Self-induced magnetic field generation in an axisymmetric isothermal plasma

SELF-INDUCED MAGNETIC FIELD GENERATION IN AN AXISYMMETRIC ISOTHERMAL PLASMA

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1. INTRODUCTION

In the description of a broad class of plasmas magnetic fields, both external and self-induced, are known to play a important role. For example in laser produced plasmas magnetic fields up to some hundred kG have been measured (McL 84, RAV 78). Also in vacuum arcs the self-induced magnetic fields may play a decisive role in the explanation of such phenomena as current constriction and the retrograde motion of cathode spots. Measurements in vacuumswitches by Schellekens (SCH 83) show that besides the azimuthal magnetic field also axial and radial components of the self-induced magnetic field occur and have to be taken into account.

The aim of this work is to describe the mechanism of self-induced magnetic field generation. This description will be quite general, but we will limit ourselves to isothermal axisymmetric plasmas. Using the result of this analysis a alternative explanation for the retrograde motion will be given. Finally the results of measurements of the self-induced axial magnetic field in a hollow cathode argon plasma will be presented.

2.1 The homopolar motor

The model of the axisymmetric plasma to be presented resembles the so-called homopolar motor model (figure 1).

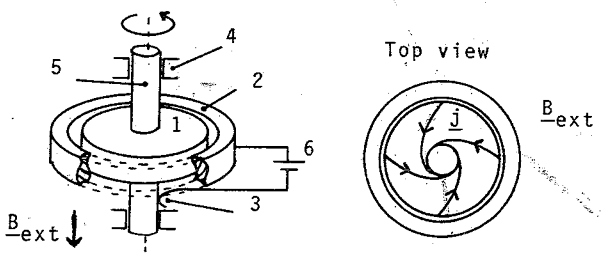

Fig 1 : The Homopolar motor.
1: conducting rotor 2: conducting stator
3: sliding contact 4: bearing 5: axis
6: voltage supply

The rotor 1 is electrically connected to the stator 2 by brushes. The externally applied voltage drives a current density $\underline{j}$. The Lorentz force exerted on the electrons by the externally applied magnetic field B_{ext}:

$$\underline{F}_{Lorentz} = - e \, \underline{w}_e \times \underline{B}_{ext} \qquad (1)$$

, induces an azimuthal component of the current density (figure 1b). This current is inhibited by the ohmic resistance of the rotor causing it in this way to rotate. The azimuthal current density will furthermore generate a meridional magnetic field which will in its turn influence the current density. In this way the self-generated magnetic field interacts with the current density causing it. It is this interaction, which will determine in the end the current density and the self-generated magnetic field. The same mechanism will operate in a plasma. In the plasma however, the electric field driving the current is determined by the combination of an externally applied electric field and a self-induced one, caused by the net charge density ρ in the plasma.

2.2. The governing equations

In this analysis we will study the electron momentum equation. We will assume that the degree of ionization is large enough to neglect the electron-neutral friction and to justify the use of Braginskiis transport coefficients (BRA 65). Furthermore, electron viscosity and electron inertia will not be taken into account. Finally we will assume that to a good approximation the electron gas can be considered as isothermal, i.e.

$$\frac{\nabla T_e}{T_e} << \frac{\nabla P_e}{P_e} \qquad (2)$$

Here T_e and P_e are the electron temperature and electron pressure. With these assumptions the electron momentum equation is given by

$$\underline{0} = - \nabla P_e - e n_e \underline{E} - e n_e \underline{w}_e \times \underline{B} + e n_e \eta \underline{j} \qquad (3)$$

In this equation w_e is the systematic velocity of the electrons. $\underline{E}$, $\underline{B}$, j and n_e are respectively electric field strength, magnetic induction, current density and electron density. The term $e n_e \eta \underline{j}$ represents the electron-ion friction; η is the resistivity which we will assume to be constant. The electromagnetic field quantities have to satisfy Maxwell's equations.

$$\nabla \cdot \underline{B} = 0 \quad (a) \qquad \nabla \times \underline{B} = \mu_o \underline{j} \quad (b)$$
$$\nabla \times \underline{E} = - \partial/\partial t \, \underline{B} \quad (b) \qquad \nabla \cdot \underline{E} = \frac{\rho}{\varepsilon_o} \quad (d) \qquad (4)$$

The neglect of the displacement current in equation (4b) has the effect of filtering electromagnetic waves from the system (4).

