Effect of different load-cases in the identification process of inhomogeneous materials

Effect of different load-cases in the identification process of inhomogeneous materials.

WFW 92.115

Dorrit van der Burgt
september 1992

Stageverslag
Begeleiders: ir. M.R. van Ratingen
dr.ir. C.W.J. Oomens

Technische Universiteit Eindhoven
Vakgroep Fundamentele Werktuigkunde
Eindhoven, september 1992

Effect of different load-cases in the identification process of inhomogeneous materials.

T.C.G. van der Burgt
Eindhoven, September 1992
Eindhoven University of Technology

Contents

Summary

1. Introduction

2. Theory of the identification method according to Hendriks (1991)

3. Results

3.1. Boundary condition change-over

3.2. Effect of different load-cases in the presence of model errors

4. Conclusions

5. Recommendations

References

Summary

With a numerical/experimental method, called the identification method, the properties of an inhomogeneous material can be characterized. Numerical simulations of experiments are used to test the identification method. Different sets of boundary conditions instead of just one set are applied during one estimation procedure. Correct models as well as wrong chosen models are used to investigate the influence of applying more sets of boundary conditions. It appears that it is very well possible that parameters converge despite the presence of model errors. This however does not "prove" that the model is correct.

1. Introduction

Soft biological materials behave inhomogeneously, i.e. they have properties that can vary with position, like the fiber direction. Particularly for inhomogeneous materials, an experimental-numerical approach offers better possibilities than traditional testing, for it no longer demands a homogeneous strain field of the loaded specimen.

Hendriks (1991) proposed an experimental-numerical approach called the identification method. Its principle is shown in figure 1.

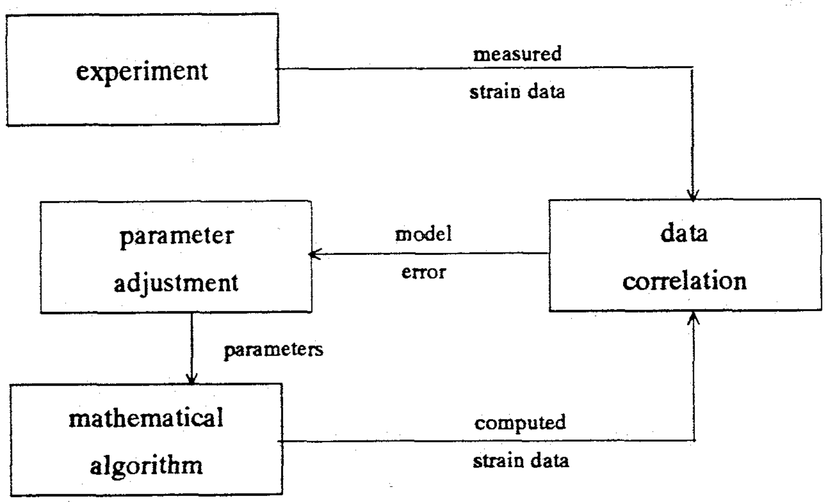


Figure 1. Diagram for the identification method

The actual inhomogeneous strain field of the specimen under investigation is measured with a digital image technique. With a finite element model a numerical analysis of the experiment is performed. With initial estimates for the material parameters the strain field of this finite element model is computed and compared with the actual strain field. The weighted difference is used for further adjustment of the parameters.

Simulated experiments are carried out by computing a set of displacements with a given model and by using these displacements as fictitious measured data for parameter estimation. The outcome of these simulation studies are useful for the selection and design of an experiment, that is best suited for material characterization. The finite element code DIANA is used for the finite element modeling and for the parameter estimations an extra module called PAREST (Courage and Hendriks,1989) is used.

In the present study the influence of a sudden change in the boundary condition on the iterative process is investigated by means of numerical simulations. Also the influence of applying successive different sets of boundary conditions in one iterative procedure on detecting model errors is studied.
In section 2 the theory of the identification method is mentioned briefly. The first part of section 3 deals with the change-over of the boundary conditions. Further section 3 describes the model used as numerical experiment. Finally the estimation results of the different estimation models will be discussed in section 3. Conclusions and recommendations are given in section 4 and 5 respectively.

2. Theory of the identification method according to Hendriks (1991)

The subject of parameter estimation of the identification method can be approached by the deterministic method of sequential minimum variance. The observational data consist of three columns y_k, $k=1,..,3$. Here k indicates a load case of the experiment under investigation. The unknown material parameters are represented in a column x. Function $h_k(x)$ describes the dependence of the k-th observation on x and the observation errors will be presented by a column v_k:

$$y_k = h_k(x) + v_k \qquad k=1,\ldots,3$$

The updating equation for the sequential estimations, based on prior knowledge of the parameters, is given by:

$$\hat{x}_{i+1} = \hat{x}_i + K_{i+1}(y_k - h_k(\hat{x}_i)) \qquad \begin{array}{l} i=1,...,5 \text{ voor } k=1 \\ i=6,...,10 \text{ voor } k=2 \\ i=11,...,15 \text{ voor } k=3 \end{array}$$

where i denotes the iteration step.
With weighting matrix $K_{i+1} = (P_i + Q_k) H_{i+1}^T (R + H_{i+1} (P_i + Q_k) H_{i+1}^T)^{-1}$
and covariance matrix $P_{i+1} = (I - K_{i+1} H_{i+1})(P_i + Q_k)(I - K_{i+1} H_{i+1})^T + K_{i+1} R K_{i+1}^T$
where

$$H_{i+1} = \left(\frac{\partial h_{i+1}(x)}{\partial x}\right)_{x=\hat{x}_i} \qquad \begin{array}{l} i=1,...,5 \text{ voor } k=1 \\ i=6,...,10 \text{ voor } k=2 \\ i=11,...,15 \text{ voor } k=3 \end{array}$$

The matrix R is the covariance matrix of the observation error v. The nonnegative symmetric matrix Q depends on the confidence in the model assumed.

3. Results

In these simulation studies a flat membrane (dimensions: 5 x 4 x 0.2) is used, the material of which is assumed to have orthotropic properties and is assumed to be linear elastic.
The chosen finite element model consists of 100 4-noded plane stress elements.

3.1. Boundary condition change-over

Assume both the exact fiber layout and the model's fiber layout are given by a bilinear function:

$$\alpha = b_1 x + b_2 y$$

The parameters

$$x^T = (b_1, b_2, E_2, \nu_{21})$$

will be estimated. The estimation results where the sample is loaded with successively three different sets of boundary conditions are compared with the estimation results where only one loading case is applied during the whole estimation procedure. Compared are three situations: first with exact measured data, then with an artificial disturbance of the measured data to simulate observation errors and after that also an artificial disturbance of the boundary conditions is performed to introduce a model error. An evaluation of the results is given in appendix 1. For the situation with noise on both the measurements and the boundary conditions the parameter values are given as function of the iteration steps, see appendix 2. We notice that a sudden change-over from one set of boundary conditions to another hardly has any influence on the parameter estimation at all. Only the kind of boundary condition appears to be of relevance.

