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November 19[illegible]4

Eindhoven University of Technology
Netherlands

Department of Electrical Engineering

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By

Dr. A. Yakimov

[illegible]

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Department of Electrical Engineering
Eindhoven University of Technology
Eindhoven, The Netherlands.

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TH-Report 74-E-55

November 1974

ISBN 90 6144 055 6

Канд.ф/м наук А.В.Якимов

Флуктуации в низкодобротных генераторах на лавинно-пролетных диодах .

Развита методика расчета естественных флуктуационных характеристик генераторов на лавинно-пролетных диодах. Принято во внимание как наличие нелинейной зависимости мощности шумов лавинообразования от уровня сигнала, так и периодическая нестационарность указанных шумов. Определены зависимости формы и величины спектров амплитудных и частотных флуктуаций, а также формы и ширины спектральной линии автоколебания от уровня сигнала. Производится сравнение полученных результатов с результатами теоретического и экспериментального анализа [I].

FLUCTUATIONS IN IMPATT-DIODE OSCILLATORS WITH LOW Q-FACTORS

A method is developed for theoretical calculations of fluctuation behaviour of IMPATT-diode oscillators. Here we calculate the dependence of the spectra both of amplitude and frequency fluctuations on the signal level, and the dependence of the shape of the spectral line as well. Comparisons are made between results calculated here and results of the theoretical and experimental analysis carried out by J.J. Goedbloed and M.T. Vlaardingerbroek [1].

Dr. A. Yakimov is now with radio-physical department of Gorky State University (prospect Gargarina 23, Gorky, USSR).

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Introduction

In 1958 Read [2] predicted theoretically the possibility for semiconducting diodes with definite doping profile to show negative differential conductivity at high frequencies. In 1959 this effect was experimentally detected by Tager and others [3]. After this dicovery it has become possible to design effective portable amplifying and generating systems; and obviously this is why such great interest of investigators is devoted to IMPATT-diodes during last years.

Before giving the description of our analitical results it seems useful to say a few words about the physics of IMPATT-diodes.

Roughly speaking an IMPATT-device is a reversely biased semiconducting diode; with an applied voltage which is big enough for developing the avalanche breakdown in the p-n junction. In this case the constant component of diode current depends only on the value of the resistance of the displacement source.

The depletion layer of the p-n junction may be divided into two regions - the avalanche region and the drift region (see figure 1). We consider here a non-symmetrical p-n junction, in which the avalanche consist only of the carriers of one kind - either electrons or holes.

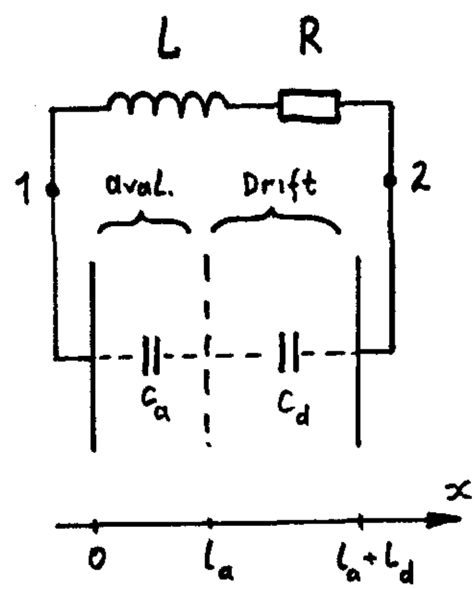

figure 1
Equivalent r.f. circuit of IMPATT-oscillator

Let us note, the same assumptions for the IMPATT-diode model were used here as in [1], particulary:

a) avalanching processes were assumed to take place only in the avalanche region;
b) the drift velocity of the carriers through the depletion layer were assumed to be constant and not dependent on the field.

Active behaviour of the device is shown only at high enough frequencies and is caused by two effects:

1) by time delay of the avalanching process; this delay takes place because the velocity of the avalanche formation is determined by instantaneous strength of the field in the avalanche region;
2) by the finite transit time of the carriers through the drift region .

It is known (see e.g. [4]) that each of these effects can cause the phase delay between the total current through the device and the applied voltage to be more than $\pi/2$. In other words each of those effects can leaa to the appearance of the negative dynamic resistance at definite frequencies. The existence of both effects in IMPATT-diode makes it possible to reach better active behaviour of the device.

1. Some notes regarding the theoretical methods for fluctuation analysis of oscillating systems

Both dynamic and fluctuation behaviour of IMPATT-oscillators were considered by many authors (see e.g. bibliography in [4,5]). The most detailed fluctuation analysis was carried out by Goedbloed and Vlaardingerbroek [1]. These authors have used in those theoretical calculations the so-called "symbolic" or impedance method. The essence of this method is the approximation of the real system under consideration by the simplest Thomson oscillator which in some cases includes the automatical displacement circuit as well. In the case of analysis of one-resonant circuit oscillators this method gives good results. But if one tries to analyse many-resonant systems (or, in another words, systems with many degrees of freedom) the applicability of the method is limited by to the frequency range not higher than the bandwidth of the resonator with the highest Q-factor used in the system. The restrictions of this method become more visible when we need to investigate the effects of the synchronization of several oscillators.

The aim of our investigation was to provide the analysis of fluctuation processes in IMPATT-oscillators by a method which is free of restrictions of the symbolic method.

But here it seems useful to make some historical remarks.

It is well-known that oscillating systems are essentially nonlinear. The simplest example of auto-oscillating system is the Thompson oscillator described by the following differential equation of the second order:

$$\frac{d^2z}{dt^2} + \omega_1^2 z = F(z,\frac{dz}{dt}) \; . \qquad (1)$$

Here z is some variable (current, voltage and so on); ω_1 is the characteristic frequency of the resonant circuit; F(z, dz/dt) is a small function which accounts for the presence of losses, nonlinearities and active behaviour of the amplifying element in the oscillator circuit. When the condition of self-excitation of the oscillator is fulfilled, the solution of (1) is

$$z = R_o\cos\psi; \; dz/dt = -\,\omega_o R_o \sin\psi; \; \psi = \omega_o t+\theta_o \quad . \qquad (2)$$

Here R_o, ω_o and θ_o are the amplitude, frequency and phase of the oscillations. The values R_o and ω_o are determined by the nature of the function F. In the common case the frequency of the oscillations can differ from the free resonant frequency: $\Delta\omega_o=\omega_o-\omega_1\neq 0$. The value of the phase θ_o is arbitrary.

For the simplest case, when there is only the quadratic nonlinearity in the function F:

$$F(z,dz/dt) = a\left[1+b(dz/dt)^2\right]\,(dz/dt), \qquad (3)$$
$$a,\; b = \text{const}$$

the solution of (1) was provided by Van der Pol . This method had gone down in history as Van der Pol's method.

For the case of an arbitrary nature of the function F the analysis has been provided by Bogolubov.

Later on these methodes were advanced and refined, and now there are no principle difficulties in analysing noise-free systems with any number of degrees of freedom (in another words, in many-resonant systems).

Introduction into (1) of a small random function G(t)*):

$$\frac{d^2z}{dt^2} + \omega_1^2 z = F(z,\frac{dz}{dt}) + \omega_1^2\, G(t) \qquad (1')$$

---

*) We shall consider only the case of the so-called natural (or additive) noise.

leads to the appearance of fluctuations of the amplitude and the phase (frequency) near their stationary states. It means that the solution of (1') is:

$$z = R_o\left[1+\varepsilon(t)\right]\cos\psi;\ \frac{dz}{dt} = -\ \omega_o R_o\left[1+\varepsilon(t)\right]\sin\psi;$$

$$\psi = \omega_o t+\theta_o+\phi(t)\ . \qquad (2')$$

The amplitude fluctuations $\varepsilon(t)$ and phase fluctuations $\phi(t)$ are slow in comparison with the oscillations because the perturbation G(t) is small. In the case of steady oscillations, the fluctuation $\varepsilon(t)$ are small as well: $\langle\varepsilon^2(t)\rangle \ll 1$.But this is not true for the phase fluctuations.If we shall fix the value of the phase at a difinite moment, for example

$$\phi(t=0)=0\ ,$$

in this case we shall find that the dispersion of these fluctuations will grow obeying a diffusion low:

$$\langle\phi^2(t)\rangle \sim t. \qquad (4)$$

The phase fluctuations are nonlimited because the free-running oscillators are not sensitive to the instanteneous value of the phase. In other words the phase can "wind" upon the limiting cycle without any restrictions. It is this circumstance that causes the "eroding" of the spectrum line of oscillations (see figure 2). Moreover the unlimited power of the phase fluctuations was one of things generating difficulties in the theoretical analysis of fluctuation phenomena in oscillating systems.

For the case of small phase fluctuations

$$\langle\phi^2(t)\rangle \ll 1 \qquad (5)$$

Blaquière worked out a method for calculations of frequency fluctuations $\nu=d\phi/dt$ and amplitude fluctuations of oscillations (see ref. [6]).

This method was the basis for the method developed by other authors which is known as the impedance method. The impedance method has many obvious advantages but it is necessary to mention its big weakness.

In this method condition (5) is essential, yet it is used for systems in which condition (5) is violated seriously. Nevertheless it was found that the theory had a good agreement with experiment. This paradox was solved by Malakhov [7,8], who worked out the exact solution of (1') by applying the mathematical apparatus developed for random Markov's processes.

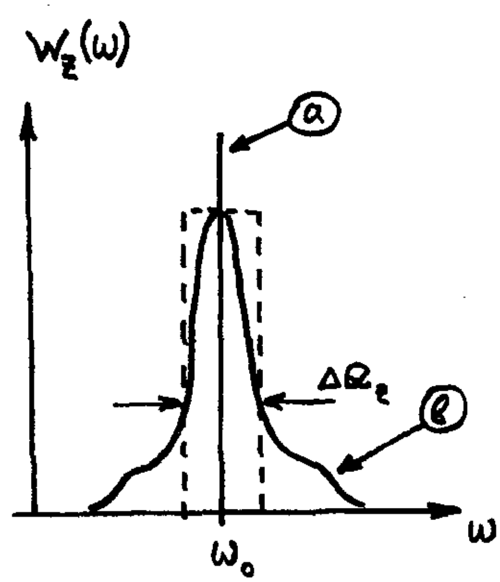

Figure 2

Shape of the spectral line of noise-free (a) and noise-perturbed (b) oscillators.

He showed that the correct nonlinear stochastic equations for amplitude and phase fluctuations and the linearized Blaquière's equations led to the similar expressions for the probability density of amplitude-phase fluctuations $P(\varepsilon_1,\phi_1;\varepsilon_2,\phi_2)$ (here sub-indices 1 and 2 mean that these values are taken at moments t_1 and t_2, respectively).

