized for variations in:

- Tabletop velocity, $\dot{q}_{l_{max}}$,
  with increasing velocity the stick-slip step increases until at a certain critical velocity, $\dot{q}_{lc}$, the load is in constant slip. With default parameters $\dot{q}_{lc} \approx 20$ (mm/s).
  The stick-slip frequency, $f_{ss}$, rises quickly to approximately $f_1$, the lowest eigen-frequency. The two components which determine this frequency, $T_{stick}$ and $T_{slip}$, however are not constant, i.e. the period of stick decreases with increasing velocity and subsequently the period of slip increases (not shown in Fig. 2.14).

- Tabletop load, $M_l$, with increasing loads $\Delta q_l$ increases, $f_{ss} \approx f_1$.

- Transmission stiffness, $c_t$, increasing the transmission stiffness by a factor 5 results in a slightly smaller stick-slip step, again $f_{ss} \approx f_1$.

- Fall in friction, $F_{stick} - F_{slip}$, doubling the difference between stick and slip leads to an increase in the stick-slip
2.5 Model analysis and simulation

step of 1 (mm), again \( f_{ss} \approx f_1 \).

![Graphs showing influence of nominal velocity, table load, transmission stiffness, and stick-slip frequency.]

Figure 2.14: simulation results: simulated (*) and predicted (o) stepszie and simulated stick-slip frequency (x).

Because the stick-slip frequency approaches the lowest eigenfrequency \( \frac{f_{stick}}{f_{slip}} \) is small, the 'stick-slip step' can be approximated by

\[
\Delta q_l \approx \frac{\dot{q}_{nom}}{f_1},
\]

(2.34)

(see Fig. 2.14). Qualitative the influence of nominal velocity, load and stiffness on the slip-stick step height is given by

\[
\Delta q_l \approx \dot{q}_{nom} \sqrt{\frac{M_i}{c_t}}
\]

Conclusions

- Below a certain critical velocity a stick-slip limit cycle is expected.
- If a stick-slip limit-cycle occurs the stick-slip amplitude can be reduced by increasing the stiffness. The effect of increased transmission stiffness however is limited.
- There are two options to avoid limit-cycling completely:
  - stay above the critical velocity, or
2.6 Friction compensation

- reduce the critical velocity.

Note that in both cases stick will still occur at the beginning and the end of every movement.

2.6 Friction compensation

In a control loop stick-slip can be a limiting factor in order to achieve accurate and smooth positioning. Possible problems are:

- large steady-state errors,
- large breakaway accelerations, and
- limit-cycling, a self-sustained vibration.

A selection of compensation techniques, found in literature, has been made. After briefly discussing the selected techniques, a choice for a first approach is made and tested by means of simulation.

2.6.1 Compensation techniques

1. Friction problem avoidance

   - Mechanical design:
     From Eq. (2.34) it can be seen that the stick-slip step can be reduced by increasing the transmission stiffness. The reduction however is small and probably requires a complete redesign of the stretcher (a stiffer cable is not enough).

2. Non-model-based compensation

   - PID-control:
     By increasing the damping or the stiffness of a system, stick-slip can be reduced. In a control context, this can be accomplished by PD control.
     For a PD-controlled single mass system Derjaguin et al. [5] illustrate this with an explicit relation for the critical velocity,

\[
\dot{q}_c = \frac{\Delta F_c}{\sqrt{2\pi m\omega k_v}}
\]  

(2.35)

where
\[
\Delta F_c = F_{stick} - F_{slip},
\]

\( m \) is the system mass,
\( \omega = \sqrt{\frac{b_v}{m}} \) the natural eigenfrequency, and
\( k_v \) velocity feedback or damping.

Indeed \( \dot{q}_c \) decreases if the stiffness, \( k_v \), but especially if the damping, \( k_v \) is increased.

Integral control on the other hand, although needed to achieve accurate positioning, can aggravate or even induce limit-cycling. To overcome limit cycling, one standard technique is to employ a dead-band as the input to the integrator block. In that case a, small, steady state-error is unavoidable.
2.6 Friction compensation

- Dither:
  Dither is a standard technique to reduce the influence of stick-slip friction in hydraulic servo systems. By adding a high frequency signal to the command the discontinuity of friction at low velocity is smoothened.

3. Model-based compensation

- Coulomb friction feedforward/feedback:
  The main drawback on this technique is the dependency on a correct model. Both structure and parameters have to be correct.

- Adaptive friction feedforward/feedback:
  By online identification the problem of parameter uncertainty can be overcome.

Initially non-model based compensation, i.e. controllers of a PID-type will be explored. Furthermore the dither present in the original controller will be maintained, i.e. \( f_{\text{dither}} = 205 \) (Hz), amplitude 6 (mA).

2.6.2 Simulations with PID-type control

For a first impression of the performance of a PID-type controller two simulations were performed.

In Fig. 2.15 the results of PD control for the load velocity trajectory of Eq. (2.33) are shown. Compared to the simulation without control (Fig. 2.12) the stick-slip steps have been reduced by almost 50%.

![Figure 2.15: Stick-slip simulation for \( q_{\text{max}} = 10 \times 10^{-3} \text{ m/s} \) with PD-control (\( K_p = 100 \) and \( K_d = 100 \)). In the top plot: \( q_{\text{max}} \) (the straight dashed curve), \( 4q_p \) and \( q_l \), in the middle: \( q_l \), and at the bottom: \( \dot{q} \).]
A second simulation aims to demonstrate a possible effect of adding an integrator, i.e.
hunting: a limit-cycle around the desired end-position.
The results are plotted in Fig. 2.16. After 6 (s) the position error is seen to oscillate
around zero.

![Simulation with a PID-controller (K_p = 60, K_i = 2.4 and K_d = 0.1042). Left: position, velocities and accelerations, right: position error and velocity error.](image)

**Figure 2.16:** Simulation with a PID-controller ($K_p = 60$, $K_i = 2.4$ and $K_d = 0.1042$). Left: position, velocities and accelerations, right: position error and velocity error.

Note that a different input trajectory was used. The choice of this input trajectory and
a more rigorous approach to come to a PID-type controller will be discussed in Chapter 5.
Up to now two conclusions can be drawn from this two simulations:

- PD-control can reduce the stick-slip step, and
- when an integrator is added hunting can occur.
Chapter 3

A flexible control environment

In order to develop and evaluate a new or improved control concept an experimental set-up has been realized in which a computer, equipped with a data acquisition (DAQ) board is used to control and/or monitor a selection of system states of the patientsupport.

3.1 Experimental set-up

The experimental set-up is equipped with a number of sensors whose analogue voltage signals are conditioned and sampled (AD conversion) by a Data Acquisition system. The digitized data can be manipulated by a computer program, i.e. scaling, storage for later analysis and/or on-line calculation of a command signal.

If desired the calculated (digital) command signal is converted into an analogue voltage signal (DA conversion) which in turn is converted to an appropriate input quantity for the system (a valve current).

Figure 3.1: The experimental set-up to be used for simple horizontal control.
3.1 Experimental set-up

3.1.1 Measurement devices

Several sensors were used for control and monitoring purposes:

1. an encoder (the standard-sensor to measure the table position) in combination with a separate DA converter,
2. a combined cable potentiometer and tachometer,
3. two pressure sensors, and
4. an accelerometer.

The experimental set-up to be used for simple control of the horizontal movement (Chapter 5) is shown in Fig. 3.1. Note that here the table position is measured with two devices but that only the analogue cable sensor is used for feedback of the table position and velocity. The accelerometer (not shown in Fig. 3.1) is used only for validation purposes.

