On the bounds of the modelling errors of black-box MIMO transfer function estimates

Research Report
ISSN 0167-9708
Coden: TEUEDE

Eindhoven
University of Technology
Netherlands

Faculty of Electrical Engineering

On the Bounds of the Modelling Errors of Black-Box MIMO Transfer Function Estimates

by
ZHU Yu-Cai

EUT Report 87-E-183
ISBN 90-6144-183-8
November 1987

Eindhoven University of Technology Research Reports

EINDHOVEN UNIVERSITY OF TECHNOLOGY

Faculty of Electrical Engineering
Eindhoven The Netherlands

ISSN 0167- 9708 Coden: TEUEDE

ON THE BOUNDS OF THE MODELLING ERRORS OF BLACK-BOX MIMO TRANSFER FUNCTION ESTIMATES

by

ZHU Yu-Cai

EUT Report 87-E-183
ISBN 90-6144-183-8

Eindhoven
November 1987

关于黑盒子多变量传递函数估计误差的上界

朱豫才

中国西安交通大学留学生

CIP-GEGEVENS KONINKLIJKE BIBLIOTHEEK, DEN HAAG

Zhu Yu-Cai

On the bounds of the modelling errors of black-box MIMO transfer function estimates / by Zhu Yu-Cai. - Eindhoven: University of Technology, Faculty of Electrical Engineering. - Fig. - (EUT report, ISSN 0167-9708; 87-E-183)
Met lit. opg., reg.
ISBN 90-6144-183-8
SISO 656 UDC 519.71.001.3 NUGI 832
Trefw.: systeemidentificatie.

CONTENTS

	Abstract	1
1.	Introduction	2
2.	The asymptotic theory	4
	2.1 Markov parameter model set	5
	2.2 Spectral analysis	11
3.	The algorithms	14
	3.1 Algorithm 1	14
	3.2 Algorithm 2	16
	3.3 Numerical test	18
4.	Other forms of Bounds	19
5.	Conclusions	25
	Acknowledgement	26
	References	27

Addres of the author:

ir. ZHU Yu-cai
Measurement and Control Group
Faculty of Electrical Engineering
Eindhoven University of Technology
P.O.Box 513
5600 MB Eindhoven
The Netherlands

ON THE BOUNDS OF THE MODELLING ERRORS OF BLACK-BOX MIMO TRANSFER FUNCTION ESTIMATES

Y.C. Zhu

Abstract: This work attempts to solve the following problem: Derive a suitable description of the modelling errors (model uncertainty) of MIMO transfer function models which are obtained by black-box identification; this description must supply the quantitative information required for robustness study of the feedback system where the black-box model is used for the controller design.

An upper bound of modelling errors will be estimated. We shall refer to the asymptotic theory on the properties of black-box transfer function estimates, developed recently by Ljung and Yuan. Their theory shows that the transfer function estimates are consistent, the errors of the estimates are asymptotically joint normal, with a very simple expression for their covariances. In this paper, their results will be extended to cases where spectral analysis is used. Based on this theory, a bound of (additive) modelling errors is defined as the sum of the absolute value of the bias part and the 3σ bound of the variance (random) part of the modelling errors. Algorithms are proposed for the computations; a numerical test is performed to validate the theory. The bounds for other forms of the modelling errors will also be derived from the bound matrix we have obtained.

1 INTRODUCTION

Robustness of a feedback controlled system has significant importance for engineering applications of identification and control theory. In order to apply the theory for robust controller design, one needs not only a nominal model of the process (plant), but also a suitable description of the modelling errors (model uncertainty).

Denote $G^0(i\omega)$ as the true frequency responses of a multi-input multi-output (MIMO) process, and $\hat{G}(i\omega)$ as the nominal model; denote $g^0_{ij}(i\omega)$ and $\hat{g}_{ij}(i\omega)$ as the (i,j) elements of $G^0(i\omega)$ and $\hat{G}(i\omega)$ respectively.

Then, $G^0(i\omega)$ can be represented as

$$G^0(i\omega) = \hat{G}(i\omega) + \Delta_a G(i\omega) \tag{1.1}$$

where $\Delta_a G(i\omega)$ are the additive modelling errors (perturbation); or as

$$G^0(i\omega) = \hat{G}(i\omega)\left[I + \Delta_m G(i\omega)\right] \tag{1.2}$$

where $\Delta_m G(i\omega)$ are the multiplicative modelling errors (perturbation). There are other ways to define the modelling errors, but (1.1) and (1.2) are most frequently used. Similarly, for the element of $G^0(i\omega)$, we have

$$g^0_{ij}(i\omega) = \hat{g}_{ij}(i\omega) + \Delta_a g_{ij}(i\omega) \tag{1.3}$$

where $\Delta_a g_{ij}(i\omega)$ are called element additive modelling errors and

$$g^0_{ij}(i\omega) = \hat{g}_{ij}(i\omega)\left[1 + \Delta_m g_{ij}(i\omega)\right] \tag{1.4}$$

where $\Delta_m g_{ij}(i\omega)$ are called element multiplicative modelling errors. Note that $\Delta_a g_{ij}(i\omega)$ are precisely the elements of $\Delta_a G(i\omega)$; but $\Delta_m g_{ij}(i\omega)$ are not the elements of $\Delta_m G(i\omega)$.

In general, it is difficult to determine the modelling errors exactly. Recent theoretical results have shown that an upper bound of the modelling errors is needed for studying the robustness of a feedback system; cf. Doyle and Stein (1981), Owens and Chotai (1984) and Vidyasagar (1985). But how to obtain this bound is still an open problem.

This work attempts to tackle the problem: we estimate the upper bounds of the modelling errors when the black-box identification technique has been used to obtain the nominal model of a MIMO process which is assumed to be linear time-invariant.

The work will concentrate on deriving a bound for element additive modelling errors.

Briefly, the idea is the following:

- The modelling error $\Delta_a g_{ij}(i\omega)$ can be considered as a random variable, due to the assumption that the disturbance of the real process (plant) is a stochastic process.
- $\Delta_a g_{ij}(i\omega)$ is the sum of the bias part and the random (variance) part

$$\Delta_a g_{ij}(i\omega) = E\{\Delta_a g_{ij}(i\omega)\} + [\Delta_a g_{ij}(i\omega) - E\{\Delta_a g_{ij}(i\omega)\}] \tag{1.5}$$

- Prove that the variance part is asymptotically normal, with a variance $\sigma(\omega)$. Then

$$|\Delta_a g_{ij}(i\omega) - E\{\Delta_a g_{ij}(i\omega)\}| \leq 3\sigma(\omega) \qquad \text{w.p. } 99.7\% \tag{1.6}$$

- Denote $UB_{ij}(\omega)$ as an upper bound of $\Delta_a g_{ij}(i\omega)$ such that

$$|\Delta_a g_{ij}(i\omega)| \leq UB_{ij}(\omega) \qquad \forall \omega \in [0,\infty] \tag{1.7}$$

and finally, let

$$UB_{ij}(\omega) = |E\{\Delta_a g_{ij}(i\omega)\}| + 3\sigma(\omega) \tag{1.8}$$

In section 2, the asymptotic theory of black-box identification is presented. Based on this theory, in section 3, a bound of $\Delta_a g_{ij}(i\omega)$ is defined and computing algorithms are proposed. In section 4, other forms of bounds are derived. Conclusions and remarks are given in section 5.

2 THE ASYMPTOTIC THEORY

In recent years, many sophisticated time domain parametric identification methods have been developed. Most of these techniques can be classified in the family of prediction error methods. Very recently, Ljung and Yuan developed an asymptotic theory on the frequency properties of the prediction error models; cf. Ljung and Yuan (1985), Ljung (1985), and Yuan and Ljung (1984). This theory not only gives us much insight into identification, but also has practical importance, due to the fact that the results have very simple expressions. We shall present this theory for the Markov parameter (impulse response) model set, and then extend the theory for spectral analysis.