In other words, the equations starting from Blaquière's method (which are valid only for small phase fluctuations)

$$\frac{d\varepsilon}{dt} = -p\varepsilon + \frac{\omega_1}{2R_o} e_{\perp} \quad ; \qquad (6)$$

$$\nu = \frac{d\varepsilon}{dt} = -q\varepsilon - \frac{\omega_1}{2R_o} e_{||}$$

have fortunately turned out to be statistically equivalent to the exact non-linear stochastic equations

$$\frac{d\varepsilon}{dt} = -p\varepsilon + \frac{\omega_1}{2R_o} \left[e_{\perp}\cos\phi - e_{||}\sin\phi\right]; \qquad (7)$$

$$\nu = \frac{d\phi}{dt} = -q\varepsilon - \frac{\omega_1}{2R_o}\left[e_{||}\cos\phi + e_{\perp}\sin\phi\right]$$

which are valid for unlimited phase fluctuations.

Thus for analysing oscillatory systems with one degree of freedom (e.g. one-resonant systems) the impedance method gives correct results for the spectra of both amplitude and phase (frequency) fluctuations.

Note: In (6,7) p and q are the stability factor of the limiting cycle and the non-isochronism of the self-excited oscillator, respectively. Both values have dimensions of the frequency. $p^{-1}=\tau_\varepsilon$ is relaxation time of the amplitude fluctuations. q defines the dependence of the frequency on the amplitude of the oscillations.

$e_\perp = e_\perp(t)$ and $e_{\parallel}=e_{\parallel}(t)$ are random functions representing sin- and cos-components of the function G(t) in a narrow frequency band centered arround ω_I.

Let us proceed now directly to the analysis of the IMPATT-oscillator of figure 1.

In order to make a comparison of our results with the results of [1] more simple we shall try to follow the notations introduced in that paper. Numerical estimations and comparison with the experiment will be given for the IMPATT oscillators described in [1].

2. Analysis of the noise-free oscillator

The circuit-diagram of the IMPATT-oscillator of figure 1 can be divided into two parts (by points 1, 2):

a) the ideal lossless IMPATT-diode;
b) the external circuit consisting of the inductance L and the loss-resistor R: the latest includes the volume resistance of IMPATT-diode, the losses in the resonator and load resistance as well.

Noise generation is assumed to take place only in the avalanche region. As far as the power of this noise depends on the value of total avalanche current, the noise-generation process in IMPATT-oscillators is a periodically nonstationary process.

Using the mathematical apparatus of Markov's random processes (see e.g. [8] §1.9) it is possible to modify the well-known Read equation by taking into account the nonlinearity of the noise generation mechanism in the avalanche region:

$$\tau_i \frac{dJ_{ca}}{dt} = J_{ca}(\alpha l_a - 1) + \zeta(t)\sqrt{J_{ca}} \quad . \tag{8}$$

Here J_{ca} is a total current through the avalanche region; $\alpha=\alpha(E)$ is an ionisation coefficient for current carriers depending on the electric field E (if we should take into account carriers of both kinds we must use the averaged ionisation coefficient instead of α, see [1,5]); τ_i is the response time of the avalanching process; $\zeta(t)$ is the stationary delta-correlated random process, its correlation function is

$$\Phi_\zeta(\tau) = \langle\zeta(t)\zeta(t+\tau)\rangle = q_e\delta(\tau) \tag{9}$$

where $q_e = 1.6\cdot10^{-19}$C is the electronic charge; $\delta(\tau)$ is Dirac delta function.

For a complete description of the IMPATT-oscillator we also need the equations for currents and voltages in the oscillator loop:

$$\left[\frac{1}{\omega_1^2}\frac{d^2}{dt^2} + \frac{2\Pi}{\omega_1^2}\frac{d}{dt} + 1\right] i_t = \frac{c_o}{c_d}\,\hat{\Phi}\left[i_{ca}(t)\right]; \tag{10}$$

$$\hat{\Phi}\left[i_{ca}(t)\right] = \frac{c_d}{c_a}\, i_{ca}(t) + \frac{1}{\tau_d}\int_{t-\tau_d}^{t} i_{ca}(t')dt' \; ; \tag{11}$$

$$i_t = i_{ca} + c_a\frac{dv_a}{dt} \quad . \tag{12}$$

Here c_a, c_d and $c_o = (c_a^{-1}+c_d^{-1})^{-1}$ are "cold" capacities of the avalanche and drift regions and the total capacity of the depleted layer of the diode. $\omega_1=(Lc_o)^{-1/2}$, $2\Pi=\omega_1/Q$ and $Q=\omega_1 L/R$ are the resonant frequency, the bandwidth and the quality-factor of the resonator. The integral operator $\hat{\Phi}$ accounts for the drift of the carriers through the drift region, τ_d is the drift time.

i_t is the signal component of the current in the oscillating loop.

The total avalanche current is devided into two components: the displacement current J_B and the signal current $i_{ca}(t)$: $J_{ca} = J_B + i_{ca}(t)$.

The electric field inside the avalanche region $E=E_o+e$ is assumed to be related to the total voltage V_a applied to this region in the following way:

$$E=E_o+e=V_a/l_a=(V_{ao}+v_a)/l_a, \tag{13}$$

or

$$E_o=V_{ao}/l_a; e=v_a/l_a \quad . \tag{13'}$$

Assuming the avalanche is developed completely

$$l_a \alpha(E_o)=1 \tag{14}$$

we can make the usual dynamic analysis of the IMPATT-oscillator. That means we can find the stationary values of amplitudes and phases for oscillations in the noise-free oscillator. Finally we shall obtain:

$$i_t=J_{10}\cos\psi;\ di_t/dt = -\omega_o J_{10}\sin\psi;$$

$$\psi=\omega_o t+\theta_o;\theta_o=\text{const};$$

$$v_a=v_{10}\sin\psi;\omega_o \int^t v_a(t')dt' = -v_{10}\cos\psi; \tag{15}$$

$$J_{ca}=J_o\left[I_o(U_o)+2\sum_{n=1}^{\infty}(-1)^n I_n(U_o)\cos n\psi\right].$$

Here U_o is the dimensionless amplitude of the voltage oscillations on the avalanche region

$$U_o=\frac{\alpha' v_{10}}{\tau_i\omega_o} \tag{16}$$

where $\alpha' = \left[d\alpha(E)/dE\right]_{E_o}$.

ω_o is the oscillating frequency $\omega_o=\omega_1+\eta\Pi$, η is a dimensionless correction factor to the oscillating frequency

$$\eta=\frac{\text{Re}\phi}{-\text{Jm}\phi}=\frac{\theta\frac{c_d}{c_a}+\sin\theta}{1-\cos\theta};\ \theta=\omega_o\tau_d . \tag{17}$$

Here we have used real and imaginary parts of the Fourier-transformed operator $\hat{\Phi}$

$$\phi(j\theta)=\text{Re}\phi+j\text{Jm}\phi=\frac{c_d}{c_a}+\frac{1-e^{-j\theta}}{j\theta}. \tag{18}$$

$I_n(U_o)(n=0,1,2,\ldots)$ are modified Bessel functions of the n^{th} order.

The integration constant J_o corresponds to a starting current J_{st} in the following way:

$$J_o=J_{st}U_o/2I_1(U_o) . \tag{19}$$

Thus, knowing J_{st} and the displacement current through the diode $J_B = J_o I_o(U_o)$, one can find the amplitudes of all currents and voltages in the oscillating circuit. Particulary for the first harmonic of the avalanche current we have:

$$i_{ca_1}(t) = -J_a \cos\psi ; \; J_a = J_o 2 I_1(U_o) \; . \qquad (20)$$

The amplitude of the a.c. current i_t in the oscillating loop equals

$$J_{10} = J_a \left(\frac{\omega_o^2}{\omega_o^2} - 1 \right) . \qquad (21)$$

Here $\omega_a = (J_{st} . \alpha' / \tau_i c_a)^{1/2}$ is the characteristic frequency above which the IMPATT-diode shows its active behaviour.

Since the phase θ_o can have any arbitrary constant value let us put $\theta_o = 0$.

3. Stability analysis of the oscillating mode

For executing the analysis of the stability of the oscillating mode it is necessary to introduce a direct current circuit-diagram (see figure 3).

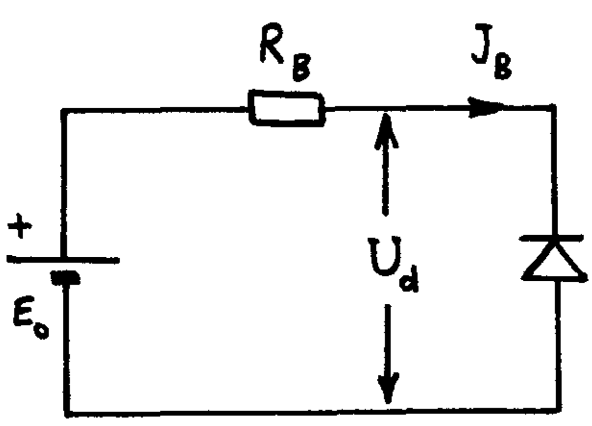

Figure 3
Equivalent d.c. circuit of IMPATT-oscillator

Roughly speaking the stabilisation effect in the IMPATT-oscillator can be explained in the following way. Let us assume that the amplitude U_o has a positive increment due to some circumstances. That leads to an increase of the displacement current J_B and to a decrease of the constant component of the total voltage V_d on the diode:

$$V_{do}=E_o-R_BJ_B \quad . \tag{22}$$

As a result the field E_o inside the avalanche region decreases and the value of the multiplication factor $\alpha(E_o)$ as well. This is equivalent to an introduction of extra losses into the oscillating loop, which leads to a decrease of U_o.

For our calculations we need the relation between the total voltage on the diode and the electric field E in the avalanche region.
Assuming the current density through the diode is small enough we put that the value E is κ times greater than the average value of electric field in the diode:

$$E=\kappa\, U_d/(1_a+1_d) \quad . \tag{23}$$

The value of κ depends only on the doping profile of the diode. In real devices κ is greater than unity and can reach values of the order of several units.

Let us give now a small variation to the displacement current δJ_B

$$J_B=J_{Bo}\left[1+\delta J_B\right] \; ; |\delta J_B|<<1 \quad . \tag{24}$$

It is obvious that the amplitudes and the phases in this case can differ from their stationary values:

$$\begin{aligned} &i_t=J_1(1+\varepsilon_1)\cos\psi;\; di_t/dt = -\,\omega_o J_1(1+\varepsilon_1)\sin\psi; \\ &\psi=\omega_o t+\phi_1;\; i_{ca_1} = -\,J_a(1+\varepsilon_a)\cos(\psi+\phi_a); \\ &v_a=v_{10}(1+\varepsilon_v)\sin(\psi+\phi_v); \\ &\omega_o\int^t v_a dt'=v_o(1+\varepsilon_v)\cos(\psi+\phi_v). \end{aligned} \tag{25}$$

The difference between the total voltage V_a in the avalanche region and its constant stationary component V_{ao} is equal to

$$V_a-V_{ao} = -\left(\kappa\,\frac{1_a}{1_a+1_d}\,R_BJ_{Bo}\right)\delta J_B+v_a \quad . \tag{26}$$

Thus for the total avalanche current we have

$$J_{ca}=J_o \exp\left\{\frac{\alpha'}{\tau_i}\int^{t}(V_a-V_{ac})dt'\right\} =$$

$$=J_o\left[1-\Omega_B\int^{t}\delta J_B dt'\right]\exp\left\{-V_o(1+\varepsilon_o)\cos(\psi+\phi_v)\right\} . \qquad (27)$$

Here $\Omega_B=\kappa c_o R_B J_{Bo}\alpha'/\tau_i c_a$. This is the characteristic frequency of the displacement current response on changing of the voltage on the avalanche region. Relaxation of the amplitudes and the phases is a slow process (compared with the auto-oscillations) due to the fact that the system under consideration is quasi Thomsonian and the perturberations are small.