**Encoder and separate DAC** The encoder (without separate DAC) is the only sensor which is present in the original control configuration. When the tabletop moves horizontally a chain wheel runs against a ribbled tape mounted on its backside. The revolutions of this chain wheel are measured by an optical encoder. Every encoder pulse corresponds to a table movement of 0.049251 (mm).

For practical reasons it was not possible to send the encoder pulses directly to the DAQ-board. So a separate 16 bits DA converter (10 V supply voltage) was used to derive an analogue position signal which in turn could be fed to the AD-converters of the DAQ-board. In short:

\[
table \text{ position (in } mm) = DC \text{ out-voltage encoder} \& DAC \times \frac{2^{16}}{10} \times 0.049251. \quad (3.1)
\]

The encoder accuracy was measured with a laser-interferometer (see Bregman [3]). In combination with the DAC the resulting maximum error becomes:

<table>
<thead>
<tr>
<th>Source</th>
<th>Error (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>encoder</td>
<td>0.1</td>
</tr>
<tr>
<td>movement stretcher and back lash</td>
<td>0.3</td>
</tr>
<tr>
<td>DA converter</td>
<td>2 LSB = 0.1</td>
</tr>
<tr>
<td>maximum error</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**Combined cable potentiometer and tachometer** This device is used to measure translational position and velocity by pulling out a cable wound on a pulley. The potentiometer and tachometer are mounted on the pulley axis.

With a power supply of 25 (V) the potentiometer returns a DC voltage of \(\frac{1}{103.3} \) (V/mm). If a lower supply voltage is used the length of the extracted cable is given by

\[
text{position (in } mm) = \text{DC out-voltage} \times 103.3 \times \frac{25.00}{\text{supply voltage}}. \quad (3.2)
\]

To avoid overload of the DAQ-board input channels the supply voltage was set to 11.40 V resulting in an output range of 0-8.82 (V) for 0-2000 (mm), the table stroke.

The tachometer returns a DC voltage of \(\frac{1}{205.5} \) (V/mm). In rest an offset voltage of 0.06 V (caused by the power supply of the potentiometer) was measured so the cable velocity is given by

\[
\text{velocity (in } mm/s) = (\text{DC out-voltage} - 0.06) \times 205.5. \quad (3.3)
\]
3.1 Experimental set-up

No exact information was available about the accuracy of both sensors. Comparative measurements with the encoder show a deviation of ±0.1 (mm) and ±0.3 (mm/s) from the encoder readings so the overall accuracy of both sensors is ±0.6 (mm) and ±0.8 (mm/s). As the found deviations in position are of a systematic nature, the accuracy to be used for control evaluation in Chapter 5 is ±0.1 (mm).

**Pressure sensors**  Two pressure sensors (type P 3MA, 100 bar, from HBM) are used to measure the (absolute) pressures on both side of the cylinder piston ($p_A$ and $p_B$ in Fig. 2.1). The sensors power supply is calibrated to return a DC voltage of 39.13 (mV/bar). The given maximum error is 0.3 % × 100 (bar) = 0.3 (bar).

**Accelerometer**  The table acceleration is obtained by off-line differentiation and filtering (cut-off frequency 10 (Hz)) of the measured velocity. An accelerometer was used to validate this procedure. Considering the accuracy of the accelerometer the maximum error in peak value proved to be 10 (mm/s²).

3.1.2 Data Acquisition

The Data Acquisition system consists of a signal conditioning (SCXI) module, which can amplify and/or filter the signals from the sensors, and a 16-bits I/O board (AT-MIO-16X) for AD and DA conversion inside a PC (AT 486, 16 MB extended memory).

In order to control the horizontal movement a sample frequency of at least 2 and preferable 10 times the tabletop eigenfrequency (6.25 to 1.82 (Hz)) is required: 12.5 to 62.5 (Hz).

**SCXI**  The SCXI-module is mainly used for filtering and overload protection. For each channel separately a second order, two stage, analogue filter is available with a cut-off frequency of 4 (Hz) or 10 (kHz). For the table sensors (the encoder and the combined potentiometer- and tachometer) the filter is set to 4 (Hz). Note that for an empty tabletop this is lower than the table eigenfrequency of 6.25 (Hz). With a load however the eigenfrequency quickly drops below 4 (Hz).

**AT-MIO-16X**  The AT-MIO-16X I/O board (National Instruments) has a 10 μsec, 16-bits, sampling ADC that can monitor up to 8 differential channels. With the chosen input range -10 to 10 (V) the smallest detectable voltage increment, 1 LSB, becomes $\frac{20.09 V}{2^{16}} = 305.2$ (μV). The maximum sample frequency using 8 channels is $\frac{100 kHz}{8} = 12.5$ (kHz). The specified accuracy for this rate (including offset error, gain error, nonlinearity and system- and quantization noise) is ± 1 LSB.

When using the potentiometer to measure the tabletop position the additional position error caused by the ADC can be found by substituting 1 LSB in Eq. (3.2). For a 16 bits ADC this results in an additional position error of ±0.0691 (mm), enabling the control to achieve an overall positioning accuracy within ±1 (mm). For comparison, a 12 bits ADC (1 LSB = $\frac{20 V}{2^{10}} = 4883$ (μV)) causes an additional error of ±1.11 (mm).

Furthermore two 16-bits output channels are available (range -10 to 10 (V)) with a maximum update rate of 100 (kHz) and an accuracy of ± 4 LSB.
3.1 Experimental set-up

3.1.3 Data manipulation software

To manipulate the digitized data the program LabVIEW for Windows was chosen because its flexibility and the available experience on site. In LabVIEW 'virtual instruments' (VI's) can be created, by means of graphical programming, to control DAQ-systems.

In each sample period the VI, created for development and evaluation of control concepts for the horizontal patient support, completes the following cycle:

- all input channels are sampled once,
- if data storage is desired the digitized measurements are appended to a file,
- one control input is determined and sent to the DAC.

Compared to VI's for continuous data acquisition and storage this VI is significantly slower and forms the slowest part of the system. On the used computer (AT 486, 16 MB extended memory) the maximum loop frequency proved to be 140 (Hz). So the Nyquist criterion can be satisfied for all sensors except the pressure sensors (the hydraulic eigenfrequency is considerably larger than 70 (Hz)).

3.1.4 Command signal

The analogue command signal generated by the DAC of the AT-MIO-16X is fed to a modified part (the PSHYDRO-board) of the existing control hardware. The analogue voltage (-10 to 10 (V)) is converted to a valve current (-200 to 200 (mA)), i.e.

\[ u(\text{mA}) = 23.5 \times u(V) \] (3.4)

Furthermore, here a dither is added to the signal \( f_{\text{dither}} = 205 \) (Hz), amplitude 6 (mA)) to reduce the influence of friction in the cylinder.
Chapter 4

Model evaluation

With the flexible control environment of Chapter 3 more refined experiments can be performed. Every desired command signal, with or without control, can be realized. The results of these experiments deviate from the model predictions at several points:

- at zero-input an offset in the cylinder pressure difference occurs,
- the cylinder does not act as a perfect integrator (from input to table position),
- the predicted stick-slip limit cycle (without control) does not occur,
- the cylinder pressures both drop when the system starts moving.

The first deviation is caused by a wrong system setting, the others are caused by (systematic) unmodelled dynamics.

In the following sections each deviation is considered in more detail. The goal here is not to perfect the model but to provide a first step(direction) towards future model improvement.