3.2. Effect of different load-cases in the presence of model errors

The continuous function over the sample surface which describes the exact fiber orientation is given by:

$$\alpha = 0.1x + 0.1y + 0.01x^2 + 0.005xy$$

and shown in figure 2. α denotes the positive angle between the material 1-direction and the model x-axis in radians.

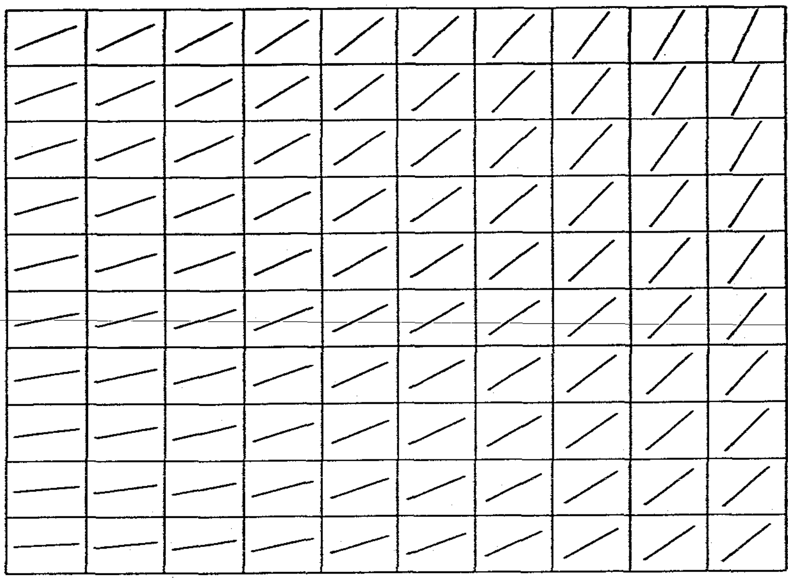


Figure 2. Sample shape and fiber layout described with $\alpha = 0.1x + 0.1y + 0.01x^2 + 0.005xy$

The exact material parameters are chosen:

$$E_1=1, E_2=0.25, \nu_{21}=0.3, G_{12}=0.25$$

The choice of these values is rather arbitrarily, although it is taken into account that a stiffness ratio of 4 is on one side large enough to face the influence of noise (Meuwissen, 1992) and gives on the other side a sufficient broad margin for the estimation of Poisson ratio ν_{21}. For the parameters have to satisfy the following condition (Oomens, 1990):

$$\nu_{21}^2 \frac{E_1}{E_2} < 1$$

Three sets of measured data are artificially obtained by applying three different sets of boundary conditions, see figure 3. The used finite element model has the exact fiber layout and the exact material parameters as given above.

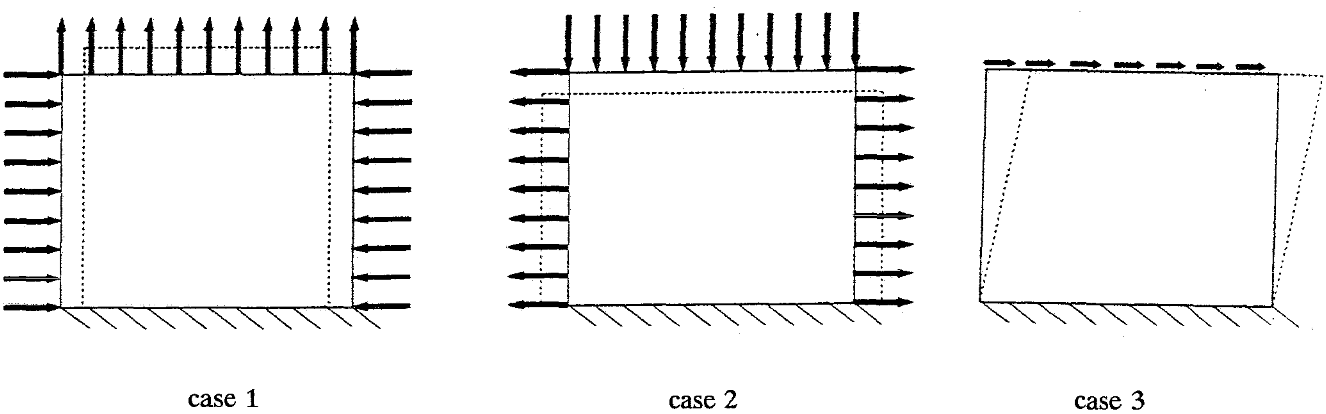


Figure 3. Sets of boundary conditions

To simulate observation errors the measured data will be disturbed. The noise is a realization of a zero mean normal distribution with a standard deviation 10^{-4}.

In the finite element model with which the estimations will be made, we introduce a model error. The fiber layout will be described by a bilinear function with unknown parameters b_1 and b_2:

$$\alpha = b_1 x + b_2 y$$

Only the ratios between the stiffness parameters will be estimated, because when dealing with real experiments the boundary conditions, in the finite element model, are based on the displacements of the edge markers on the specimen. This means only displacements can be used as boundary conditions, but no forces. A consequence of this so-called local approach (Hendriks, 1991) is that no stiffness parameters can be determined, but only their ratios.
The Young's modulus in the material 1-direction and the shear modulus are assumed to be known: $E_1 = 1.0$, $G_{12} = 0.25$.
The column of unknown parameters is given by:

$$x^T = (b_1, b_2, E_2, \nu_{21})$$

The initial guesses are:

$$x_0^T = (0.01, 0.04, 0.5, 0.2)$$

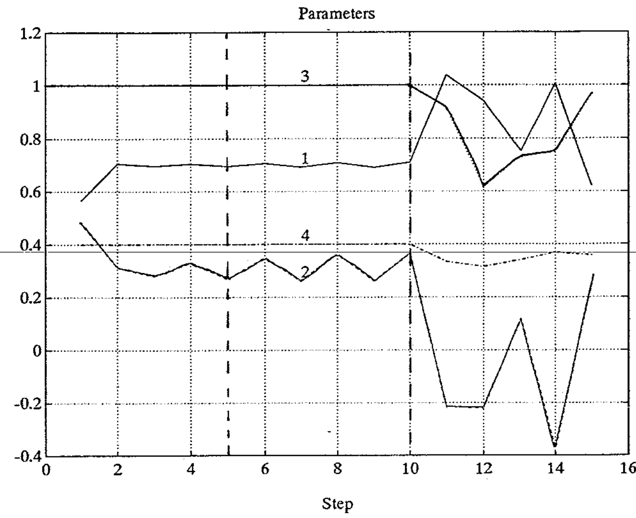
The confidence in the initial guesses is expressed by setting matrix P_0. P_0 is considered to be diagonal:

$$P_0 = \lceil 10^{-3}, 10^{-3}, 10^{-2}, 10^{-3} \rfloor$$

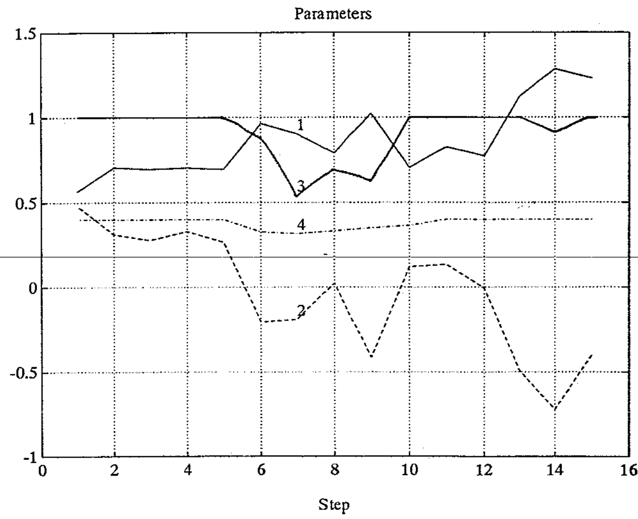
Q prevents the parameter error covariance P_i to become too small. Q is also taken diagonal:

$$Q = \lceil 10^{-3}, 10^{-3}, 10^{-2}, 10^{-3} \rfloor$$

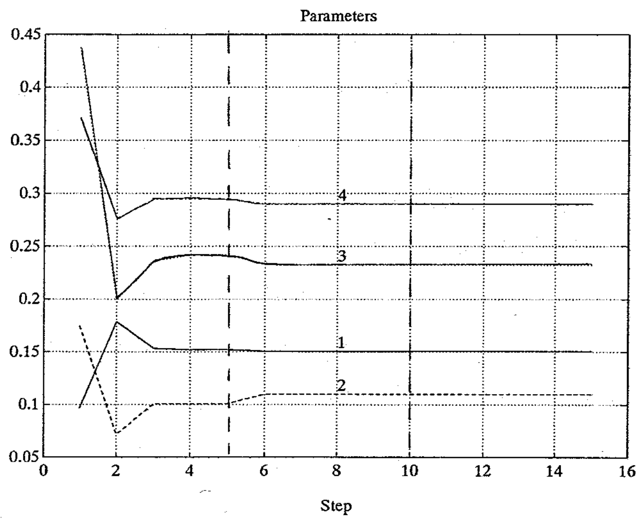
The elements of the diagonal matrix R equal $\sigma^2 = 10^{-8}$.
During one estimation procedure case 1, case 2 and case 3 are applied successively. In all three simulations with the cases applied in different order, will be tested:
simulation A with case 1, case 2, case 3
simulation B with case 2, case 3, case 1
simulation C with case 3, case 1, case 2.
Each case comprises 5 iterations. After each iteration the computed data are compared with the measured data, which have been obtained by applying the concerned case on the exact finite element model. The difference between the measured and computed data leads to adjustments of the parameter values. The parameter estimations obtained after the first 5 iterations are used as initial guesses for the parameter values as the second case is being applied, the same counts for the change-over from case 2 to 3. This also holds for the initial guesses for the error covariance of the parameter estimations.
The estimation results for the three simulations are shown in figure 4.



simulation A



simulation B



simulation C

par 1= bilin 1
par 2= bilin 2
par 3= Young
par 4= Poisson

Figure 4. Parameter estimations as function of the iteration step

Only for simulation C the parameter estimations converge to stable values. From the parameter estimation courses, see figure 4, we see a clear difference between case 3 on one side and case 1 and case 2 on the other side. This also holds for the courses of the mean and standard deviation of the residuals of the displacement field, as shown in figure 5.

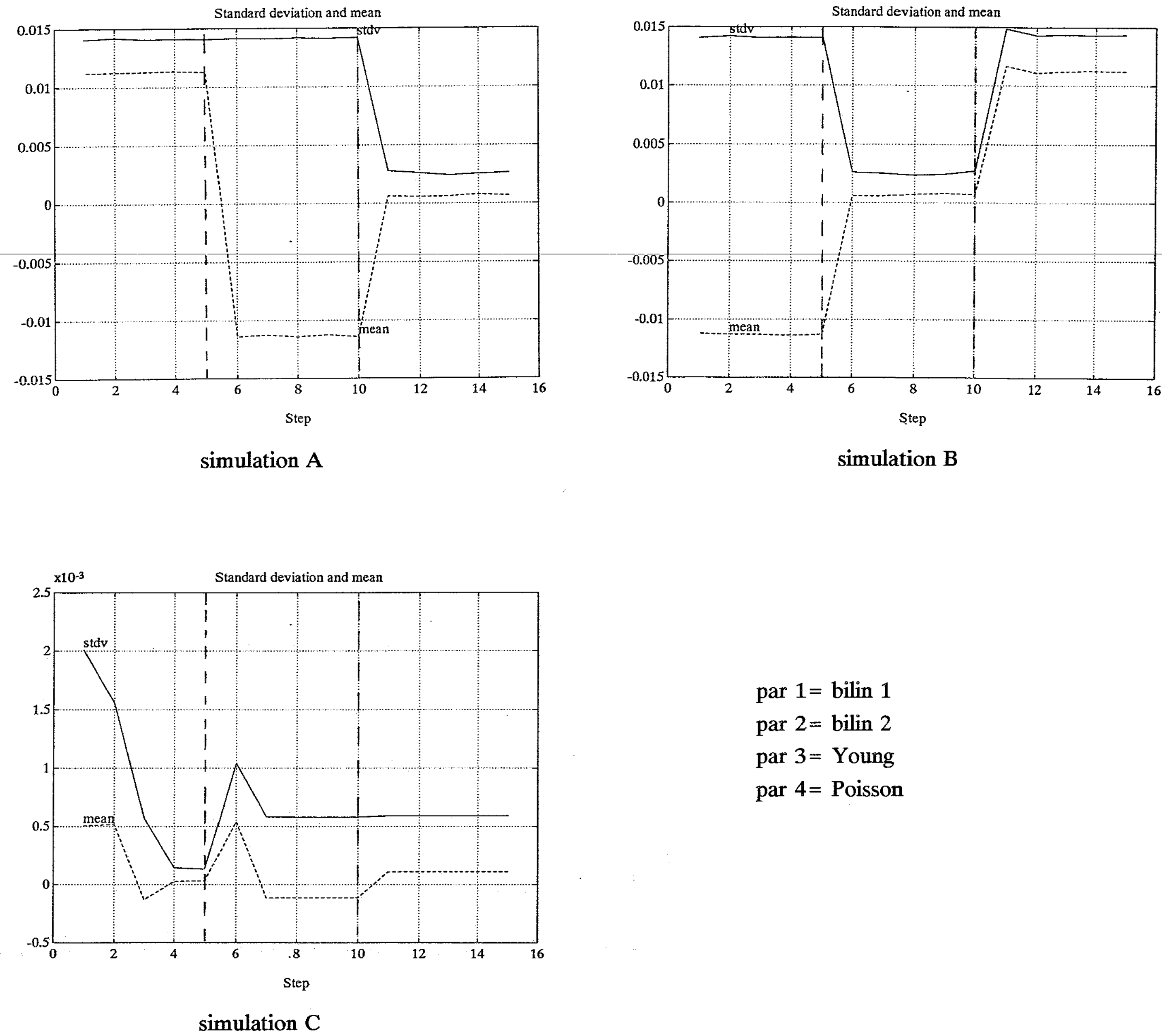


Figure 5. Mean and standard deviation of the residuals vs. iteration steps

Simulations A and B show identical features, namely a constant standard deviation and mean during one case with a minimum standard deviation and the mean closest to zero during case 3. Evidently the initial values of the parameter at each new case are significant for the further course of the parameter estimations. Clearly case 3 gives good results when it is being applied as first, i.e. with the original initial guesses, but when case 3 is applied as second or third apparently the initial values are too far from the exact values. Case 1 and 2 only give good results when applied after case 3. It appears that the initial values reached after applying case 3 as first are that close to the exact values that convergence appears. When we look at the stable parameter values reached after 11 iterations in simulation C we notice that the fiber layout described with the estimated coefficients:

$$\alpha = 0.15x + 0.11y$$

and shown in appendix 3, agrees very good with the actual fiber layout shown in figure 2. The estimation of Young's modulus E_2 and Poisson ratio ν_{21} also approach the exact value.

However the plot of the mean and standard deviation of the residuals during the parameter estimations, see figure 5, show clear deflections from the desired course. The standard deviation of the residuals does not descend step by step to a minimum value that approaches the standard deviation of the noise (which is in fact the measurement error), but increases when case 2 is being applied. Also the mean of the residuals, which has to go to zero, shows a peak when case 2 is being applied. The course of the mean as function of the iteration steps while case 2 or 3 is being applied shows a constant value that differs from zero.

Figure 6 is a plot of the residuals. The little points represent the positions of the markers when the sample is unloaded, the cross-signs when it is loaded. The open circles represent the calculated positions of the markers, based on the final parameter estimates.

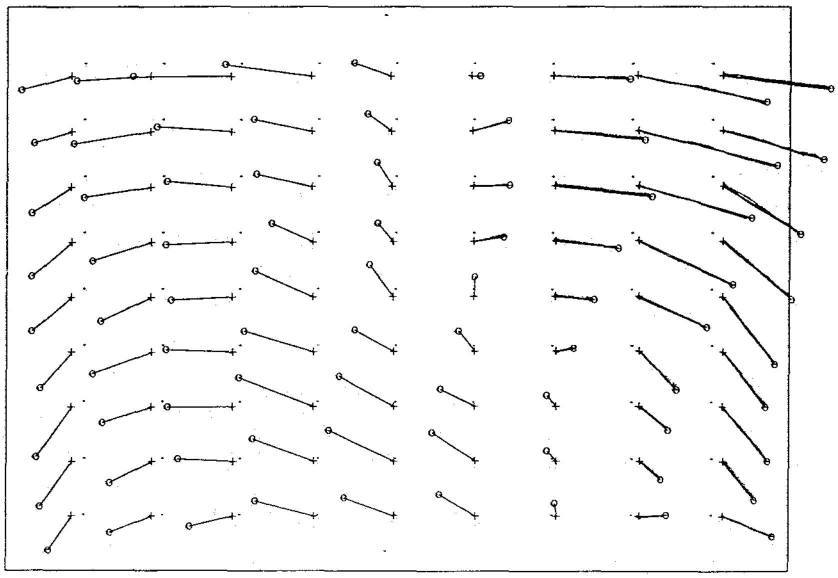


Figure 6. Residuals (x500) of the displacement field after 11 iterations (simulation C)

Despite of the well executed parameter estimations we clearly are able to distinguish a structure in the residuals; this suggests model deficiencies, what is of course in agreement with the initial wrong chosen function for the fiber layout.

Now only one case instead of three cases in one estimation process is being applied. The same initial guesses as before are chosen for the parameter values and the same bilinear function for the fiber layout is taken. With case 2 the parameters don't converge to stable values and the plot of the residuals shows a strong place-structured design, see appendix 4. However with case 3 as the only boundary condition the estimates converge very fast, see appendix 5. Already after 5 iterations the material parameters reach stable values that approach the exact values very well, see appendix 5. Appendix 6 shows the estimated fiber direction field given by the equation: $\alpha = 0.15x + 0.10y$. It approaches the exact fiber layout almost identically. In appendix 7 the mean and standard deviation of the residuals show a descending course during the parameter estimations. In appendix 7 it is also shown that the residuals appear to be very small and at random, despite the existence of a model error.

In table 1 the statistics of the residuals after 15 iterations are given for the different models.

simulation	stdev of residuals	mean of residuals
A	$0.2614 \cdot 10^{-2}$	$0.7296 \cdot 10^{-3}$
B	$0.1435 \cdot 10^{-1}$	$0.1121 \cdot 10^{-1}$
C	$0.5904 \cdot 10^{-3}$	$0.1103 \cdot 10^{-3}$
case 2	$0.1414 \cdot 10^{-1}$	$-0.1124 \cdot 10^{-1}$
case 3	$0.1325 \cdot 10^{-3}$	$0.3134 \cdot 10^{-4}$

Table 1. Statistics of the residuals after 15 iterations.

It can be observed that for the simulations A and B and for the simulation with only case 2 the standard deviation of the residuals differ significantly from the standard deviation of the measurement error, which was set to 10^{-4}. For a random distribution of the residuals the mean has to go to zero, which isn't true for the simulations B and case 2.

4. Conclusions

From the simulations above it has been made clear that an inhomogeneous strain field does not automatically mean that a good identifiability is possible. Another conclusion is that small and random-structured residuals are not a guarantee for the absence of model deficiencies. Applying successively different sets of boundary conditions in only one simulation procedure offers advantages above applying just one loading case, because the chances of misinterpreting the estimation results are much smaller with more loading cases applied. It leads to a faster and more reliant way to detect whether or not model errors are at issue. Shear loading appears to be a very good test. It leads to a fast convergence of the parameter values and to a good fit of the fiber layout. A bilinear function with only two parameters can describe a fiber layout given by a function with four parameters very well.

5. Recommendations

It is recommended to use more different sets of boundary conditions when the confidence in the model used for the estimations is less. It is a faster and more reliant way to detect model errors and to find the best set of boundary conditions for fast convergence of the parameters.
For the fiber layout can often be approximated by a bilinear function it is recommended to estimate no more than two geometrical parameters, kept in mind that a model error can be at presence. It is better to put more effort in estimating the material parameters.
Another recommendation is to investigate with help of numerical simulations the detection of other model errors; again by applying more than just one set of boundary conditions in one estimation procedure.

References

Courage, W.M.G., Hendriks, M.A.N., Module Parest. Internal reports WFW 89.38 and WFW 89.39, Eindhoven University of Technology, 1989.

DIANA Finite Element Analysis, Release 4.1. TNO Building and Construction Research, July 1, 1991.

Hendriks, M.A.N., Identification of the mechanical behaviour of solid materials. Ph.D.-thesis, Eindhoven University of Technology, 1991.

