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Resonant Scattering of Trapped Particles by Toroidal Plasma Modes (*).

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(ricevuto l'8 Maggio 1972)

In magnetically confined plasmas where the collisional mean free paths are much longer than the periodicity length of the lines of force, the major transport parameters are expected to be only partially accounted for by the effects of single-particle collisions (1). An aspect of this is the observed absence of current skin layers in diffuse-pinch experiments, which is contrary to what would be expected from considering only the effects of collisions on the plasma electrical resistivity and electron thermal conductivity. Another related question is whether trapped particles, in a toroidal magnetic configuration, can be lost by large-scale instabilities in the high-temperature regimes where the effects of collisional scattering become negligible as indicated by the theoretical analysis. The fraction of trapped particles is in fact more than half of the total population in most of the diffuse pinch experiments now being realized.

Here we find that collective modes capable of causing resonant scattering of trapped ions can in fact be excited, under realistic conditions on the ion temperature and density gradients, in two-dimensional plasma confinement configurations. In addition, a significant asymptotic limit exists in which the influence of the excited modes on the trapped particle orbits can be evaluated analytically.

We represent, as usual, the toroidal magnetic field by $B_\zeta = B_0[1 + (r/R_0)\cos\theta]^{-1}$, where r labels the magnetic surface and θ and ζ the poloidal and toroidal angle, respectively. An important class of modes which can be excited, in the limit where $8\pi nkT \ll B_0^2$, is electrostatic ($\boldsymbol{E}_1 = -\nabla\tilde{\Phi}$). Since the equilibrium configuration is nonhomogeneous in θ, the normal mode solutions for $\tilde{\Phi}$ of the linearized plasma equations are of the form $\Phi = \tilde{\varphi}_{m^0,n^0}(r,\theta)\exp[-i\omega t + im^0\theta - in^0\zeta]$, where m^0 and n^0 are integers. Indicating by $\varkappa$ the rationalized rotational transform $(\varkappa \approx R_0B_\theta/(rB_\zeta))$, we consider, in particular, modes which are localized in the radial direction around a rational surface $r = r_0$ such that $\varkappa(r_0) = n^0/m^0$. The electric field $\tilde{E}_\parallel = \boldsymbol{E}_1\cdot\boldsymbol{B}/B$ associated with these modes is $\tilde{E}_\parallel \approx -(1/r_0)(\partial/\partial\theta)\tilde{\varphi}_m(\theta)$, where we define $\tilde{\varphi}_m(\theta) \equiv \tilde{\varphi}_{m^0,n^0}(r_0,\theta)$. There-

(1) J. CALLEN, B. COPPI, R. DAGAZIAN, R. GAJEWSKI and D. SIGMAR: in *Plasma Physics and Controlled Nuclear-Fusion Research 1971*, vol. II, paper F-9 (International Atomic-Energy Agency, Vienna, 1972).

fore the θ-dependence of $\tilde{\varphi}_m$ is an important feature in determining the resonant interaction between particles and excited modes. Notice that the magnetic field $B \approx B_\zeta$ is minimum about $\theta = 0$. Then, referring to this point, the modes under consideration can be distinguished into *odd* and *even* ([2]). The corresponding field $\tilde{E}_\parallel$ is, respectively, even and odd about $\theta = 0$. Here we do not consider flute modes for which $(\partial\tilde{\varphi}_m/\partial\theta)/\tilde{\varphi}_m$, and therefore $\tilde{E}_\parallel$, is insignificant. So all the modes of interest propagate along the toroidal (ζ) direction with phase velocity $v_{\text{ph}} = \omega R/n^0$.

It is clear from the previous considerations that *odd* modes tend to affect more the dynamics of deeply trapped particles than even modes. For this we have made a survey of the modes which can be excited in a typical diffuse pinch experiment and seen that their frequency of oscillation ω is likely to be closer to the ion average bounce frequency $\langle\omega_b\rangle_i$ than to the electron's. Therefore we have been led to consider resonant scattering of trapped ions and focus attention on modes which are not heavily damped by wave-particle resonances when $\omega \approx \langle\omega_b\rangle_i$. Modes of this kind can be visualized as a kind of negative-energy waves ([3]), in a proper frame of reference, whose instability can be associated with dissipative effects of Landau dampings in the case of collisionless plasmas.

