Markovian models of a transactional system supported by checkpointing and recovery strategies, Part 1: A model with state-dependent parameters

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EUT Report 82-E-128
ISBN 90-6144-128-5
ISSN 0167-9708
August 1982

Errata to: EUT Report 82-E-128
Markovian models of a transactional system supported by checkpointing and recovery strategies, Part 1, by V. F. Nicola.

page	line or equation	(...) to be replaced with (...)		
9	eq. (3.8)	+ p(c,i-1)	→	p(c,i-1)
9	l. 17	θ_i	→	θ_j
18	l. 3	(2.23)	→	(3.23)
21	l. 25	p(a,o)	→	p(a,i)
27	l. 16	and a failure	→	or a failure
27	l. 17	checkpointing	→	processing

Eindhoven University of Technology Research Reports
EINDHOVEN UNIVERSITY OF TECHNOLOGY

Department of Electrical Engineering
Eindhoven The Netherlands

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August 1982

CIP-gegevens

Nicola, V.F.

Markovian models of a transactional system supported by
checkpointing and recovery strategies / by V.F. Nicola. -
Eindhoven: University of technology.
Part 1: A model with state-dependent parameters. -
(Eindhoven university of technology research reports;
82-E-128)
Met lit. opg., reg.
ISBN 90-6144-128-5
ISSN 0167-9708
SISO 656 UDC 519.71
Trefw.: regeltechniek / meettechniek.

Contents

Abstract:

A Markovian model of a transactional system supported with checkpointing and recovery strategies to guarantee reliable operation is considered. The model allows representations with state-dependent parameters. Algorithms for the computation of the state probabilities (and thus the performance variables) and their sensitivities with respect to the model parameters are presented. In the case of state-independent parameters, a state-space analysis approach is demonstrated for the derivation of analytic expressions for the performance variables.

The optimization of some important performance criterions, such as the system availability and the mean response time of a transaction, is discussed.

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Department of Electrical Engineering, Eindhoven University of Technology, 1982.
EUT Report 82-E-128

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1 Introduction

This paper introduces a state-space approach to the analysis of a class of models which may serve as a tool in the performance analysis of certain kinds (or components) of computer systems. A single server may be switched to different modes of operation depending on the occurrence of certain events, the arrival and service rates of customers may depend on the state of the system, e.g. on the operation mode of the server and on the number of customers in the system (customers in service and waiting for service). It is of much interest to consider models which allow representations with state-dependent parameters and aids to determine (or control) important performance criterions.
In particular, we consider a model of a file-oriented (or database) transactional system supported with checkpointing and rollback recovery strategies. The system is assumed to have a finite waiting room.
A checkpoint is an operation which is performed at consecutive time stages, during which a copy of the relevant system files is saved in a secondary storage device. Checkpointing is a common technique to restore the integrity of information in critical database applications subject to information destructive failures and to enhance the reliability of the system operation for serving the users.

In the following we describe the system operation (assumptions concerning the mathematical model will follow in the next chapter).
The system can be operating in one of three modes, labelled as a, c and r.

Mode 'a' (available):

In this mode the system is available for processing transactions (by a transaction we mean one or more tasks generated at the same time by a single user to be executed by the computer system). These transactions arrive at a rate depending on the state of the system (i.e. the number of transactions in the system). This dependency exists, for instance, in systems with a limited number of users or in cases of discouraged arrivals.

Transactions are processed at a rate depending on the state of the system; such a dependency exists in multiprocessing environments. Checkpoints are performed at predefined time instants (according to a checkpointing strategy) during mode 'a' of operation. When checkpoints are performed, transitions to mode 'c' of operation take place.
A state-dependent checkpointing rate is a realistic requirement since it is preferable to perform a checkpoint when the system is lightly loaded. Failures (due to hardware, software, ... etc.) may occur during mode 'a' of operation. When a failure is detected a recovery action is initiated and a transition to mode 'r' of operation takes place. In certain circumstances failures may increase with the number of transactions in the system and thus the failure rate may be state-dependent. Transactions which have caused modifications in the system files since the last checkpoint, are recorded in a file called an "audit trail".

Mode 'c' (checkpointing):

In this mode, transaction processing is blocked and a valid non-erroneous copy of the relevant system files (files and information needed to restore the system to its state just before the initiation of the checkpoint) is saved in a secondary storage device. Transactions keep arriving at the system at a state-dependent rate. The checkpoint duration may increase with the load on the system and thus it may be state-dependent. When a checkpoint operation is completed a transition to mode 'a' takes place and the system becomes available for transaction processing.

Mode 'r' (recovery):

Transition from mode 'a' to mode 'r' occurs with the initiation of a recovery action after the detection of a failure. Transaction processing is blocked during recovery and a valid copy of the relevant system files (which was saved at the most recent checkpoint) is loaded into primary storage. This restores the system files to their status just before the initiation of the most recent checkpoint. The modifying transactions which were recorded in the audit trail (after being processed) since the last checkpoint, are reprocessed. The recovery action is completed when reprocesing reaches the point at which the failure

occurred. With the completion of a recovery action a transition to mode 'a' takes place and the system resumes useful processing of transactions. It is obvious that the duration of a recovery action depends on the amount of modifying processing in the time interval between the instant of failure occurrence and the last checkpoint. This implies the dependency of the mean duration of a recovery action on the mean time interval between successive checkpoints. Transactions keep arriving at the system at a state-dependent rate during recovery actions.
Obviously, the shorter the mean time interval between successive checkpoints, the more time spent by the system in performing checkpoints, and, similarly, the longer the mean time interval between successive checkpoints, the more time spent by the system in recovery actions after failures. Thus there is an optimum strategy for determining the time intervals between successive checkpoints which minimizes the time spent by the system in checkpointing and recoveries after failures (or maximizes the available time for transactions processing).

The determination of the optimum time interval between checkpoints has been considered previously in several papers [3,4,5,6,7,9].
Young and Chandy [9,3] considered models of checkpointing and rollback-recovery in which the queueing and the backlog of transactions are not taken into account. They determined an optimum constant value for the time between checkpoints which maximizes the system availabilty.
Gelenbé et al. [5] introduced a stochastic model in which the queueing and the backlog of transactions are taken into account. They assumed an exponential distribution for the available time between checkpoints. They obtained analytic expressions for the system availability and for the mean response time of a transaction and considered their optimization with respect to the mean available time between checkpoints. In [6] Gelenbé assumed a general distribution for the available time between checkpoints. He showed that the optimum checkpoint interval (which maximizes the system availability) must be deterministic and obtained an explicit expression for its value which is a function of the system load.
Bacelli [1] continued the work of Gelenbé to derive useful relations for the numerical computation of the average number of transactions in the system under general assumptions concerning the available time

between checkpoints and the checkpointing duration, with the restrictive assumption of constant recovery periods. In [2] Bacelli considered queueing analysis of an M/G/1 system subject to Poisson breakdowns of exponential duration, with an application to the modelling of checkpointing and recovery in database systems.