Meuwissen, M.H.H., The identification of inhomogeneous materials. Eindhoven University of Technology, 1992.

Oomens, C.W.J., Constitutieve modellen. Lecture Notes, Eindhoven University of Technology, Department of Mechanical Engineering, January 1991.

Ratingen, M.R. van, Identification of orthotropic membranes with high stiffness ratio. Eindhoven University of Technology, 1992.

Appendix 1

The exact parameter values are: $\alpha = 0.1x + 0.1y$
$E_1 = 1$
$E_2 = 0.25$
$\nu_{21} = 0.30$
$G_{12} = 0.25$

$P_0 = \lceil 10^{-3}, 10^{-3}, 10^{-2}, 10^{-3} \rfloor$

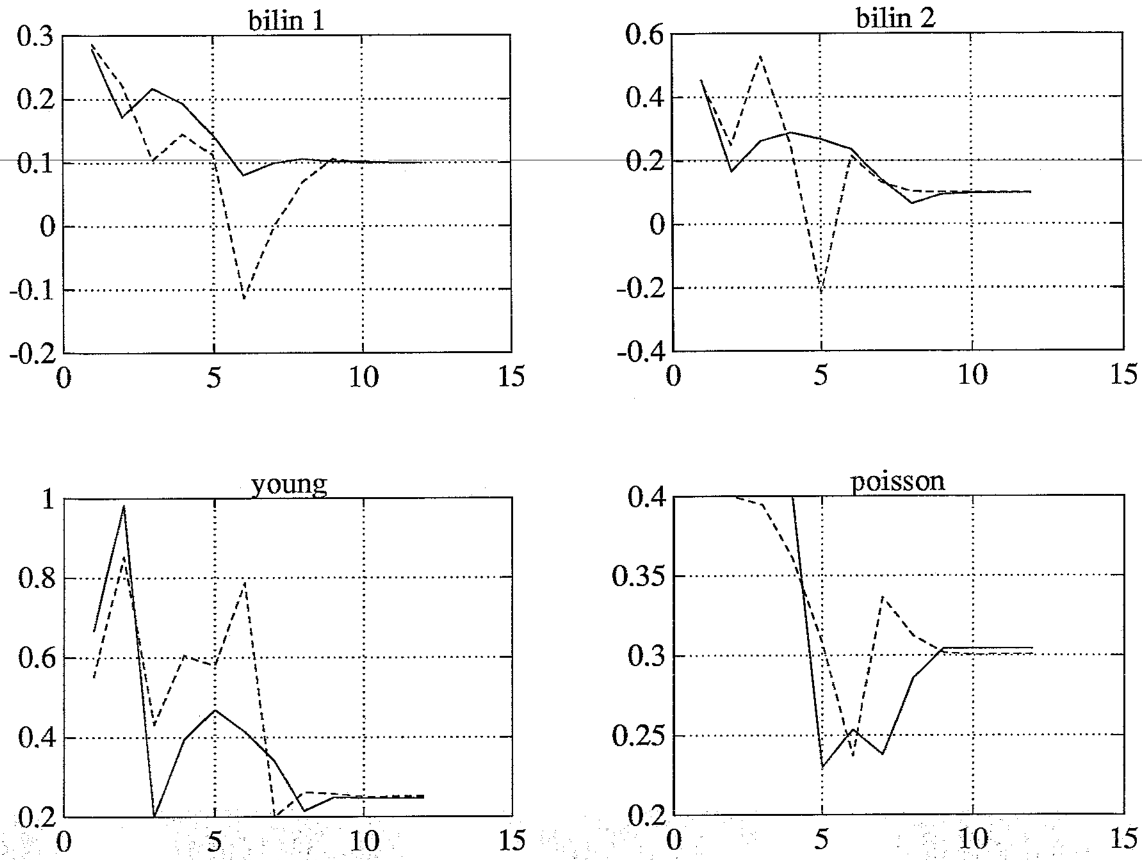
$Q = \lceil 10^{-3}, 10^{-3}, 10^{-2}, 10^{-3} \rfloor$

No noise added to data: diagonal elements of matrix R equal $\sigma^2 = 10^{-12}$
Noise added to data: diagonal elements of matrix R equal $\sigma^2 = 10^{-8}$

para-meter	exact value	initial guess	estimations					
			no model error				model error	
			no noise		$\sigma = 10^{-4}$		$\sigma = 10^{-4}$	
			1 case	3 cases	1 case	3 cases	1 case	3 cases
b_1	0.10	0.01	0.1000	0.1000	0.1001	0.1002	0.1001	0.0999
b_2	0.10	0.04	0.1000	0.1000	0.9993	0.1002	0.0973	0.1003
E_2	0.25	0.50	0.2500	0.2500	0.2500	0.2500	0.2471	0.2511
ν_{21}	0.20	0.30	0.3000	0.3000	0.2996	0.2999	0.3043	0.3008
number of iterations			10	10	10	10	11	11