3.1. Deriviation of the induction equation

Taking the curl of the electron momentum equation (3) and combination with equation (4c) gives the so-called induction equation, describing the time-dependence of the magnetic induction $\underline{B}$:

$$\partial/\partial t \, \underline{B} = \nabla \times \{\underline{w}_e \times \underline{B}\} + \frac{\eta}{\mu_o} \nabla^2 \underline{B} \qquad (5)$$

As we have assumed the electron gas to be isothermal the pressure term drops out. The relative importance of the two terms on the RHS of equation (5) is given by the magnetic Reynolds number R_m :

$$R_m = \frac{\mu_o w_{eo} l_o}{\eta} \qquad (6)$$

Here w_{eo} is a typical magnitude for the velocity field $\underline{w}_e$ and l_o the typical scale length over which it varies. If $R_m >> 1$ the magnetic field lines can be considered to be "frozen" in the electron fluid. For $R_m << 1$ the induction equation describes the free decay of the magnetic field through ohmic dissipation. When the plasma under consideration shows axisymmetry the induction equation (5) can be seperated in a meridional component (r,z component in cylindrical coordinates) and an azimuthal component (ϕ- component). To that purpose we write :

$$\underline{B}_m = B_r \underline{e}_r + B_z \underline{e}_z \quad ; \quad \underline{B}_\phi = B_\phi \underline{e}_\phi \qquad (7a)$$

$$\underline{w}_e = w_{er} \underline{e}_r + w_{ez} \underline{e}_z \quad ; \quad \underline{w}_{e\phi} = w_{e\phi} \underline{e}_\phi \qquad (7b)$$

Substitution of these equation in (5) yields the meridional, respectively azimuthal components of the induction equation :

meridional:

$$\partial/\partial t \; \underline{B}_m = \nabla \times (\underline{w}_{em} \times \underline{B}_m) + \frac{\eta}{\mu_o} \nabla^2 \underline{B}_m \qquad (8a)$$

azimuthal:

$$\partial/\partial t \; \underline{B}_\phi = \nabla \times (\underline{w}_{em} \times \underline{B}_\phi + \underline{w}_{e\phi} \times \underline{B}_m) + \frac{\eta}{\mu_o} \nabla^2 \underline{B}_\phi \qquad (8b)$$

Equation (8a) shows that when $\underline{B}_m (t=0) = \underline{0}$, $\underline{B}_m$ will remain zero, i.e. there is a "source term missing" in the meridional component of the induction equation. This conclusion leads to the wellknown laminar anti-dynamo theories (MOF 78).

3.2. Stationary induction equation

In this section we take a more quantitative approach in evaluating the induction equation. Here we will consider stationary axisymmetric situations only. Furthermore we will assume that the current is carried almost completely by the electrons i.e.

$$\underline{j} \simeq -e n_e \underline{w}_e \qquad (9)$$

Translation of this assumption to the homopolar motor model implies that the rotor is kept fixed. In fact we neglect the influence of the ion motion on the current density and thus on the self-generated magnetic field. In this way the electrons determine the current density and the magnetic field in which the ions and neutrals move.
With equation (9) we can transform the electron momentum equation to

$$\underline{0} = \frac{-1}{e n_e} \nabla p_e - \underline{E} + \frac{1}{e n_e} \underline{j} \times \underline{B} + \eta \underline{j} \qquad (10)$$

Using the azimuthal component of this equation we can express the electron density in terms of $\underline{j}$ and $\underline{B}$:

$$\frac{1}{e n_e} = \frac{-\eta j_\phi}{(\underline{j} \times \underline{B})_\phi} \qquad (11)$$

substitution of this expression in the meridional component of (10) gives after combination with the stationary Maxwell equation $\nabla \times \underline{E} = \underline{0}$:

$$\nabla \times \{ \frac{j_\phi}{(\underline{j} \times \underline{B})_\phi} (\underline{j} \times \underline{B})_m \} = \nabla \times \underline{j}_m \qquad (12)$$

Equation (12) is an equation in the components of $\underline{j}$ and $\underline{B}$ only. To analyse the set of equations formed by equation (12) and the Maxwell equations (4a,b,c). We have introduced so-called flux functions Φ_j and Φ_B for respectively the current density and the magnetic induction :

$$\underline{j}_m = \frac{1}{2\pi r} \nabla \Phi_{\underline{j}} \times \underline{e}_\phi \qquad B_\phi = \frac{+\mu_o}{2\pi r} \Phi_{\underline{j}} \qquad (13a)$$

$$B_m = \frac{1}{2\pi r} \nabla \Phi_{\underline{B}} \times \underline{e}_\phi \qquad j_\phi = \frac{-1}{\mu_o} \frac{1}{2\pi r} S \Phi_{\underline{B}} \qquad (13b)$$