Consider a process with m inputs and p outputs. A general linear time-invariant discrete model for the relationship between inputs and outputs can be written

$$y(t) = \sum_{k=1}^{\infty} G(k)\, u(t-k) + v(t) \qquad (2.1)$$

where $y(t)$ is a p-dimensional column output vector at time t; $u(t)$ is an m-dimensional column input vector at time t; $\{G(k)\}$ is a sequence of $p \times m$ matrices; and $\{v(t)\}$ is assumed to be a p-dimensional stationary stochastic process with zero mean values.
When the delay operator q^{-1} is introduced as

$$q^{-1} u(t) = u(t-1)$$

the model (2.1) can also be written

$$y(t) = G(q^{-1})u(t) + v(t) \qquad (2.2)$$

where

$$G(q^{-1}) = \sum_{k=1}^{\infty} G(k)q^{-k} \qquad (2.3)$$

The transfer function matrix for the model is then

$$G(e^{i\omega}) = \sum_{k=1}^{\infty} G(k) \cdot e^{-ik\omega} \qquad -\pi \leq \omega \leq \pi \tag{2.4}$$

Note that we use $G(e^{i\omega})$ instead of $G(i\omega)$ to indicate discrete models. The real process is assumed to be linear time-invariant, with

$$G^0(e^{i\omega}) = \sum_{k=1}^{\infty} G^0(k) \cdot e^{-ik\omega} \qquad -\pi \leq \omega \leq \pi \tag{2.5}$$

being the true transfer function matrix.

Suppose that the input-output data have been collected from the real process until time N:

$$Z^N : \{ y(1),\ u(1),\ \ldots,\ y(N), u(N) \} \tag{2.6}$$

Then the problem is twofold: firstly to estimate the transfer function matrix from data set Z^N; secondly to assess the modelling errors defined in section 1.

2.1 Markov Parameter Model Set

Basically the estimation of a transfer function is a non-parametric problem. In practice, however, the estimation is carried out via a finite-dimensional parameter vector, so the techniques are parametrical.
But the parameters are only the vehicles for arriving at a transfer function estimate. The validation is done in the frequency domain.

One of the parametrizations is the Markov parameter model (finite impulse response), given by

$$G^n(q,\theta) = \sum_{k=1}^{n} G_k q^{-k} \tag{2.7}$$

so

$$G_k = G(k)\ , \quad k=1,\ldots,n$$

is a parametrization of model (2.1); and

$$\theta^n = [G_1\ G_2\ \ldots\ G_n]^T \quad \text{(an (mn) x p matrix)} \tag{2.8}$$

where θ^n is the parameter matrix and n is the order of the model. If n is big enough, this model set will give a good transfer function estimate, in the sense of a small bias, which is

$$G_N^n(e^{i\omega}) = \sum_{k=1}^{n} G_k \cdot e^{-ik\omega} \qquad -\pi \leq \omega \leq \pi \qquad (2.9)$$

If the objective is to estimate $G_N^n(e^{i\omega})$, rather than to determine θ^n, it is natural to let the order n depend on N, i.e. n=n(N).
For black-box models, the "best" order will tend to infinity as N tends to infinity.

If we introduce

$$\phi(t) = \begin{bmatrix} u(t-1) \\ u(t-2) \\ \vdots \\ u(t-n) \end{bmatrix} \qquad ((mn) \times 1 \text{ vector}) \qquad (2.10)$$

the model can be rewritten as

$$y(t) = (\theta^n)^T \phi(t) + \hat{v}(t) \qquad (2.11)$$

where $\{\hat{v}(t)\} \to \{v(t)\}$ as $n \to \infty$.

Let us introduce the parameter vector η

$$\eta^n = \mathrm{col}[(\theta^n)^T] \qquad ((mpn) \times 1 \text{ vector}) \qquad (2.12)$$

where col(.) denotes the column operator which stacks the columns of the matrix on top of each other.

Then

$$(\theta^n)^T \phi(t) = \mathrm{col}[(\theta^n)^T \phi(t)] = [\phi^T(t) \otimes I_p] \cdot \mathrm{col}[(\theta^n)^T]$$

$$= [\phi^T(t) \otimes I_p] \eta^n \qquad (2.13)$$

where $\otimes$ denotes Kronecker products; I_p is a p x p identity matrix.

Let $A = (a_{ij})$, $B = (b_{ij})$ be m x n and p x r matrices, then the Kronecker product of A and B is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ \vdots & & & \\ a_{m1}B & \cdots & \cdots & a_{mn}B \end{bmatrix} \tag{2.14}$$

and the second equality in (2.13) is the property of Kronecker products.

The parameters of θ can be estimated by the well known prediction error method, which minimizes the sum of the squared prediction errors associated with a parametrization of the model (2.1). If the parametrization is given in (2.7) and (2.8), and no further assumption about $\{v(t)\}$ is included in the model except that which is given in (2.1), then the prediction error method will take the form

$$\hat{\eta}_N^n = \arg\min_{\eta} \frac{1}{N} \sum_{t=1}^{N} \varepsilon^T(t,\theta^n)\varepsilon(t,\theta^n) \tag{2.15}$$

$$\begin{aligned} \varepsilon(t,\theta) &= y(t) - G(q,\theta)u(t) \\ &= y(t) - \left[\phi(t)\otimes I_p\right]\cdot\eta^n \end{aligned} \tag{2.16}$$

The estimate $\hat{\eta}_N^n$ is the well-known linear least squares estimate

$$\hat{\eta}_N^n = \left[R_n(N)\right]^{-1} \frac{1}{N} \sum_{t=1}^{N} \tilde{\phi}(t)y(t) \tag{2.17}$$

where

$$\tilde{\phi}(t) = \phi(t)\otimes I_p$$

and

$$R_n(N) = \frac{1}{N} \sum_{t=1}^{N} \tilde{\phi}(t)\tilde{\phi}^T(t)$$

The corresponding transfer function estimate is

$$\hat{G}_N^n(e^{i\omega}) = \sum_{k=1}^{n} \hat{G}_k \cdot e^{-ik\omega} = \left(\hat{\theta}_N^n\right)^T \cdot W(\omega) \tag{2.18}$$

where the parameters of $\hat{\theta}_N^n$ are determined by (2.17) and

$$W(\omega) = \begin{pmatrix} e^{-i\omega} \cdot I_m \\ e^{-2i\omega} \cdot I_m \\ \vdots \\ e^{-ni\omega} \cdot I_m \end{pmatrix} \tag{2.19}$$

Formal assumptions:

The results on the properties of the estimate (2.18) are given under the following conditions:

C1: The real process is given by (2.5) and

$$\sum_{k=1}^{\infty} k \cdot |g_{ij}^0(k)| < \infty . \qquad \forall\, i,j$$

(The real process is stable).

C2: $$E\{v(t)v^T(t+\tau)\} = R_v(\tau), \quad \sum_{\tau=-\infty}^{\infty} |\tau| \cdot \| R_v(\tau) \| < \infty$$

where E denotes the mean value expectation, and $\| \cdot \|$ denotes the matrix norm.

C3: $$\Phi_v(\omega) = \sum_{\tau=-\infty}^{\infty} R_v(t) e^{-i\tau\omega} \; ; \quad \| \Phi_v(\omega) \| < c \quad \forall \omega .$$

C4: $$v(t) = \sum_{k=0}^{\infty} H(k) e(t-k)$$

where $\{e(t)\}$ is white noise with unitary covariance matrix, and uniformly bounded fourth moment, and

$$\sum_{k=0}^{\infty} |h_{ij}(k)| < \infty \qquad \forall i,j$$

Let

$$R_u^N(\tau) = \frac{1}{N}\sum_{t=1}^{N} u(t)u^T(t+\tau) \tag{2.20}$$

$$\Phi_u^N(\omega, n) = \sum_{\tau=-n}^{n} R_u^N(\tau)\, e^{-i\tau\omega} \tag{2.21}$$

C5: $\{u(t)\}$ is independent of $\{v(t)\}$ and $\|u(t)\| \leq C_1 \quad \forall t$

C6: $\Phi_u^N(\omega, n(N)) \to \Phi_u(\omega) \quad$ as $N \to \infty$

C7: $C_2 \geq \|\Phi_u(\omega)\| \geq \delta > 0 \quad \forall\omega$

C8: $\frac{1}{\sqrt{n(N)}} \sum_{\tau=-2n(N)}^{2n(N)} |\tau| \, \|R_u^N(\tau)\| \to 0 \quad$ as $N \to \infty$

C9: $\lambda_{min}(\Phi_u(\omega)) \geq \delta > 0 \quad \forall\omega$

where $\lambda_{min}(A)$ denotes the smallest eigenvalue of the matrix A.