4.1 Offset pressure difference

![Figure 4.1](image)

*Figure 4.1:* Table velocity and pressure difference versus slowly increasing input, $u(t) = 0.1 \, t$ (and $u(t) = -0.1 \, t$), for $M_l = 165.5$ and $17 \,(kg)$ without correction. The table acceleration $|\ddot{q}_l| \leq 0.01 \,(m/s^2)$ so $|M_l \ddot{q}_l| \leq 1.65 \,(N)$. 

- 28 -
4.2 Pressure dynamics

When the pump is turned on a pressure difference, $p_A - p_B$ of approximately -6 (bar) exists in the cylinder at zero input (see Fig. 4.1). As a result the table dynamics are strongly asymmetrical. For positive input voltages (or valve currents) the tabletop performs much better (e.g. smaller steady-state error) than for negative inputs.

The offset is caused by a wrong (factory) setting of the valve mid-position. After correction, with the use of the pressure sensors a more symmetrical figure is seen (Fig. 4.2).

\[ u(t) = 0.1t \text{ (and } u(t) = -0.1t), \text{ for } M_l = 165.5 \text{ and } 17 \text{ (kg) after correction of valve mid-position.} \]

The table acceleration $|\ddot{q}_I| \leq 0.01 \text{ (m/s}^2\text{)}$ so $|M_l \ddot{q}_I| \leq 1.65 \text{ (N)}$.

**Conclusion** A significant deviation in valve setting was found and corrected. As the standard procedure for setting the valve mid-position does not involve the use of pressure sensor(s), this offset in pressure difference is possible a systematic error.

4.2 Pressure dynamics

Ideally the cylinder acts as an integrator from input to table displacement, i.e. if the valve is opened the pressure difference will be built up until the resulting force is big enough for the table to start moving.

The Figures 4.1 and 4.2 suggest however that the pressure build-up is limited. To verify this for a stationary situation an input voltage inside the dash-dotted lines was applied for a considerable time (Fig. 4.3, top). The table velocity (not plotted) remained zero.
4.2 Pressure dynamics

![Graph of input voltage and cylinder pressure difference versus time for 17 (kg).](image)

**Figure 4.3:** Input voltage (top) and pressure difference (bottom) versus time for 17 (kg).

Although it takes 10 to 15 seconds until a stationary value is reached, the ultimate pressure difference indeed is limited. A possible explanation for the limited pressure can be a leakage in the cylinder or in the servo-valve.

![Graph of table velocity and pressure difference versus input voltage.](image)

**Figure 4.4:** Table velocity and pressure difference versus slowly increasing input, \( u(t) = 0.005 \ t \ (|M_l \ \dot{q}_l| \leq 0.034 \ (N)) \) and \( u(t) = 0.1 \ t \ (|M_l \ \dot{q}_l| \leq 0.17 \ (N)) \), for \( M_l = 17 \ (kg) \).

The relative slow pressure dynamics of this leakage flow can also be seen in Fig. 4.4 where two different voltage acceleration rates were used. In both cases the inertia of load and piston can be neglected. Note the considerable deviations in both pressure difference and table velocity.

**Conclusion** The, unmodelled, pressure dynamics, probably caused by (systematic) valve leakage, have an important influence on the overall system behaviour.
4.3 Friction

From the simulations a stick-slip limit-cycle is expected for low velocities, i.e. below the critical velocity (with default parameters approximately 20 (mm/s)).

Measurements however show that even at very low velocities the predicted limit-cycle does not occur (see Fig. 4.5 left).

![Graphs showing table velocity and input voltage, and pressure difference versus input table velocity.]

*Figure 4.5: Left: table velocity and input voltage (top) and pressure difference (bottom) for 165.5 (kg) load, without control. Right: the pressure difference versus the tabletop velocity for a load of 17 (kg), $u(t) = 0.005 \, t$ and without control.*

Apparently the critical velocity is much lower than predicted by the model, so after sticking once (see also 4.2) the load stays in motion.

A possible explanation is the presence of more damping in the system (see also Eq. (2.35)). With more damping the oscillation amplitude of the cable forces is reduced, in other words the load does not 'overtake' the piston and sign changes in table velocity do not occur.

A possible damping source is viscous friction in the cylinder (see Fig. 4.5, right). Especially for $M_l = 17$ (kg) a strong velocity dependent friction force occurs. Without a significant stick the friction force increases first to approximately 10 (bar) $\equiv 300$ (N), subsequently the friction force decreases to 170 (N).

**Conclusions** In combinations with additional observations from Fig. 4.2 the following conclusions can be drawn with regard to friction:

- A stick-slip limit cycle does not occur probably due to the presence of extra damping,
- Stick-slip is caused only by the load, i.e. for an empty tabletop hardly any stick occurs, probably due to the influence of dither,
- The pressures which are ultimately reached (for $|u| > 2(V)$ or after more than 15 (s)) concur with the measured slip forces (see section 2.4):  
  \[
  \begin{array}{|c|c|c|}
  \hline
  M_l \, (kg) & |F_{slip}| \, (N) & A_c \, |P_A - P_B| \, (N) \\
  \hline
  17 & 187 & 184 \\
  165.5 & 420 & 493 \\
  \hline
  \end{array}
  \]
4.4 Pressure deviations

In simulations the pressures $p_A$ and $p_B$ are seen to behave symmetrically around half the supply pressure (leaving the initial condition out of consideration). The measured pressures however show a quite different behaviour (see for instance Fig. 4.6, right, from the same measurement as Fig. 4.2).

Note that the impact of this deviation on the overall system behaviour is small, i.e. the pressure difference is determined by the load only.

![Simulated pressures in the cylinder chambers](image1)

<table>
<thead>
<tr>
<th>$u(t)$</th>
<th>$P_A$</th>
<th>$P_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 t</td>
<td>1.5</td>
<td>3.5</td>
</tr>
<tr>
<td>0.3 t</td>
<td>4.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Figure 4.6: Simulated (left) and measured (right) pressures versus slowly increasing input, $u(t) = 0.1$ t for $M_t = 165.5$ (kg). The table acceleration $|\dot{q}_t| \leq 0.01$ (m/s²) so $|M_t \dot{q}_t| \leq 1.65$ (N).

The deviation can be considered more closely for the system in stationary state. The model then results in

$$\dot{q}_t = \dot{q}_p = 0$$
$$\dot{q}_t = 4 \dot{q}_p = \text{constant} \Rightarrow Q_A = Q_B$$

The stationary pressures can be obtained from the flow equations (see section 2.1), i.e.

$$p_A + p_B = p_0 + p_r,$$

where $p_0 + p_r = 80$ bar.