Here S is the Stokes operator given by

$$S := \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} \qquad (14)$$

By introducing these flux functions we have identically satisfied Maxwell's equations (4a, b, c). The flux functions have a simple interpretation. For example $\Phi_B(r_o, z_o)$ represents the magnetic flux through a surface bounded by the anisymmetric curve C at $r = r_o$, $z = z_o$. An analogous interpretation can be ascribed to Φ_j. Equation (12) links the flux functions $\Phi_{\underline{j}}$ and Φ_B. Evaluation of this relation near the axis of symmetry gives :

$$B_z(r=0) \frac{d}{dz} (\Omega_{elec}(r=0) = \frac{-\mu_o e n_e}{4} \frac{d}{dz}$$

$$\{(w_{ez}(r=0))^2\} - \frac{\eta}{2} (\frac{\partial^2}{\partial z^2} j_z |_{r=0} + 2 \frac{\partial^2}{\partial r^2} j_z |_{r=0}) \qquad (15)$$

Here Ω_{elec} is the rotational frequency of the electron gas near the axis. From equation (15) we expect magnetic field generation to occur in plasmas where the electron gas is axially accelerated. Such an acceleration may be caused by an electric field or by a pressure gradient. So, for example magnetic field generation may play a role in laser produced plasmas or in the cathode spots of a vacuum arc.

4. APPLICATION TO THE RETROGRADE MOTION OF CATHODE SPOTS

In this section we will show that the observed repulsion between cathode spots may be due to the self-generated meridional magnetic field. Consider two cathode spots at an inter spot distance R. As the internal forces in a spot will be much greater than the mutual forces, the internal structure of a spot is not influenced by the other, thus leaving the symmetric structure of the spots intact. The magnetic field generated by a cathode spot consists of an azimuthal part and a meridional part. The azimuthal part results in an attracting force between the spots. As the cathode spots are observed to repel each other this motion is called retrograde. However, the meridional component of the magnetic field will also induces a force between the spots. Assuming that the inter spot distance R is much greater than the spot diameter, the meridional magnetic field can be represented by a magnetic dipole field (orientation of the dipole perpendicular to the cathode surface). If the orientation of the dipoles is the same, the repelling force due to the dipole-dipole interaction is given by

$$F_{rep} = \frac{3 m_1 m_2}{R^4} \qquad (16)$$

Here m_1 and m_2 are the dipole strengths of respectively spot 1 and spot 2. Characterizing the axial dimensions of the magnetic dipole (i.e. the region where meridional field generation occurs) by δ_z, the attracting force on a dipole due to the azimuthal magnetic field is given by

$$F_{attr.} = \frac{+\mu_o I^2 \delta_z}{2\pi r} \qquad (17)$$

In order to explain the retrograde motion we must demand :

$$\left|\frac{F_{rep}}{F_{attr}}\right| > 1; \; m_1 m_2 > \frac{\mu_0 \; I^2 R^3 \; \delta_z}{6\pi} \qquad (18)$$

Rough estimates of the quantities in equation (18) show that the retrograde motion of the cathode spots can be explained for distances between spots up to 5 cm by assuming a pitch of the current lines equal to one.
In order to explain the retrograde motion of a single spot in an external magnetic field more sophisticated models should be considered which essentially incorporate the lack of axisymmetry of the physics of the spot. The (distorted) meridional magnetic field can however also play an important role in the explanation of this type of retrograde motion.

5. THE SELF-GENERATED FIELD OF A HOLLOW CATHODE ARC

Using diamagnetic loops we have demonstrated the existence of a self-generated magnetic field in a hollow cathode Ar arc shown in figure 2.

