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How to compose mechanisms with parallel moving bars

(with application on a level-luffing jib-crane consisting of a
four-bar linkage and exploiting a coupler-point curve)

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Abstract: Application of the principle of stretch rotation gives an easy way of finding the 3 four-bar cognates and also the 6 six-bar linkages of Watt's type with translating bars. With the same principle an adjoined linkage is built to hold a platform, attached to the end point of a jib-lever of a level-luffing jib-crane, in a horizontal position during luffing. A measuring apparatus attached to this platform enables the crane-driver to weigh the load while it is suspended from the crane.

1. Introduction

Six-bar linkages of *Watt*'s type having turning-pairs only, and having one parallel moving (or translating) bar, have been designed by *K. Hain* [1]. His design, with which he found six different solutions, was based on the theorem[1] of *Roberts-Chebyshev* [2].

In the present paper the same result will be obtained using the principle of *stretch rotation* as introduced by *H. Pflieger-Haertel* [3] in his derivation of the mentioned four-bar cognates.

An example is worked out as an application for a level-luffing crane, enabling the crane-driver to weigh the load while the latter is hanging from the crane.

2. Application of the principle of stretch rotation in the derivation of four-bar cognates

A four-bar coupler-curve may be generated by an arbitrarily chosen coupler-point (K) in the coupler-plane (ABK) which is attached to the coupler (AB) of some four-bar (A_0ABB_0).

In Fig. 1 a source mechanism of this kind is drawn, generating a curve of order 6 and genus 1. The moving links in this mechanism are numbered from 1 to 3. The frame-link A_0B_0 is indicated by the number 0.

The so-termed 2/3-*cognate* of the source mechanism may now be obtained following the design instructions given next (see Fig. 2):

a. Form the linkage-parallelograms ABB_0B'' and B_0BKB'.
b. Frame the rigid[2] triangle $B'B_0B'$ which is identically equal to the coupler-triangle ABK.
c. Turn the four-bar $A_0AB''B_0$ about B_0 over β = angle ABK and multiply the four-bar geometrically by the factor $f_\beta = \overline{B'B_0}/\overline{B'B_0}$. (One is thus using the principle of *stretch rotation*, with which one obtains the four-bar $B_0B'C''C_0$ which is similar to the four-bar $B_0B''AA_0$.)
d. Form the similar and rigid triangles $KB'C''$ and $A_0B_0C_0$. (Both triangles are similar to the coupler-triangle ABK.)

The initial four-bar mechanism is now supplemented by another one, $B_0B'C''C_0$, of which the coupler-point K also traces the same four-bar coupler-curve.

The obtained alternative mechanism is called a *cognate* of the source mechanism. And, since the angular velocities of the links 2 and 3 are interchanged, one may speak of the 2/3-*cognate* for short. It is clear that the 1/2-cognate as shown in Fig. 3 may be obtained in a similar way.

And, finally, it may be shown that the 1/3-cognate of the 1/2-cognate of the source mechanism, turns out to be identical

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[1] This theorem states that three different planar four-bar linkages will trace identical coupler curves.

[2] Throughout the paper, triangles are made rigid as long as two sides of such a triangle may move at the same angular velocity at any point of time.
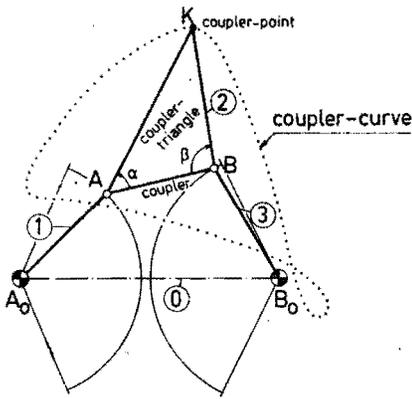
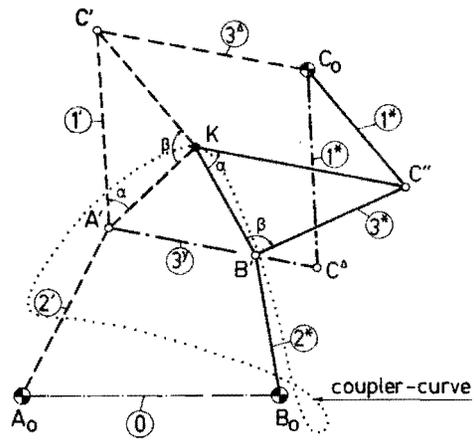**Fig. 1.** Initial four-bar linkage generating a unicursal coupler-curve.

