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THE RADIATIVE PRODUCTION AND DESTRUCTION OF ATOMIC STATES.

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In recombining plasmas of low electron density the lower part of an atomic energy scheme is completely determined by radiative processes. Atoms of a certain level are produced by decay of atoms of higher bound levels (cascade) or two particle recombination (capture) and destroyed by radiative decay. In this contribution the kinetics of these radiative processes will be discussed as functions of the energy of the level. We confine ourselves to hydrogenic systems where levels can be 'labeled' by their principal quantum number (pqn) p or by the energy $E_p = -RZ^2p^{-2}$ (R=13.6 eV and Z is the charge number of the core). We assume that the p-value is high enough to formulate several quantities in a quasi-continuous way. One of these quantities is the density of states with respect to energy which is related to the statistical weight $g(p) = 2p^2$ and reads

$$G(E_p) \simeq g(p)\frac{dp}{dE} = p^5/(Z^2R) \qquad (1)$$

The decay frequency [1] of an upper (energy) level u to a lower level l

$$A(u,l) = \gamma Z^4 g(l)u^{-5}l^{-5}(l^{-2} - u^{-2})^{-1}h(l,u) \qquad (2)$$

can, employing eq(1), be written as

$$A(u,l) = (\gamma Z^2/R)g(l)G(E_u)^{-1}G(E_l)^{-1}\Delta E^{-1} \qquad (3)$$

Here $\Delta E = E_u - E_l$, the Gauntfactor h(u,l) is put equal to unity and $\gamma = 7.87\cdot 10^9 s^{-1}$. The number of decay processes of the level p for which radiation is emitted with energy values between ΔE and $\Delta E + d\Delta E$ can now be approximated by

$$n(p)A(p,\Delta E)d\Delta E = n(p)t(p)\Delta E^{-1}d\Delta E \qquad (4)$$

in which $t(p) = \gamma Z^2R^{-1}G(E_p)^{-1}$. Eq. (4) expresses that the decay of the level p is less probable if the density of states at location p is high and that 'landing' to nearby levels is dominant. Integration of eq.(4)

$$n(p)A(p) = n(p)\int_{E_{p-1,p}}^{E_p} A(p,\Delta E)d\Delta E = n(p)t(p)\ln(E_{1p}/E_{p-1,p}) \qquad (5)$$

gives an approximation of the total number of decay processes of atoms with initial energy E_p. Since $E_p \simeq Z^2R$ and $E_{p-1,p} \simeq 2Z^2Rp^{-3}$ the logarithm tends to $3\ln p - \zeta$, in which ζ is a correction term to equate this global result to its actual value and depends on the opacity. For a transparant plasma $\zeta = 0.25$. The number of decay processes which leads from an upper level in the energy band dE to level p can be written as

$$n(u)\frac{du}{dE}\,dE\;A(u,p) = (n(u)/g(u)g(p)t(p)\Delta E^{-1}dE \tag{6}$$

If the upper level population is not far from equilibrium, we may substitute $n(u)/g(u) \simeq (n^S(p)/g(p))\exp(-\Delta E/kT_e)$ in which $n^S(p)$ is the density according to Saha and T_e the electron temperature. For the total number of cascade processes populating the level p we find

$$\sum_{u>p} n(u)A(u,p) \simeq n^S(p)t(p)\int_{E_{p,p+1}}^{E_{pi}} \Delta E^{-1}\exp(-\Delta E/kT_e)\,d\Delta E \tag{7}$$

in which $E_{pi}=-E_p$ is the ionization potential of the level p.
Using the energy representation has the advantage that we can extrapolate eq.(7) to capture, which can be regarded as a decay of a free electron to a bound state. The number of capture processes thus found is

$$n_e n_+\alpha(p) \simeq n^S(p)t(p)\int_{E_{pi}}^{\infty} \exp(-\Delta E/kT_e)\,d\Delta E/\Delta E \tag{8}$$

A difference between eq.(5) at one hand and eqs.(7,8) at the other hand is that the latter contain the electron temperature. But the influence of T_e is limited provided that $E_{pi}/kT_e \ll 1$. In that case the integral in eq.(7) approaches $\ln(E_{pi}/E_{p,p+1}) \simeq \ln p$, while the integral in eq.(8) approaches $\ln(kT_e/E_{pi}) \simeq 2\ln p - \ln(R/(kT_e Z^{-2}))$.
So if only radiative processes are of importance and the number density of p-level atoms is determined by the balance

$$n(p)\sum_{l<p} A(p,l) = n_e n_+\alpha(p) + \sum_{u>p} n(u)A(u,p) \tag{9}$$

we find for the equilibrium departure b(p) of the level p

$$b(p) = n(p)/n^S(p) = (3\ln p - \xi(T_e))/(3\ln p - \zeta) \tag{10}$$

This analytical approximation can, after a proper fit of $\xi(T_e)$ and ζ, be used to describe results of extensive numerical calculations [2]. Besides it gives insight in the kinetics of radiative processes. For instance we see that

- the relative importance of decay, capture and cascade processes is approximately as 3:2:1.
- the deviation from equilibrium is limited over a large T_e-range (if $E_{pi}/kT_e \ll 1$). In a first order approximation $\xi \simeq \ln(Z^2R/kT_e) + 0.64$
- for low T_e-values we get an underpopulation ($b(p)<1$); for high T_e-values an overpopulation. At a critical temperature $T_e=T_c$ it is found that $b(p)=1$. The value of T_c is related to the equation $\zeta - \xi(T_c)=0$ and depends on the opacity of the plasma. For an optically thin plasma it can be found that $T_c = 20\,Z^2$ eV.

[1] Johnson L.C. Astr. J. (1972) 174, 227
[2] Mullen J.A.M van der (1986) Thesis, Eindhoven University of Technology.