In this paper an M/M/1/N system subject to Poisson breakdowns of exponential durations is considered as a model of a transactional database system, supported with checkpointing and recovery strategies (as described earlier).
The state-transition parameters depend on the number of transactions in the system (for state-independent transition parameters and infinite waiting room [$N \to \infty$], this model is equivalent to the model in [5]).

We present algorithms for the computation of the state probabilities (and the performance variables) as well as their sensitivities with respect to the model parameters (the sensitivities are employed in the numerical optimization of the performance variables). In the case of state-independent parameters we demonstrate a state-space approach (as an alternative to the generating function approach) to derive analytic expressions for the performance variables (they agree with Gelenbé's results [5] as $N \to \infty$).
The maximization of the system availability yields an expression for the optimum checkpointing rate as a function of the system load.
The minimization of the mean response time of a transaction yields a different optimum for the checkpointing rate. The relation between the two optima is discussed in some detail.

In chapter 2 we introduce the mathematical model and the underlying assumptions, together with some notations and definitions. Sections 3.1 and 3.2 are devoted to the presentation of algorithms for the recursive computation of the state probabilities and their sensitivities with respect to the state-transition parameters. The numerical optimization of the performance variables is considered in section 3.3. Chapter 4 is devoted to the analysis of the model in the case of state-independent transition parameters. Analytic expressions for the performance variables are derived in section 4.1. Analytic optimization of the performance variables is considered in section 4.2.

2. The model:

In this chapter we introduce a mathematical model (and the underlying assumptions) of the system described in chapter 1. We also introduce some notations and definitions which will be used in the following chapters.

The system is modelled as an M/M/1/N system, subject to two different types of interrupts (checkpoints and failures). The following assumptions will be made in the model analysis:

i) Transaction requests arrive according to a Poisson process at a state-dependent rate λ_i, i ($0 \leq i \leq N$) is an index to indicate the number of transactions present in the system. They require processing time which is exponentially distributed with a state-dependent mean μ_i^{-1}. Transaction processing is blocked during an interrupt and is resumed at the end of an interrupt.

ii) Checkpoints occur according to a Poisson process at a state-dependent rate α_i (thus α_i^{-1} is the mean "available" time between checkpoints with i transactions present in the system). Checkpointing periods are exponentially distributed with a state-dependent mean β_i^{-1}.

iii) Failures occur according to a Poisson process at a state-dependent rate γ_i (thus γ_i^{-1} is the mean "available" time between failures with i transactions present in the system). It is assumed that the detection of a failure coincides with its occurrence. Recovery periods are exponentially distributed with a state-dependent mean ϕ_i^{-1} (the ϕ_i's depend on the α_i's; this dependence will be considered when performance optimization is discussed).

Figure (2.1) shows the state transition diagram of the considered model. The following are some basic notations and definitions related to the model.

The index "m" (m = a,c,r) indicates the mode of the system operation (as described in chapter 1), "a" stands for the available mode, "c" stands for the checkpointing mode and "r" stands for the recovery mode.

Let p(m,i), m = a,c or r and $0 \leq i \leq N$, be the probability that the system is operating in mode m with i transactions present in the

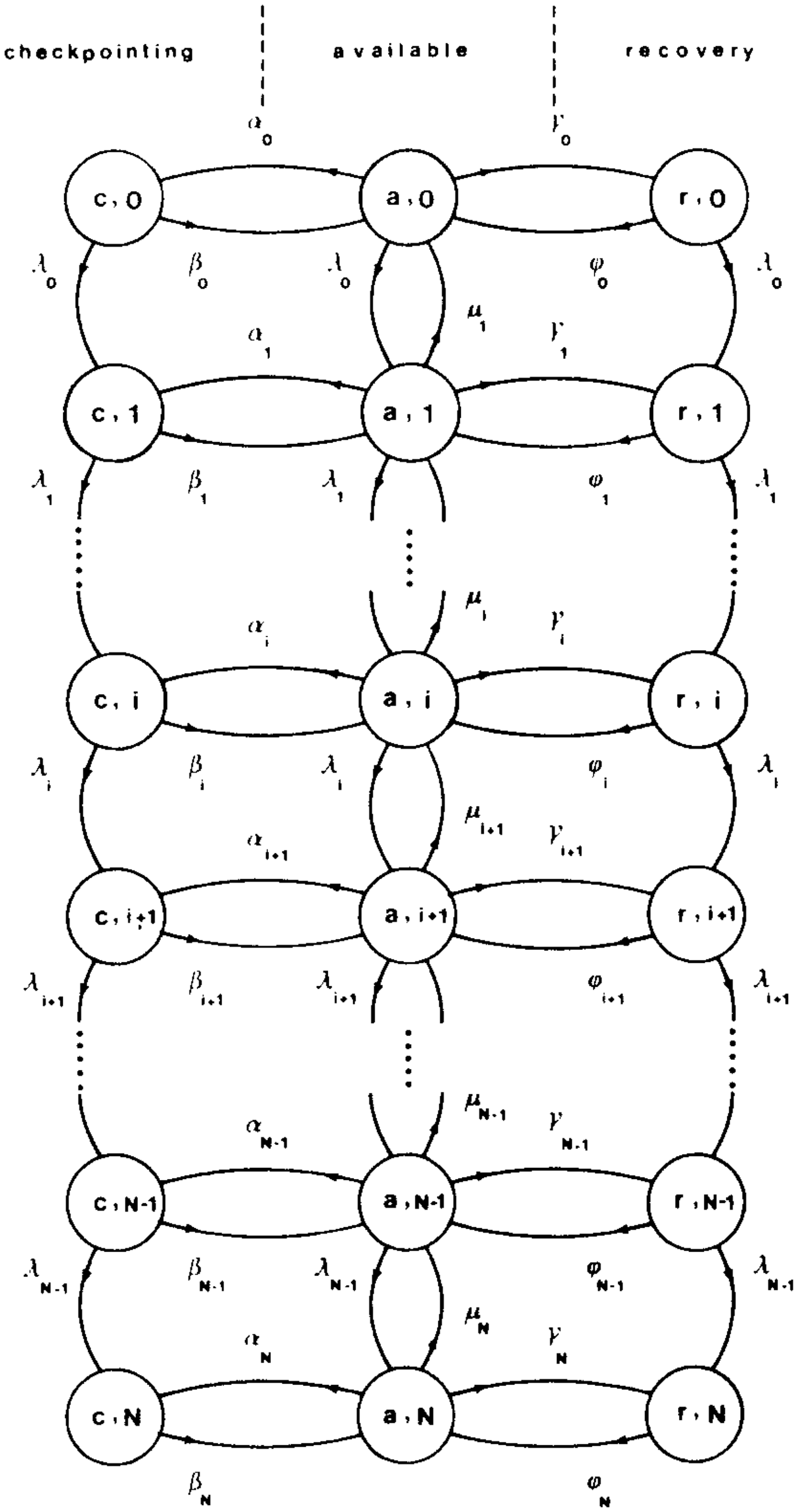

Fig.(2.1) State transition diagram of a finite continuous-time Markov chain representing the system considered system. Define the following probabilities:

(2.1) $$p(i) \triangleq \sum_m p(m,i), \quad m = a,c \text{ and } r, \quad 0 \leq i \leq N ,$$

$p(i)$ is the probability that i transactions are present in the system.