Fig. 2. The 2/3-cognate of the initial four-bar linkage.

Fig. 3. The 1/2-cognate of the initial four-bar linkage.

Fig. 4. The 1/3-cognate of the 1/2-cognate (of the initial four-bar) identical to the 2/3-cognate of the initial four-bar.
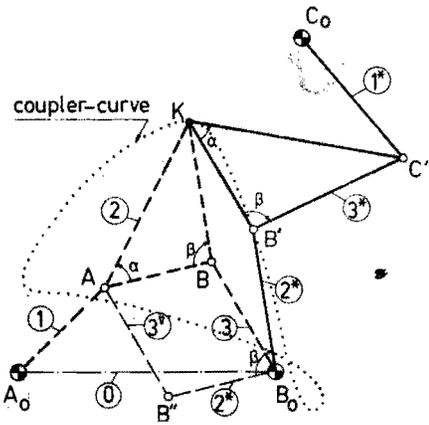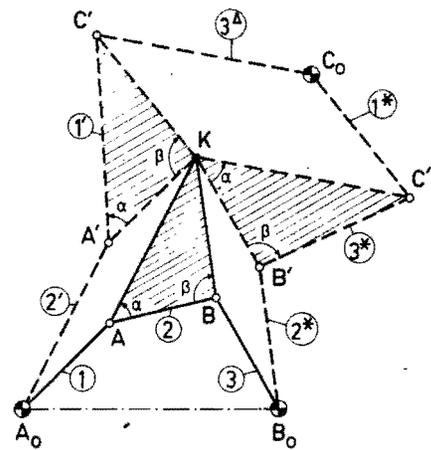

Fig. 5. Roberts' Law demonstrated.
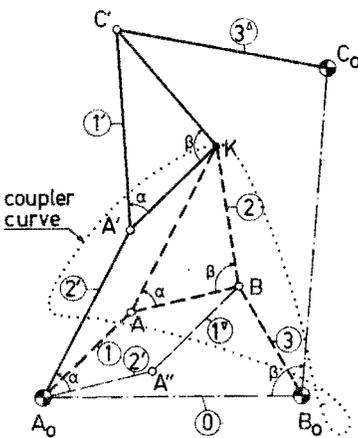
to the 2/3-cognate of the source mechanism (see Fig. 4). The entire configuration of Roberts is illustrated in Fig. 5 and contains the three cognates, including the source mechanism.

A quick way of finding the link-lengths of the cognate four-bars has been given by *Cayley*. His plan to do so is illustrated in Fig. 6 and needs no further explanation. (The cognate frame-link lengths are, in addition to Cayley's plan, found by the similarity of the frame-triangle and the coupler-triangle of the source mechanism.)

3. Composition of six-bar translating mechanisms through stretch rotation

The first six-bar with translating rod is to be designed through the following instructions (see Fig. 7):

a. Start the design with an arbitrarily chosen four-bar A_0ABB_0 and coupler-triangle ABK.

b. Form the linkage-parallelogram $A_0B_0BB_0'$.

c. Turn the four-bar A_0ABB_0' about A over the angle α = angle BAK and multiply the four-bar geometrically by the factor $f_\alpha = \overline{KA}/\overline{BA}$.

(One thus obtains the four-bar $\Box$ EAKF which is similar to $\Box A_0ABB_0'$.)

d. Frame the rigid triangle A_0AE which is similar to the coupler-triangle BAK.