After putting eq. (25) into eqs. (10-(12) and taking into account the stationary solution one can find relations for amplitude and phase (frequency) perturberations.

Fourier representations of these relations are:

$$\begin{bmatrix}\hat{\varepsilon}_1 & \begin{bmatrix} 1+\frac{j\Omega}{\Pi} ; & 0 \\ 1 ; & \frac{1}{\eta\Pi}\end{bmatrix} \\ \hat{\nu} & \end{bmatrix}=\begin{bmatrix}\begin{bmatrix} 1 ; & -\eta \\ 1 ; & 1/\eta\end{bmatrix} & \hat{\varepsilon}_a \\ & \hat{\Phi}_a\end{bmatrix} ; \qquad (28)$$

$$\omega_a^2\begin{bmatrix}\hat{\varepsilon}_a \\ \hat{\Phi}_a\end{bmatrix}=\omega_o^2\begin{bmatrix}\begin{bmatrix}1 ; & 0 \\ 0 ; & 1\end{bmatrix} & \hat{\varepsilon}_v \\ & \hat{\Phi}_v\end{bmatrix}-\begin{bmatrix}\omega_o^2-\omega_a^2 \\ 0\end{bmatrix}\hat{\varepsilon}_1 . \qquad (29)$$

From eq. (27) one can find

$$\delta\hat{J}_B(\Omega_B+j\Omega)=j\Omega\,\frac{U_o I_1(U_o)}{I_o(U_o)}\,\hat{\varepsilon}_v ; \qquad (30)$$

$$\hat{\varepsilon}_a=\hat{c}\,\hat{\varepsilon}_v ; \; \hat{\Phi}_a=\hat{\Phi}_v \qquad (31)$$

where

$$\hat{c}=B+\frac{A-B}{1+j\Omega/\Omega_B} ; \qquad (32)$$

$$A=\frac{U_o}{I_1(U_o)}\left[\frac{dI_1(U)}{dU}\right]_o ; \; B=U_o\left[\frac{d}{dU}\ln\frac{I_1(U)}{I_o(U)}\right]_o .$$

Here sub index "o" means that corresponding expressions are taken for $U=U_o$. Let us note that the operator $\hat{c}$ has an absolute value of the order of unity: $|\hat{c}| \sim 1$.

Comparing eq. (29) with eq. (31) and taking into account that $\omega_o \neq \omega_a$ one can find:

$$\hat{\Phi}_a=\hat{\Phi}_v=0 \quad .$$

In other words, in the approximation used here the phase relations between currents and voltages in the oscillating loop do not depend on the value of the displacement current J_B.

From the same equations (29) and (31) it follows that

$$\hat{\varepsilon}_a = \frac{\omega_o^2-\omega_a^2}{\omega_o^2-\hat{c}\omega_a^2}\,\hat{c}\,\hat{\varepsilon}_1 \quad . \tag{33}$$

In real IMPATT-oscillators the oscillating frequency is usually chosen rather high $\omega_o^2 >> \omega_a^2$.
That is why instead of eq. (33) it is possible to use a simple relation:

$$\hat{\varepsilon}_a=\hat{c}\,\hat{\varepsilon}_1 \quad . \tag{34}$$

Taking into account eq. (34) and relation $\hat{\Phi}_a=0$ it is possible to transform eq. (28) into the final relation :

$$\begin{bmatrix} \hat{\varepsilon}_1 \\ \hat{v} \end{bmatrix} \begin{bmatrix} 1-\hat{c}+j\Omega/\Pi & ; & 0 \\ 1 & ; & (\eta\Pi)^{-1} \end{bmatrix} = 0 \tag{35}$$

or

$$\begin{bmatrix} \hat{\varepsilon}_1 \\ \hat{v} \end{bmatrix} \hat{\Delta}(j\Omega)=0 \quad .$$

Replacing $j\Omega$ by the parameter λ in $\hat{\Delta}(j\Omega)$ one finds the characteristic equation for the IMPATT-oscillator:

$$|\hat{\Delta}(\lambda)|=0 \quad .$$

After taking into account eq. (32) this can be transformed into

$$\lambda^2 + \left[\Omega_B-\Pi(A-1)\right]\lambda + (1-B)\Pi\Omega_B=0 \quad . \tag{36}$$

The stationary solution (15) is steady if equation (36) has solutions for λ with only a negative real part, or, this is the same, if the third term and the factor before λ in eq. (36) are positive:

$$(1-B)\Pi > 0 \;;$$
$$\Omega_B > (A-1)\Pi. \qquad (37)$$

The first inequality of (37) is always valid due to the properties of Bessel functions (see figure 4).

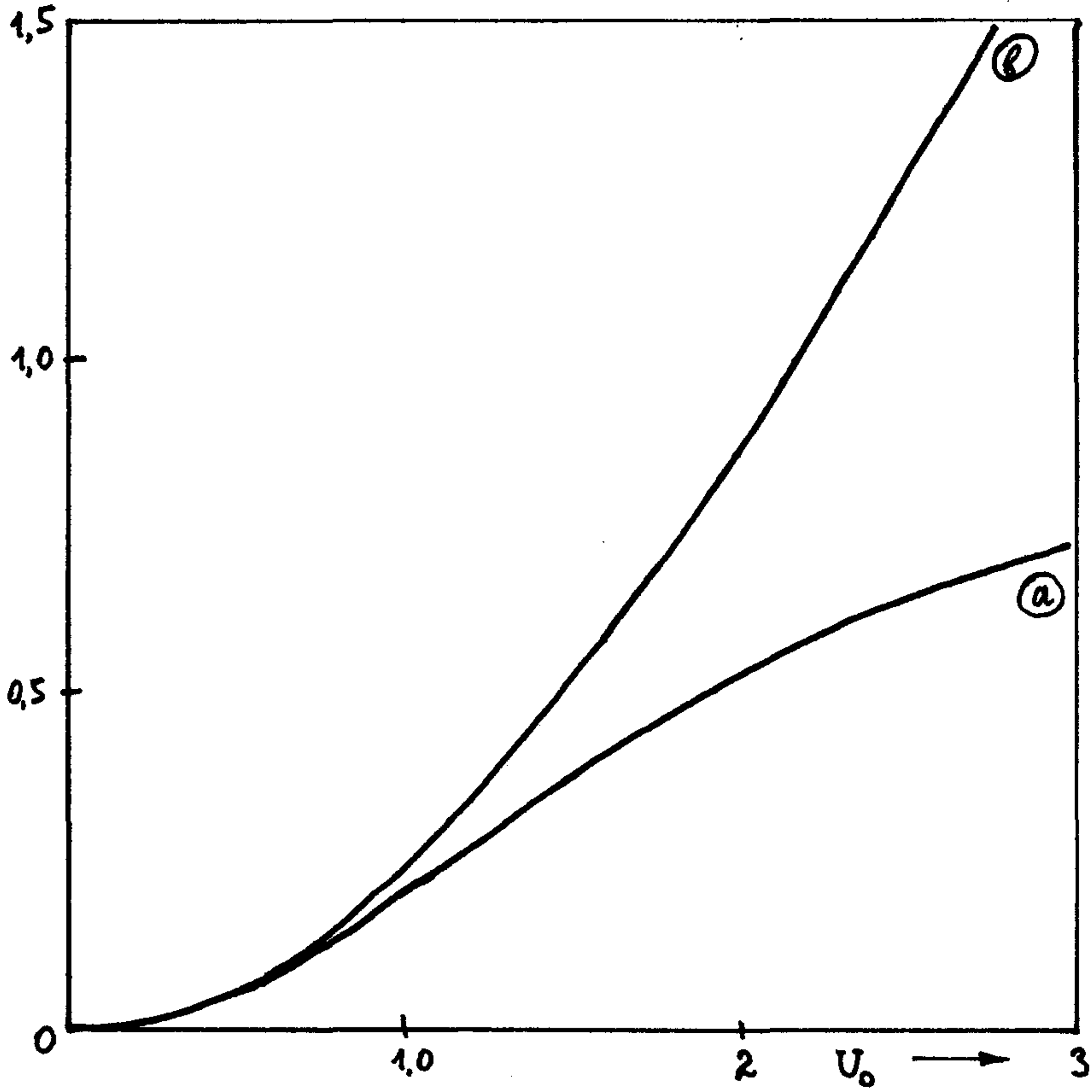

Figure 4

Dependence of the stability factor p and of a minimal value Ω_B^* of the response frequency Ω_B on the amplitude U_o:

(a) the value $1-B=p/\Pi$ versus U_o;

(b) the value $A-1=\Omega_B^*/\Pi$ versus U_o.

The stability consequence is $\Omega_B>\Omega_B^*;p>0$.

The second inequality of (37) determines the minimum value of the resistor R_B required by the parameters of the oscillating circuit. It is possible to find that this inequality is valid if

$$\frac{1}{\tau_i}\frac{R_B J_o}{U_{do}}\frac{E_o}{\alpha}\left[\frac{d\alpha}{dE}\right]_o > \Pi \frac{U_o^2}{4} \, . \qquad (38)$$

For example for the diodes described in [1] the value of R_B must be higher than 2Ω if $U_o \sim 1$. It was reported in [1] that the value of the resistor used was equal to 1,5KΩ. It means that for the IMPATT-oscillators described in [1] the stability condition was fulfilled within a wide margin.

We can estimate that for IMPATT-oscillators [1] the value of the frequency Ω_B is very high: $\Omega_B \gtrsim \omega_o$. From this it becomes obvious that it is absurd to take into account the inertiality of the displacement circuit.

Now it is possible to rewrite eq. (36) in a simpler manner:

$$\lambda + (1-B)\Pi = 0 \, .$$

By using the analogy with other oscillators [8] we can introduce the stability factor of the limiting cycle:

$$p = (1-B)\Pi \, , \qquad (39)$$

and transform the characteristic equation in the following manner:

$$\lambda + p = 0 \, . \qquad (40)$$

As far as for $U_o > 0$ we have $p > 0$, the oscillation mode (15) is always stable. Besides this, from eq. (40) it is possible to find that the oscillating system has practically one degree of freedom only and the system is characterized by the soft *) exitation condition.

In conclusion of the stability analysis let us note that for oscillations with small amplitude ($U_o < 1$) it is possible to transform the conditions (37) on the following manner:

$$\Omega_B > p > 0 \, . \qquad (41)$$

---

*) It means that when the displacement current increases smoothly from its threshold value I_{st}, the auto-oscillations start immediately and their amplitude U_o increases monotonously from zero.