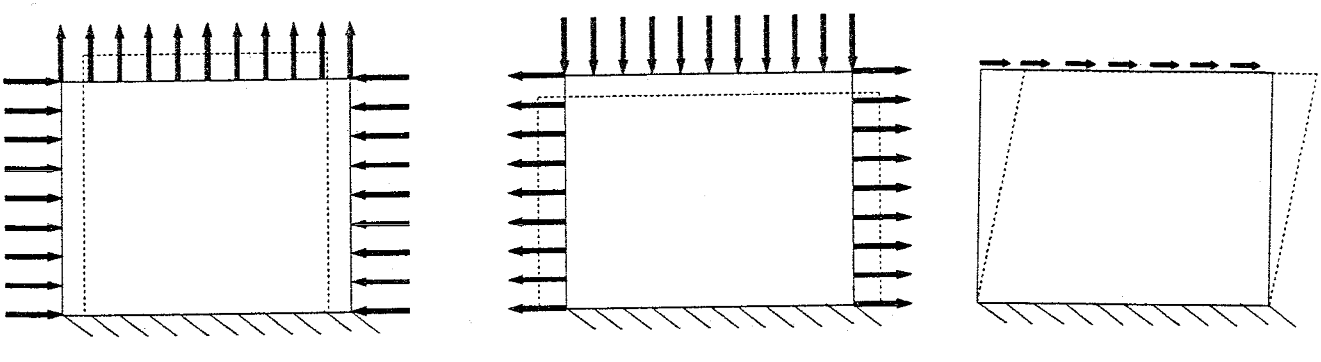
Estimation results

Appendix 2



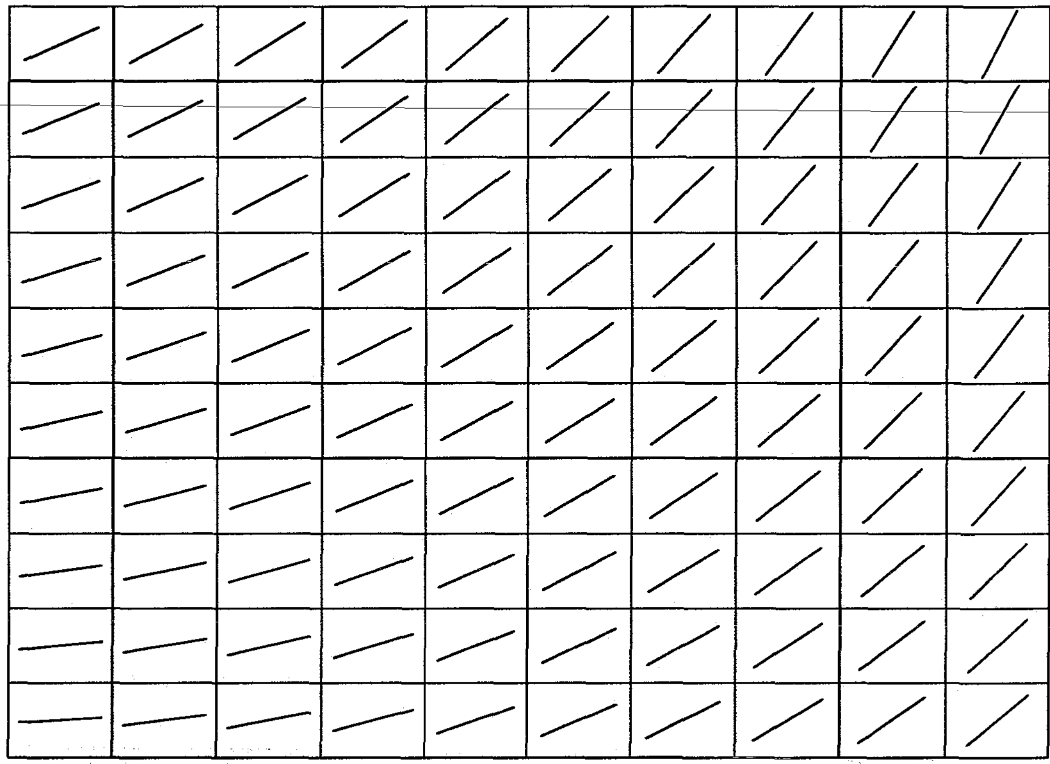
Parameter values vs. iteration step, for the situation with noise on the measurements and on the boundary conditions.

- - - = case 2
—— = case 2 for iteration step 1,...,4
case 3 for iteration step 5,...,8
case 1 for iteration step 9,...,12