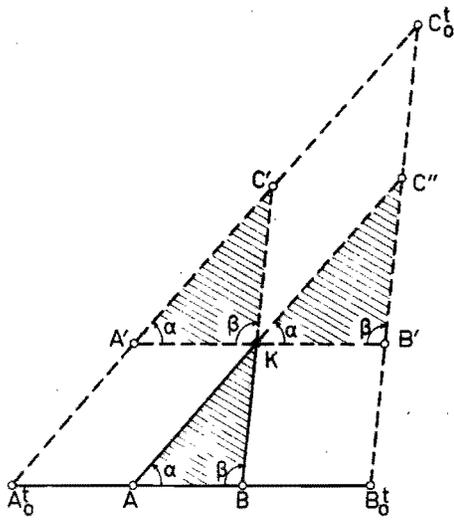Since FK and $B_0'B$ enclose the fixed angle α, rod FK will move

Fig. 6. Cayley's plan for determining the lengths of cognate links.

Fig. 7. First Watt's six-bar with translating bar.

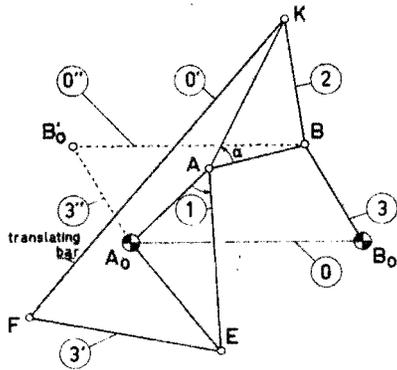**Fig. 8.** Second Watt's six-bar with translating bar.
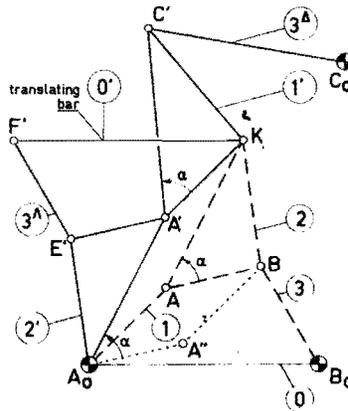
Fig. 9. Third Watt's six-bar with translating bar.

parallel to the frame. The obtained mechanism has one degree of freedom of movement and since A_0B_0' and $B_0'B$ may be considered as redundant bars, the linkage resembles a six-bar mechanism.

A second solution, as shown in Fig. 8, is obtained by interchanging the functions of the turning-joints A and B.

A third solution to the problem of finding six-bar mechanisms with translating rods is demonstrated in Fig. 9. The design of this mechanism follows the next pattern of instructions:
a. Start the design with the same initial four-bar A_0ABB_0 as chosen with the first two solutions. (Here, too, the coupler-point K is similarly situated in the coupler-plane attached to AB.)
b. Form the linkage-parallelograms A_0AKA' and A_0ABA''.
c. Make the four-bar $\square$ KA'E'F' identically equal to the four-bar $\square A_0ABB_0$.
d. Frame the rigid triangle $\triangle A_0A'E'$ which is similar to the initial coupler-triangle $\triangle$ KAB.
e. Turn the four-bar $\square A_0A''BB_0$ about A_0 over the angle $\alpha =$ angle $A''A_0A' =$ angle BAK and multiply the four-bar
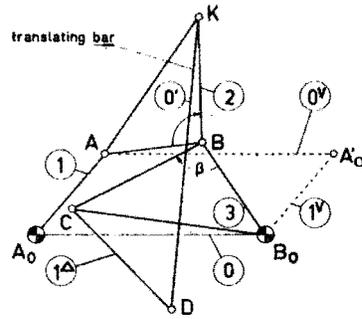
geometrically by the factor $f_\alpha = \overline{A_0A'}/\overline{A_0A''} = \overline{KA}/\overline{BA}$. (One obtains the four-bar $\square A_0A'C'C_0$ which is similar to the four-bar $\square A_0A''BB_0$.)
f. Frame the rigid and similar triangles $KA'C'$ and $B_0A_0C_0$. Since F'K remains parallel to A_0B_0, the rod F'K is a translating bar. Thus the six-bar mechanism, as shown in the figure in solid lines, resembles a mechanism with a parallel moving rod.