This does not agree with stationary measurements. In Table 4.1 the results of a series of stationary measurements are summarized. As can be seen $p_A + p_B$ is considerably larger than 80 bar. This phenomenon did also occur after exchanging the horizontal valve by another one. A possible explanation for this deviation is the occurrence of an external flow, $Q_{x1}$, out of the system (leakage to reservoir or environment) or a flow, $Q_{x2}$, into the system (see Fig. 4.7 for $u < 0$). As a result $Q_A \neq Q_B$. As the pressure deviations are reproduced for both positive and negative inputs, the flow should be of symmetrical origin, i.e. the place where the flow leaves or enters the systems should change with the input sign.
4.4 Pressure deviations

<table>
<thead>
<tr>
<th>$u$ (V)</th>
<th>$q_i$ (mm/s)</th>
<th>$p_A$ (bar)</th>
<th>$p_B$ (bar)</th>
<th>$p_A - p_B$</th>
<th>$p_A + p_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.0</td>
<td>-281.1</td>
<td>56.1</td>
<td>51.0</td>
<td>5.1</td>
<td>107.1</td>
</tr>
<tr>
<td>-3.0</td>
<td>-192.9</td>
<td>57.7</td>
<td>52.3</td>
<td>5.4</td>
<td>110.0</td>
</tr>
<tr>
<td>-2.0</td>
<td>-106.3</td>
<td>61.4</td>
<td>56.7</td>
<td>4.7</td>
<td>118.1</td>
</tr>
<tr>
<td>-1.0</td>
<td>-26.4</td>
<td>69.1</td>
<td>62.8</td>
<td>6.3</td>
<td>131.9</td>
</tr>
<tr>
<td>1.0</td>
<td>36.6</td>
<td>61.6</td>
<td>66.1</td>
<td>-4.5</td>
<td>127.7</td>
</tr>
<tr>
<td>2.0</td>
<td>121.1</td>
<td>56.3</td>
<td>60.2</td>
<td>-3.9</td>
<td>116.5</td>
</tr>
<tr>
<td>3.0</td>
<td>216.8</td>
<td>51.6</td>
<td>55.7</td>
<td>-4.1</td>
<td>107.3</td>
</tr>
<tr>
<td>4.0</td>
<td>320.1</td>
<td>50.3</td>
<td>54.3</td>
<td>-4.0</td>
<td>104.6</td>
</tr>
</tbody>
</table>

Table 4.1: Stationary pressure and velocity measurements at constant input (load $M_i = 98 \text{ kg}$). Measured before the valve adjustment.

Figure 4.7: Hypothetic stationary flows for $u < 0$.

A small external flow can have a substantial impact because flows during normal operation are relatively small.

Possible (symmetrical) candidate sources causing an external flow are:

- leakage ($Q_{z1}$) through black-white valve (used for opening the by-pass),
- flow summation ($Q_{z2}$) in the no-return valves.

Conclusion A considerable deviation is seen between simulated and measured cylinder pressures. The impact of this deviation on other system states however is small.

Additional experiments are needed to test two hypotheses to explain the deviation.
Chapter 5

Simple horizontal control

In this Chapter the possibility of simple horizontal control, i.e. at most a PID-type controller, will be considered. Initially two signals will be used for feedback: the table position and the table velocity (see Fig. 3.1).

For comparison first the performance of the original controller is analyzed. After describing the control configuration and choosing a smooth input trajectory (sections 5.2 and 5.3) the need for PID controller with modified integrator is discussed (section 5.4). In section 5.5 two of those modifications are considered. Finally, the results of the various controllers presented are summarized in section 5.6 (see Table 5.1). In this last section also the influence of sensor accuracy will be taken into account.

5.1 Original controller

The original controller is implemented in an EPROM-chip and uses the encoder for a ‘PD like’ control of the horizontal movement.

![Graphs of valve current, table position, velocity and acceleration for the original controller.](image)

*Figure 5.1: Valve current, table position, velocity and acceleration for the original controller, moving into the magnet, total load $M_1 = 98 \text{ kg}$.***
5.2 Control configuration

A two stage trapezium-form velocity trajectory is specified (see Fig. 5.1 for the resulting velocity). Initially the table moves at low velocity until the gap between table and magnet has been bridged. Next the velocity is increased to the maximum velocity of 180 (mm/s).

This controller does not satisfy the specifications:

- The steep rise in valve current, intended to compensate for friction and valve dead-zone, causes a step-like velocity response at the beginning and the end of the movement. At these points the specified maximum acceleration of 100 (mm/s²) is exceeded.
- Furthermore according to Bregman [3], the required positioning accuracy of ±1 (mm) is not achieved:
  - the maximum error, reproducing a desired position is 1.53 (mm),
  - performing a Travel To Scan (829 ± 1 (mm)) the realized steplength is between 829.0 to 831.4 (mm).

5.2 Control configuration

A simple PID controller can be realized using the combined cable potenti- and tachometer to measure the table position and velocity (see Fig. 5.2).

\[ u(t) = K_p (e + K_i \int_0^t e(\tau) \, d\tau + K_d \dot{e}). \]  \hspace{1cm} (5.1)

Figure 5.2: PID with tachometer feedback.

Figure 5.3: PID with tachometer feedback.
5.3 Input trajectory

For future reference the transfer function, $K(s)$, is also be expressed in terms of an over all gain factor $K_r$ and two frequencies, $f_i$ and $f_d$, i.e.

$$K(s) = K_p (1 + \frac{K_i}{s} + K_d s) = K_r (1 + \frac{2\pi f_i}{s}) (1 + \frac{s}{2\pi f_d}) ,$$

where

$$K_p = K_r (1 + \frac{f_i}{f_d}), \quad K_i = \frac{1}{2\pi f_i + 2\pi f_d} \quad \text{and} \quad K_d = \frac{1}{2\pi f_i + 2\pi f_d} .$$

**Initial parameter setting** The Ziegler-Nichols rules of thumb were used to obtain an initial PID-parameter setting for positioning of a high load ($M_l = 165.5$ (kg)). In this case the smallest bandwidth and the largest accelerations are expected. Setting $K_d$ (and $K_i$) to zero the system reaches the stability limit (steady oscillation, frequency $f_{zn}$) at $K_{zn} \approx 100$ and $f_{zn} \approx 1.21$ (Hz). So

$$K_p = 0.6 K_{zn} = 60$$

$$K_d = \frac{K_{zn}}{K_p} = 0.1047 \Rightarrow f_d = 1.52 \text{ (Hz)} \quad \text{(for PD-control)}$$

$$K_i = 2 K_{zn} f_{zn} \frac{1}{K_p} = 2.4 \Rightarrow f_i = 0.38 \text{ (Hz)} \quad \text{(for PI-control)} .$$

5.3 Input trajectory

In order to achieve a smooth motion and to avoid steps in the acceleration a combined ramp and sine curve is chosen as input trajectory, i.e.

$$r(t) = q_o + (q_d - q_o) \frac{t}{T} - \frac{q_d - q_o}{2\pi} \sin \left( \frac{2\pi}{T} t \right) .$$

![Figure 5.4: Left: desired position, $r$, velocity $\dot{r}$ and acceleration $\ddot{r}$. Right: period time reference trajectory versus the step length.](image)

The period $T$ is determined by the step length $q_d - q_o$ and the specified maximum velocity (180 mm/s) or acceleration (100 mm/s²),

$$\max(\ddot{r}) = \frac{2(q_d - q_o)}{T} \Rightarrow T = \frac{2(q_d - q_o)}{0.180} .$$

- 36 -
\[ \text{max}(\tau) = \frac{2\pi(q_f - q_0)}{T^2} \Rightarrow T_r = \sqrt{\frac{2\pi(q_f - q_0)^2}{0.100}} \] (5.7)

Which period is chosen depends on the step length, \( T = \text{max}(T_r, T_x) \) (see Fig. 5.4, right).

### 5.4 New control strategy

The main drawback of the original control concept is apparently a too rigid form of friction compensation. Without friction compensation a PD-controller in general can not achieve the desired position accuracy. In Fig. 5.5 this can be seen for P and PD-control. Note the slightly smaller steady state-error for PD-control.

When however a rigid compensation is used, variations in load and exemplary deviations are likely to cause over- or undercompensation (depending on the load for which the compensation is set) resulting in a jerky motion and overshoot and/or an unacceptable steady state-error.