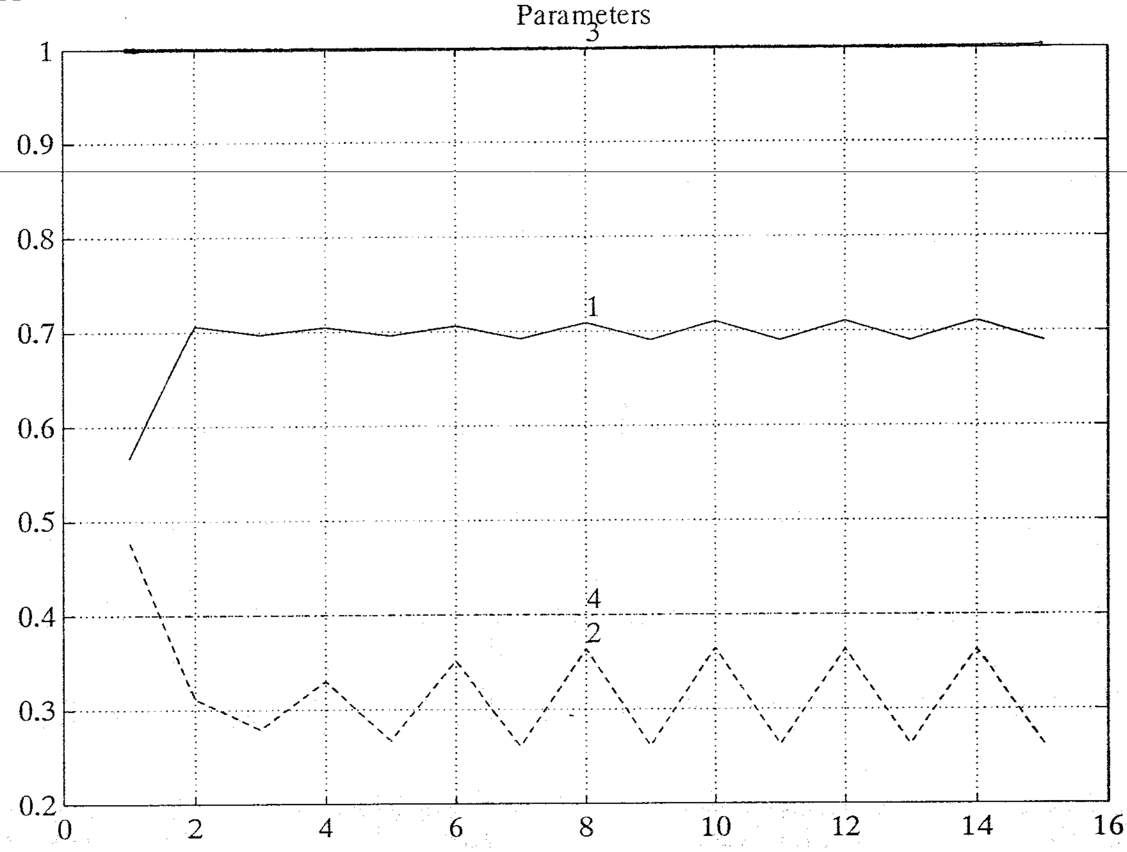
case 1 case 2 case 3

Appendix 3

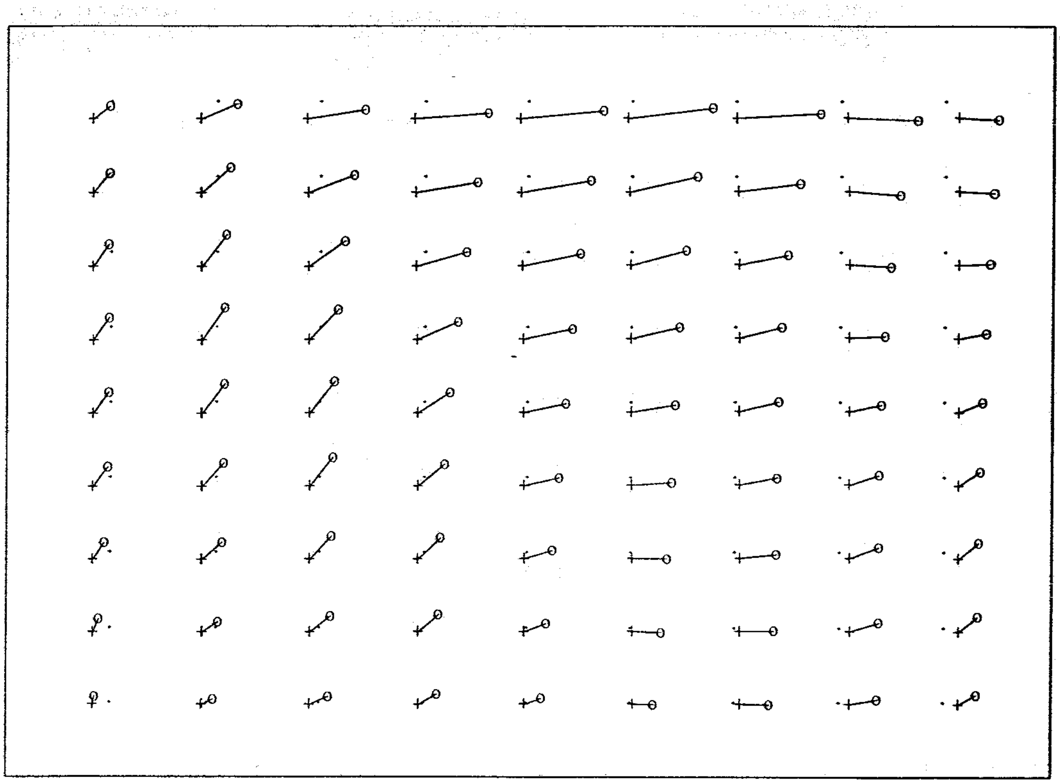


Fiber layout described with $\alpha = 0.15x + 0.11y$.

Appendix 4



Parameter estimations vs. iteration step (1=bilin 1, 2=bilin 2, 3=Young, 4=Poisson).

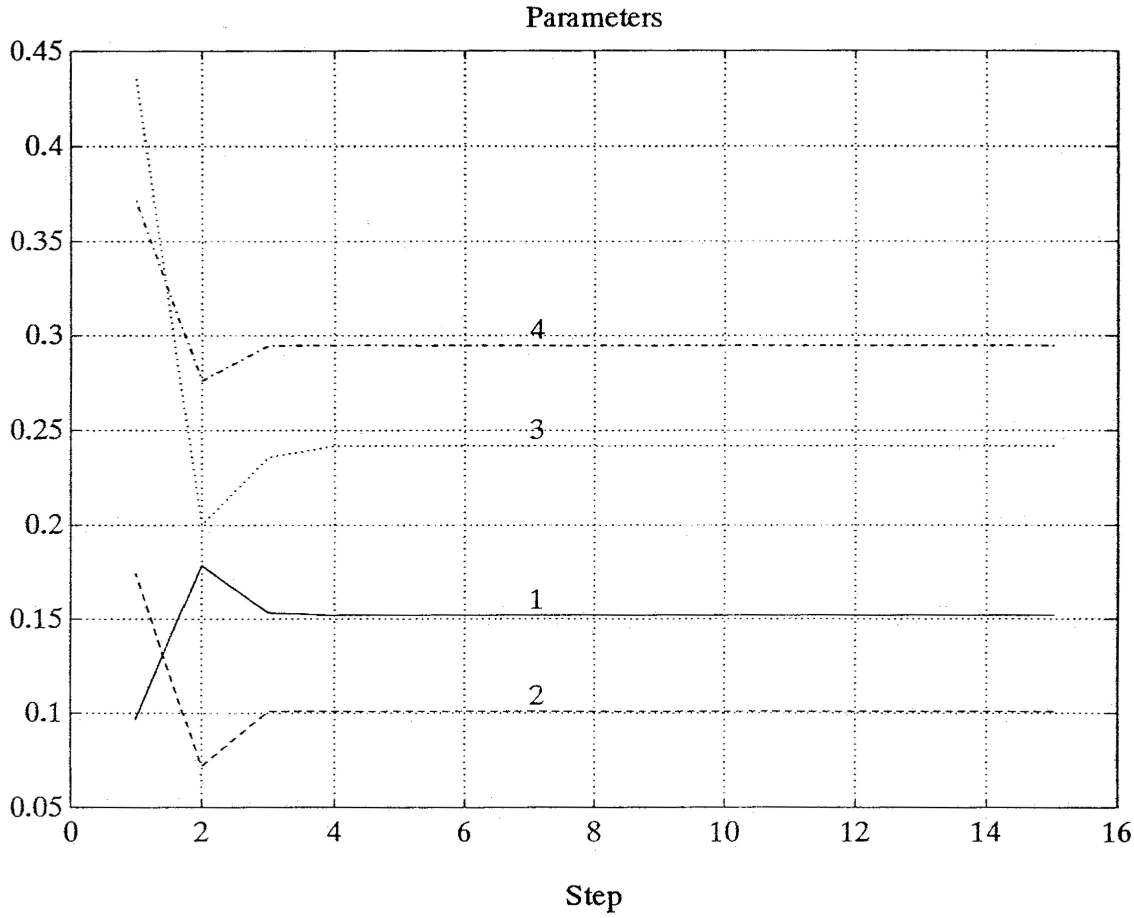


Residuals (x10) of the displacement field after 15 iterations with case 2 as the only set of boundary conditions.

Appendix 5

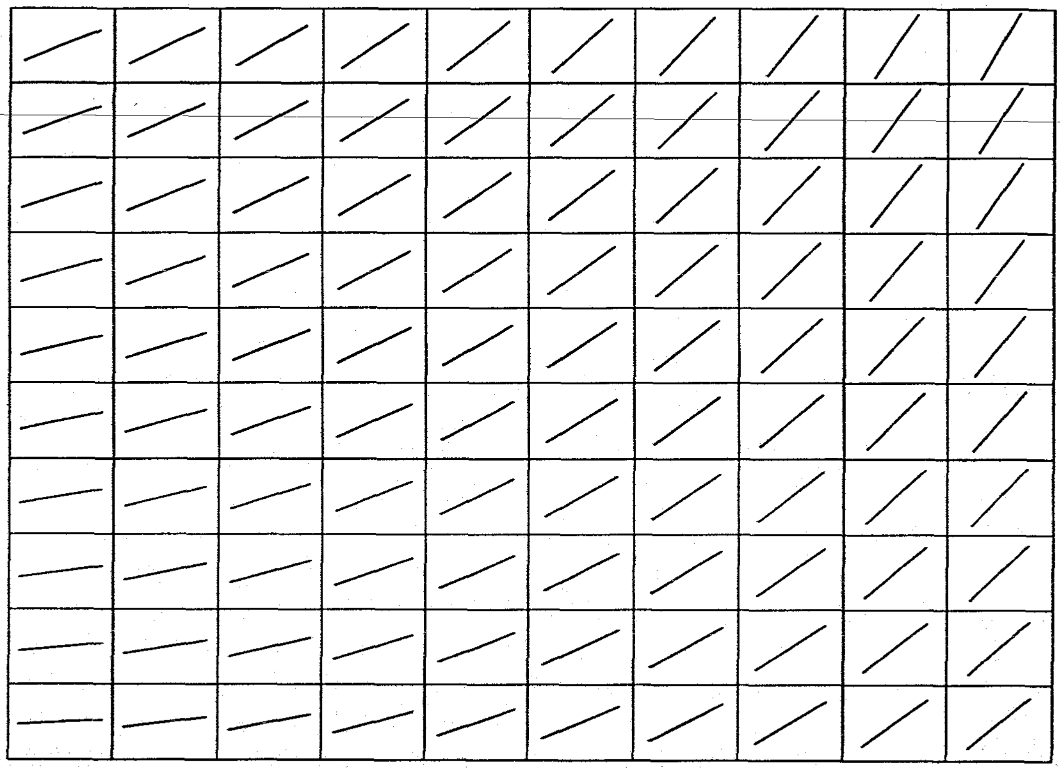
parameter	exact value	initial guess	estimations
b_1	-	0.01	0.1521
b_2	-	0.04	0.1008
E_2	0.25	0.50	0.2419
ν_{21}	0.30	0.20	0.2946

Parameter estimation results.