A similar translating rod will be obtained if the functions of the turning-joints A and B are interchanged. The resulting mechanism is shown in Fig. 10 as a fourth solution.

The fifth solution to the problem, as illustrated in Fig. 11, may be found by following the next pattern of design instructions:
a. Start the design with the same source mechanism as before (i.q. four-bar A_0ABB_0 and coupler-triangle ABK).
b. Form the linkage-parallelograms A_0ABA'', ABB_0B'' and B_0BKB'.
c. Turn the four-bar $A_0AB''B_0$ about B_0 over the angle $\beta =$ angle $B''B_0B' =$ angle ABK and multiply the four-bar by the factor $f_\beta = \overline{B'B_0}/\overline{B''B_0} = \overline{BK}/\overline{BA}$.
(One thus obtains the four-bar $\square B_0B'C'C''_0$ which is similar to the four-bar $\square B_0B''AA_0$.)
d. Frame the rigid and similar triangles $A_0B_0C_0$ and $KB'C''$.
e. Make the four-bar $KC''A^*D^*$ similar to the four-bar $B_0BA''A_0$.
f. Frame the rigid triangle $\triangle A^*C_0C''$ which is similar to $\triangle KB'C''$.
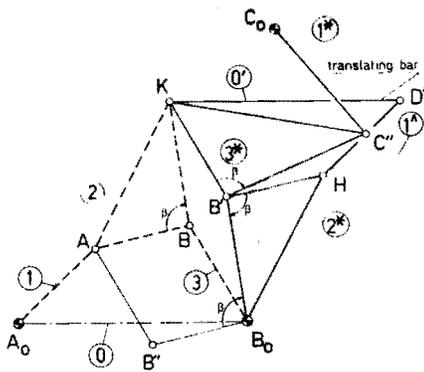
Fig. 10. Fourth Watt's six-bar with translating bar.

Fig. 11. Fifth Watt's six-bar with translating bar.

Fig. 12. Sixth Watt's six-bar with translating bar.

Since KD^* moves at the angular velocity of the fixed link A_0B_0, a six-bar mechanism is obtained having a translating rod. The resulting mechanism is indicated by the full-drawn lines of Fig. 11.

The sixth and last solution to the problem is illustrated in Fig. 12. This translating mechanism may be obtained by using the design instructions of the just-mentioned 5th translating six-bar, while interchanging the functional characters A and B. (One may remark, finally, that all solutions generate the same parallel moving plane 0' and that all points of this plane describe the same coupler-curve as produced by the source mechanism, which is the arbitrarily chosen four-bar A_0ABB_0 with coupler-triangle ABK.)

4. The design of a translating platform attached to the coupler-point (summit-point) of a level-luffing jib-crane

Most European level-luffing cranes, used in shipbuilding and cargo handling, feature a double-rocker mechanism. With these jib-cranes the motion of the jib-lever is identical to the motion of the coupler-plane of the double-rocker mechanism and the motion will be controlled by some luffing gear attached to the foremost leg of the crane. The hoisting rope of such a crane passes over a single pulley at the jib-head (A), and then over another single pulley at the end (K) of the jib-lever, directly to the lifting hook attachments.

In order to move the suspended load inwards or outwards along a level path between maximum and minimum position of the jib-lever, one aims to move the coupler-point K of this lever along a horizontal path. This objective may be approached in letting K coincide with Ball's point (or undulation point) of the jib-lever in the midposition of the jib [4].

In this paper, however, the main features of such a jib-crane will be taken for granted. The principal objective of this paper is to find the means of moving a *platform* attached to the coupler-point (K) parallelwise, destined to hold a loadcell for measuring suspended loads in a vertical position during the motion of the crane.