In order to obtain a robust control, rigid friction compensation is abandoned. Obviously then integral action is needed to minimize the steady-state error. Using integral action however can introduce limit-cycling around the desired end position (hunting). In Fig. 5.6 can be seen that PI-control does indeed achieve the desired accuracy but with limit-cycling (when using Ziegler-Nichols settings).

---

*Figure 5.5: Controlled table position error (solid line, top) and velocity error (dotted line, top) and table acceleration (bottom) for \( M_I = 165.5 \text{ (kg)} \) with Ziegler-Nichols settings. Left: P-control, right: PD-control.*
5.4 New control strategy

Many studies (a survey was made by Armstrong-Hélouvry et al. [1]) have shown that if limit-cycling occurs the amplitude of stick-slip can be reduced by decreasing the mass, increasing the damping or increasing the stiffness of the system. Decreasing the mass obviously is not feasible. The stiffness however can be increased by proportional feedback. More damping can be introduced using derivative feedback. This also increases system stability (possibly needed in combination with an integrator) and noise reduction. In Fig. 5.7 can be seen that adding a differentiator reduces acceleration and limit-cycling (the error after 42 (s) with PID control is smaller and is reached in fewer cycles).

**Figure 5.6:** PI-control of 165.5 (kg) load with Ziegler-Nichols settings. Left: table position error (solid line) and velocity error (dotted line) and table acceleration, right: the three components of the control input (top) and the pressure difference in the cylinder (bottom).

**Figure 5.7:** PID-control of 165.5 (kg) load with Ziegler-Nichols settings. Left: table position (solid line) and velocity error (dotted line) and table acceleration, right: table position and velocity.

In summary, without compensation integral control is necessary to achieve the position accuracy, derivative feedback is included for damping, noise reduction and possibly stick-
5.5 Modified PID-control

The modifications concern limiting or resetting the integrator input or output in order to prevent integral wind-up. Two modified integrators will be considered:

- an anti-windup integrator, and
- a switching and resetting integrator.

5.5.1 PID with anti-windup integrator

This controller (see 5.8) limits the output of the integral action in time by subtracting a surplus error-type signal when this signal exceeds a certain limit. Two extra parameters have to be set, the anti-windup gain, \( K_a \), and the saturation limit, \( s_l \).

![PID-controller with anti-windup integrator.](image_url)

Starting-point for setting this controller is the PID controller of Fig. 5.7. Experimentally a reasonable setting, although only just satisfying the specifications, was found, i.e.

\[
K_p = 60, \quad K_i = 20, \quad K_d = 0.15, \quad K_a = 1 \quad \text{and} \quad s_l = 0.010.
\]

(see Fig. 5.9, the steady-state error only just remains inside the \( \pm 1 \) (\( mm \)) tolerance\(^1\)).

\(^1\)This is only true under the assumption that the position sensor is perfect. The issue of sensor accuracy will be addressed further in section 5.6.
5.5 Modified PID-control

![PID-controller with anti-windup integrator](image)

**Figure 5.9:** PID-controller with anti-windup integrator (165.5 (kg) load) with $K_p = 60$, $K_i = 20$, $K_d = 0.15$, $K_a = 1$ and $s_I = 0.010$. Left: table position (solid line) and velocity error (dotted line) and table acceleration, right: the three components of the control input (top) and the pressure difference in the cylinder (bottom).

A serious drawback of this controller is its very critical nature, after refilling the oil for instance, a different setting was necessary to prevent limit-cycling.

5.5.2 PID with switching and resetting integrator

The controller in the previous subsection was set to follow the desired trajectory as best as possible. Minimizing position and velocity error at all time however is not a priority, i.e. a considerable lagging behind is allowed if the final positioning accuracy is achieved. This freedom is used to design a sluggish PD-controller with an integrator used only in the last stage of the trajectory. To maintain stability and achieve enough damping, even when the integrator switches on, the derivative feedback is increased.

A sluggish PD-controller

Although the system is clearly non-linear a measured transfer function\(^2\) (see Fig. 5.10) for medium range velocities and high load was used to aid setting this PD-controller. First $f_d$ is lowered to 0.5 (Hz), i.e. $K_d = 0.32$ (Eq. (5.3)), subsequently the proportional gain $K_p$ is lowered to 30 to obtain a gain margin of approximately 0.4 and a phase margin of approximately 100 degrees (see Fig. 5.10).

\(^2\)The transfer function (with $M_l = 165.5$ kg) was obtained by adding a white noise with an amplitude of 1 (V) on a constant control signal of 2 (V). With the use of FFT $H(f)$ can be determined by dividing the cross spectral density of input and the relevant output by the auto spectral density of the input, i.e.

$$H(f) = \frac{G_{yx}}{G_{xx}}.$$
5.5 Modified PID-control

\[ H_{p1} = \frac{q1}{u} \quad \text{and} \quad H_{p2} = \frac{q2}{u} \]

\[ K(s) = K_p \cdot \frac{1}{s} \quad \text{and} \quad H_{r} = K' \cdot H_{p2} \]

\[ Kr = 30, \quad f_i = 0.05 \quad \text{and} \quad f_d = 0.5 \]

Figure 5.10: Left: bode plots of measured transfer function, \( H_{p1} = \frac{q1}{u} \) and \( H_{p2} = \frac{q2}{u} \), and the open loop transfer function, \( K(s) \cdot H_{p2} \).

Right: bode plots of the PD-controller.

Using this PD-setting indeed results in a sluggish system behaviour and a large steady-state error (see Fig. 5.11 left).

By adding a little integral action \( (f_i = 0.05 \quad \text{Hz}) \), see Fig. 5.11 (right)) the required accuracy is achieved without limit-cycling. Because of the constant lag of the tabletop however, a normal integrator accumulates too much energy and causes overshoot and a high initial acceleration. Note that more integral action would again result in limit-cycling.
Switching the integrator

A possible remedy is the use of a 'switching' integrator which is active in the last phase of the trajectory and resets when the desired tolerance is achieved, i.e.

\[ u = K_p \left( e + K_i \int_t \, \dot{e} \right) + K_d \dot{e}, \quad \text{for} \quad \int_t e(t) \, dt = e_{\text{low}} \leq |e(t)| \leq e_{\text{high}} \quad \text{and} \quad \left| q_i - q_i^0 \right| \geq 0.5 \left| q_{\text{des}} - q_i^0 \right| \]

where

\[ I(t) = \begin{cases} \int_{t_0}^t e(\tau) \, d\tau & \text{for} \quad e_{\text{low}} \leq |e(t)| \leq e_{\text{high}} \quad \text{and} \quad \left| q_i - q_i^0 \right| \geq 0.5 \left| q_{\text{des}} - q_i^0 \right| \\ 0 & \text{else} \end{cases} \]

Thus the integrator switches on when entering a certain error band after completing at least half the trajectory. In contrast with the anti-windup integrator the integral action (inside the error band) is not limited in time. This is especially advantageous if a small displacement is needed.

Although resetting the integrator is necessary to avoid overshoot or limit cycling in all cases it also causes the control input, \( u \), to be discontinuous. When \( e_{\text{low}} \) is set to the specified position accuracy of 1 (mm) the step in \( u \) and thus its impact on the system (e.g. peak in the acceleration) is determined by the time in which the integrator accumulates energy, determined by \( e_{\text{high}} \), and the multiplication factor of this energy amount \( K_i \times K_p \).