The condition $\Omega_B > p$ is the usual stability condition for the auto-oscillating systems with an automatical displacement circuit.

This condition means that the displacement circuit must be able to follow the variations of the currents in the loop, which are caused by variations in the amplitude of the oscillations.

Let us proceed now to the fluctuation analysis of the IMPATT-oscillator.

4. To the solution of Read's equation taking into account the presence of the avalanche generation noise

Let us only take into account the presence of a shot noise of the avalanche generation process. It is known that this noise gives the main contribution to the amplitude and phase fluctuations at frequencies higher than 1 KHz (see e.g. [9]).

The behaviour of an IMPATT-oscillator is described by the system of equations (8)-(14), (23). We assume that the process of building up of the auto-oscillations is completed and we try to find a solution of the equations mentioned above. This solution will be analogous to the solution (24)-(25).

Owing to the smallness of the shot-noise in the diode we assume that the fluctuations of all amplitudes and phases are slow compared to the auto-oscillations:

$$\begin{aligned} &\langle \dot{\varepsilon}_1^2 \rangle , \langle \dot{\varepsilon}_a^2 \rangle, \langle \dot{\varepsilon}_v^2 \rangle, \langle \delta \dot{J}_B^2 \rangle \ll \omega_o^2 ; \\ &\langle \dot{\phi}_a^2 \rangle , \langle \dot{\phi}_v^2 \rangle, \langle \nu^2 \rangle \ll \omega_o^2 . \end{aligned} \tag{42}$$

As far as the oscillating mode is stable we assume that the fluctuations of all amplitudes are small as well:

$$\langle \varepsilon_1^2 \rangle, \langle \varepsilon_a^2 \rangle, \langle \varepsilon_v^2 \rangle, \langle \delta J_B^2 \rangle \ll 1 . \tag{43}$$

We set just the same restrictions to fluctuations of the phases ϕ_a and ϕ_v which are deviations from the phase $\psi = \omega_o t + \phi_1$ of the oscillating current in the inductive branch of the loop:

$$\langle \phi_a^2 \rangle, \langle \phi_v^2 \rangle \ll 1 . \tag{44}$$

The main difficulty of the theoretical analysis of the fluctuation phenomena in IMPATT-oscillators is the problem of finding a solution of the Read's equation (see e.g. [5]) taking into account the presence of the noise:

$$\tau_i \frac{dJ_{ca}}{dt} = J_{ca}\left[\alpha(E)1_a - 1\right] + g'(t) .$$

The essence of this problem is that this equation is not only nonlinear for the current J_{ca} but that the process of the noise generation g'(t) depends nonlinearly upon the instantaneous value of the avalanche current J_{ca}.

The problem mentioned above is solved in this paper in the following way. First of all, on the basis of the analysis [5] and §1.9 in [8] the Read equation is displayed in the form (8) which obviously represents the dependence of the noise-generation process on the value of the avalanche current. Such representation of the Read's equation allows us to find its exact solution by the method of variation of the arbitrary constant. The solution reached is

$$J_{ca} = e^{U(t)}\left[J_o + \frac{\sqrt{J_o}}{2\tau_i}\int_{-\infty}^{t} 2\zeta(t')e^{-\frac{1}{2}U(t')}\,dt'\right] ;$$
$$U(t) = \frac{\alpha'}{\tau_i}\int_{-\infty}^{t}\left[V_a - V_{ao}\right]dt' . \qquad (45)$$

Since the integrants in eqs. (45) have oscillating components it is necessary for the calculation of the integrals to use the methods of summarizing (see. e.g. [10, 11]). In our case it means that the prototypes corresponding to the infinite low limit of integration must be put equal to zero.

Using the method of the statistically equivalent equations (see ch. 5 in [8]) let us select from the random function $\zeta(t)$ the following components:

$$\zeta(t)=\zeta_o(t) + \sum_{k=1}^{\infty}\left[\zeta_{k\parallel}(t)\cos k\psi - \zeta_{k\perp}(t)\sin k\psi\right] + \ldots \qquad (46)$$

where

$$\zeta_o(t) = \frac{1}{T} \int_{t-T}^{t} \zeta(t')dt' \; ;$$

$$\zeta_{k\{{}^{\shortparallel}_{\perp}\}}(t) = \frac{1}{T} \int_{t-T}^{t} 2\zeta(t') \begin{Bmatrix} [\cos k\psi(t')] \\ [\sin k\psi(t')] \end{Bmatrix} dt' \; ; \; T = 2\pi/\omega_o \; . \tag{47}$$

The random processes ζ_o, $\zeta_{k\shortparallel}$, $\zeta_{k\perp}$ ($k = 1,2,3, \ldots$) are statistically independent and delta-correlated.

Their spectra densities are equal to

$$S_{\zeta o}(\Omega)=S_\zeta(0) \; ;$$

$$S_{\zeta k\shortparallel}(\Omega)=S_{\zeta k\perp}(\Omega)=2\; S_\zeta(k\omega_o) \quad (k = 1,2, \ldots) \; ;$$

$$S_{\zeta k\shortparallel,\zeta n\perp}(\Omega)=S_{\zeta k\shortparallel,\zeta m\shortparallel}(\Omega)=S_{\zeta k\perp.\zeta m\perp}(\Omega)=0 \; ; \tag{48}$$

$$(k,n,m = 0,1,2, \ldots, k\neq m).$$

The spectrum of $\zeta(t)$ in realistic cases (see e.g. [12]) can be found as

$$S_\zeta(\omega) = \frac{1}{2\pi}\, q_e \left[\frac{\sin\omega\tau_i}{\omega\tau_i}\right]^2 . \tag{49}$$

From (45) one can see that the total avalanche current may be devided into two components:

a) a regular component which depends only on the field in the region;

b) a fluctuating component caused by the noise generation process $\zeta(t)$ modulated by oscillations.

In other words, using eqs. (24)-(25) it is possible to represent eqs. (45) in the following manner:

$$J_{ca}(t)=J_{ca,reg}(t)+J_{ca,fl}(t)+\text{higher harmonics};$$

$$J_{ca,reg}=J_o\left[1-\Omega_B \int^{t} \delta J_B dt'\right] \exp\left[-U_o(1+\varepsilon_a)\cos(\psi+\phi_a)\right] \; ; \tag{50}$$

$$J_{ca,fl}=J_{Bo}\,\delta J_B^{*}-J_a\left[\varepsilon_a^{*}\cos\psi-\phi_a^{*}\sin\psi\right] .$$

Here δJ_B^*, ε_a^*, ϕ_a^* are the small slow random processes representing noise generation in the oscillator. For calculation of these processes it is useful to put $U(t)= -U_o\cos\psi$ into eq. (45). This means that we neglect the modulation of the noise generation process by amplitude and phase fluctuations.

By taking into account eqs. (47) it is possible now to find relations between δJ_B^*, ε_a^* and ϕ_a^* and the components ζ_o, $\zeta_{k\parallel}$, $\zeta_{k\perp}$ $(k = 1,2, \ldots)$ of the noise source $\zeta(t)$.

5. The spectral characteristics of the auto-oscillations

Let us proceed now to the calculations of spectral characteristics of the auto-oscillations.

Since the frequency Ω_B is very high ($\Omega_B >> \Pi$), the operator $\hat{c}$ given by eq. (32) can be presented in a simpler form:

$$\hat{c} = B = 1 - \frac{p}{\Pi} . \tag{51}$$

In other words, the transformation of the amplitude fluctuations on the nonlinearity of the IMPATT-diode obeys the low:

$$\varepsilon_a(t)=(1 - \frac{p}{\Pi})\varepsilon_v(t) \tag{52}$$

which is typical for any quasi-Thomsonian auto-oscillating system [13].

After executing the standard calculating procedure one finds all spectral and correlation characteristics of the auto-oscillations.

First of all it is possible to find the following relations between the Fourier-representation of the amplitude $\varepsilon_1(t)$, frequency $\nu(t)$ and current $\delta J_B(t)$ fluctuations and the Fourier-representations of the random processes $\zeta(t)=\varepsilon_a^*(t)-\delta J_B^*(t), \phi_a^*(t)$ and $\delta J_B^*(t)$:

$$\hat{\varepsilon}_1 = \frac{\Pi}{p+j\Omega}\,\hat{e}_1 ;$$

$$\hat{\nu} = -\eta p\hat{\varepsilon}_1+\Pi\hat{e}_2 ;$$

$$\delta\hat{J}_B = \frac{j\Omega}{\Omega_B}\,\delta\hat{J}_B^* .$$

Here $\hat{e}_1$ and $\hat{e}_2$ are Fourier-representations of the functions

$$e_1(t)=\xi_a(t)-\eta\phi_a^*(t) ;$$

$$e_2(t)=\eta\xi_a(t)+\phi_a^*(t) .$$

The correlation between $e_1(t)$ and $e_2(t)$ is negligible when U_o is small, but it becomes large for large values of $U_o(U_o \sim 1)$:

$$<e_1(t)\ e_2(t)> = \eta\left[<\xi_a^2> - <\phi_a^{*2}>\right].$$

Hence for small values of U_o the correlation between the amplitude and frequency fluctuations has an even character and is determined only by the non-isochronism of IMPATT-oscillator. Refering to (7) let us note that this non-isochronism is equal to

$$q=\eta p\ .$$

<u>Note</u>: Now we can see that the parameter η (see eq. (17)) has a double meaning:

a) the correction factor for the oscillator frequency, relative to the bandwidth;

b) the non-isochronism factor of the IMPATT-oscillator, relative to the stability factor of the limiting cycle p.

When U_o is large enough $(U_o \sim 1)$, the processes $e_1(t)$ and $e_2(t)$ are not statistically independent any more due to the effect of the modulation of the noise generation mechanism by oscillations. That leads to the appearance of the extra-correlation between the amplitude and phase fluctuations. This correlation will have an add component as well. In other words, the presence of the periodical nonstationarity in the noise generation mechanism leads to the appearance of the extra-correlation between the amplitude and phase fluctuations. Obviously we can provide the same conclusions for the current fluctuations $\delta J_B(t)$ as well.