Should the loadcell be attached to the coupler-plane (KA), in a manner as shown in Fig. 15a, the equilibrium of forces on the coupler-point K would cause the loadcell to stand the varying force $G\cos\varphi$ instead of some constant force proportional to the load force G. Here, the angle φ is the varying angle
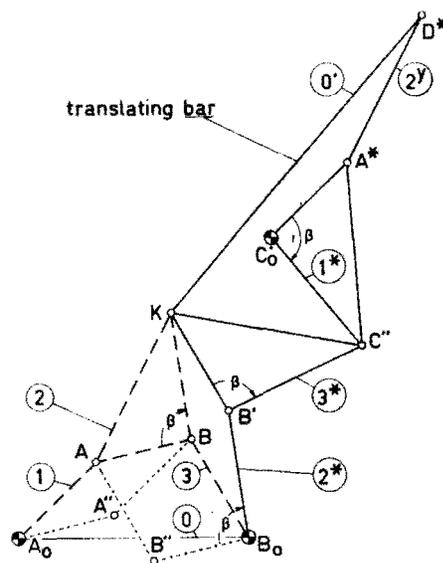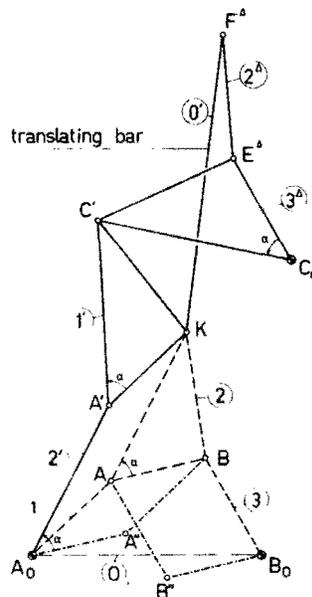
between the moving jib-line KA and the horizon. Therefore, a translating platform is necessary.

The design of the additional mechanism may be carried out in several ways. One of these may be obtained through the next sequence of design instructions (see Fig. 13):

a. The jib-crane consists of the double-rocker A_0ABB_0 and the jib-lever (coupler-triangle) ABK.

b. Form the linkage-parallelogram $A_0B_0BB_0'$.

c. Turn the four-bar A_0ABB_0' about A over the angle $\alpha =$ angle BAK and multiply the four-bar by the factor $f_\alpha = \overline{KA}/\overline{BA}$. (One thus obtains the four-bar EAKF which is similar to $\Box A_0ABB_0'$.)