(2.2) $$A_m \triangleq \sum_i p(m,i), \quad i = 0,1,\ldots,N, \quad m = a,c \text{ or } r ,$$

A_m is the probability that the system is operating in mode m.

(2.3) $$g(m,i) \triangleq \frac{p(m,i)}{p(a,0)} , \quad m = a,c \text{ or } r, \quad 0 \leq i \leq N$$

(2.4) $$g(i) \triangleq \frac{p(i)}{p(a,0)} = \sum_m g(m,i), \quad m = a,c \text{ and } r, \quad 0 \leq i \leq N$$

The $g(m,i)$'s and $g(i)$'s, $m = a,c$ or r, $0 \leq i \leq N$, are scaled probabilities (with a factor $(p(a,0))^{-1}$).

Define the following vectors:

$$\underline{P}_m \triangleq \left[p(m,0),\ldots,p(m,i),\ldots,p(m,N)\right]^T, \quad m = a,c \text{ or } r$$

$$\underline{G}_m \triangleq \left[g(m,0),\ldots,g(m,i),\ldots,g(m,N)\right]^T, \quad m = a,c \text{ or } r$$

$$\underline{P} \triangleq \left[p(0),\ldots,p(i),\ldots,p(N)\right]^T$$

$$= \sum_m \underline{P}_m \quad , \quad m = a,c \text{ and } r$$

$$\underline{G} \triangleq \left[g(0),\ldots,g(i),\ldots,g(N)\right]^T$$

$$= \sum_m \underline{G}_m \quad , \quad m = a,c \text{ and } r$$

It follows that

(2.5) $$\underline{P}_m = p(a,0)\, \underline{G}_m \quad ,$$

and

(2.6) $$\underline{P} = p(a,0)\, \underline{G}$$

3. Computational aspects:

In the case of state-dependent transition parameters, the limiting state-probabilities can only be determined by numerical means. Fortunately, for the model we introduced in chapter 2, it is possible to develop recursive schemes for the computation of the limiting state probabilities. These schemes will be developed in section 3.1. In section 3.2 we show that the partial derivatives (or the sensitivities) of the limiting state probabilities, with respect to the transition parameters, can be computed in a similar fashion to the computation of of the limiting state probabilities. Section 3.3 is devoted to the numerical optimization of the performance variables.

3.1 Recursive computation of the limiting state probabilities

The limiting state probabilities of the the continuous-time Markov chain in fig. (2.1) can, in general, be determined using the transition balance equations at each of the 3(N+1) states. These equations contain 3N+2 independent euqations, together with the normalizing condition

$$(3.1) \qquad \sum_{m,i} p(m,i) = 1 \quad , \quad m = a,c \text{ and } r, \quad i = 0,1,\ldots,N$$

they form a set of linear equations which can be solved for the 3(N+1) unknown state probabilities. Due to the model structure we are able to determine the state probabilities recursively in terms of the state probability p(a,0). Then p(a,0) can be determined from the condition (3.1).

Transition balance at state (c,0) yields

$$(3.2) \qquad p(c,0) = \left(\frac{\alpha_0}{\lambda_0+\beta_0}\right) p(a,0)$$

Transition balance at state (r,0) yields

$$(3.3) \qquad p(r,0) = \left(\frac{\gamma_0}{\lambda_0+\phi_0}\right) p(a,0)$$

It follows from definition (2.1) that

$$(3.4) \qquad p(0) = \left(1 + \frac{\alpha_0}{\lambda_0+\beta_0} + \frac{\gamma_0}{\lambda_0+\phi_0}\right) p(a,0)$$

Transition balance between the i-th and the (i-1)-th set of states yields

$$(3.5) \qquad p(a,i) = \rho_i \; p(i-1) \qquad , \qquad 1 \leq i \leq N$$

with

$$\rho_i \triangleq \frac{\lambda_{i-1}}{\mu_i} \qquad , \qquad 1 \leq i \leq N$$

Transition balance at state (c,i) yields

$$(3.6) \qquad p(c,i) = \left(\frac{\alpha_i}{\lambda_i+\beta_i}\right) p(a,i) + \left(\frac{\lambda_{i-1}}{\lambda_i+\beta_i}\right) p(c,i-1), \quad 1 \leq i \leq N$$

Transition balance at state (r,i) yields

$$(3.7) \qquad p(r,i) = \left(\frac{\gamma_i}{\lambda_i+\phi_i}\right) p(a,i) + \left(\frac{\lambda_{i-1}}{\lambda_i+\phi_i}\right) p(r,i-1), \quad 1 \leq i \leq N$$

It follows that

$$(3.8) \qquad p(i) = \left(1 + \frac{\alpha_i}{\lambda_i+\beta_i} + \frac{\gamma_i}{\lambda_i+\phi_i}\right) p(a,i)$$

$$+ \left(\frac{\lambda_{i-1}}{\lambda_i+\beta_i}\right) + p(c,i-1) + \left(\frac{\lambda_{i-1}}{\lambda_i+\phi_i}\right) p(r,i-1), \quad 1 \leq i \leq N$$

with $\lambda_N = 0$.

The state probabilities p(c,i), p(r,i) and p(i), $0 \leq i \leq N$, can be expressed in terms of all p(a,j), $j \leq i$, as follows

$$p(c,i) = \sum_{j=0}^{i} \left(\prod_{k=j}^{i-1} \theta_{k+1} \lambda_k\right) \theta_i \; \alpha_j \; p(a,j) \quad ,$$

$$p(r,i) = \sum_{j=0}^{i} \left(\prod_{k=j}^{i-1} \psi_{k+1} \lambda_k\right) \psi_j \; \gamma_j \; p(a,j) \quad ,$$

$$p(i) = p(a,i) + \sum_{j=0}^{i} \left(\left(\prod_{k=j}^{i-1} \theta_{k+1}\lambda_k \right) \theta_j\alpha_j + \left(\prod_{k=j}^{i-1} \psi_{k+1}\lambda_k \right)\psi_j\gamma_j \right) p(a,j)$$

with

$$\theta_i \triangleq \frac{1}{\lambda_i+\beta_i} \quad , \quad 0 \le i \le N$$

$$\psi_i \triangleq \frac{1}{\lambda_i+\phi_i} \quad , \quad 0 \le i \le N$$

(Note that $\lambda_N = 0$ and $\left(\prod_{k=i}^{i-1} \ldots \right) = 1$, in the above equations).