C10: $n(N) \to \infty \quad$ as $N \to \infty$

C11: $n^2(N)/N \to 0 \quad$ as $N \to \infty$

C12: $\sum_{k=1}^{\infty} \left(n(k^2)/k\right)^2 < \infty$

<u>Theorem 2.1</u>: Consider the transfer function estimate $\hat{G}_N^n(e^{i\omega})$ defined in (2.17) and (2.18). Denote

$$\hat{\mathcal{G}}_N^n(e^{i\omega}) = \text{col } \hat{G}_N^n(e^{i\omega}), \quad \mathcal{G}^0(e^{i\omega}) = \text{col } G^0(e^{i\omega})$$

Assume that C1 - C12 hold. Then

- $$\| G^0(e^{i\omega}) - E\{\hat{G}_N^n(e^{i\omega})\} \| \to 0 \qquad \text{as } N \to \infty \tag{2.22}$$

  (Consistent estimate)

  where $\| \cdot \|$ means the matrix norm.

- $$\frac{N}{n(N)} \text{cov}[\hat{\mathcal{G}}_N^n(e^{i\omega})] \to \Phi_u^{-1}(\omega) \otimes \Phi_v^T(\omega) \quad \text{as } N \to \infty \tag{2.23}$$

  (Asymptotic covariance)

- $$\sqrt{\frac{N}{n(N)}} [\hat{\mathcal{G}}_N^n(e^{i\omega}) - E\{\hat{\mathcal{G}}_N^n(e^{i\omega})\}] \in \text{As } N(0, \Phi_u^{-1}(\omega) \otimes \Phi_v^T(\omega)) \tag{2.24}$$

  (Asymptotic normal distribution)

The proof of the asymptotic covariance can be found in Yuan and Ljung (1984); the consistency and asymptotic normality will follow the SISO case analogously as in Ljung and Yuan (1985).

We now need the properties of each transfer function estimate; these follow directly from Theorem 2.1. Let

$$\hat{G}_N^n(e^{i\omega}) = \{\hat{g}_{ij}^n(e^{i\omega})\} \qquad \forall\ i,j \tag{2.25}$$

<u>Corollary 2.1</u>: Consider the estimate $\hat{g}_{ij}^n(e^{i\omega})$, as in (2.25). Assume that C1 - C12 hold. Then

- $$| g_{ij}^0(e^{i\omega}) - E\{\hat{g}_{ij}^n(e^{i\omega})\} | \to 0 \qquad \text{as } N \to \infty \tag{2.26}$$

- $$\frac{N}{n(N)} \operatorname{var}\left[\hat{g}^{n}_{ij}(e^{i\omega})\right] \to \left[\Phi_u^{-1}(\omega)\right]_{jj} \cdot \Phi_{v_i}(\omega) \quad \text{as } N \to \infty \tag{2.27}$$

  where $\left[\Phi_u^{-1}(\omega)\right]_{jj}$ is the (j,j) element of $\Phi_u^{-1}(\omega)$.

- $$\sqrt{\frac{N}{n(N)}}\left[\hat{g}^{n}_{ij}(e^{i\omega}) - E\{\hat{g}^{n}_{ij}(e^{i\omega})\}\right] \in \text{As } N(0,\left[\Phi_u^{-1}(\omega)\right]_{jj}\cdot\Phi_{v_i}(\omega) \tag{2.28}$$

From (2.27) we have

$$\operatorname{var} \hat{G}^{n}_{ij}(e^{i\omega}) \approx \frac{n}{N}\left[\Phi_u^{-1}(\theta)\right]_{jj}\cdot\Phi_{v_i}(\omega) \tag{2.29}$$

This remarkably simple expression will have much practical importance. In this paper, for instance, this result is used to obtain a bound of the modelling errors.

2.2 Spectral analysis

For the estimation of transfer functions, the spectral analysis technique can be used. The transfer function matrix of a MIMO process can be estimated row by row. The i-th row of the transfer matrix can be estimated by the following steps:

- Calculate the auto- and cross-correlation functions:

$$R_u^N(\tau) = \frac{1}{N}\sum_{t=1}^{N} u(t)\, u^T(t+\tau) \qquad \text{(m x m matrix)}$$

$$\tau = -n,\ldots,-1,0,1,\ldots,n \tag{2.30}$$

$$R_{uy_i}^N(\tau) = \frac{1}{N}\sum_{t=1}^{N} u(t)\, y_i(t+\tau) \qquad \text{(m x 1 vector)}$$

- Fourier transform

$$\Phi_u^N(\omega) = \sum_{\tau=-n}^{n} R_u^N(\tau)\, e^{-i\tau\omega} \qquad (m \times m \text{ matrix})$$

$$-\pi \leqslant \omega \leqslant \pi \qquad (2.31)$$

$$\Phi_{uy_i}^N(\omega) = \sum_{\tau=-m}^{m} R_{uy_i}^N(\tau)\, e^{-i\tau\omega} \qquad (m \times 1 \text{ vector})$$

- Calculate the transfer function estimate

$$\begin{pmatrix} \hat{g}_{i1}^n(\omega) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hat{g}_{im}^n(\omega) \end{pmatrix} = \left[(\Phi_u^N(\omega))^T \right]^{-1} \Phi_{uy_i}^N(\omega) \qquad (2.32)$$

Here $R_u^N(\tau)$, $R_{uy_i}^N(\tau)$, $\tau = -n,\dots,0,\dots,n$, are the parameters of the spectral analysis. The order of this model is 2n+1. Note that here the n is not necessarily equal to the n in the Markov parameter model set (2.7).

The computations of $R_u^N(\tau)$ and $R_{uy_i}^N$ may involve inputs and outputs prior to the first, and after the N-th sample moment. These values are supposed to be unknown. For the analysis, these values can either be replaced by zeros or by other bounded numbers. They will not influence the asymptotic result. In practice, however, these values are taken to be the real measurements. This can be realized by collecting the data set Z^{N+2n} instead of Z^N in (2.6):

$$Z^{N+2n}:\{ y(-n+1), u(-n+1), \dots y(N+m), n(N+m)\}$$

and computing $R_u^N(\tau)$ and $R_{uy_i}^N(\tau)$ according to (2.30).

<u>Theorem 2.2</u>: Consider the estimate $\hat{G}_i^n(e^{i\omega}) = [\hat{g}_{i1}^n(e^{i\omega}) \dots \hat{g}_{im}^n(e^{i\omega})]^T$ obtained from (2.30)-(2.32). Assume that C1-C11 hold.

Then

- $\| E\{ \hat{G}_i^n(e^{i\omega})\} - G_i^0(e^{i\omega})\| \to 0 \qquad \text{as } N \to \infty \qquad (2.33)$

(Consistent estimate)
where $G_i^0(e^{i\omega})$ is the i-th row of $G^0(e^{i\omega})$.

- $\frac{N}{2n(N)+1} \operatorname{var}[\hat{G}_i^n(e^{i\omega})] \to (\Phi_u^T(\omega))^{-1} \Phi_{v_i}(\omega) \qquad \text{as } N \to \infty \qquad (2.34)$

(Asymptotic covariance)

- $\sqrt{\frac{N}{2n(N)+1}} \left[\hat{G}_i^n(e^{i\omega}) - E\{\hat{G}_i^n(e^{i\omega})\}\right] \in \text{As } N(0, [\Phi_u^T(\omega)]^{-1}\Phi_{v_i}(\omega)) \qquad (2.35)$

(Asymptotic normal distribution)

This is the straightforward MIMO extension of theorem 2.2 in Zhu (1987a). Note that C12 is not necessary for spectral analysis. We found that the time domain estimation technique and the spectral analysis are asymptotically equivalent for the transfer function estimates.