The modes which most closely fulfill these requirements are driven by the ion temperature gradient. We point out that the stability properties of these modes in the two-dimensional equilibrium configuration, which we consider, are substantially different from those which are found for a one-dimensional configuration. Notice that ion temperature driven modes have also been suggested as playing an important role in determining the ion thermal-energy transport across the magnetic field in Tokamak experiments ([4]). We analyse the limit $\omega < \langle\omega_b\rangle_i < \langle\omega_b\rangle_e$ and decompose $\tilde{\varphi}_m(\theta)$ in harmonics of the orbit periodicity $\tilde{\varphi}_m = \sum \Phi_m^{(p)}(\lambda) \exp[i2p\omega_b \hat{t}]$, where $\hat{t} = R\int^{\theta} \mathrm{d}\theta'/v_\parallel \varkappa)$, $\lambda = \varepsilon/\mu$, $\varepsilon = \frac{1}{2}m(v_\perp^2 + v_\parallel^2)$ is the particle kinetic energy, $\mu = \frac{1}{2}mv_\perp^2/B$ the magnetic moment, $\omega_b = 2\pi/\tau$, $\tau = R\oint \mathrm{d}\theta/(v_\parallel \varkappa)$ for trapped particles, and $\tau = R\int_0^{4\pi} \mathrm{d}\theta/(v_\parallel \varkappa$ for circulating particles ([1]). In the latter case ω_b has the sign of $v_\parallel$. In particular, for odd modes the average potential over a particle orbit $\Phi_m^{(0)} = \tau^{-1} R\oint \mathrm{d}\theta\, \tilde{\varphi}_m(\theta)/(\varkappa v_\parallel) = 0$. Therefore ([2]) the perturbed electron density is simply $\tilde{n}_e = en\tilde{\varphi}/T_e$. Notice that, since $\tilde{n}_e$ and $\tilde{\varphi}$ are in phase, no particle loss is caused by these modes. The assumed ion equilibrium distribution is of the form $f_i = f_{Mi}(1 + \hat{f}_i)$,

$$\hat{f}_i = -(v_\parallel/\Omega_{\theta i})[(\mathrm{d}n_i/\mathrm{d}r)/n_i - (\mathrm{d}T_i/\mathrm{d}r)(\tfrac{3}{2} - \varepsilon/T_i)/T_i]\,,$$

$\Omega_{\theta i} = eB_\theta/(m_i c)$ and f_{Mi} is the Maxwellian. The expression for $\tilde{n}_i$ then is obtained by integration of the Vlasov equation along particle orbits in the zero Larmor-radius limit:

$$(1)\qquad \tilde{n}_i = -(en/T_i)\Big\{\tilde{\varphi}_m - n^{-1}\int \mathrm{d}^3\boldsymbol{v} f_{0i}[\omega - \omega_{*i} + \\ + \omega_{T_i}(\tfrac{3}{2} - \varepsilon/T_i)] \sum_p [\tilde{\Phi}_m^{(p)}(\lambda) \exp[i2p\omega_b \hat{t}]]/(\omega - 2p\omega_b)\Big\},$$

where $\omega_{*i} = -(m^0/r)(\mathrm{d}n/\mathrm{d}r)\, cT_i/(eBn)$ and $\omega_{T_i} = \omega_{*i}(\mathrm{d}\ln T_i/\mathrm{d}r)/(\mathrm{d}\ln n/\mathrm{d}r)$. Then we consider the quadratic form which results from the quasi-neutrality condition expres-

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([2]) B. COPPI: *Rivista Nuovo Cimento*, **1**, 357 (1969).

([3]) B. COPPI, M. ROSENBLUTH and R. SUDAN: *Ann. of Phys.*, **55**, 207 (1969).

([4]) L. A. ARTSIMOVICH, A. V. GLUKHOV and M. P. PETROV: *Žurn. Èksp. Teor. Fiz. Pis. Red.*, **11**, 449 (1970) [*Sov. Phys. JETP Lett.*, **11**, 304 (1970)].

sed as $\oint \mathrm{d}l\,\tilde{\varphi}_m^*(\tilde{n}_i - \tilde{n}_e)/B = 0$, where $\mathrm{d}l = R\,\mathrm{d}\theta/\chi$:

$$(1 + T_i/T_e)\,n\oint \mathrm{d}l|\tilde{\varphi}_m|^2/B - (\pi/2)(2/m_i)^2\iint \mathrm{d}\varepsilon\,\mathrm{d}\mu f_{Mi}|2\pi/\omega_b|\cdot$$
$$\cdot[\omega - \omega_{*i} + \omega_{T_i}(\tfrac{3}{2} - \varepsilon/T_i)]\sum_{p\neq 0}[|\tilde{\varphi}_m^{(p)}(\lambda)|^2/(\omega - 2p\omega_b)]\,.$$