In a vector-matrix form we can write (using the definitions of chapter 2)

(3.9) $\quad \underline{P}_c = \Theta D_\alpha \underline{P}_a$

where Θ is a triangular matrix with elements $\Theta\{i,j\}$, $0 \le i, j \le N$,

$$\Theta\{i,j\} = \begin{cases} 1 & , \text{ for } i = j \\ \prod_{k=j}^{i-1} \theta_{k+1}\lambda_k & , \text{ for } i > j \end{cases}$$

and D_α is a diagonal matrix with elements $D_\alpha\{i,j\}$, $0 \le i,j \le N$,

$$D_\alpha\{i,i\} = \theta_i\alpha_i \quad , \quad 0 \le i \le N$$

Similarly,

(3.10) $\quad \underline{P}_r = \Psi D_\gamma \underline{P}_a$

where Ψ is a triangular matrix with elements $\Psi\{i,j\}$, $0 \le i,j \le N$,

$$\Psi\{i,j\} = \begin{cases} 1 & , \text{ for } i = j \\ \prod_{k=j}^{i-1} \psi_{k+1}\lambda_k & , \text{ for } i > j \end{cases}$$

and D_γ is a diagonal matrix with elements $D_\gamma\{i,j\}$, $0 \le i,j \le N$,

$$D_\gamma\{i,j\} = \psi_i\gamma_i \quad , \quad 0 \le i \le N$$

It follows that

(3.11) $\quad \underline{P} = \underline{P}_a + \underline{P}_c + \underline{P}_r$

$$= (I + \Theta D_\alpha + \Psi D_\gamma)\, \underline{P}_a$$

where I is the identity matrix.

If we employ the relation given by equation (3.5) then we can rewrite equation (3.11) in the following form

$$(3.12) \qquad \begin{bmatrix} p(0) \\ p(1) \\ \vdots \\ p(N) \end{bmatrix} = Q \begin{bmatrix} p(a,0) \\ p(0) \\ \vdots \\ p(N-1) \end{bmatrix}$$

with $Q \triangleq (I + \Theta D_\alpha + \Psi D_\gamma) D_\rho$,

and D_ρ is a diagonal matrix with elements $D_\rho\{i,j\}$, $0 \leq i,j \leq N$,

$$D_\rho\{i,i\} = \begin{cases} 1 & , \text{ for } i = 0 \\ \rho_i = \dfrac{\lambda_{i-1}}{\mu_i} & , \text{ for } 1 \leq i \leq N \end{cases}$$

Q is a triangular matrix and thus the system of equations (3.12) can be solved recursively to obtain all state probabilities $p(i)$, $0 \leq i \leq N$, in terms of the state probability $p(a,0)$.

If $p(a,0)$ is made equal to one in (3.12), then the recursive solution of the system equations yields values for $g(i)$, $0 \leq i \leq N$ ($g(i)$ is defined in (2.4)), which, if substituted in the normalizing condition $\sum_{i=0}^{N} p(i) = 1$ yields a value for $p(a,0)$

$$(3.13) \qquad p(a,0) = \Big(\sum_{i=0}^{N} g(i) \Big)^{-1}$$

The values of the state probabilities $p(i)$, $0 \leq i \leq N$, immediately follow

$$(3.14) \qquad p(i) = g(i) \Big(\sum_{i=0}^{N} g(i) \Big)^{-1} \quad , \quad 0 \leq i \leq N$$

Figure (3.1) shows the recursive scheme for the computation of $g(i)$, $0 \leq i \leq N$.