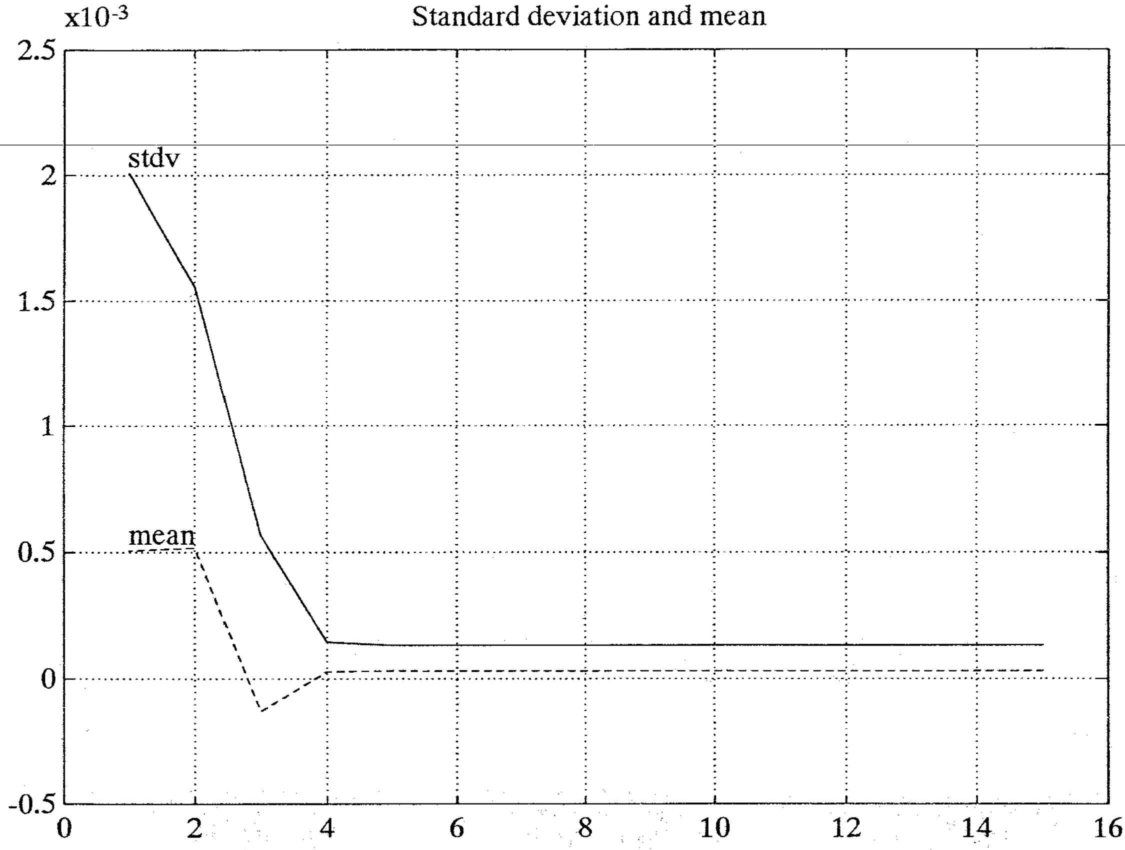
Parameter estimation as function of the iteration step (1=bilin 1, 2=bilin 2, 3=Young, 4=Poisson).

Appendix 6

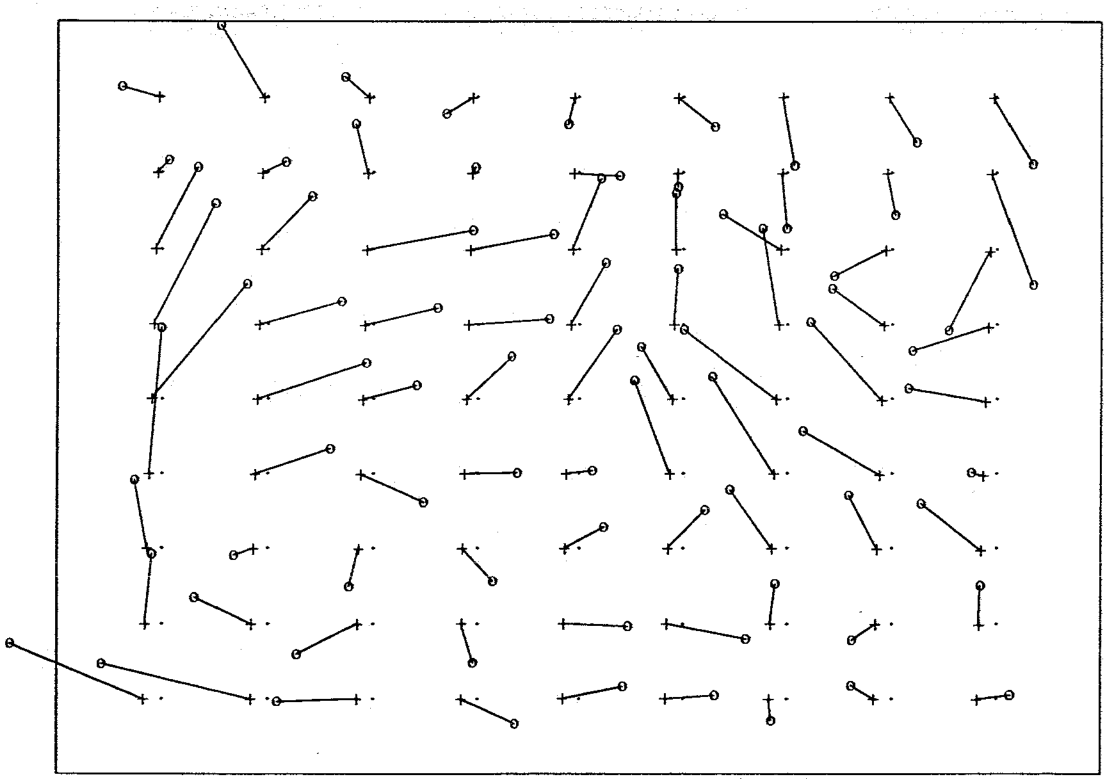


Fiber layout described with $\alpha = 0.15x + 0.10y$.

Appendix 7



Mean and standard deviation of the residuals vs. iteration step



Residuals (x2000) of the displacement field after 5 iterations with case 3 as the only set of boundary conditions.