In the limit $\omega < \langle\omega_b\rangle_i < \omega_{T_i} \sim \omega_{*i}$ we obtain

$$(2)\qquad (1 + T_i/T_e)\oint \mathrm{d}l|\tilde{\varphi}_m|^2/B + (\omega_{*i} - \omega_{T_i})\,\omega/4\pi^2\int \mathrm{d}\lambda\big(L_0^3(\lambda)/v_{thi}^2\big)\sum_{p\neq 0}|\tilde{\Phi}_m^{(p)}(\lambda)|^2/p^2 -$$
$$- i\omega^2(3\omega_{T_i}/2 - \omega_{*i})/8\pi^{\frac{5}{2}}\int \mathrm{d}\lambda\big(L_0^4(\lambda)/v_{thi}^3\big)\sum_{p\neq 0}|\tilde{\Phi}^{(p)}(\lambda)|^2/|p|^3 = 0\,,$$

where $2L_0(\lambda) = \oint \mathrm{d}l/(1 - \lambda B)^{\frac{1}{2}}$, and $v_{thi} = (2T_i/m_i)^{\frac{1}{2}}$. This equation shows that instability occurs for

$$(3)\qquad \frac{1}{T_1}\frac{\mathrm{d}T_i}{\mathrm{d}r} > \frac{2}{3}\frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}r}\,.$$

We recall that in a one-dimensional configuration ([5]), instability can be found only if $\mathrm{d}\ln T_i/\mathrm{d}r > \mathrm{d}\ln n/\mathrm{d}r$ and by considering very short wavelengths, of the order of the ion gyroradius ([6]). For the present case the frequency ω can no longer be considered smaller than $\langle\omega_b\rangle_i$ and the expansion leading to eq. (2) is not valid in the limit where $\mathrm{d}\ln T_i/\mathrm{d}r$ tends to become equal to $\mathrm{d}\ln n/\mathrm{d}r$. The different nature of the dispersion relation for a one-dimensional geometry, as given for instance in ref. ([5]), and the one presented here is related to the fact that in the former case Landau resonances $\omega = k_\parallel v_\parallel$ are involved, with no constraint, on the value of $v_\perp$ ($k_\parallel$ is the wave number in the direction of $\boldsymbol{B}$). In the present case the relevant resonances are of the form $\omega = \omega_b(\varepsilon, \mu)$ and involve a considerably smaller portion of velocity space ([2]) (in practice surfaces of constant ε). The instability criterion (3) would be slightly modified had we considered a collision dominated equilibrium ([7]) in which the expression for $\hat{f}_i$, relevant to circulating particles, is different from the one assumed here.

The particle equation of motion in θ under the influence of the fluctuating electric field can be derived from

$$(\mathrm{d}\varepsilon/\mathrm{d}t) = -e\boldsymbol{v}\cdot\nabla\tilde{\Phi} = -e(\mathrm{d}\tilde{\Phi}/\mathrm{d}t - \partial\tilde{\Phi}/\partial t)\,,$$

where $v_\parallel = (\mathrm{d}/\mathrm{d}t)(R\theta(t)/\varkappa)$. Neglecting the magnetic curvature drift, as allowed by a realistic choice of parameters, we then have

$$(4)\qquad (\mathrm{d}^2\theta/\mathrm{d}t^2) + \omega_{0b}^2\sin\theta = -(e/m)(\varkappa/R)^2(\mathrm{d}\theta/\mathrm{d}t)^{-1}(\mathrm{d}\tilde{\Phi}/\mathrm{d}t - \partial\tilde{\Phi}/\partial t) =$$
$$= \omega_{0b}^2(\hat{\alpha}/\tilde{\varphi}_e)\big(\mathrm{d}\tilde{\varphi}_m(\theta)/\mathrm{d}\theta\big)\cos(\omega t)\,,$$

([5]) A. A. GALEEV, V. N. ORAEVSKII and R. Z. SAGDEEV: *Žurn. Èksp. Teor. Fiz.*, **44**, 903 (1963) [*Sov. Phys. JETP*, **17**, 615 (1963)].

([6]) The relatively mild temperature gradient required by eq. (3) is such that the relevant instability can take place well inside the plasma column where the density is not too low. This combined with the fact that the instability transverse wave-lengths can be relatively long, leads to expect a considerable thermal energy transport from it, contrary to the conclusions expressed by B. COPPI, M. ROSENBLUTH and R. Z. SAGDEEV: *Phys. Fluids*, **10**, 582 (1967).