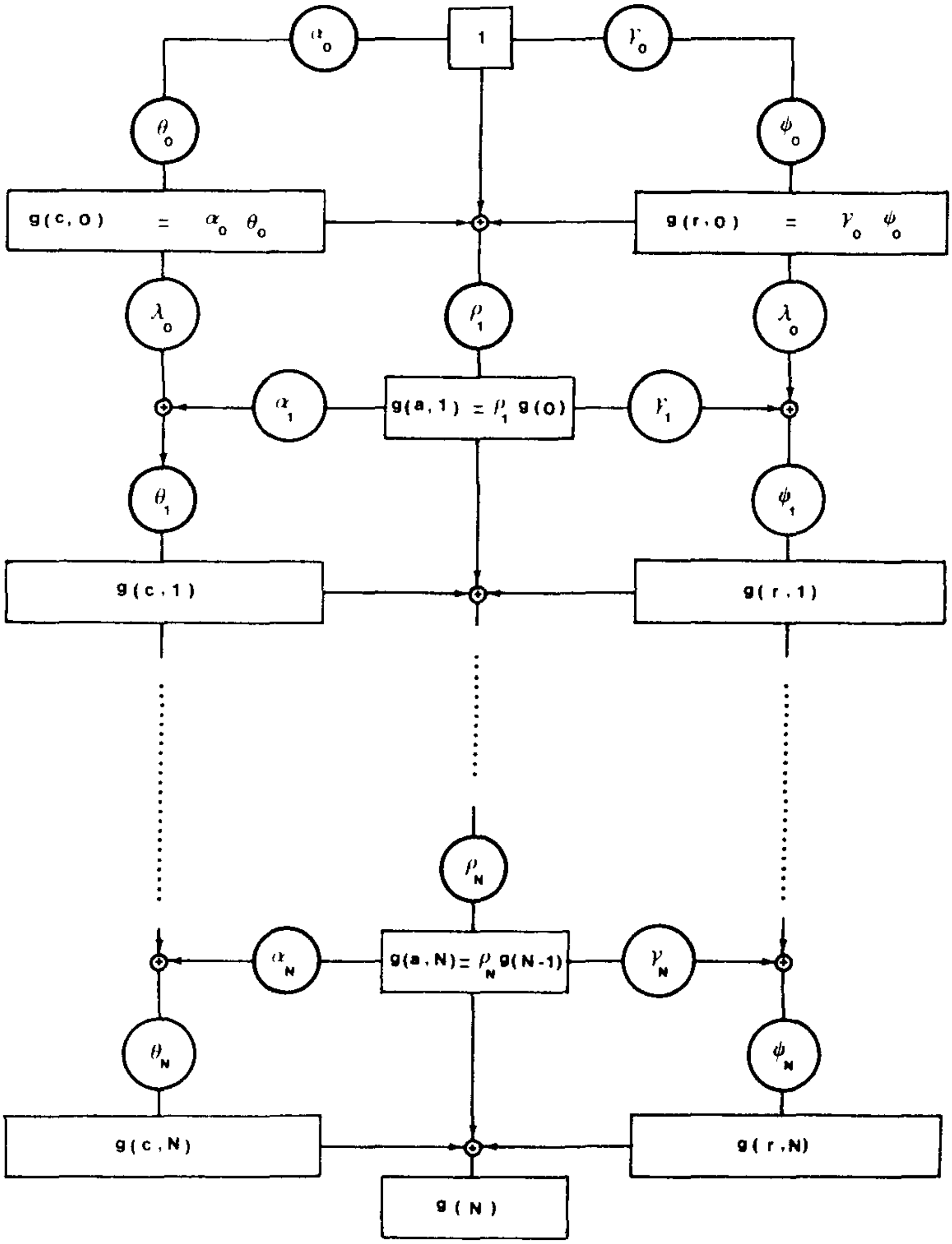

Fig.(3.1) Recursive computation of the
state probabilities

3.2 Recursive compuation of the sensitivities of the limiting state probabilities with respect to the transition parameters

It is of much interest to determine the effect of varying the transition parameters on the limiting state probabilities. This will allow numerical optimization of the performance variables with respect to the transition parameters under control.

For the specific case considered in this paper, we are interested in the values of α_j, $0 \leq j \leq N$, which optimize some performance criterion; this will require the determination of the partial derivatives and the sensitivities of the limiting state probabilities, with respect to the parameters α_j, $0 \leq j \leq N$, as well as the partial derivatives, with respect to the parameters ϕ_j, $0 \leq j \leq N$ (since the ϕ_j's depend on the α_j's in our specific model).

In the following, we derive some important relations to proceed with the determination of the partial derivatives and the sensitivities.

Differentiating equation (3.14) with respect to α_j and ϕ_j and making use of equation (3.13) yields

$$(3.15) \qquad \frac{\partial}{\partial \alpha_j} p(i) = p(a,0) \frac{\partial}{\partial \alpha_j} g(i) - (p(a,0))^2 g(i) \frac{\partial}{\partial \alpha_j} \sum_{\ell,k}^{k \geq j} g(\ell,k),$$

$$0 \leq i \leq N \quad , \quad 0 \leq j \leq N \quad ,$$

$$(3.16) \qquad \frac{\partial}{\partial \phi_j} p(i) = p(a,0) \frac{\partial}{\partial \phi_j} g(i) - (p(a,0))^2 g(i) \frac{\partial}{\partial \phi_j} \sum_{\ell,k}^{k \geq j} g(\ell,k),$$

$$0 \leq i \leq N \quad , \quad 0 \leq j \leq N$$

Now, let

$$(3.17) \qquad \phi_q = \phi_q(\alpha_0, \alpha_1, \ldots, \alpha_N) \quad , \quad 0 \leq q \leq N$$

The sensitivities with respect to the parameters α_j, $0 \leq j \leq N$, can be determined a follows:

$$(3.18) \qquad \frac{d}{d\alpha_j} p(i) = \frac{\partial}{\partial \alpha_j} p(i) + \sum_{q=0}^{N} \frac{\partial}{\partial \phi_q} p(i) \cdot \frac{d\phi_q}{d\alpha_j} \quad ,$$

$$0 \leq i \leq N \quad , \quad 0 \leq j \leq N$$

For the evaluation of the partial derivatives $\frac{\partial}{\partial \alpha_j} p(i)$ and $\frac{\partial}{\partial \phi_j} p(i)$, $0 \leq i \leq N$, we need to determine the partial derivatives $\frac{\partial}{\partial \alpha_j} g(k)$ and $\frac{\partial}{\partial \phi_j} g(k)$, $k = j,\ldots,N$ (as shown by equations (3.15) and (3.16)).

In the remainder of this section we show that the partial derivatives $\frac{\partial}{\partial \alpha_j} g(k)$ and $\frac{\partial}{\partial \phi_j} g(k)$, $j \leq k \leq N$, $0 \leq j \leq N$, can be computed recursively in a similar fashion to the computation of the state probabilities.

<u>The partial derivatives with respect to α_j, $0 \leq j \leq N$:</u>

The partial derivatives $\frac{\partial}{\partial \alpha_j} g(k)$, $j \leq k \leq N$ (Note that $\frac{\partial}{\partial \alpha_j} g(k) = 0$, for $k < j$), can be determined recursively as will be shown in the following.

From equation (3.11) it follows that

(3.19) $\quad \underline{G} = (I + \Theta D_\alpha + \Psi D_\gamma)\, \underline{G}_a$

Differentiating equation (3.19) with respect to α_j yields

(3.20) $\quad \frac{\partial}{\partial \alpha_j} \underline{G} = (I + \Theta D_\alpha + \Psi D_\gamma) \frac{\partial}{\partial \alpha_j} \underline{G}_a + \Theta D_{\theta j} \underline{G}_a$

where $D_{\theta j}(= \frac{\partial}{\partial \alpha_j} D_\alpha)$ is a matrix with elements $D_{\theta j}\{\ell,k\}$, $0 \leq \ell,k \leq N$, and all elements are equal to zero except $D_{\theta j}\{j,j\} = \Theta_j$.

It follows from equation (3.5) and the definitions (2.3) and (2.4) that

(3.21) $\quad g(a,j) = \begin{cases} 1 & , \text{ for } j = 0 \\ \rho_j g(j-1) & , \text{ for } 1 \leq j \leq N \end{cases}$

Using equation (3.21), equation (3.20) can be written in the following form:

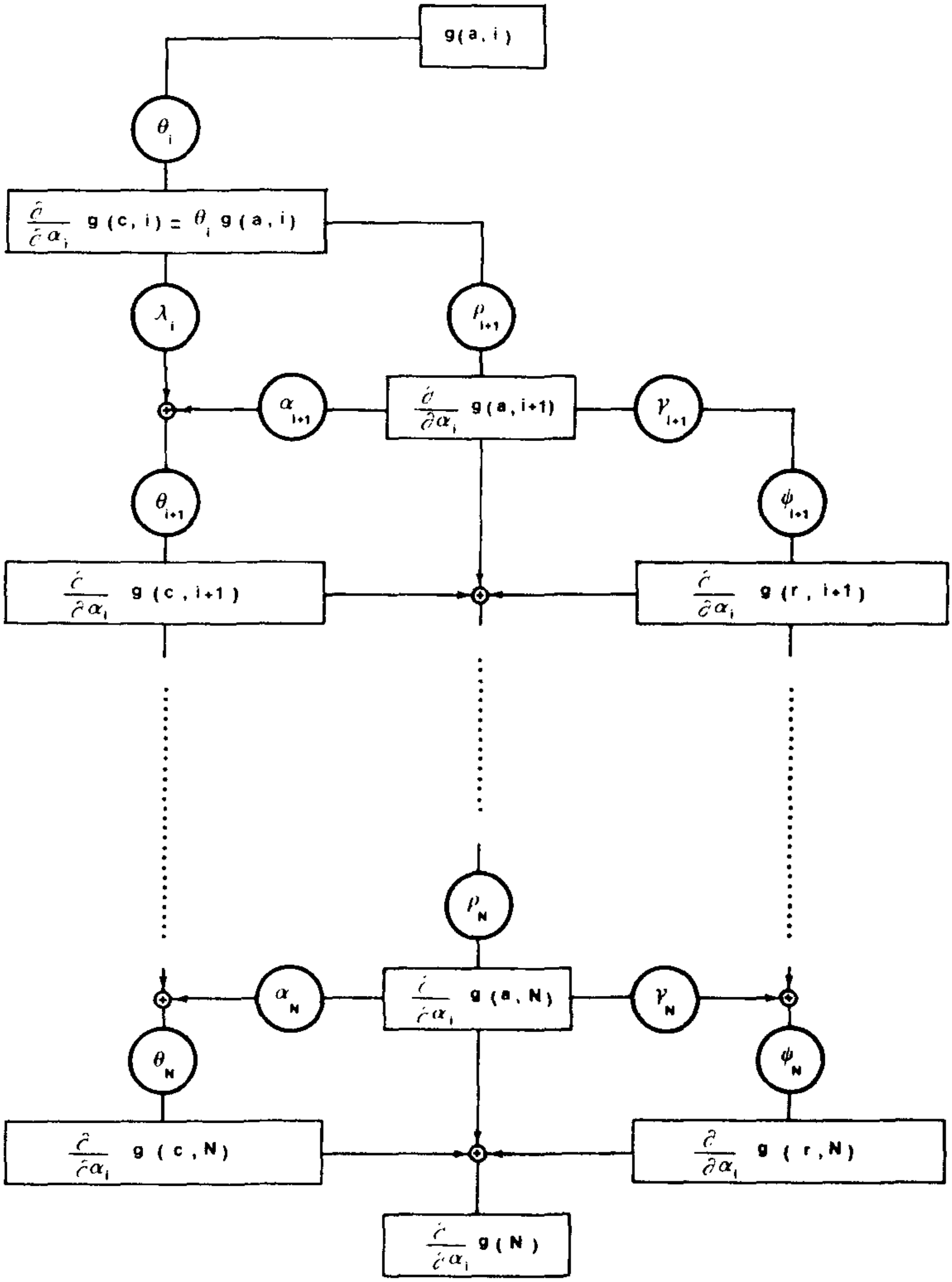

Fig.(3.2) Recursive computation of the sensitivities with respect to α_i

$$(3.22)\quad \frac{\partial}{\partial\alpha_j}\begin{bmatrix}0\\0\\g(i)\\g(j+1)\\ \cdot\\ \cdot\\ g(N)\end{bmatrix} = Q\cdot\frac{\partial}{\partial\alpha_j}\begin{bmatrix}0\\0\\0\\g(j)\\g(j+1)\\ \cdot\\ g(N-1)\end{bmatrix} + \Theta\begin{bmatrix}0\\0\\ \theta_j\rho_j g(j-1)\\0\\0\\0\\0\end{bmatrix}$$

where Q and Θ (as defined in equations (3.12) and (3.9)) are triangular matrices, and the system of equations (3.22) can be solved recursively to obtain $\frac{\partial}{\partial\alpha_j} g(k)$, $j \leq k \leq N$. Similar systems can be solved for $\frac{\partial}{\partial\alpha_j}\underline{G}$, $0 \leq j \leq N$. Figure (3.2) shows the recursive scheme for the computation of $\frac{\partial}{\partial\alpha_j} g(\ell,k)$, $\ell = a,c$ and r, $k = i,\ldots,N$.

The partial derivatives with respect to γ_j, $0 \leq j \leq N$ can be computed in exactly the same way.

The partial derivatives with respect to ϕ_j, $0 \leq j \leq N$:

The partial derivatives $\frac{\partial}{\partial\phi_j} g(k)$, $j \leq k \leq N$ (Note that $\frac{\partial}{\partial\phi_j} g(k) = 0$, for $k < j$), can be determined recursively as will be shown in the following.

Differentiating equation (3.19) with respect to ϕ_j yields

$$(3.23)\qquad \frac{\partial}{\partial\phi_j}\underline{G} = (I + \Theta D_\alpha + \Psi D_\gamma)\,\frac{\partial}{\partial\phi_j}\underline{G}_a + (S_{\psi j}D_\gamma + \Psi D_{\psi j})\,\underline{G}_a$$

where $S_{\psi j}(= \frac{\partial}{\partial\phi_j}\Psi)$ is a matrix with elements $S_{\psi j}\{\ell,k\}$, $0 \leq \ell,k \leq N$,

$$S_{\psi j}\{\ell,k\} = \begin{cases} -\psi_j\left(\prod_{q=k}^{\ell-1} \psi_{q+1}\lambda_q\right), & \text{for } j \leq \ell \leq N,\ 0 \leq k \leq j-1\\ 0, & \text{otherwise}\end{cases}$$