Corollary 2.2: Consider the estimate $\hat{g}_{ij}^n(e^{i\omega})$ obtained from (2.32). Assume that C1-C11 hold. Then

- $|E\{ \hat{g}_{ij}^n(e^{i\omega})\} - g_{ij}^0(e^{i\omega})| \to 0 \qquad \text{as } N \to \infty \qquad (2.36)$

- $\frac{N}{2n(N)+1} \operatorname{var}[\hat{g}_{ij}^n(e^{i\omega})] \to [\Phi_u^{-1}(\omega)]_{jj} \cdot \Phi_{v_i}(\omega) \qquad \text{as } N \to \infty \qquad (2.37)$

- $\sqrt{\frac{N}{2n(N)+1}} \left[\hat{g}_{ij}^n(e^{i\omega}) - E\{\hat{g}_{ij}^n(e^{i\omega})\}\right] \in \text{As } N(0, [\Phi_u^{-1}(\omega)]_{jj} \cdot \Phi_{v_i}(\omega))$

(2.38)

The same result has been proved by Ljung (1985) for the general prediction error models, e.g. ARMA models, ARMAX models. Zhu (1987b) has proved the MIMO extension of the theory.

3. The algorithms

Based on the asymptotic theory, there are several possibilities for estimating a bound of the modelling errors. We will present two of them.

3.1 Algorithm 1:

According to Corollary 2.1, we propose

1) Perform Markov parameter estimation as in (2.17), with the order n sufficiently large. In practice, n will have to be determined according to the engineering knowledge about the process.

2) Calculate $\hat{G}_N^n(e^{i\omega})$ as in (2.18). Then, Corollary 2.1 tells us that $(\hat{g}_{ij}^n - g_{ij}^0)$ is asymptotically normal and

$$\mathrm{var}\left[\hat{g}_{ij}^n - g_{ij}^0\right] \approx \frac{n}{N}\left[\Phi_u^{-1}(\omega)\right]_{jj}\Phi_{v_i}(\omega)$$

Hence, asymptotically, we have the 3σ bound for $(\hat{g}_{ij}^n - g_{ij}^0)$:

$$|\hat{g}_{ij}^n - g_{ij}^0| < 3\sqrt{\frac{n}{N}\left[\Phi_u^{-1}(\omega)\right]_{jj}\cdot\Phi_{v_i}(\omega)} \qquad \text{w.p. } 99.7\% \tag{3.1}$$

3) Estimate $\Phi_u(\omega)$ and $\Phi_v(\omega)$ by

$$\hat{\Phi}_u(\omega) = \sum_{\tau=-n}^{n}\left[\frac{1}{N}\sum_{t=1}^{N} u(t)\, u^T(t+\tau)\right]e^{-i\omega\tau} = \Phi_u^N(\omega) \tag{3.2}$$

and

$$\hat{\Phi}_v(\omega) = \sum_{\tau=-n}^{n}\left[\frac{1}{N}\sum_{t=1}^{N} \hat{e}(t)\, \hat{e}^T(t+\tau)\right]e^{-i\omega\tau} = \Phi_{\hat{e}}^N(\omega) \tag{3.3}$$

where $\hat{e}(t)$ is the vector of output residuals

$$\hat{e}(t) = y(t) - \hat{G}_N^n(q^{-1})u(t)$$

$$= \left[G^0(q^{-1}) - \hat{G}_N^n(q^{-1}) \right] u(t) + v(t) \tag{3.4}$$

4) Perform a model reduction on $\hat{G}_N^n(e^{i\omega})$ to obtain the <u>low order model</u> $\hat{G}_N^\ell(e^{i\omega})$, where ℓ denotes the <u>low order</u>. In practice, it is the low order model that is used as the nominal model for the controller design. The theory and techniques of model reduction are well-established (see Glover (1984)).

5) Define the modelling error as

$$\Delta_a g_{ij}(e^{i\omega}) = g^0_{ij}(e^{i\omega}) - \hat{g}^\ell_{ij}(e^{i\omega}) \qquad \forall i,j \tag{3.5}$$

where $\hat{g}^\ell_{ij}(e^{i\omega})$ is the (i,j) element of $\hat{G}_N^\ell(e^{i\omega})$.

Then

$$\Delta_a g_{ij}(e^{i\omega}) = \left[\hat{g}^n_{ij}(e^{i\omega}) - \hat{g}^\ell_{ij}(e^{i\omega}) \right] + \left[g^0_{ij}(e^{i\omega}) - \hat{g}^n_{ij}(e^{i\omega}) \right]$$

We call the first term the bias part and the second term the random (variance) part of the modelling error $\Delta_a g_{ij}(e^{i\omega})$.

Now

$$|\Delta_a g_{ij}(e^{i\omega})| \leq |\hat{g}^n_{ij}(e^{i\omega}) - \hat{g}^\ell_{ij}(e^{i\omega})| + |g^0_{ij}(e^{i\omega}) - \hat{g}^n_{ij}(e^{i\omega})|$$

From (3.1) we have

$$|\Delta_a g_{ij}(e^{i\omega})| \leq |\hat{g}^n_{ij}(e^{i\omega}) - \hat{g}^\ell_{ij}(e^{i\omega})| + 3\sqrt{\frac{n}{N}\left[\hat{\Phi}_u^{-1}(\omega)\right]_{jj} \cdot \hat{\Phi}_{v_i}(\omega)}$$

where $\left[\hat{\Phi}_u^{-1}\right]_{jj}$ and $\hat{\Phi}_{v_i}(\omega)$ can be obtained from (3.2) and (3.3).

Denote $UB_{ij}(\omega)$ as an upper bound of $\Delta g_{ij}(e^{i\omega})$, then

$$UB_{ij}(\omega) = |\hat{g}^{n}_{ij}(e^{i\omega}) - \hat{g}^{\ell}_{ij}(e^{i\omega})| + 3\sqrt{\frac{n}{N}[\hat{\Phi}_u^{-1}(\omega)]_{jj}\cdot\hat{\Phi}_{v_i}(\omega)} \quad (3.6)$$

Remarks:

- From (3.4) we have

$$\hat{\Phi}_v(\omega) \rightarrow [G^0(e^{i\omega}) - \hat{G}^n_N(e^{i\omega})]\Phi_u(\omega)[G^0(e^{-i\omega}) - \hat{G}^n_N(e^{-i\omega})]^T + \Phi_v(\omega)$$

  From Theorem 2.1 and the large number theorem,

  $\hat{\Phi}_v(\omega) \rightarrow \Phi_v(\omega)$ as $N \rightarrow \infty$, $n \rightarrow \infty$

  This means that $\hat{\Phi}_v(\omega)$ is a consistent estimate of $\Phi_v(w)$.

- Algorithm 1 is a combination of identitification and model reduction. This method is proposed not only for obtaining a bound of modelling error; it is a new method for finding a black-box-low-order-nominal model. We have at least two reasons for stating this. Firstly, we have the theories which give us clear insight into identification and model reduction - the asymptotic theory of Ljung and Yuan, and the model approximation theory of Glover and others; secondly, numerically reliable algorithms are available. See Wahlberg (1985) for more discussions on the point.

If one already has a low-order-nominal-model, the computations can be done by spectral analysis, as shown in the following algorithm.

3.2 Algorithm 2:

1) Calculate the output residual from the low order model,

$$\hat{e}(t) = y(t) - \hat{G}^{\ell}_N(q^{-1})\, u(t) \quad (3.7)$$

$$\Longrightarrow$$

$$\hat{e}(t) = \Delta_a G(q^{-1})\, u(t) + v(t) \quad (3.8)$$

where

$$\Delta_a G(q^{-1}) = G^0(q^{-1}) - \hat{G}_N^{\ell}(q^{-1})$$

Note that $\hat{e}(t)$ here is different from the one in (3.4) where $\hat{e}(t)$ was the output residual from the high order model $\hat{G}_N^n(q^{-1})$.

2) Perform spectral analysis to estimate $\Delta_a G(e^{i\omega})$,

$$\begin{bmatrix} \Delta_a \hat{g}_{i1}^n(e^{i\omega}) \\ \\ \\ \Delta_a \hat{g}_{im}^n(e^{i\omega}) \end{bmatrix} = \left[\left(\Phi_u^N(\omega)\right)^T\right]^{-1} \Phi_{u\hat{e}_i}^N(\omega) \tag{3.9}$$

where $\Phi_{u\hat{e}_i}^N(\omega)$ is calculated as in (2.30) and (2.31) by replacing $y_i(t)$ by $e_i(t)$.