([7]) M. ROSENBLUTH, R. HAZELTINE and F. HINTON: *Phys. Fluids*, **15**, 116 (1972).

where $\omega_{0b}^2 = (\mu B_0 \varkappa_0^2/(mR_0^2)(r/R_0))$ represents the bounce frequency of trapped particles with amplitudes $\theta \ll (6)^{\frac{1}{2}}$, $\tilde{\varphi}_c$ is a typical amplitude of the fluctuating potential and $\hat{\alpha} = e\tilde{\varphi}_c R/(\mu B_0 r)$. We consider the case of deeply trapped particles with excursion amplitudes such that $(\mathrm{d}/\mathrm{d}\theta)\,\tilde{\varphi}_m \approx (\tilde{\varphi}_c/\theta_{0d})\cdot(1 + \alpha_2\theta^2 + \alpha_4\theta^4 + \dots)$ over them. Then, for $\alpha_2\theta^2 < 1$, eq. (4) becomes

$$\ddot{\theta} + \theta - \theta^3/6 = \alpha \cos \omega_0 t_0 , \tag{5}$$

where $t_0 = t\omega_{0b}$, $\omega_0 = \omega/\omega_{0b}$ and $\alpha = (\hat{\alpha}/\theta_{0d}) \sim (e\tilde{\varphi}_c/T_i)(R_0/r\theta_{0d})$, as we refer to resonances with trapped ions. Note that if, for instance, $e\tilde{\varphi}_c/T_e \approx 2\cdot 10^{-2}$, $R_0/(r\theta_{0d}) \sim 6$ and $T_e \sim 2T_i$, the parameter α is not a really small number. The solution of eq. (5) can be written (8) in the form $\theta = 2\chi(t_0)\cos[\omega_0 t_0 - \psi(t_0)]$, where the amplitude $\chi(t_0)$ satisfies the equation

$$\chi^4/4 + 2\chi^2(\omega_0 - 1) + \alpha\chi\cos\psi = c . \tag{6}$$

The constant c is determined by the initial conditions and the phase ψ is given by the equation $\mathrm{d}\psi/\mathrm{d}t_0 = (\frac{1}{4})\chi^2 + (\omega_0 - 1) + (\alpha/4\chi)\cos\psi$. The limit $\alpha = 0$ where $\psi = (\omega_0 - 1)t_0$ and $\chi = \text{const}$, corresponds to the unperturbed « banana » orbit in the torus. In the same limit, if we introduce Van der Pol co-ordinates $a = 2\chi\cos\psi$ and $b = 2\chi\sin\psi$, these orbits correspond to circles centered about 0. The amplitude of the oscillation is represented by the polar vector rotating in time around 0. We recall that trapped particles correspond to a pitch angle range $-r/R_0 < \Lambda - 1 < r/R_0$, where $\Lambda = \mu B_0/\varepsilon$ and that $2\chi^2 R_0/r \approx 1 + r/R_0 - \Lambda$ when $\alpha = 0$. When $\alpha > 8|\omega_0 - 1|^{\frac{3}{2}}$ the resulting orbits (« quasi-bananas ») correspond to the representation given in Fig. 1. In particular, dif-

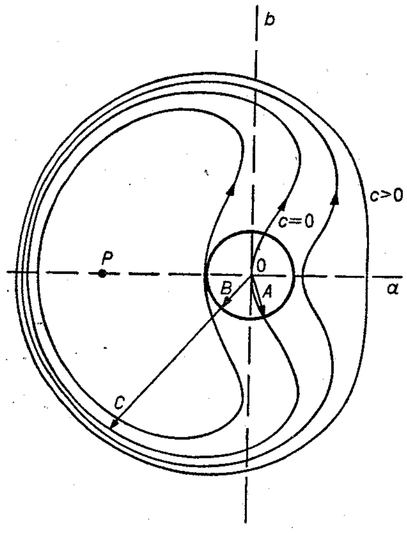

Fig. 1.

ferent orbits correspond to different initial conditions. The point P is such that $OP \sim 2\alpha^{\frac{1}{3}}$ and represents the amplitude that the oscillator would reach in the presence of damping. An amplitude of the same order of magnitude is obtained, in the absence of damping when the time average of $\chi^2(t)$ is performed. In particular, this average can be evaluated

(8) R. A. STRUBLE: *Nonlinear Differential Equations* (New York, 1962).