Note that the the band \( \pm e_{\text{low}} \) around the desired position, used to bring the load to a full stop, actually limits the accuracy to some value within this band.

An acceptable setting proved to be \( K_i = 0.4 \) and \( e_{\text{high}} = 0.035 \) (m) (see Fig. 5.12).

\[ \text{Figure 5.12: PID-control of 165.5 (kg) load with switching integrator: } K_p = 33, K_i = 0.4, \quad f_d = 0.5 (Hz) \quad \text{or} \quad K_d = 0.3121 \text{ and } e_{\text{high}} = 0.035 \text{ (m). Left: table position and velocity error (top) and table acceleration (bottom). Right: the three components of the control input (top) and the pressure difference in the cylinder (bottom).} \]

This controller performs also satisfactorily for smaller loads as can be seen in Fig. 5.13.
5.5 Modified PID-control

Finally the controller performance for several other cases is considered:

- The maximum (desired) velocity is increased to 350 (mm/s) in order to reduce the duration of the motion to 14 (s) (Fig. 5.14, left), as expected the acceleration increases.

- The desired trajectory is changed to a low speed ramp: \( r = 0.010 \) t (Fig. 5.14, right), the realized velocity \( q_t = 10 \pm 0.5 \) (mm/s).

- The steplength is reduced to 10 (mm) (Fig. 5.15), the desired position is reached after some time.

Figure 5.13: PID-control of 17 (kg) load with switching integrator: \( K_p = 33, K_i = 0.4, f_d = 0.5 \) (Hz) (or \( K_d = 0.3121 \)) and \( \epsilon_{\text{high}} = 0.035 \) (m). Left: table position and velocity error (top) and table acceleration (bottom). Right: the three components of the control input (top) and the pressure difference in the cylinder (bottom).

Figure 5.14: PID-control of 165.5 (kg) load with switching integrator: \( K_p = 33, K_i = 0.4, f_d = 0.5 \) (Hz) (or \( K_d = 0.3121 \)) and \( \epsilon_{\text{high}} = 0.035 \) (m). Left: table position and velocity error (top) and table acceleration (bottom) when the maximum desired velocity is 350 (mm/s) Right: table velocity (top) and table acceleration (bottom) with a ramp-trajectory, \( r = 0.010 \) t.
5.5 Modified PID-control

Figure 5.15: PID-control of 165.5 (kg) load with switching integrator: \( K_p = 33, K_i = 0.4, f_a = 0.5 \) (Hz) (or \( K_d = 0.3121 \)) and \( e_{high} = 0.035 \) (m). Required steplength: 10 (mm), left: table position (top) and velocity (bottom), right: the three components of the control input (top) and the pressure difference in the cylinder (bottom).

5.5.3 PI with switching and resetting integrator

A serious drawback of derivative feedback is the need for velocity measurement or reconstruction (e.g. digital or analogue filtering).

Derivative action was included for stability, noise reduction, damping and possible stick-slip reduction. Indeed it was shown that the differentiator reduces

- the steady-state error,
- the acceleration, and
- the amplitude of the limit-cycle (caused by the integrator).

All these effects however are relatively small. Not surprisingly considering the small derivative contribution in the control law (see for instance Fig. 5.12, top, right).

Experiments show (see Fig. 5.16) that if the differentiator is left out, the acceleration remains just within specifications.
5.5 Modified PID-control

Figure 5.16: PI-control of 165.5 (kg) load with switching integrator: $K_p = 33$, $K_i = 0.4$ ($K_d = 0$) and $e_{\text{high}} = 0.035$ (m). Left: table position and velocity error (top) and table acceleration (bottom). Right: the three components of the control input (top) and the pressure difference in the cylinder (bottom).

Decreasing the proportional gain to $K_p = 25$ the peak value of the starting acceleration can be slightly reduced making the controller even more sluggish (see Fig. 5.17).

Figure 5.17: Table position and velocity error (top) and table acceleration for PI-control of 17 (left) and 165.5 (kg) load (right) using a switching integrator, $K_p = 25$, $K_i = 0.4$ ($K_d = 0$) and $e_{\text{high}} = 0.035$ (m)).

Further acceleration reduction however is not possible without a differentiator.
Rigid friction compensation, used in the original controller, results in over- or under-compensation (depending on the load for which the compensation is set) causing a jerky motion and overshoot and/or an unacceptable steady-state error.

Without compensation, integral control is necessary to achieve the desired position accuracy. Classic integral control induces limit-cycling (hunting) for high gains, $K_i$, or overshoot for lower gains.

Two modified integrators were implemented and tested, an anti-windup integrator and a switching and resetting integrator resulting in $\text{PI}_{\text{awD}}$- and $\text{PI}_{\text{sD}}$- / $\text{PI}_{\text{S}}$-controllers. The performance of both controller is within specifications if the position sensor is perfect (see Table 5.1).

Compared with the $\text{PI}_{\text{awD}}$, $\text{PI}_{\text{sD}}$ (or even $\text{PI}_{\text{S}}$) is less sensitive for changes in operation conditions (like the amount of oil in the system). It is expected therefore that a controller with a switching integrator can also cope with the considerable exemplary deviations between different patientsupports.

In contrast with the anti-windup integrator the integral action of a switching integrator is not limited in time. The controller will therefore always achieve the desired accuracy even if a small displacement is desired.

| controller | deviation from default conditions | within spec. | $e(t_o)$ | $\max(|\dot{e}|)$ | further comment | Figure |
|------------|----------------------------------|-------------|---------|------------------|----------------|-------|
| Original   | $M_i = 98$ (kg)                  | ---         | 1.53    | 0.40             | see [3]         | 5.1   |
| P (ZN)     |                                 | ---         | 9.8     | 0.15             |                | 5.5, left |
| PD (ZN)    |                                 | ---         | 9.1     | 0.12             |                | 5.5, right |
| PI (ZN)    |                                 | ---         | 0.9     | 0.21             | limit-cycle    | 5.6   |
| PID (ZN)   |                                 | ---         | 0.6     | 0.14             | limit-cycle    | 5.7   |
| $\text{PI}_{\text{awD}}$ | +? | 1.0 | 0.050 |        |                | 5.9   |
| PD (sluggish) |                  | ---         | 10.9    | 0.080           | overshot       | 5.11, left |
| PID (sluggish) |                  | ---         | 0.3     | 0.15             |                | 5.11, right |
| $\text{PI}_{\text{sD}}$ | $M_i = 17$ (kg) | + | 0.4 | 0.075 |                | 5.12 |
|                 | max($\dot{r}$) = 350 (mm) | + | 0.3 | 0.24 |                | 5.13 |
|                 | $\dot{r}(t) = 10$ (mm/s) | + | - | 0.02 | [\dot{e}] \leq 0.5 (mm/s) | 5.14, left |
|                 | $\Delta r = 10$ (mm) | + | 0.4 | - | | 5.15 |
| $\text{PI}_{\text{s}}$ | +? | 0.3 | 0.10 |        |                | 5.16 |
| $K_p \perp, M_i = 17$ (kg) | + | 0.6 | 0.06 |        |                | 5.17, left |
| $K_p \perp, M_i = 165.5$ (kg) | + | 0.1 | 0.08 |        |                | 5.17, right |

Table 5.1: A summary of the presented measurements (+? means: only just within specifications).

A resetting integrator however can achieve only a limited accuracy, it requires an error band, $\pm \varepsilon_{\text{low}}$, around the desired position to bring the load to a full stop. A band of $\pm 1$ (mm) proved to be wide enough.