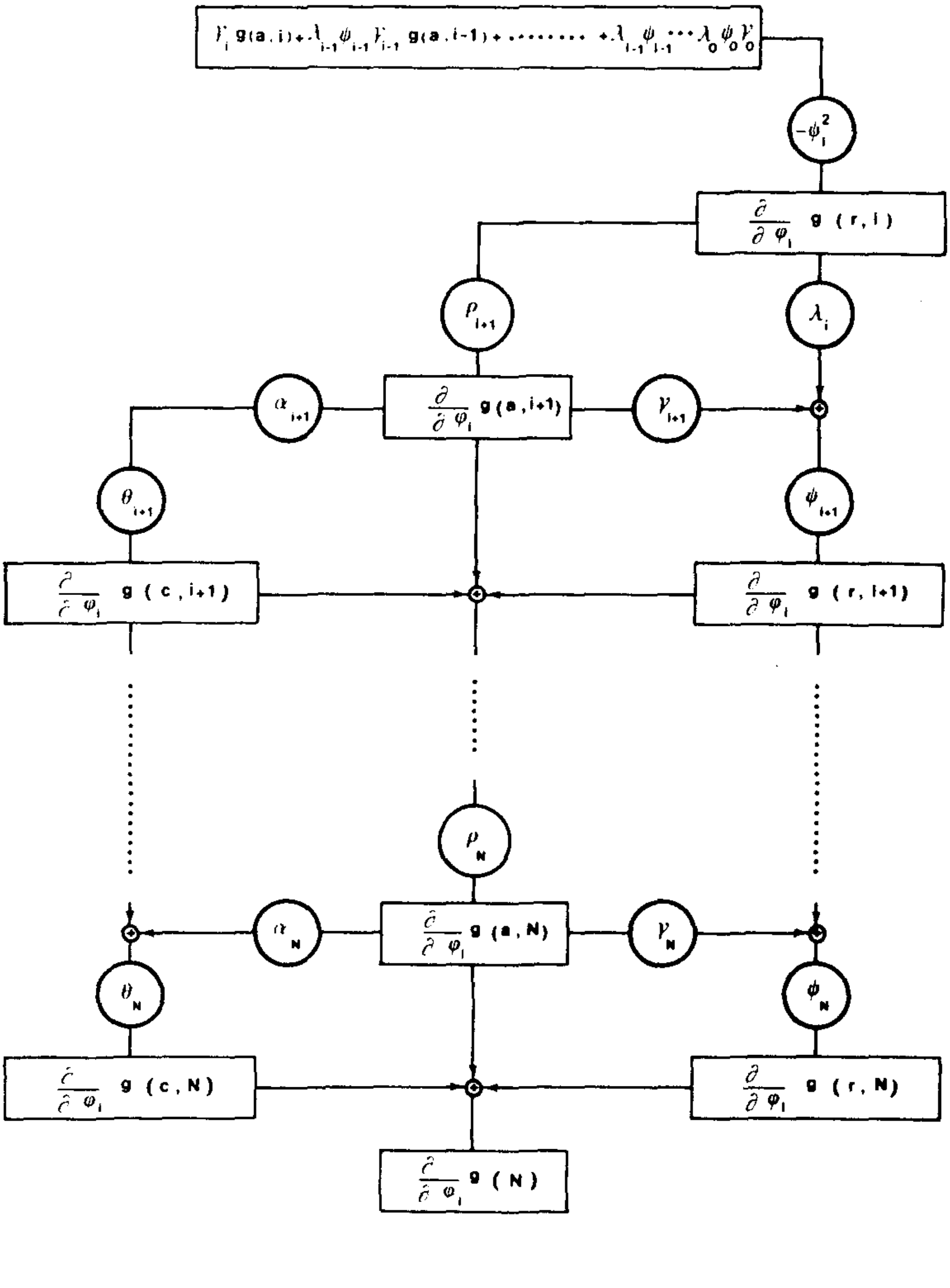

Fig.(3.3) Recursive computation of the sensitivities with respect to ϕ_i

and $D_{\psi j}(=\frac{\partial}{\partial\phi_j} D_\gamma)$ is a matrix with elements $D_{\psi j}\{\ell,k\}$, $0 \leq \ell,k \leq N$

all elements are equal to zero except $D_{\psi j}\{j,j\} = -\psi_j^2\ \gamma_j$.

Using equation (3.21), we can rewrite equation (2.23) in the following form:

$$(3.24)\quad \frac{\partial}{\partial\phi_j}\begin{bmatrix} 0 \\ 0 \\ g(j) \\ g(j+1) \\ \vdots \\ g(N) \end{bmatrix} = Q\cdot\frac{\partial}{\partial\phi_j}\begin{bmatrix} 0 \\ 0 \\ 0 \\ g(j) \\ g(j+1) \\ \vdots \\ g(N-1) \end{bmatrix} + Z_{\psi j}\begin{bmatrix} 0 \\ 0 \\ \rho_j g(j-1) \\ \rho_{j+1} g(j) \\ \vdots \\ \rho_N g(N-1) \end{bmatrix}$$

where Q is a triangular matrix (defined in (3.12)) and $Z_{\psi j}$ is a matrix with elements $Z_{\psi j}\{\ell,k\}$, $0 \leq \ell,k \leq N$,

$$Z_{\psi j}\{\ell,k\} = \begin{cases} -\psi_j\psi_k\gamma_k \left(\prod_{q=k}^{\ell-1} \psi_{q+1}\ \lambda_q\right), & \text{for } j \leq \ell \leq N,\ 0 \leq k \leq j \\ 0 & \text{, otherwise} \end{cases}$$

The system of equations (3.24) can be solved recursively to obtain $\frac{\partial}{\partial\phi_j} g(k)$, $j \leq k \leq N$. Similar systems can be solved for $\frac{\partial}{\partial\phi_j} \underline{G}$, $0 \leq j \leq N$. Figure (3.3) shows the recursive scheme for the computation of $\frac{\partial}{\partial\phi_j} f(\ell,k)$, ℓ = a,c and r, $k = 1,\ldots,N$.

The partial derivatives with respect to β_j, $0 \leq j \leq N$ can be computed in exactly the same way.

3.3 Numerical optimization

In this section we will consider the numerical optimization of two performance criterions; namely, the maximization of the system availability (A), given by

$$(3.25) \qquad A = \sum_{i=0}^{N} p(a,i)$$

and the minimization of the average number of transactions in the system ($\bar{N}$), given by

$$(3.26) \qquad \bar{N} = \sum_{i=1}^{N} i\ p(i)$$

We are interested in the values of the checkpointing rates α_0, $\alpha_1,\ldots,\alpha_N$ (or a subset of them) which optimize a chosen performance criterion. Due to the possibly complicated dependence of the parameters ϕ_i, $0 \leq i \leq N$, on the parameters α_i, $0 \leq i \leq N$, it is reasonable to employ numerical (iterative) optimization techniques. We start with an acceptable guess $\underline{\alpha}_0$ $(=[\alpha_{00},\alpha_{10},\ldots,\alpha_{N0}]^T)$ and generate a sequence $\underline{\alpha}_1$, $\underline{\alpha}_2$, ... which should converge to $\hat{\underline{\alpha}}$ at which the chosen criterion is optimized.

The model is adjusted to the new parameters $\underline{\alpha}_n$ after each iteration and the chosen criterion, as well as its derivatives with respect to $\underline{\alpha}_n$ are computed; they are used to perform the next iteration.