In practical cases the most important fluctuation parameters of IMPATT-oscillators are the spectra of the amplitude $S_\varepsilon(\Omega)$ and frequency $S_\nu(\Omega)$ fluctuations, the shape $W_t(\Omega)$ and the width $\Delta\Omega_t$ of the spectral line of the auto-oscillations in the inductive branch of the oscillating loop. Let us note (see [14]) that $W_t(\Omega)$ is determined by the spectra of the amplitude and frequency fluctuations and by the even component $S^o_{\varepsilon\nu}(\Omega)$ of the mutual spectrum of the amplitude-frequency fluctuations:

$$S_{\varepsilon\nu}(\Omega)=S^o_{\varepsilon\nu}(\Omega)+jS^1_{\varepsilon\nu}(\Omega). \qquad (53)$$

Here the last term represents the odd component of the mutual spectrum. The calculations give us the following expressions for these spectra:

$$S_\varepsilon(\Omega) = \frac{1}{p^2+\Omega^2}\left[C+\eta^2 D\right]_{U_o} \quad ; \tag{54}$$

$$S_\nu(\Omega) = \frac{1}{p^2+\Omega^2}\left[\eta^2\Omega^2 C + ((1+\eta^2)^2 p^2+\Omega^2)D\right]_{U_o} \quad ; \tag{55}$$

$$S^o_{\varepsilon\nu}(\Omega) = \frac{-\eta p}{p^2+\Omega^2}(1+\eta^2)D(U_o) \quad ; \tag{56}$$

$$S^1_{\varepsilon\nu}(\Omega) = \frac{-\eta p}{p^2+\Omega^2}\left[C-D\right]_{U_o} \quad ; \tag{57}$$

$$W_t(\Omega) = \frac{S_\nu(\Omega)+2S^o_{\varepsilon\nu}(\Omega)\Omega}{(\Delta\Omega_t/\pi)^2+\Omega^2} + S_\varepsilon(\Omega) \quad ; \tag{58}$$

$$\Delta\Omega_t(\sec^{-1}) = \pi^2 S_\nu(0) \quad . \tag{59}$$

Here $C=C(U_o)$ and $D=D(U_o)$ characterize the spectral densities of the random processes *) $\xi(t)$ and $\phi^*_a(t)$:

$$C = \Pi^2 S_\xi(\Omega) = \frac{\Pi^2 q_e}{2\pi J_{st}(\omega_o\tau_i)^2}\cdot\frac{2I_1(U_o)}{U_o} \cdot \frac{1}{2}\sum_{l=1}^{\infty} A_l^2(U_o)\left(\frac{\sin\omega_o\tau_i l}{\omega_o\tau_i l}\right)^2 ; \tag{60}$$

$$D = \Pi^2 S_{\phi a}(\Omega) = \frac{\Pi^2 q_e}{2\pi J_{st}(\omega_o\tau_i)^2}\,\frac{2I_1(U_o)}{U_o} \cdot \left[B_o^2(U_o) + \frac{1}{2}\sum_{l=1}^{\infty} B_l^2(U_0)\left(\frac{\sin\omega_o\tau_i l}{\omega_o\tau_i l}\right)^2\right] ; \tag{61}$$

---

*) Let us note that the processes $\xi(t)$ and $\phi^*_a(t)$ are statistically independent and delta-correlated.

where

$$A_e(U_o) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left[I_{e-n} - I_{e+n} \right]_{\frac{U_o}{2}} \left[\frac{I_{n-1}+I_{n+1}}{2I_1} - \frac{I_n}{I_o} \right]_{U_o} ; \qquad (62)$$

$$B_e(U_o) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left[I_{e-n} + I_{e+n} \right]_{\frac{U_o}{2}} \left[\frac{I_{n-1}-I_{n+1}}{2I_1} \right]_{U_o} . \qquad (63)$$

The expression (39) for the stability factor of the limiting cycle can be represented as

$$p = \Pi \left\{ 2 + U_o \left[\frac{I_1}{I_o} - \frac{I_o}{I_1} \right]_{U_o} \right\} . \qquad (64)$$

Sometimes it may become necessary to know the spectrum of the current fluctuations $\Delta J_B = J_B . \delta J_B$ and the mutual spectra for the current-amplitude and current-frequency fluctuations as well. These spectra are given by the following expressions:

$$S_{\Delta J_B}(\Omega) = \frac{q_e S(U_o)}{2\pi J_{st} (\kappa R_B \alpha' c_o / c_a)^2} ; \qquad (65)$$

$$S^{\left\{ {o \atop 1} \right\}}_{\Delta J_B, \epsilon}(\Omega) = \frac{-\eta \Pi}{p^2+\Omega^2} \begin{Bmatrix} p \\ \Omega \end{Bmatrix} \frac{J_B}{\Omega_B \tau_i^2 \omega_o J_{st}} \frac{q_e}{2\pi} . H(U_o) ;$$

$$S^{\left\{ {c \atop 1} \right\}}_{\Delta J_{B,\nu}}(\Omega) = \frac{\Pi}{p^2+\Omega^2} \begin{Bmatrix} (1+\eta^2)p^2+\Omega^2 \\ \Omega p \eta^2 \end{Bmatrix} \frac{J_B}{\Omega_B \tau_i^2 \omega_o J_{st}} . \frac{q_e}{2\pi} . H(U_o) .$$

Here

$$S(U_o) = \frac{2I_1(U_o)}{U_o} \left[I_o^2 \left(\frac{U_o}{2} \right) + \frac{1}{2} \sum_{l=1}^{\infty} I_1^2 \left(\frac{U_o}{2} \right) \left(\frac{\sin \omega_o \tau_i l}{\omega_o \tau_i l} \right)^2 \right] \qquad (66)$$

and

$$H(U_o) = \frac{2I_1(U_o)}{U_o} \left[I_o B_o + \frac{1}{2} \sum_{l=1}^{\infty} I_1 B_1 \left(\frac{\sin \omega_o \tau_i l}{\omega_o \tau_i l} \right)^2 \right] .$$

In the expression for $H(U_o)$ the argument of the Bessel functions in the square brackets is assumed to be equal to $U_o/2$ and the functions $B_n (n=0,1,2,\ldots)$ are determined by expression (63) (i.e. for $U=U_o$).

6. Numerical estimations and comparison with experimental data of reference [1]

Let us proceed now to the analysis of the results obtained above and to comparison of these results with the theory and the experimental data [1].

Since there are no data on the shape of the curves for the analysed spectra in the paper [1], we have calculated these curves following to the method [1] as well *).

All calculations have been executed for the IMPATT-oscillators described in [1].

In all figures mentioned further on there are plotted so called technical spectra, i.e. spectra determined only for positive frequencies (having dimension "Hz")and related to physical spectra in the following way:

$$\langle x^2 \rangle_F = S_x(2\pi.F).4\pi \ .$$

All numerical calculations were executed with the aid of Philips time-sharing system P9200.

Let us note that for numerical calculations the infinite upper limit in the expressions (60)-(66) was replaced into N_{max} - the number of harmonics (of the avalanche current) taken into account. We chose N_{max}=3 as a value which provided rather good accuracy of the calculations and rather quick execution of the computing program.

6.1. Fluctuations of the displacement current

The spectrum of the fluctuations of the displacement current is described by relation (65).

First of all let us note that this spectrum does not depend on the frequency within the range of validity of the abridged equations used here ($\Omega \ll \omega_0$). In other words the fluctuations of the displacement current have a character of white noise.

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*) For executing the calculations after the method [1] it is necessary to correct a misprint. In item V it is mentioned that "the load impedance ... could be described by R_L+j L ...". Instead of R_L it must actually be written here as $R_L+R_S+R_C=R$ - the total loss resistance of the loop.

In figure 5 there is plotted the dependence of a power of current fluctuations (in the bandwidth $\Delta F = 100Hz$) for Si p^{+} -n IMPATT-oscillator versus modulation depth m (the modulation depth m is the ratio between the amplitude of voltage oscillations over the p-n junction and the threshold voltage of the IMPATT-diode).

The theoretical result [1] is represented by the dashed line *), and experimental data are plotted by dots.

Let us note it was assumed here (and in [1] as well) that $\kappa=1$. It means that the electric field in the avalanche region is assumed to be equal to the average field inside the depletion layer of the IMPATT diode. This approximation is true only for IMPATT diodes having Read's structure. The difference of the value κ from unity leads to a reduction by a factor of κ^2 of the theoretical results mentioned above.

It follows from figure 5 that under the assumptions were made the agreement of the theory with the experimental data is rather good.

6.2. The amplitude fluctuations

Let us note first of all that the amplitude fluctuations have a spectrum (54) with a simple resonant form. The width of this spectrum is equal to the value of the stability factor of the limiting cycle p (on the level 0.5). Such resonant form of this spectrum is typical for quasi - Thomsonian auto-oscillating systems [8,13].

In figure 6 there is represented the power of the amplitude fluctuations (in the frequency band $\Delta F = 100Hz$) versus the modulation depth. There are curves in this picture for IMPATT-oscillators with n-GaAs and Si n^{+} -p diodes. The full and dashed lines represent theoretical results (54) and [1] respectively. The experimental data [1] are plotted by dots.

In figure 7 there are plotted spectra for the Si n^{+} -p IMPATT-oscillator for some fixed values of the modulation depth. Here, just like in figure 6, the results are represented which were calculated from eq. (54) and on the basis of theory [1].

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*) Owing to an inaccuracy made in [1] the theoretical curve was reduced in [1] by a factor of 2. In this paper this error is corrected.