We note that (3.8) is a MIMO process with u(t) being the input, $\hat{e}(t)$ the output and v(t) the disturbance. Therefore, Theorem 2.2 can be applied to the estimate (3.9). Then, according to Corollary 2.2, we have

$$\left[\Delta_a \hat{g}_{ij}^n(e^{i\omega}) - \Delta_a g_{ij}(e^{i\omega})\right] \in \text{As } N\left(0, \frac{2n+1}{N}\left[\Phi_u^{-1}(\omega)\right]_{jj} \cdot \Phi_{v_i}(\omega)\right) \tag{3.10}$$

and

$$\Delta_a \hat{g}_{ij}^n(e^{i\omega}) - \Delta_a g_{ij}(e^{i\omega}) \leq 3\sqrt{\frac{2n+1}{N}\left[\Phi_u^{-1}(\omega)\right]_{jj} \Phi_{v_i}(\omega)} \quad \text{w.p. } 99.7\% \tag{3.11}$$

3) Estimate the noise spectrum.

Because the input and the noise are independent:

$$\Phi_v(\omega) = \Phi_{\hat{e}}(\omega) - \Delta_a G(e^{i\omega})\, \Phi_u(\omega)\, \Delta_a G^T(e^{-i\omega})$$

we can have a consistent estimate of $\Phi_v(\omega)$ by

$$\hat{\Phi}_v(\omega) = \Phi^N_{\hat{e}}(\omega) - \Delta_a\hat{G}_N(e^{i\omega})\,\Phi^N_u(\omega)\,\Delta_a\hat{G}^T(e^{-i\omega}) \tag{3.12}$$

4) Calculate the estimate of the bound,

$$\Delta_a g_{ij}(e^{i\omega}) = \left[\Delta_a\hat{g}^n_{ij}(e^{i\omega})\right] + \left[\Delta_a g(e^{i\omega}) - \Delta_a\hat{g}^n_{ij}(e^{i\omega})\right]$$

Again, we call the first term the bias part and the second term the random (variance) part of the modelling error $\Delta_a g_{ij}(e^{i\omega})$.

$$|\Delta_a g_{ij}(e^{i\omega})| \leq |\Delta_a\hat{g}^n_{ij}(e^{i\omega})| + |\Delta_a g_{ij}(e^{i\omega}) - \Delta_a\hat{g}^n_{ij}(e^{i\omega})|$$

From (3.11) we have

$$\Delta_a g_{ij}(e^{i\omega}) \leq |\Delta_a\,\hat{g}^n_{ij}(e^{i\omega})| + 3\sqrt{\frac{2n+1}{N}\left[\left(\Phi^N_u(\omega)\right)^{-1}\right]_{jj}\cdot\hat{\Phi}_{v_i}(\omega)} \tag{3.13}$$

where $\Delta_a\hat{g}^n_{ij}(e^{i\omega})$ is obtained from (3.9) and $\hat{\Phi}_{v_i}(\omega)$ is determined by (3.12).

Now, if we denote $UB_{ij}(\omega)$ as an upper bound of the modelling error $|\Delta_a g_{ij}(e^{i\omega})|$, we have

$$UB_{ij}(\omega) = |\Delta_a\hat{g}^n_{ij}(e^{i\omega})| + 3\ \frac{2n+1}{N}\left[\left(\Phi^N_u(\omega)\right)^{-1}\right]_{jj}\cdot\hat{\Phi}_{v_i}(\omega) \tag{3.14}$$

3.3 Numerical Test

The numerical test is performed for Algorithm 1 using the Markov parameter estimates of an industrial process. This is a process of 2 inputs and 2 outputs. The length of the Markov parameters is 50, and we will consider it to be the real process $G_0(t)$. We let $G^0(t) = 0$ for $t > 50$.

First, the simulation is performed in order to generate I/O data, where

the input u(t) and the output disturbance both consist of white noise signals. The power of the disturbances is about 11% of the power of the outputs.
Then a 50-th order parameter model is estimated by using 1000 samples of the I/O data from the simulation. The Markov parameters of the process and of the model are shown in Fig. 1. According to Algorithm 1, the upper bound is computed. Because the order of the model is equal to the order of the process, the bias part of the modelling errors is zero. The estimated 3σ bounds and the errors are plotted in Fig. 2, and the 2σ bounds and the errors are in Fig. 3. We find that the 2σ bounds (w.p. 95.5%) are good enough for the testing example.
More information about the numerical test can be found in van Beuningen (1988).

4. OTHER FORMS OF BOUNDS

So far, we have derived an upper bound of the element additive modelling error $\Delta_a g_{ij}(e^{i\omega})$, which can be calculated by (3.6) or by (3.14). It is easy to verify that for the element multiplicative modelling error $\Delta_m g_{ij}(e^{i\omega})$,

$$|\Delta_m g_{ij}(e^{i\omega})| < \frac{UB_{ij}(\omega)}{|\hat{g}_{ij}(e^{i\omega})|} \qquad \forall i,j \tag{4.1}$$

Hence we can take the right-hand side of (4.1) as an upper bound of $\Delta_m g_{ij}(e^{i\omega})$.

The bounds of the element (additive and multiplicative) modelling errors are also called <u>structured model uncertainty</u>, and they can be used to analyse the robust stability of a closed-loop system (see Owens and Chotai (1984)).

The recently developed so-called singular value analysis has proven to be effective for studying the robust properties of a MIMO feedback controlled system (See Doyle and Stein (1981) and Vidyasagar (1985)). This technique needs an upper bound of the matrix norm of the modelling error