d. Frame the rigid triangle A_0AE which is similar to the rigid coupler-triangle BAK.
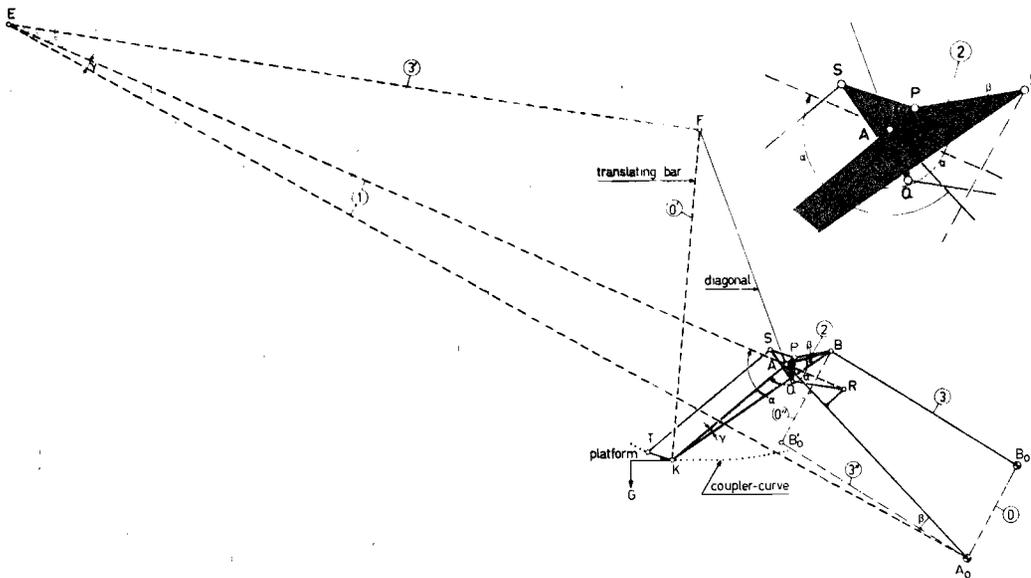
Fig. 13. Level-luffing crane with adjoined linkage holding some weighing apparatus in horizontal position during motion.
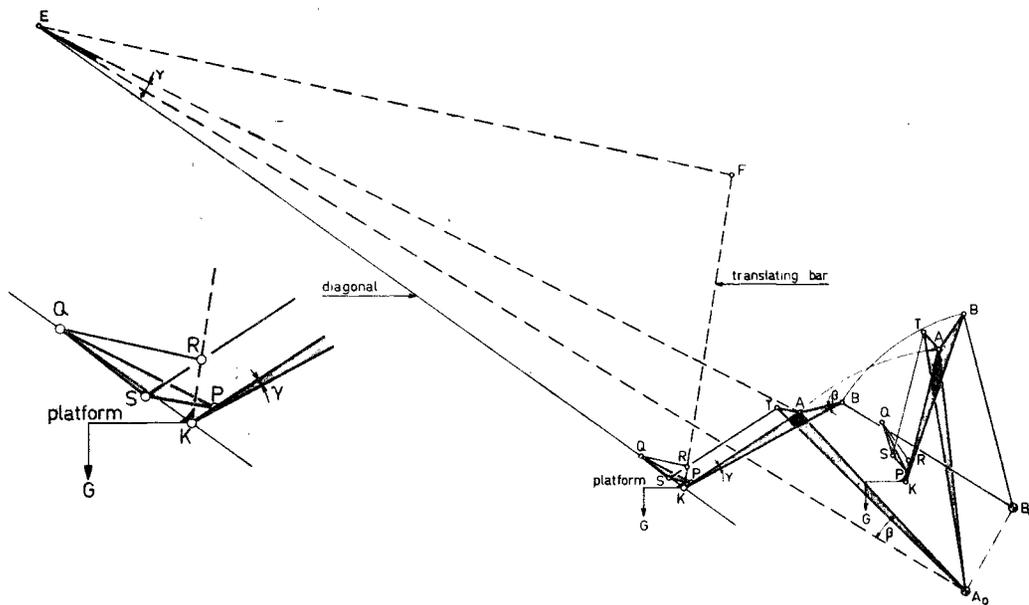

Fig. 14. Level-luffing crane with adjoined linkage holding some weighing apparatus in horizontal position during motion.

e. Choose a turning-joint Q on the diagonal FA of the four-bar EAKF in the neighbourhood of jib-head A.

f. Turn the four-bar EAKF about A over π radians and multiply the four-bar so as to obtain the four-bar RAPQ. (Thus $\square A_0ABB'_0 \sim \square EAKF \sim \square RAPQ$.)

g. Attach turning-joint R to the foremost leg A_0A of the crane.

h. Attach turning-joint P to the jib-lever ABK. (So the dyad PQR is added to the crane of which PQ moves parallel to the frame.)

i. Choose a random turning-joint S in the neighbourhood of jib-head A and frame the rigid triangle PQS (see also under k).

j. Form the linkage-parallelogram PSTK. (Then TK controls

the motion of the parallel moving platform [3]) and is rigidly attached to it.)

k. In order to keep the varying angle TSP in the neighbourhood of $\pi/2$ rad during luffing, it is advisable to choose the angle TSP $\approx \pi/2$ in the midposition of the jib-lever. (This may be done in choosing the turning-joint S accordingly.)

l. The crane and adjoined linkage are indicated in Fig. 13 by solid lines.