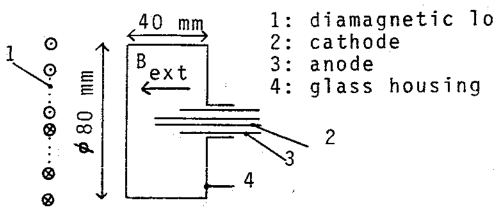

Fig 2: The experimental set up

To prevent distortion of the magnetic field by metallic parts of the housing of the arc has been made of pyrex glass. The cathode is a hollow tantalum pipe (inner diameter 1 mm, outer diameter 2 mm) through which argon gas is fed. The flow is adjusted to give a filling pressure of approximately 0.7 Torr. The arc as a whole is placed in an external magnetic field variable from -8.10^{-2} T to 8.10^{-2} T. In the experiment we operated the arc in the DC-mode (I_{DC} = 100 mA) and superimposed a current pulse of some hundred Ampères during .6 ms. The associated flux change induces a voltage in the diamagnetic loop. Integration of this voltage with a RC-integrator yields the magnetic flux. We used 12 diamagnetic loops diameters from 9 to 146 mm. Furthermore, the symmetry of the discharge could be checked by using four quarter section loops. Finally, the use of a bifilarly wound loop showed that indeed the measured signals were associated with flux changes and did not result from e.g. common mode problems. Figure 3 shows a typical current pulse and the associated magnetic flux.

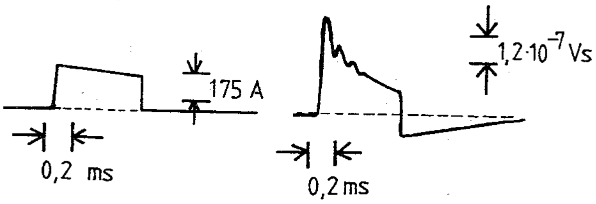

Fig 3 : Current pulse and magnetic flux pulse

The measurement shows that the plasma behaves paramagnetic near the axis : close to the axis the generated field strengthens the applied field.
Varying the external magnetic field the relation between the generated flux and external magnetic field has been measured.

The result for the four smallest loops are shown in figure 4.

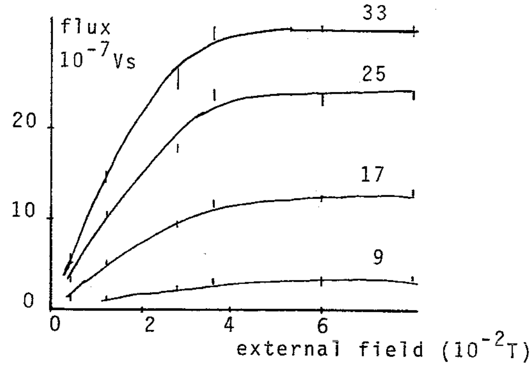

Fig 4 : Generated flux versus external field.

The curves are labelledby the loop diameters (mm). From this figure we see that the generated flux saturates for high external magnetic fields and tends to zero when this field is reduced to zero. However, for these low external fields the discharge is highly asymmetric as can been seen by comparing the signals from the quarter section loops. Variation of the distance between the diamagnetic loops and the hollow cathode arc yields the possibility of constructing the flux pattern generated by the plasma. The results of such a measurement are given in figure 5.

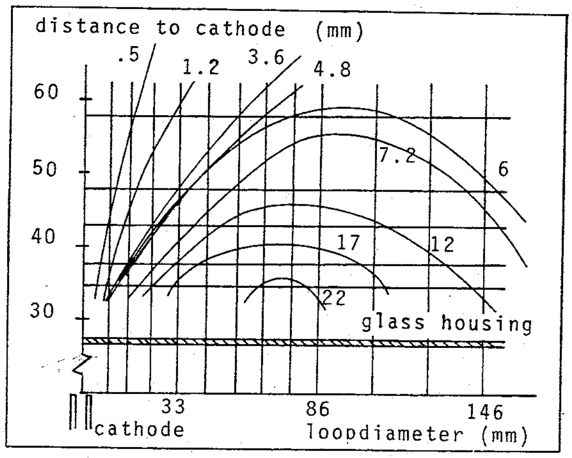

Fig 5 : Measured fluxpattern (10^{-8} Vs)
B_{ext}=0.06 T I_{pulse}=220 A

To a good approximation this flux pattern can be described by assuming a magnetic dipole with a dipole strength of 10^{-2} Am^2 near the vessel wall.

6. CONCLUSIONS

Using the induction derived from the electron momentum equation we have shown that magnetic field generation is likely to occur in plasmas where the electron gas is accelerated. Based on this conclusion we have shown that in vacuum arcs the self-generated magnetic field can be of importance in the explanation of e.g. the retrograde motion of cathode spots. Finally, the self-generated magnetic field has been determined in a hollow cathode Ar arc.

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