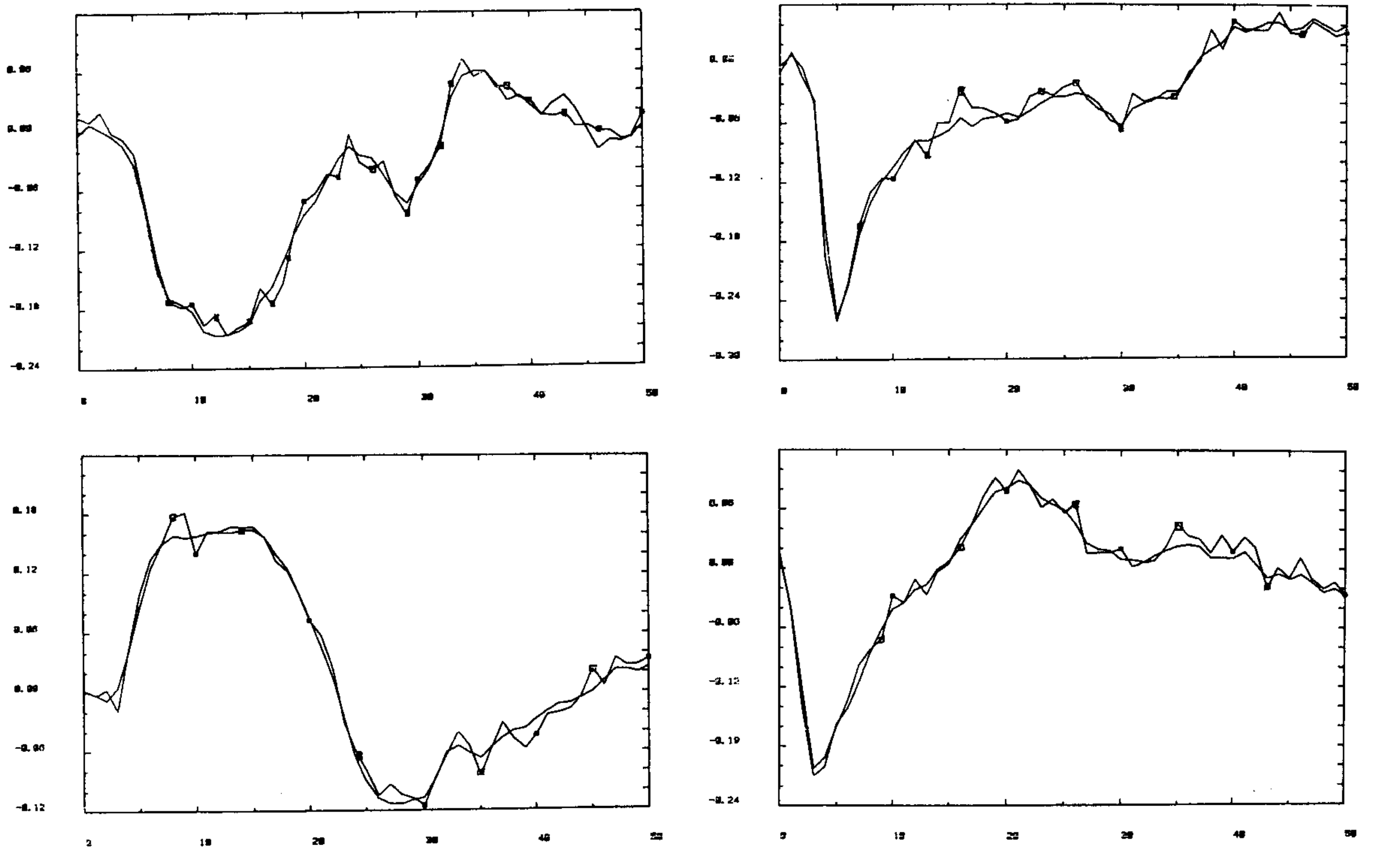

Fig. 1. Markov parameters. ——— $G^{\circ}(t)$, —□—□— $\hat{G}(t)$

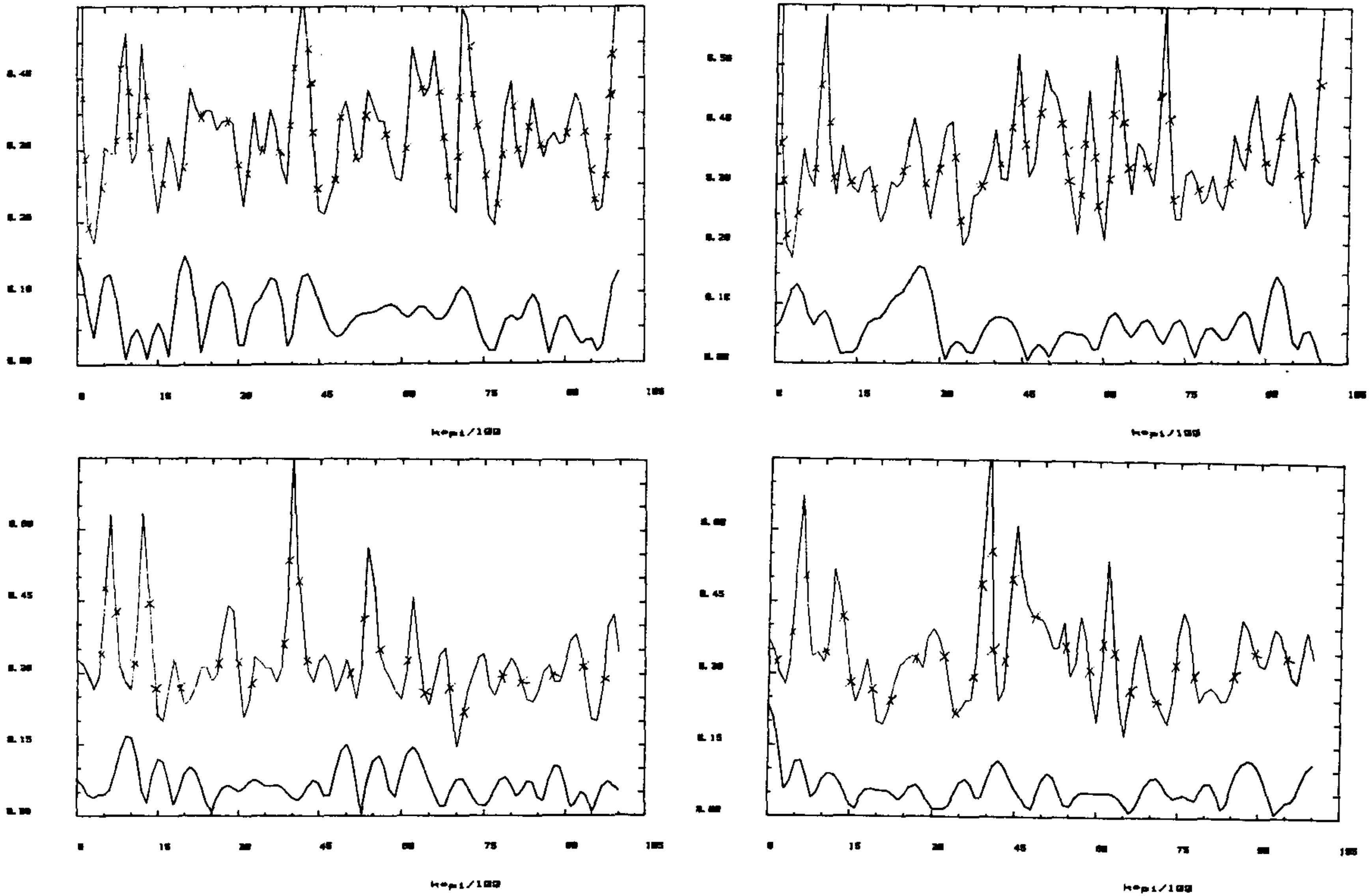

Fig. 2. Modelling errors and 3σ bounds. ——— $|\Delta G(e^{i\omega})|$, —×—×— $UB(\omega)$