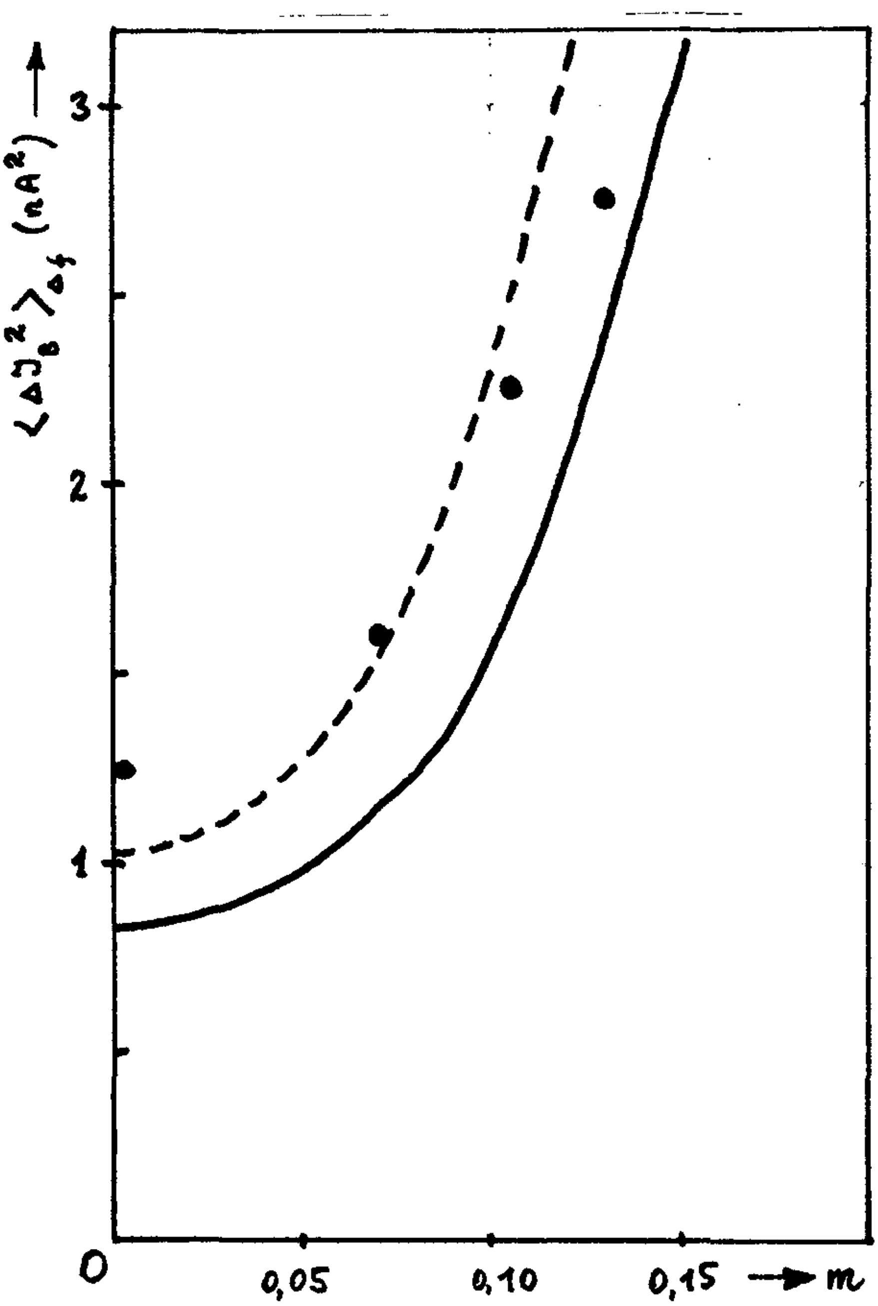

Figure 5

The power of the displacement current fluctuations (in a bandwidth 100 Hz) versus the modulation depth m for the Si n^+ -p IMPATT oscillator [1]. The dashed line and the dots represent theoretical and experimental data [1], respectively.

The full line represents our theoretical result.

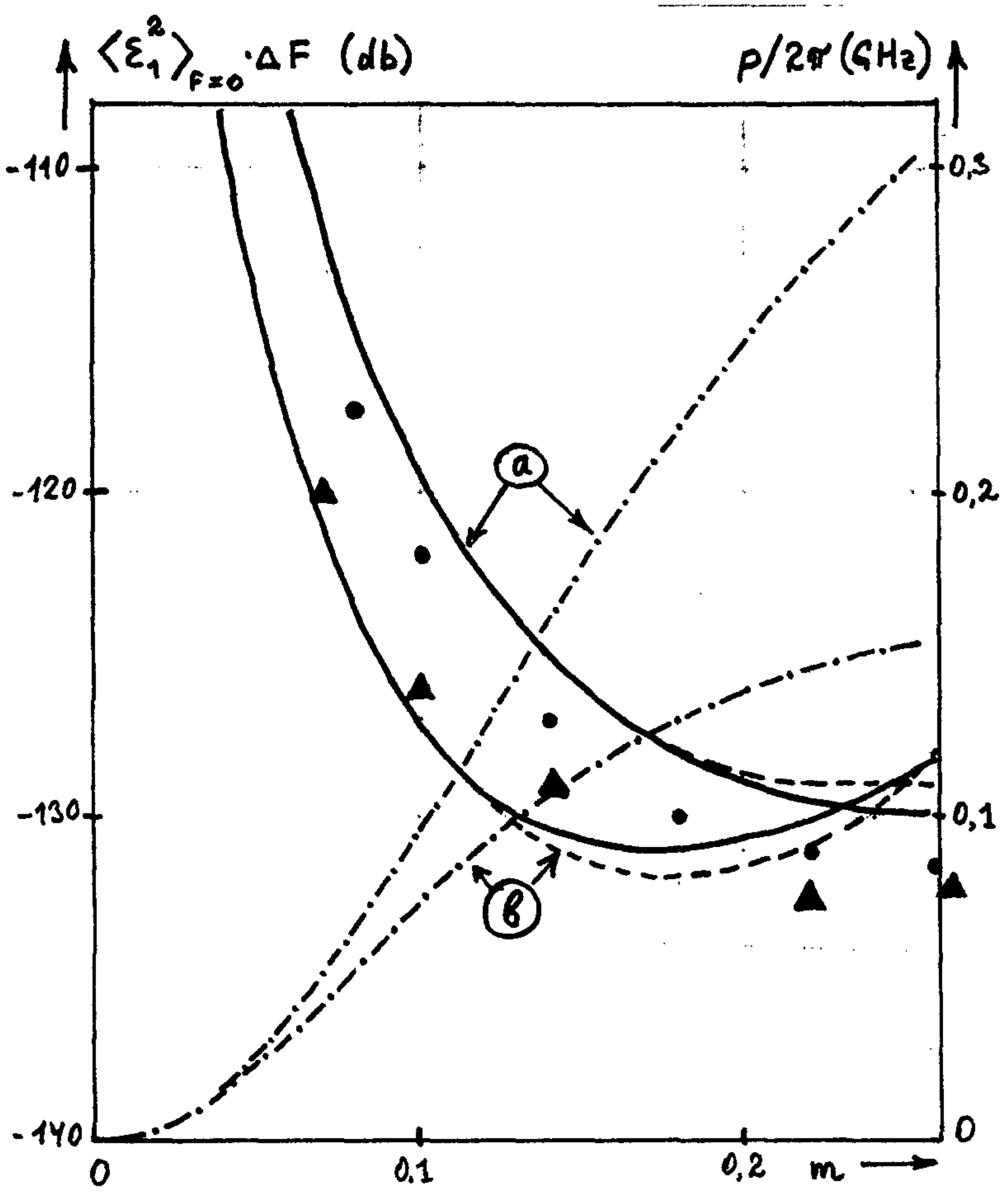

Figure 6

Dependence of the low-frequency AM-noise (ΔF=100Hz) and the stability factor p (dash-dot lines) on the modulation depth m for n-GaAs (curves "a") and Si n^{+} -p (curves "b") IMPATT oscillators [1]. Experimental data [1] for the AM-noise are presented by circles and triangles, respectively.

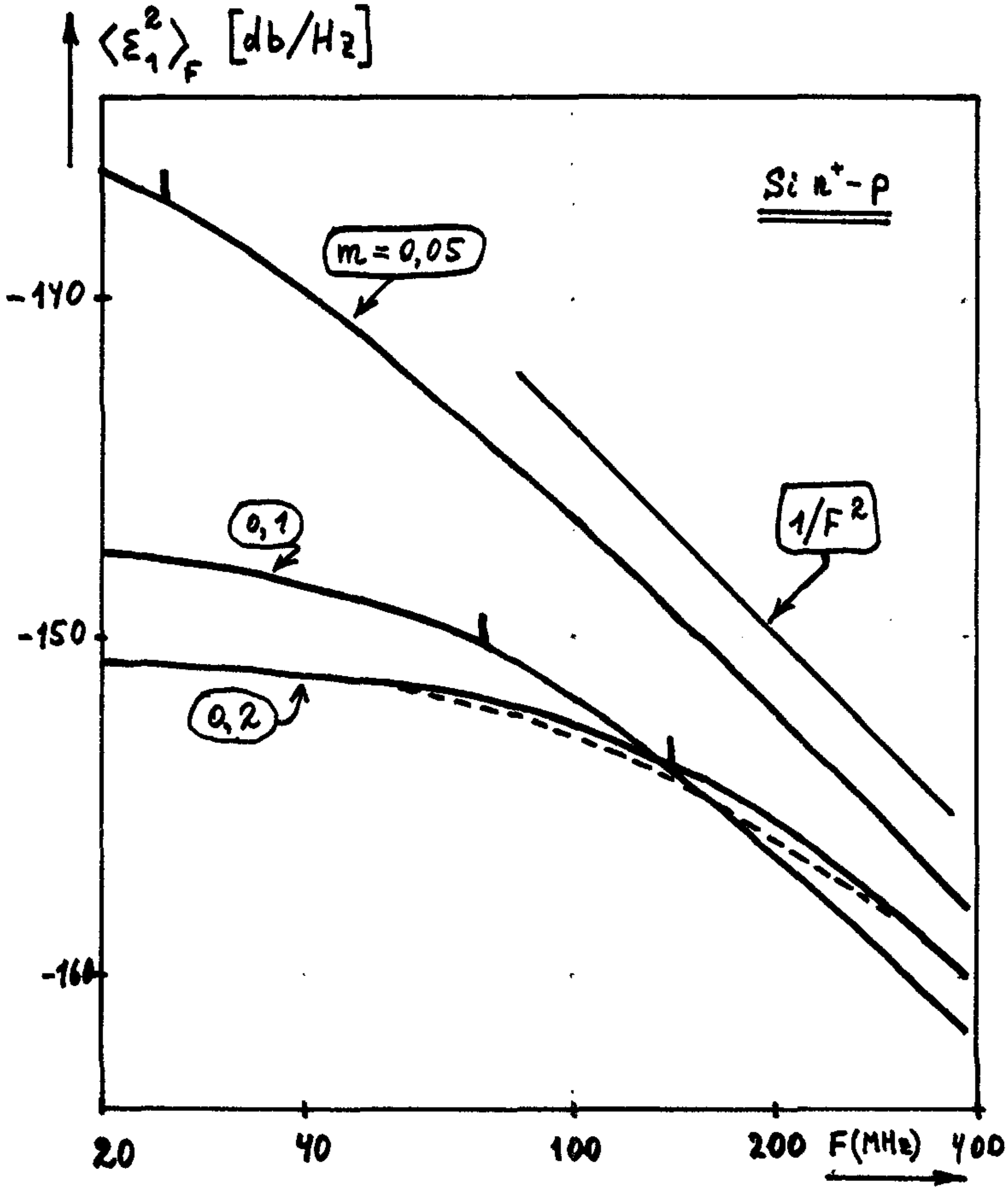

Figure 7

Spectra of the amplitude fluctuations for the Si n^+ -p IMPATT oscillator [1]. Here full and dashed lines represent our results and results after [1], respectively. Values of the stability factor p ar marked on the curves by vertical lines. There is also depicted here the curve labeled "$1/F^2$" which characterizes the dependence of the tailes of these curves on the frequency F.

6.3. The frequency fluctuations

Let us proceed now to an analysis of the frequency fluctuations which have the spectrum given by expression (55).

First of all we can see the presence of a very strong correlation between the amplitude and frequency fluctuations. This follows from [8,14] and it is possible to find from eqs. (54)-(56) that this correlation leads to an increase of the spectrum $S_\nu(\Omega)$ at low frequencies ($\Omega<<p$) by a factor of approximately

$$1+(q/p)^2=1+\eta^2$$

compared to the ideal case of noncorrelated amplitude-frequency fluctuations.

In figure 8 (in the same notations introduced above) there are plotted the theoretical (55), [1] and experimental [1] data for the low frequency fluctuations of the auto-oscillations ($\Omega<<p$). The data are given for three diodes Si p^+ -n, Si n^+ -p and n-GaAs.