With $\text{PI}_{\text{s}}$-control the design specifications can be met without the need for derivative feedback. Thus making the controller simpler and cheaper at the cost of slightly higher accelerations.

---

$^5\text{PI}_{\text{awD}}$: PID-control with an anti-windup integrator.

$^4\text{PI}_{\text{sD}}$ / $\text{PI}_{\text{s}}$: PID-/PI-control with a switching and resetting integrator.
5.6 Summary

Note that the steady-state errors presented in Table 5.1 are the errors as seen by the controller, assuming a perfect position sensor (in a series of repeated measurements of an equal distance the potentiometer readings showed no deviation in excess of ±0.1 (mm)).

With a sensor accuracy of ±0.6 (mm) all the controllers with switching integrator (except PI8 with 17 (kg) load) still perform within the specifications (e.g. |ε(t)| = 0.4 ± 0.6 (mm)). Unfortunately this can not be guaranteed, i.e. with an error band of ±1 (mm) a maximum error of 1.6 (mm) can occur. Using the more accurate encoder (without the DA converter) this maximum error is reduced to 1.4 (mm).

Compared to the original controller both the controllers with switching integrator are seen to have a higher positioning accuracy (although the guaranteed accuracy is only slightly higher) and result in a much smoother movement (lower accelerations).

Further research is necessary to determine whether a smaller error band can be used without an unacceptable amount of overshoot. The ‘breaking distance’ observed during experiments is between 0.6 - 1.4 (mm) indicating that a small decrease of the error band is possible without overshoot.
Chapter 6

Conclusions and recommendations

6.1 Conclusions

6.1.1 Model

A model for the horizontal movement is constructed and used for analysis of the system. The results, mainly regarding mechanical improvements, are summarized below. With the use of a more refined experimental set-up a number of unmodelled effects was found. These unmodelled dynamics inhibit the use of the present model for setting a PID-type controller or developing more advanced control concepts.

A first step towards improving the model was made by describing the observed effects and offering possible explanations or directions for model improvement. Considerable extra effort will be needed however to realize an adequate simulation model.

6.1.2 Mechanical improvements

From model analysis and measurements, conclusions can be drawn regarding transmission stiffness and valve configuration.

Transmission stiffness Measurements have shown that an increase in the transmission stiffness can not be realized by simple modifications like a stiffer cable. To achieve this probably a complete redesign is necessary.

Model analysis furthermore indicates that the influence of such an increase would be limited, i.e. when the transmission stiffness is doubled only a 17 % increase of the lowest eigenfrequency, \( f_i \), is seen. The reduction in ‘stick-slip step’ is even smaller.

Valve configuration In hydraulic systems the servo valves are usually placed directly on the cylinders, especially when rigid oil supply pipes can not be used. By model analysis however it was shown that the flexibility of the supply hoses has only a moderate influence on \( f_i \) (Placement direct on the cylinder results in a 18 % increase).
6.2 Recommendations

6.1.3 Control improvements

Dither Measurements show that the used dither signal eliminates stick-slip in the cylinder.

Simple horizontal control Rigid friction compensation, used in the original controller, results in over- or undercompensation (depending on the load for which the compensation is set) causing a jerky motion and overshoot and/or an unacceptable steady state-error.

Without compensation, integral control is necessary to achieve the desired position accuracy. Classic integral control induces limit-cycling for high gains, $K_i$, or overshoot for lower gains.

Two modified integrators were implemented and tested, an anti-windup integrator and a switching and resetting integrator resulting in $\text{PI}_{awD}$ and $\text{PI}_5D$ / $\text{PI}_5S^2$-controllers. The performance of both controller is within specifications if the table position sensor is assumed to be perfect.

Compared with the $\text{PI}_{awD}$, $\text{PI}_5D$ (or even $\text{PI}_s$) is less sensitive for changes in operation condition (like the amount of oil in the system). It is expected therefore that a controller with a switching integrator can also cope with the considerable exemplary deviations between different patientsupports.

In contrast with the anti-windup integrator the integral action of a switching integrator is not limited in time. The controller will therefore always achieve the desired accuracy even if a small displacement is desired.

A resetting integrator however can achieve only a limited accuracy, it requires an error band, $\pm \epsilon_{\text{low}}$, around the desired position to bring the load to a full stop. An error band of $\pm 1$ (mm) proved to be wide enough.

With $\text{PI}_s$-control the design specifications can be met without the need for derivative feedback. Thus making the controller simpler and cheaper at the cost of slightly higher accelerations.

If the sensor accuracy ($\pm 0.6$ (mm)) is also taken into account the $\text{PI}_5S^2$-controller still just performs within the specifications, i.e. the steady-state error $\leq 0.4$ (mm). Unfortunately this can not be guaranteed, i.e. with an error band of $\pm 1$ (mm) a maximum error of 1.4 (mm) can occur when the encoder (without the separate DA converter) is used. The 'breaking distance' observed during experiments is between 0.6 - 1.4 (mm) indicating that a small decrease of the error band is possible without overshoot.

Compared to the original controller both the controllers with switching integrator are seen to have a higher positioning accuracy (although the guaranteed accuracy is only slightly higher) and result in a much smoother movement (lower accelerations).

6.2 Recommendations

6.2.1 Mechanical improvements

Valve mid-position The patientsupport used in the experimental set-up suffered from an incorrect setting of the valve mid-position, resulting in a strongly assymetrical behaviour. An important question is whether this incorrect setting is the result of an wrong procedure or a coincidental error.

$^1\text{PI}_{awD}$: PID-control with an anti-windup integrator.

$^2\text{PI}_5D$ / $\text{PI}_5S$: PID-/PI-control with a switching and resetting integrator.
6.2 Recommendations

6.2.2 Control improvements

Accuracy of a resetting integrator Although it can not be guaranteed, the PI\textsubscript{S}D-controller is seen to perform just within specifications if a sensor accuracy of ± 0.6 (mm) is taken into account. To guarantee performance, even for a PI\textsubscript{S}-controller, further experiments are needed to determine if the error band can be made smaller without overshoot or without an unacceptable overshoot.

An alternative approach is to perform an extensive series of measurements to verify that the steady-state error with PI\textsubscript{S}D indeed always ≤ 0.4 (mm).

Control configuration The controllers up to now use the combined cable potential-and tachometer for feedback. Eventually the standard table position sensor, the encoder, should be used. For PI\textsubscript{S}-control little extra effort is needed. PI\textsubscript{S}D-control however requires velocity reconstruction, i.e. digital or analogue filtering of the measured velocity. Especially in the case of digital filtering, additional research is needed.

More advanced control concepts The controller with switching and resetting integrator just performs within the present design specifications. But if a higher accuracy or an even ‘smoother’ motion is required (without a complete redesign of the system) a more advanced control concept is necessary.

In literature two interesting options were found:

- In the line of modified PID-control is the work of Hanson \textit{et al} (see [1]). They apply a fuzzy rule system for controlling windup in a PID controller.
- When a more reliable model is available a control scheme with adaptive friction compensation would be possible. Brandenburg and Schäfer [4] for example describe a model reference position control of an elastic two-mass system with friction compensation (measuring only the position).