m. The fastening of the loadcell to the platform is indicated in

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[3]) Any point K' of this platform describes the same coupler-curve or load-trajectory as the coupler-point K of the jib-lever.
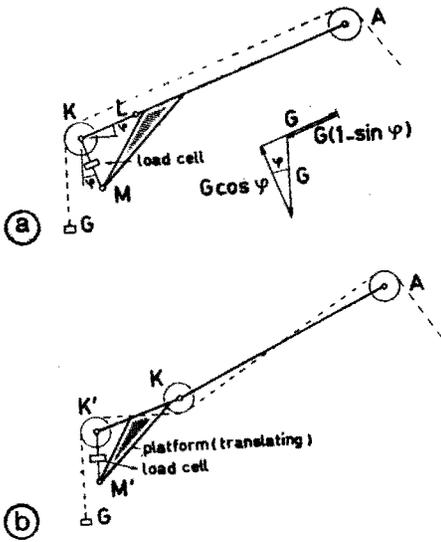
Fig. 15. Arrangement of loadcell for weighing purposes.
a. Inadequate solution; b. Correct solution.

Fig. 15b. The measuring will then be proportional to the load-force G and is independent of the position of the jib-lever. Another method in securing the parallel motion of the platform may be obtained through following the next altered design-instructions (see Fig. 14):

e. Choose a turning-joint Q on the diagonal EK of the fourbar EAKF in the neighbourhood of the end (K) of jib-lever ABK.
f. Multiply the four-bar EAKF geometrically from K by the factor $\overline{QK}/\overline{EK}$. (Thus the four-bar QPKR is obtained which is similar to EAKF and A_0ABB_0'.)
g. Attach turning-joint P to the jib-lever ABK. (Then RK moves parallel to the frame as long as PQ moves parallel to the leg A_0A.)
h. Choose a random turning-joint S in the neighbourhood of coupler-point K while making the angle SPA $\approx \pi/2$ rad in the midposition of the jib-lever.
i. Frame the rigid triangle PQS.
j. Form the linkage-parallelogram SPAT.
k. Fix turning-joint T to the leg A_0A and fix the platform to the link RK.
l. The crane and adjoined linkage are indicated in Fig. 14 by solid lines.
m. The fastening of the loadcell to the platform is indicated again in Fig. 15a and b. The electrical measuring equipment attached to the loadcell may then measure the load in any luffing position of the crane.
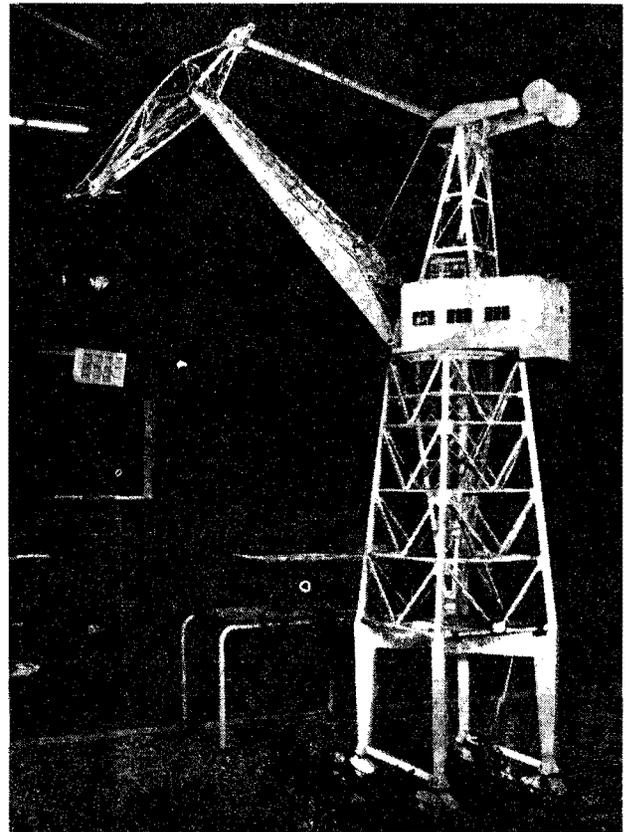

Fig. 16. Model of the crane.

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