In figure 8 one can see a typical discrepancy between theory and all experimental data (for large values of the modulation depth). The following conclusion suggests itself. Namely, the data given in [1] for the loss resistances are overestimated. Putting for example for the n-GaAs IMPATT-oscillator R=2,13Ω (instead of R=3,6Ω as mentioned in [1]) we can reach excellent agreement of our theory with the experiment (see dotted line in figure 8). Let us note, that in this case the low-frequency ($\Omega<<p$) value of the amplitude fluctuations spectrum will not change. The value of the stability factor p will decrease by a factor of $3,6/2,13\doteq 1,6$ due to an increase of the Q-factor of the resonator.

Thus before one will reach conclusions about the restrictions on the range of validity of the method developed here and of method [1] as well it will be necessary to carry out some further experiments, namely:

a) to measure the amplitude and frequency spectra untill the frequencies of the order of the bandwidth of the resonator;
b) to determine the parameter η from the measurement of the difference between the oscillation frequency ω_0 and the resonant frequency ω_1.

In figure 9 there are plotted the frequency fluctuation spectra for different values of the modulation depth m. In these spectra one can easily see the influence of the amplitude fluctuations represented by the typical hump at low frequencies.

An important conclusion follows from analysis of figures 7, 9. For treating the problem of minimization of the fluctuations in IMPATT-oscillators the frequency dependence of the spectra must be taken into account.

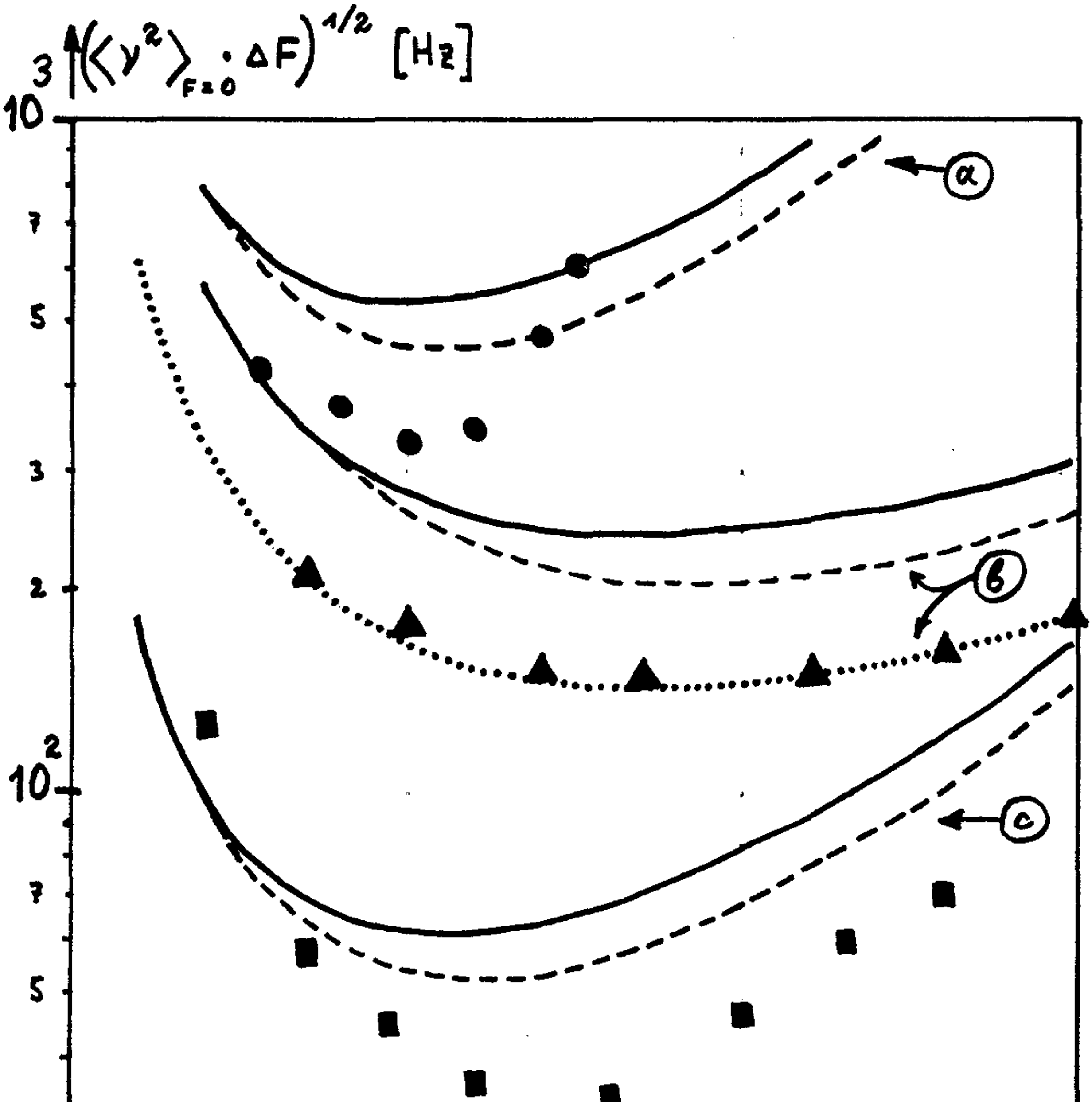

Figure 8

Dependence of the low-frequency FM-noise (ΔF=100Hz) on the modulation depth m for Si n^+-p, n-GaAs and Si p^+-n IMPATT oscillators [1] (curves "a", "b" and "c" respectively). Experimental data [1] for these curves are presented by circles, triangles and rectangles respectively. The dotted line "b" gives the best fit by using R=2.13Ω instead of R=3.6Ω as was done by [1].

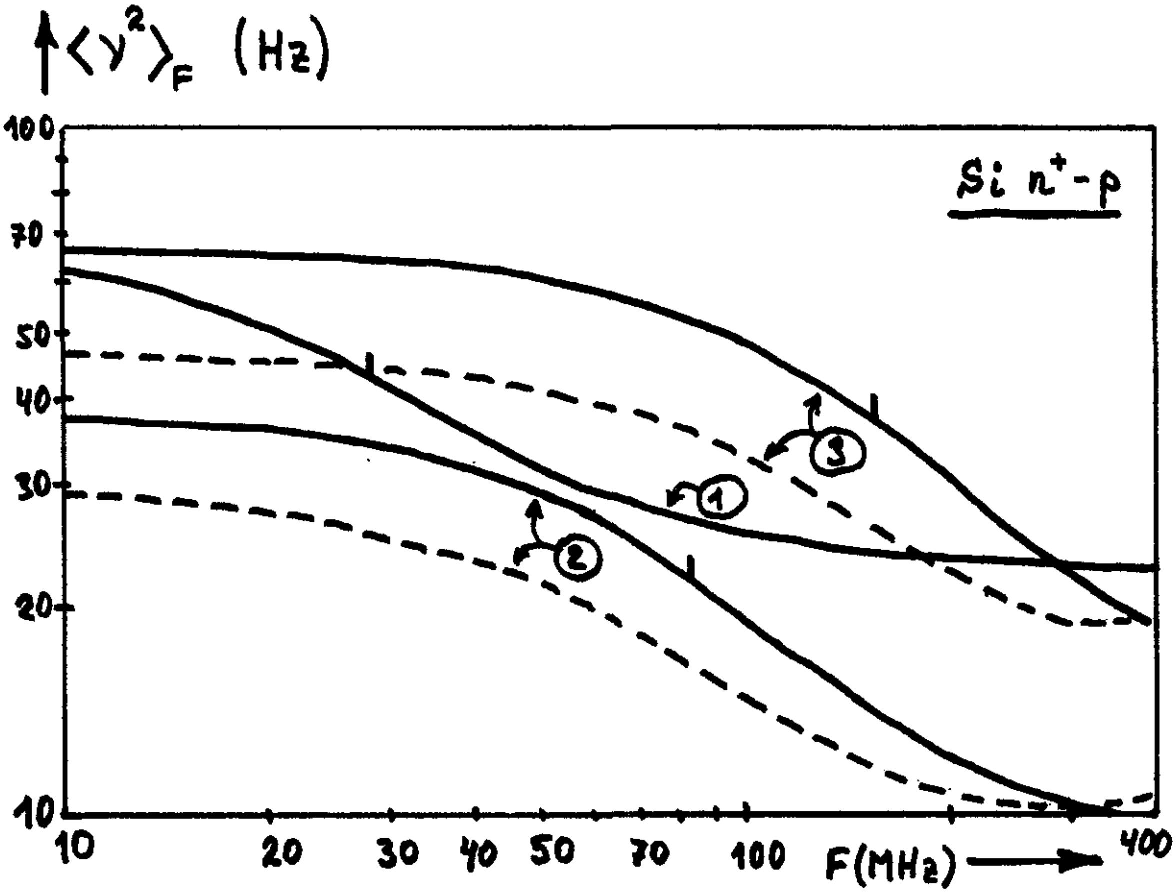

Figure 9

Spectra of the frequency fluctuations for Si n^+ -p IMPATT oscillator [1]. Here the full and dashed lines represent our results and results after [1], respectively.

Values of the non-isochronism factor q are marked on the curves by vertical lines. The labels "1", "2" and "3" correspond to the values of the modulation depth which are equal to 0,05, 0,1 and 0,2 respectively.

In some cases the information about the dependence of the power of low-frequency ($\Omega \ll p$) fluctuations (see [1]) is insufficient. For example for Si n^+ -p IMPATT-oscillator one can see that at low frequencies the value of the amplitude fluctuations spectrum for m=0,1 is higher than the value of this spectrum for m=0,2. But at the frequencies higher than 140MHz the situation is reversed.

6.4. The shape of the spectral line

In figure 10 there is given the pedestal *) of the spectral line (58). In this figure frequency F is a distance from the central frequency $f_o = \omega_o / 2\pi$. The curve labeled WH represents the upper wing ($f = f_o + F$) of the pedestal and the curve labeled WL represents the lower wing ($f = f_o - F$) of the pedestal. For the sake of explicity there is given the amplitude spectrum as well.

First of all one can see that in the frequency range considered in figure 10 the value of the lower wing WL is higher than the value of the upper wing WH. Moreover this difference can reach a value of the order of 6dB. Such asymmetry of the spectral line of the auto-oscillations is caused by the fact that the IMPATT-oscillator has a positive nonisochronism $q = \eta p$. Roughly speaking it means that increasing of the amplitude of oscillations leads to decreasing of their frequency.

Let us note that the problem of determining of the shape of the spectral line has been treated earlier by other authors. For example the results of such analysis are reported in [15,16] from which it follows that this line has a symmetrical pedestal. This result contradicts not only our analysis but also data obtained by the same authors in [1] as well. In [1] the existence of a strong even and odd correlation between the amplitude and frequency fluctuations of the auto-oscillations in IMPATT-oscillator is reported. But it is known [8,14] that the existence of an even correlation leads to the appearance of the asymmetry in the shape of the spectral line of the oscillations.

*) The pedestal of the spectral line of auto-oscillations is $W_t(\Omega)$ for $|\Omega| \gg \Delta\Omega_t$. When $|\Omega| \sim \Delta\Omega_t$ in this case $W_t(\Omega)$ describes the peak of the spectral line. This peak has a symmetrical resonant shape.