by the cyclic integral $\oint \chi^2(\psi)\,\mathrm{d}\psi$ and, in conditions of strong resonance, for orbits which initially are small $[\theta_0 \equiv \theta(t=0) \ll 2\alpha^{\frac{1}{3}}]$, one finds $\overline{\chi^2} \approx 2\alpha^{\frac{2}{3}}$. Other quantities of interest are the time t_m needed by a small orbit to become large (more precisely to reach the point $\psi \approx \pi/2$) and the time t_M during which the orbit remains large. These quantities can be estimated as $t_m \approx \pi\theta_0/(\alpha\omega_{0b})$ and $t_M \approx 2\pi/(\alpha^{\frac{2}{3}}\omega_{0b})$. In conclusion, the small orbits become large in few bounce periods and then they stay large for a longer time, as $t_M > t_m$.

A survey of the even modes [9,10] which can be excited in the limit $\omega \lesssim \langle\omega_b\rangle_i$ points to those which can be associated with the toroidal magnetic-field curvature drift of the guiding centers. The eigenfunctions $\tilde{\varphi}_m(\theta)$ which characterize these modes, are peaked in the region where the magnetic curvature is favorable. Therefore the effect on the mode stability of trapped particles with large excursion angle, tends to prevail over that of the trapped particles with small excursion angle, which are subject to unfavorable magnetic curvature. This is indicated by a quadratic form, which can be derived in substitution of eq. (2) and can be written as

$$\pi(2/m_i)^2\int \mathrm{d}\varepsilon\,\mathrm{d}\mu\, f_{M_i}|2\pi/\omega_{bi}|\Big[\sum_{p\neq 0}|\tilde{\Phi}_m^{(p)}|^2\omega^2 - T_e\,\omega_{*i}\,\omega_{Di}^{(0)}|\tilde{\Phi}_m^{(0)}|^2/T_i\Big] + \\ + i\omega^4\omega_{*i}/4\pi^{\frac{5}{2}}\int \mathrm{d}\lambda\big(L_0^4(\lambda)/v_{thi}^3\big)\sum_{p\neq 0}|\tilde{\Phi}_m^{(p)}(\lambda)|^2/|p|^3\,, \tag{7}$$

where $\omega_{Di}^{(0)} = (1/\tau)\oint \mathrm{d}l\,\omega_{Di}/|v_\parallel|$, $\omega_{Di} = n^0(\mathrm{d}\zeta/\mathrm{d}t)_D$ and $(\mathrm{d}\zeta/\mathrm{d}t)_D$ is the angular drift velocity due to magnetic curvature [1]. The presence of a temperature gradient has not been included for simplicity and $\langle\omega_D\rangle_i < \omega < \omega_{*i}, \langle\omega_b\rangle_i$. The relevant profile of the eigenfunction $\tilde{\varphi}_m(\theta)$ is such as to make the contribution of the term $\omega_{*i}\omega_{Di}^{(0)}$ positive. It appears that it is not possible to study the influence of these modes on the orbits of trapped particles with relatively large excursions by reducing eq. (4) to a simple form.

In conclusion, the orbits of trapped ions which in the absence of fluctuations would be « bananas » localized in the region of unfavorable curvature, can acquire a considerably larger amplitude and reach the region where the magnetic curvature is favorable when the modes with $\omega \lesssim \langle\omega_b\rangle_i$ we have considered earlier are excited. As a result the average magnetic curvature, that is seen by the majority of trapped particles, over their orbits, may prevent the appearance of magnetic curvature driven and purely growing modes [9], with characteristic times considerably longer than $\langle\omega_b\rangle_i^{-1}$.

Another suggestion coming from this analysis is that it may be possible to interact with trapped particles by attempting to enhance the amplitudes of the considered modes from the outside. There are evident experimental difficulties for this. An oscillating voltage applied between two probes along the torus could produce the desired excitation locally, thus enabling the modes to select their own toroidal wave number. However, a radial voltabe drop may require too large a current in order to produce a sufficient electric field inside the plasma. Alternatively, the electric field could be created by induction, *e.g.*, by a local bar system, but could still suffer an appreciable radial voltage drop.

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(9) B. B. KADOMTSEV and O. P. POGUTZE: *Žurn. Èksp. Teor. Fiz.*, **51**, 1734 (1966) [*Sov. Phys. JETP*, **24**, 1172 (1967)].

(10) M. ROSENBLUTH, D. ROSS and D. KOSTOMAROV: *Nucl. Fusion*, **12**, 3 (1972).