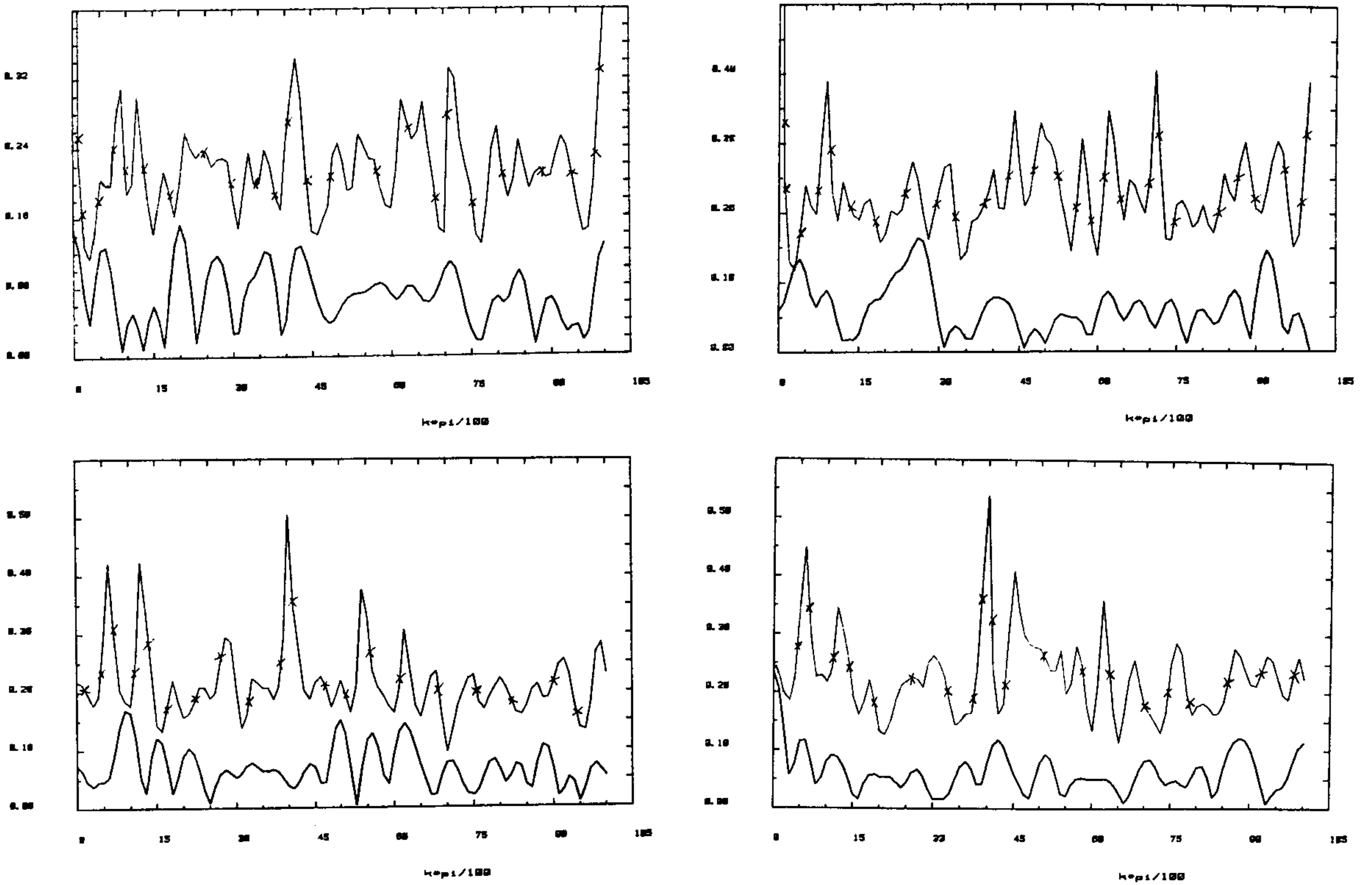

Fig. 3. Modelling errors and 2σ bounds. ——— $|\Delta G(e^{i\omega})|$, —×—×— $UB(\omega)$

$\Delta_a G(e^{i\omega})$ or $\Delta_m G(e^{i\omega})$ (see (1.1) and (1.2)). Following Doyle, we denote $\ell_a(\omega)$ and $\ell_m(\omega)$ as the bounds such that

$$\bar{\sigma}(\Delta_a G(e^{i\omega})) \leq \ell_a(\omega) \qquad \forall\, \omega \tag{4.2}$$

$$\bar{\sigma}(\Delta_m G(e^{i\omega})) \leq \ell_m(\omega) \qquad \forall\, \omega \tag{4.3}$$

where $\bar{\sigma}(\cdot)$ denotes the matrix norm, $\ell_a(\omega)$ and $\ell_m(\omega)$ are the additive and multiplicative bounds respectively. For a matrix A with complex variables, the matrix norm is defined as

$$\bar{\sigma}(A) \triangleq \max_{\|x\|=1} \|Ax\| \equiv \sqrt{\lambda_{max}[A^*A]} \tag{4.4}$$

where $\|\cdot\|$ is the Euclidean norm, $\lambda[\cdot]$ denotes eigenvalues, and * denotes conjugate transpose. $\bar{\sigma}(\cdot)$ is also called the <u>maximum singular value</u> of a matrix.

Denote $UB(\omega)$ as the matrix of the upper bounds of the element additive modelling errors which are obtained from section 3,

$$UB(\omega) = \{UB_{ij}(\omega)\} \tag{4.5}$$

Then we can ask ourselves the question: can we derive $\ell_a(\omega)$ and $\ell_m(\omega)$ from $UB(\omega)$? Let us derive $\ell_a(\omega)$ first.

<u>Theorem 4.1</u>: For $\Delta_a G(e^{i\omega})$ defined in (1.1) and $UB(\omega)$ defined in (4.5), we have

$$\bar{\sigma}[\Delta_a G(e^{i\omega})] \leq \bar{\sigma}[UB(\omega)] \qquad \forall\, \omega \tag{4.6}$$

Therefore, we can let

$$\ell_a(\omega) = \bar{\sigma}[UB(\omega)] \qquad \forall\, \omega \tag{4.7}$$

where $\ell_a(\omega)$ is called <u>unstructured uncertainty</u>.

<u>The proof of Theorem 4.1:</u>

Denote A as a matrix with complex variables, B as a matrix with real positive variable. Assume that A and B have the same dimensions, and

$$|A_{ij}| \leq B_{ij} \qquad\qquad i,j$$

From (4.4) we have

$$\bar{\sigma}^2(A) = \max_{\|x\|=1} \|Ax\|^2$$

$$= \max_{\|x\|=1} \sum_i (\sum_j A_{ij} x_{ij})^2 \tag{4.8}$$

But,

$$\sum_i (\sum_j A_{ij} x_{ij})^2 \leq \sum_i (\sum_j |A_{ij}| \cdot |x_j|)^2$$

$$\leq \sum_i (\sum_j B_{ij} |x_j|)^2 \qquad x \in C^n \tag{4.9}$$

Because $B_{ij} \geq 0$, there exists a $\tilde{x}$ with $\tilde{x}_j \geq 0$, $\|\tilde{x}\| = 1$ such that

$$\|B\tilde{x}\| = \bar{\sigma}(B)$$

Hence

$$\sum_i (\sum_j B_{ij} x_j)^2 \leq \sum_i (\sum_j B_{ij}\tilde{x}_{ij})^2 = \sigma^2(B) \qquad x \in C^n \tag{4.10}$$

From (4.9) and (4.10)

$$\bar{\sigma}^2(A) \leq \bar{\sigma}^2(B) \tag{4.11}$$

Therefore

$$\bar{\sigma}(A) \leq \bar{\sigma}(B) \tag{4.12}$$

If we substitute A and B by $\Delta_a G(e^{i\omega})$ and $UB(\omega)$, we will obtain (4.6).

Now let us try to derive the upper bound for multiplicative modelling error $\Delta_m G(e^{i\omega})$. Assume that the nominal model $\hat{G}(e^{i\omega})$ is square and invertible, then

$$\Delta_m G(e^{i\omega}) = \hat{G}^{-1}(e^{i\omega}).\Delta_a G(e^{i\omega}) \tag{4.13}$$

It is easy to show that

$$\bar{\sigma}[\Delta_m G(e^{i\omega})] < \bar{\sigma}[\hat{G}^{-1}(e^{i\omega})].\ell_a(\omega) \tag{4.14}$$

and

$$\bar{\sigma}[\Delta_m G(e^{i\omega})] < \bar{\sigma}[\,|\hat{G}^{-1}(e^{i\omega})|.UB(\omega)] \tag{4.15}$$

where $\ell_a(\omega)$ is from (4.7), $UB(\omega)$ is from (4.5), and $|.|$ means taking the absolute value of each element of the matrix.

Therefore we can determine the upper bound of $\Delta_m G(e^{i\omega})$ by

$$\ell_m(\omega) = \min\{\bar{\sigma}[\hat{G}^{-1}(e^{i\omega})].\ell_a(\omega)\ ,\ \bar{\sigma}[\,|\hat{G}^{-1}(e^{i\omega}).UB(\omega)]\} \tag{4.16}$$

5. CONCLUSIONS

The problem of defining and estimating the bounds of model uncertainty (modelling errors) of the transfer function estimates has been studied. We have shown that it is possible to represent the uncertainty of black-box transfer function models stochastically by using the asymptotic theory developed by Ljung and Yuan. This new theory has been extended here for the spectral analysis technique and applied to obtain a bound of the element additive errors of the transfer function models. Two algorithms have been proposed for the computations. The result of the numerical test has shown that the theory is adequate. The bounds of the matrix norm of (additive and multiplicative) modelling errors have been derived and they can readily be used to analyse the robustness of the feedback system.

Note that we can only derive the multiplicative modelling error $\Delta_m G$ from Δ_a if $\hat{G}$ is square and invertible.

Our result here is based on the assumption that the real process is linear, which is sometimes impractical.
The modelling and estimation of the model uncertainty for the processes with some non-linearity requires further research.

ACKNOWLEDGEMENT

I would like to express my thanks to my supervisor, Professor Eykhoff, for his encouragement, guidance and valuable help throughout the work; to my colleagues Dr. Damen and Dr. van den Boom for the stimulating discussions we had. I would also like to thank Professor Hautus of the Department of Mathematics and Informatics, Eindhoven University of Technology, who provided the proof of Theorem 4.1. The numerical test was performed by van Beuningen.