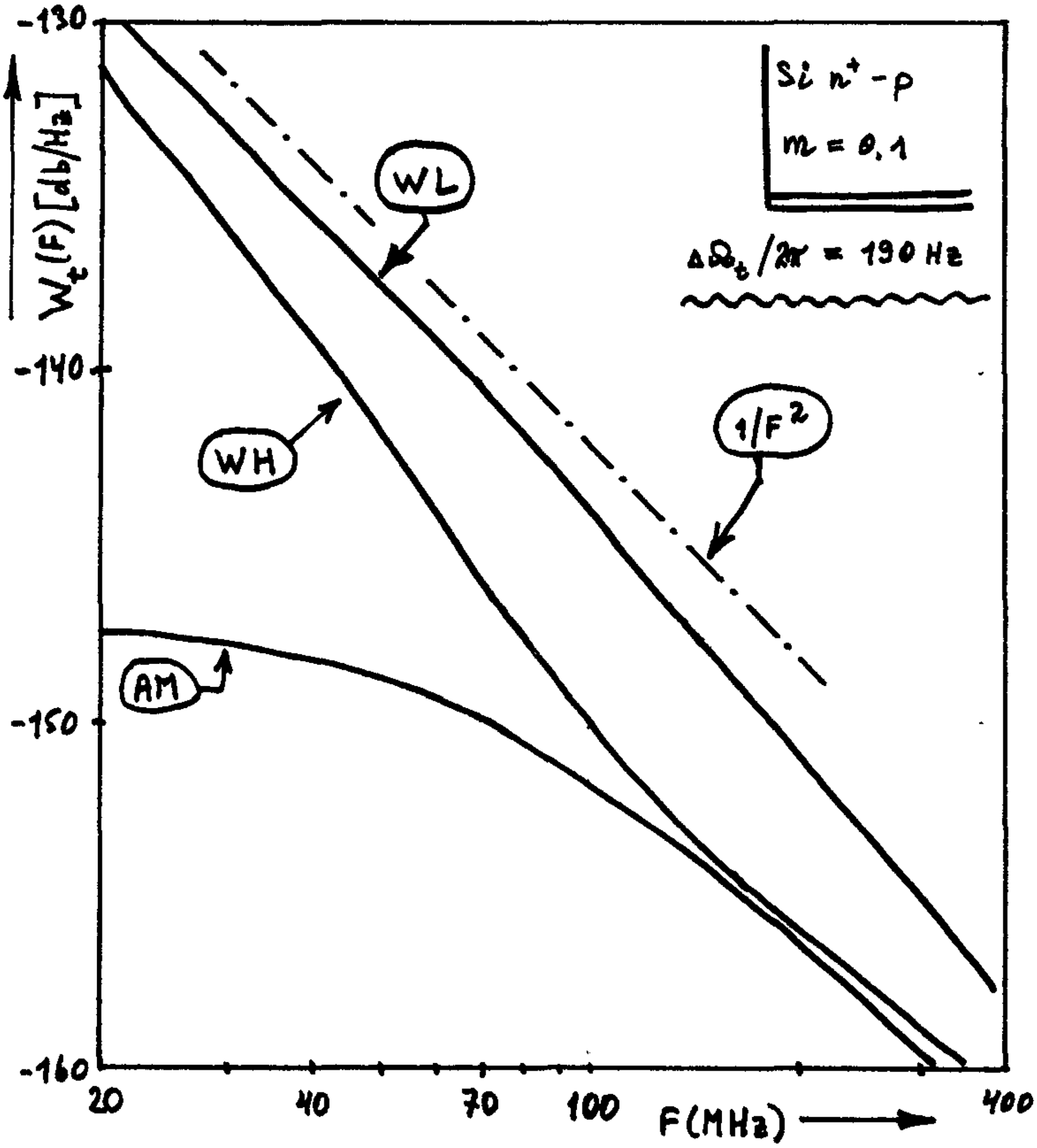

Figure 10

The shape of the spectral line versus frequency. The curves labeled "WH" and "WL" represent the upper-wing and the lower-wing of this line. The curve labeled "AM" represents the spectra of amplitude fluctuations. The curve labeled "$1/F^2$" characterizes the dependence of the tailes of these curves on the frequency and the dependence of the shape of the spectral line on frequency within an intermediate region ($\Delta\Omega_t/2\pi \ll F \ll p/2\pi$).

6.5. Influence of higher harmonics

In order to examine the influence of higher harmonics of the current of the IMPATT-diode the calculations mentioned above have been executed taking into account different values of N_{max}. For these values the numbers 1,2,3 and 10 were used. The results for n-GaAs IMPATT-oscillator are presented in figure 11 (a,b). The curves for $N_{max}=3$ and for $N_{max}=10$ have practically no difference. Besides that, one can see that it is enough to take into account only the first harmonic (i.e. $N_{max}=1$). The accuracy reached in this case will be rather reasonable.

Conclusion

In the presented paper the method is described which makes it possible to determine the spectral characteristics of the natural fluctuations (caused by additional noise)of auto-oscillations in IMPATT-oscillators.

In this method not only the dependence of the noise generation mechanism upon the signal level (like it was done in [1]) was taken into account but also the periodical nonstationarity of this noise as well.

This method allows us to take into consideration any number of harmonics of IMPATT-diode current without losing the physical picture of the final results. The only restriction is the assumption that the voltage of the signal on the diode is sinusoidal. In other words only the current non-linearity of IMPATT-diode was taken into account.

It follows from [4] that this non-linearity plays the most important role in IMPATT-oscillators when the modulation depth has value smaller than unity.

The numerical calculations of our results need less computing time than calculations following method [1].

Besides this, the execution of the computing program after [1] leads to a great inaccruracy when the analysing frequencies are less than 100KHz and sometimes the execution of this program even becomes impossible. Our method is completely free from this disadvantage.

On the basis of the method delivered in this paper it is possible to carry out the theoretical analysis for more complicated IMPATT-oscillators (having the high Q-external resonator, phase-locked or mutualy synchronized and so on).

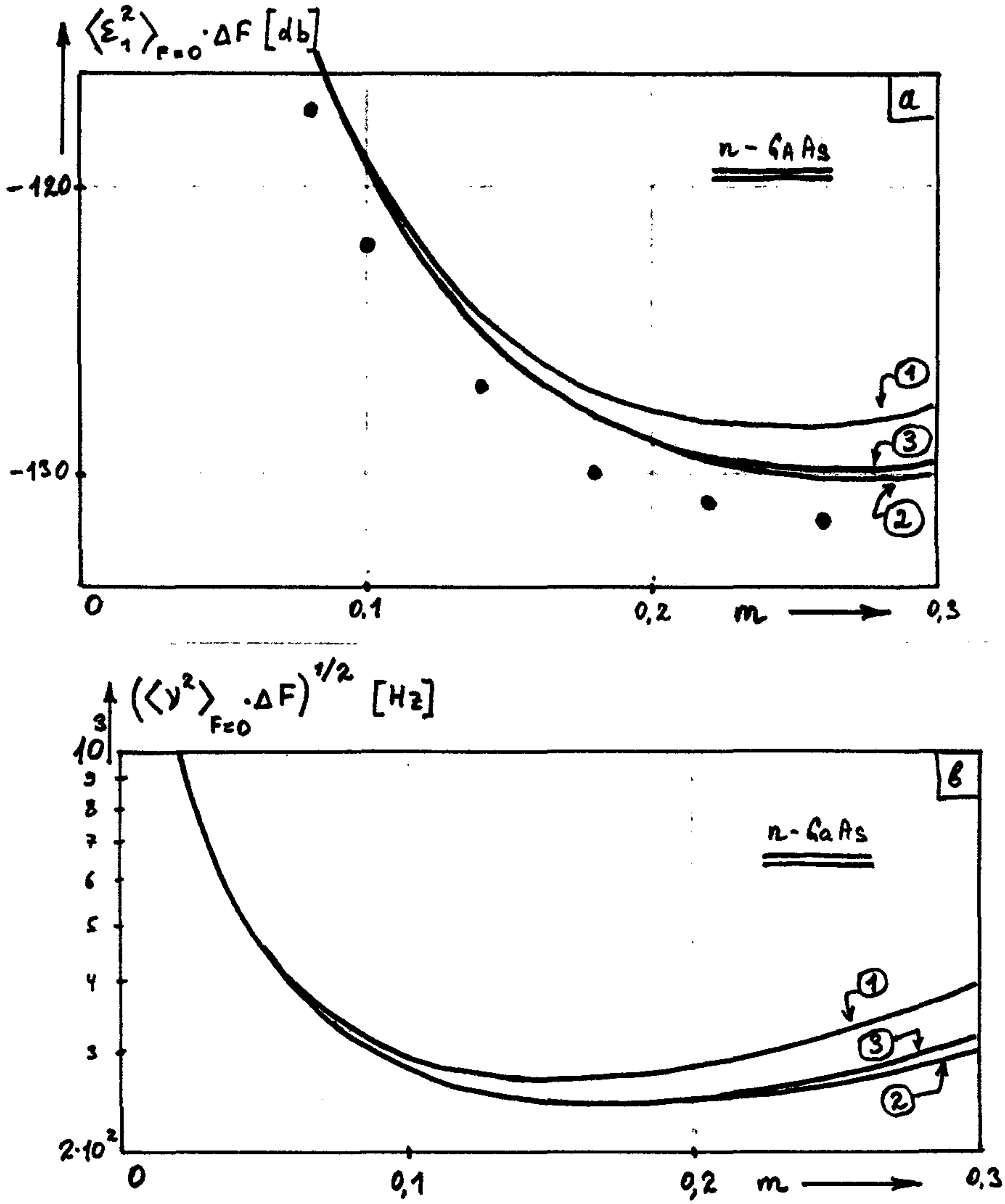

Figure 11 (a,b)

The influence of higher harmonics on the spectra of amplitude and frequency fluctuations. The curves labeled "1", "2" and "3" are plotted for $N_{max}=1$, $N_{max}=2$ and $N_{max} \geq 3$, respectively.

This is so because the most specific difficulty of such an analysis is taking into account the non-linearity of the noise generation mechanism in the IMPATT-diode.

All calculations necessary for this purpose were carried out here in the most general manner and put into a form useful for both qualitative and quantitative analysis.

The author considers it as a very pleasant duty to express his acknowledgement to Prof.Dr. F.N. Hooge for the help and support during the whole period of work in the Eindhoven University of Technology .

He is also thankful to Dr. M.T. Vlaardingerbroek and Dr. J.J. Goedbloed for the fruitful discussions of some problems in the theory of the IMPATT-diode oscillators.

He thanks Mr. O. Koopmans for his help during the execution by the author of the computer calculations.

He is grateful to Miss J.H.W.M. van der Linden who performed the difficult job of typing this report.

He likes to express his acknowledgements to all staff of the Eindhoven University of Technology and especially of the group EV who supported him during his work at the University of Technology.

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