Defining comfort In the design specifications and in this report, patient comfort was measured by the maximum absolute acceleration. This criterion however offers only a rough approximation of the experienced comfort. For a better comparison of the presented and future controllers a more realistic criterion is needed.
Bibliography


## Appendix A

### The nominal model parameters

<table>
<thead>
<tr>
<th>Valve</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{nom}$</td>
<td>nominal valve current</td>
<td>200 mA</td>
</tr>
<tr>
<td>$\omega_v$</td>
<td>natural frequency valve piston</td>
<td>$160 \ 2\pi$</td>
</tr>
<tr>
<td>$\beta_v$</td>
<td>damping factor valve piston</td>
<td>0.8 - 1.1</td>
</tr>
<tr>
<td>overlap</td>
<td>valve overlap in % of $I_{nom}$</td>
<td>2 %</td>
</tr>
<tr>
<td>$C_{v_{max}}$</td>
<td>maximal flow gain</td>
<td>$1.575 \ 10^{-8}$</td>
</tr>
<tr>
<td>$p_0$</td>
<td>supply pressure</td>
<td>80 bar</td>
</tr>
<tr>
<td>$p_r$</td>
<td>return pressure</td>
<td>0 bar</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supply hoses</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_h$</td>
<td>hose diameter</td>
<td>$4.8 \ 10^{-3} \ m$</td>
</tr>
<tr>
<td>$A_h$</td>
<td>hose area</td>
<td>$1.81 \ 10^{-5} \ m^2$</td>
</tr>
<tr>
<td>$l_h$</td>
<td>length both hoses separately</td>
<td>$760 \ 10^{-3} \ m$</td>
</tr>
<tr>
<td>$V_{h_{0}}$</td>
<td>volume both hoses separately ($p=0$)</td>
<td>$1.38 \ 10^{-5} \ m^3$</td>
</tr>
<tr>
<td>$C_h$</td>
<td>hydraulic capacity</td>
<td>$2.18 \ 10^{-13} \ \frac{m^3}{Pa \ \text{m}}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_p$</td>
<td>mass piston</td>
<td>1.8 kg</td>
</tr>
<tr>
<td>$D$</td>
<td>exterior diameter cylinder</td>
<td>$25.4 \ 10^{-3} \ m$</td>
</tr>
<tr>
<td>$d$</td>
<td>interior diameter cylinder</td>
<td>$15.9 \ 10^{-3} \ m$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>‘effective’ piston area</td>
<td>$3.08 \ 10^{-4} \ m^2$</td>
</tr>
<tr>
<td>$L$</td>
<td>cylinder stroke</td>
<td>0.5 m</td>
</tr>
<tr>
<td>$V_0$</td>
<td>halve of total cylinder volume ($p=0$)</td>
<td>$8.32 \ 10^{-5} \ m^3$</td>
</tr>
<tr>
<td>$E_{oil}$</td>
<td>oil compressibility</td>
<td>$1.2 \ 10^9 \ \frac{N \ \text{m}}{m^2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transmission</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>reference spring</td>
<td>$2.94 \ 10^4 \ \frac{N}{m}$</td>
</tr>
<tr>
<td>$F_{cr}$</td>
<td>reference force</td>
<td>217 N</td>
</tr>
<tr>
<td>$A_{sc}$</td>
<td>area cable</td>
<td>$3.2 \ 10^{-6} \ m^2$</td>
</tr>
<tr>
<td>$l_{A0}$</td>
<td>length cable part A</td>
<td>2.0 m</td>
</tr>
<tr>
<td>$k = \frac{E_{A_{sc}}}{l_{A0}}$</td>
<td>cable stiffness</td>
<td>$3.36 \ 10^5 \ \frac{N}{m}$</td>
</tr>
<tr>
<td>$c_t$</td>
<td>actual transmission stiffness</td>
<td>$3.7 \ 10^4 \ \frac{N}{m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table/load</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_l$</td>
<td>total load: table and patient</td>
<td>150 kg</td>
</tr>
</tbody>
</table>
Appendix B

The model equations

Servo valve
\[ \dot{q}_v + \omega_0^2 q_v + 2\beta_0 \omega_0 \dot{q}_v = \omega_0^2 K_u u \]

\[ \begin{cases} 
    -q_{vo} + w_v \leq q_v \leq -q_{vo}; & C_v = -C_{v_{\text{max}}}; \quad C_v = \frac{q_{vo} + w_v}{q_{vo}} \\
    -q_{vo} \leq q_v \leq q_{vo}; & C_v = 0 \\
    q_{vo} \leq q_v \leq q_{vo} + w_v; & C_v = \frac{q_{vo} + w_v}{q_{vo}}; \quad C_v = C_{v_{\text{max}}} \\
    q_{vo} + w_v \leq q_v; & C_v = C_{v_{\text{max}}} 
\end{cases} \]

\[ \begin{cases} 
    Q_A = C_v \sqrt{P_A - P_r} \quad \text{and} \quad Q_B = C_v \sqrt{P_0 - P_B} \\
    Q_A = 0 \quad \text{and} \quad Q_B = 0 \\
    Q_A = C_v \sqrt{P_0 - P_A} \quad \text{and} \quad Q_B = C_v \sqrt{P_B - P_r} 
\end{cases} \]

Cylinder and hoses

\[ \dot{p}_A = \frac{E_{\text{out}}}{V_0 + A_c \rho_p + C_{\lambda_{\text{h}}}(E_{\text{out}} + p_A)} (-A_c \dot{q}_p + Q_A - Q_I) \]

\[ \dot{p}_B = \frac{E_{\text{out}}}{V_0 - A_c \rho_p + C_{\lambda_{\text{h}}}(E_{\text{out}} + p_B)} (A_c \dot{q}_p - Q_B + Q_I) \]

\[ M_p \ddot{q}_p = A_c (p_A - p_B) + 4(F_B - F_A) - F_{fp} \]
APPENDIX B. THE MODEL EQUATIONS

Transmission

\[ c_t = k_A + \frac{k_B c}{k_B + c} \]

\[ F_B - F_A = c_t (q_l - 4q_p) \]

Load

\[ M_l \ddot{q}_l = -(F_B - F_A) - F_{fi} \]
Appendix C

Simulink model

On the following pages the main components of the Simulink model are displayed. In order of execution:

- Fig. C.1: the hydraulics\textsuperscript{1}
- Fig. C.3 (left): the pressure difference,
- Fig. C.2: the piston dynamics,
- Fig. C.3 (right): the transmission and
- Fig. C.4: the table dynamics.

In Fig. C.5 a short description is given of the used Simulink blocks.

\textsuperscript{1}Notation:

- subscripts are put on equal with the rest of the symbol,
- the addition of a 'd' stands for derivative of time (dot), e.g. $q_{dd} \equiv \ddot{q}$.
Figure C.1: Hydraulics: servo valve and cylinder.
APPENDIX C. SIMULINK MODEL

Figure C.2: Piston dynamics

Figure C.3: Pressure difference (left) and transmission (right).

Figure C.4: Table dynamics.

- 57 -
Figure C.5: Simulink blocks.
Acknowledgements

In October '94, I started with the last phase of my study at the Systems and Control group. During the last year the following people contributed particularly to this work.

In the first place, I am grateful to my direct coaches at the TU, dr. ir. Frans Veldpaus, and at Philips Medical Systems, ir. Wiely van Groninge and ir. Twan van den Oetelaar for there advice, inspiring support and constructive criticism.

I want to thank ing. John Bregman for his practical help in realizing the various experimental set-ups, ir. Erwin Engelsma for his helpful LabVIEW hints and ir. Renz Kodde for his help with the pressure sensors.

Furthermore I want to thank all the other people at PMS who tried to help me with all my problems and questions about the ins and outs of the patientsupport.

Bert van Beek