REFERENCES

Broekhoven, E. van (1987)
ON A BOUND OF BLACK-BOX MODELLING ERRORS.
Report of a "stage" project. Measurement and Control Group, Faculty of Electrical Engineering, Eindhoven University of Technology, The Netherlands, 1987.

Beuningen, E. van (1988)
THE ESTIMATION OF AN UPPER BOUND OF IDENTIFICATION ERRORS AND ROBUSTNESS ANALYSIS OF FEEDBACK SYSTEMS. Master's Thesis, Measurement and Control Group, Faculty of Electrical Engineering, Eindhoven University of Technology, The Netherlands, (1988).

Doyle, J.C. and G. Stein (1981)
MULTIVARIABLE FEEDBACK DESIGN: CONCEPTS FOR A CLASSICAL/MODERN SYNTHESIS.
IEEE Trans. Autom. Control, Vol. AC-26, p. 4-16.

Glover, K. (1984)
ALL OPTIMAL HANKEL-NORM APPROXIMATIONS OF LINEAR MULTIVARIABLE SYSTEMS AND THEIR L^{∞}-ERROR BOUNDS.
Int. J. Control, Vol. 39(1984), p. 1115-1193.

Ljung, L. (1985)
ASYMPTOTIC VARIANCE EXPRESSIONS FOR IDENTIFIED BLACK-BOX TRANSFER FUNCTION MODELS.
IEEE Trans. Autom. Control, Vol. AC-30(1985), p. 834-844.

Ljung, L. and Z.D. Yuan (1985)
ASYMPTOTIC PROPERTIES OF BLACK-BOX IDENTIFICATION OF TRANSFER FUNCTIONS.
IEEE Trans. Autom. Control, Vol. AC-30(1985), p. 514-530.

Owens, D.H. and A. Chotai (1984)
ON EIGENVALUES, EIGENVECTORS AND SINGULAR VALUES IN ROBUST STABILITY ANALYSIS.
Int. J. Control, Vol. 40, p. 285-296.

Vidyasagar, M. (1985)
CONTROL SYSTEM SYNTHESIS: A factorization approach.
Cambridge, Mass.: MIT Press, 1985.
MIT Press series in signal processing, optimization and control, Vol. 7.

Wahlberg, B. (1985)
CONNECTIONS BETWEEN SYSTEM IDENTIFICATION AND MODEL REDUCTION.
Department of Electrical Engineering, Linköping University, Linköping, Sweden, 1985.
Technical Report LiTH-ISY-I-0746.

Yuan, Z.D. and L. Ljung (1984)
BLACK-BOX IDENTIFICATION OF MULTIVARIABLE TRANSFER FUNCTIONS: Asymptotic properties and optimal input design.
Int. J. Control, Vol. 40 (1984), p. 233-256.

Zhu, Y.C. (1987a)
ON A BOUND OF THE MODELLING ERRORS OF BLACK-BOX TRANSFER FUNCTION ESTIMATES.
Faculty of Electrical Engineering, Eindhoven University of Technology, Eindhoven, the Netherlands, EUT-Report 87-E-173.

Zhu, Y.C. (1987b)
BLACK-BOX IDENTIFICATION OF MIMO TRANSFER FUNCTIONS - Asymptotic Properties of Prediction Error Models.
Faculty of Electrical Engineering, Eindhoven University of Technology, The Netherlands, 1987. EUT-Report 87-E-182.

ISSN 0167-9708
Coden: TEUEDE

(162) Meer, A.C.P. van
TMS32010 EVALUATION MODULE CONTROLLER.
EUT Report 86-E-162. 1986. ISBN 90-6144-162-5

(163) Stok, L. and R. van den Born, G.L.J.M. Janssen
HIGHER LEVELS OF A SILICON COMPILER.
EUT Report 86-E-163. 1986. ISBN 90-6144-163-3

(164) Engelshoven, R.J. van and J.F.M. Theeuwen
GENERATING LAYOUTS FOR RANDOM LOGIC: Cell generation schemes.
EUT Report 86-E-164. 1986. ISBN 90-6144-164-1

(165) Lippens, P.E.R. and A.G.J. Slenter
GADL: A Gate Array Description Language.
EUT Report 87-E-165. 1987. ISBN 90-6144-165-X

(166) Dielen, M. and J.F.M. Theeuwen
AN OPTIMAL CMOS STRUCTURE FOR THE DESIGN OF A CELL LIBRARY.
EUT Report 87-E-166. 1987. ISBN 90-6144-166-8

(167) Oerlemans, C.A.M. and J.F.M. Theeuwen
ESKISS: A program for optimal state assignment.
EUT Report 87-E-167. 1987. ISBN 90-6144-167-6

(168) Linnartz, J.P.M.G.
SPATIAL DISTRIBUTION OF TRAFFIC IN A CELLULAR MOBILE DATA NETWORK.
EUT Report 87-E-168. 1987. ISBN 90-6144-168-4

(169) Vinck, A.J. and Pineda de Gyvez, K.A. Post
IMPLEMENTATION AND EVALUATION OF A COMBINED TEST-ERROR CORRECTION PROCEDURE FOR MEMORIES WITH DEFECTS.
EUT Report 87-E-169. 1987. ISBN 90-6144-169-2

(170) Hou Yibin
DASM: A tool for decomposition and analysis of sequential machines.
EUT Report 87-E-170. 1987. ISBN 90-6144-170-6

(171) Monnee, P. and M.H.A.J. Herben
MULTIPLE-BEAM GROUNDSTATION REFLECTOR ANTENNA SYSTEM: A preliminary study.
EUT Report 87-E-171. 1987. ISBN 90-6144-171-4

(172) Bastiaans, M.J. and A.H.M. Akkermans
ERROR REDUCTION IN TWO-DIMENSIONAL PULSE-AREA MODULATION, WITH APPLICATION TO COMPUTER-GENERATED TRANSPARENCIES.
EUT Report 87-E-172. 1987. ISBN 90-6144-172-2

(173) Zhu Yu-Cai
ON A BOUND OF THE MODELLING ERRORS OF BLACK-BOX TRANSFER FUNCTION ESTIMATES.
EUT Report 87-E-173. 1987. ISBN 90-6144-173-0

(174) Berkelaar, M.R.C.M. and J.F.M. Theeuwen
TECHNOLOGY MAPPING FROM BOOLEAN EXPRESSIONS TO STANDARD CELLS.
EUT Report 87-E-174. 1987. ISBN 90-6144-174-9

(175) Janssen, P.H.M.
FURTHER RESULTS ON THE McMILLAN DEGREE AND THE KRONECKER INDICES OF ARMA MODELS.
EUT Report 87-E-175. 1987. ISBN 90-6144-175-7

(176) Janssen, P.H.M. and P. Stoica, T. Söderström, P. Eykhoff
MODEL STRUCTURE SELECTION FOR MULTIVARIABLE SYSTEMS BY CROSS-VALIDATION METHODS.
EUT Report 87-E-176. 1987. ISBN 90-6144-176-5

(177) Stefanov, B. and A. Veefkind, L. Zarkova
ARCS IN CESIUM SEEDED NOBLE GASES RESULTING FROM A MAGNETICALLY INDUCED ELECTRIC FIELD.
EUT Report 87-E-177. 1987. ISBN 90-6144-177

(178) Janssen, P.H.M. and P. Stoica
ON THE EXPECTATION OF THE PRODUCT OF FOUR MATRIX-VALUED GAUSSIAN RANDOM VARIABLES.
EUT Report 87-E-178. 1987. ISBN 90-6144-178-1

(179) Lieshout, G.J.P. van and L.P.P.P. van Ginneken
GM: A gate matrix layout generator.
EUT Report 87-E-179. 1987. ISBN 90-6144-179-X

(180) Ginneken, L.P.P.P. van
GRIDLESS ROUTING FOR GENERALIZED CELL ASSEMBLIES: Report and user manual.
EUT Report 87-E-180. 1987. ISBN 90-6144-180-3

(181) Bollen, M.H.J. and P.T.M. Vaessen
FREQUENCY SPECTRA FOR ADMITTANCE AND VOLTAGE TRANSFERS MEASURED ON A THREE-PHASE POWER TRANSFORMER.
EUT Report 87-E-181. 1987. ISBN 90-6144-181-1