If A is to be maximized, then the (n+1)-th iteration is given by

$$(3.27) \qquad \underline{\alpha}_{n+1} = \underline{\alpha}_n + \Delta\underline{\alpha}_n$$

where, if a gradient method is used, then $\Delta\underline{\alpha}_n$ is given by

$$(3.28) \qquad \Delta\underline{\alpha}_n = \delta_n \left.\frac{dA}{d\underline{\alpha}}\right|_{\underline{\alpha} = \underline{\alpha}_n}$$

where δ_n is a scalar such that $A(\underline{\alpha}_{n+1}) > A(\underline{\alpha}_n)$, and

$$(3.29) \qquad \frac{dA}{d\underline{\alpha}} = \frac{\partial A}{\partial \underline{\alpha}} + \frac{d\underline{\phi}^T}{d\underline{\alpha}} \cdot \frac{\partial A}{\partial \underline{\phi}}$$

$$= \sum_{i=0}^{N} \left(\frac{\partial p(a,i)}{\partial \underline{\alpha}} + \frac{d\phi^T}{d\underline{\alpha}} \cdot \frac{\partial p(a,i)}{\partial \underline{\phi}}\right)$$

where $\frac{\partial p(a,i)}{\partial \underline{\alpha}}$ and $\frac{\partial p(a,i)}{\partial \underline{\phi}}$, $0 \leq i \leq N$, can be computed using the schemes presented in section 3.2 and the matrix $\frac{d\underline{\phi}^T}{d\underline{\alpha}}$ is determined from the dependency of the ϕ_i's on the α_i's.

The iterations are terminated according to a stopping criterion. For example, if the value of $\left| \frac{dA}{d\underline{\alpha}} \right|_{\underline{\alpha} = \underline{\alpha}_n}$ or $\left| \underline{\alpha}_n - \underline{\alpha}_{n-1} \right|$ approaches zero, then we accept $\underline{\alpha}_n$ as an approximation to $\hat{\underline{\alpha}}$ which maximizes A. If $\bar{N}$ is to be minimized, then $\Delta\underline{\alpha}_n$ is given by

$$(3.30) \qquad \Delta\underline{\alpha}_n = \delta_n \left. \frac{d\bar{N}}{d\underline{\alpha}} \right|_{\underline{\alpha} = \underline{\alpha}_n}$$

where δ_n is a scalar such that $\bar{N}(\underline{\alpha}_{n+1}) < \bar{N}(\underline{\alpha}_n)$, and

$$(3.31) \qquad \frac{d\bar{N}}{d\underline{\alpha}} = \frac{\partial\bar{N}}{\partial\underline{\alpha}} + \frac{d\underline{\phi}^T}{d\underline{\alpha}} \frac{\partial\bar{N}}{\partial\underline{\phi}}$$

$$= \sum_{i=0}^{N} i \left(\frac{\partial p(i)}{\partial\underline{\alpha}} + \frac{d\underline{\phi}^T}{d\underline{\alpha}} \cdot \frac{\partial p(i)}{\partial\underline{\phi}} \right)$$

The computation of the partial derivations, the iteration steps and the stopping criterion are carried out in a manner similar to maximizing A.

4. Analytical aspects:

In this chapter we consider the Markovian model presented in chapter 2. When the transition parameters are state-independent ($\alpha_i = \alpha$, $\beta_i = \beta$, $\gamma_i = \gamma$, $\phi_i = \phi$, $\lambda_i = \lambda$, and $\mu_i = \mu$, $0 \leq i \leq N$), it is possible to derive analytic expressions for important system performance quantities such as the ergodicity condition, the system availability and the average number of transaction in the system. A similar system with infinite waiting room was analysed by Gelenbé [5]. In section 4.1 we present a state-space analysis approach as an alternative to the generating function approach which is widely used in the analysis of queueing systems [8]. Section 4.2 is devoted to discussion on the analytic optimization of the performance quantities.

4.1 State-space analysis and performance variables

The considered model (shown in fig. (2.1)) is a continuous-time irreducible Markov chain. For a finite state-space (corresponding to a system with a finite waiting room), it can be shown that all states are ergodic [8], i.e. there exists a limiting stationary probability distribution for which all state probabilities have positive finite values. For an infinite state-space (corresponding to a system with an infinite waiting room), the system is ergodic if and only if the state probability $p(a,o) > 0$ [5].

Balance of downward transitions and upward transitions (fig. (2.1)) yields

(4.1) $\lambda(1-p(N)) = \mu(A-p(a,o))$

where $A \left(= \sum_{i=0}^{N} p(a,o)\right)$ is the system availability.

It follows that for a system with an infinite waiting room,

(4.2) $p(a,0) = A - \dfrac{\lambda}{\mu}$

since for a stable system, $p(N) \to 0$ as $N \to \infty$.

The system availability A can easily be derived as follows.
Transition balance at the states (c,i), $0 \leq i \leq N$ and at the states (r,i), $0 \leq i \leq N$, yield

(4.3) $$A_c = \frac{\alpha}{\beta} A$$

(4.4) $$A_r = \frac{\gamma}{\phi} A$$

with A_c and A_r as defined in (2.2).

But since $A + A_c + A_r = 1$, A follows immediately,

(4.5) $$A = (1 + \frac{\alpha}{\beta} + \frac{\gamma}{\phi})^{-1}$$

Note that A is independent of the system load and the size of the waiting room.
Now, using (4.2), we can write the ergodicity condition in terms of the system parameters,

(4.6) $$\lambda/\mu < (1 + \frac{\alpha}{\beta} + \frac{\gamma}{\phi})^{-1}$$

The average number of transactions in the system $\bar{N}$ $\left(= \sum_{i=1}^{N} i\ p(i)\right)$ will be derived by making use of the following definitions

(4.7) $$\bar{N}_a \triangleq \sum_{i=1}^{N} i\ p(a,i)$$

(4.8) $$\bar{N}_c \triangleq \sum_{i=1}^{N} i\ p(c,i)$$

(4.9) $$\bar{N}_r \triangleq \sum_{i=1}^{N} i\ p(r,i)$$

and, thus

(4.10) $$\bar{N} = \bar{N}_a + \bar{N}_c + \bar{N}_r$$

For convenience we rewrite the recursive relations (3.5), (3.6) and (3.7)

(4.11) $$\mu\ p(a,i) = \lambda\ p(i-1)\ , \quad 1 \leq i \leq N$$

$$(4.12) \qquad (\lambda + \beta)\ p(c,i) = \alpha\ p(a,i) + \lambda\ p(c,i-1)\ , \quad 1 \leq i \leq N-1,$$

with

$$\beta\ p(c,N) = \alpha\ p(a,N) + \lambda\ p(c,N-1)$$

$$(4.13) \qquad (\lambda + \phi)\ p(r,i) = \gamma\ p(a,i) + \lambda\ p(r,i-1)\ , \quad 1 \leq i \leq N-1$$

with

$$\lambda\ p(r,N) = \gamma\ p(a,N) + \lambda\ p(r,N-1)$$

Multiplication of equation (4.11) by i and summation for $i = 1,2,\ldots,N$, yields

$$(4.14) \qquad \bar{N}_a = \frac{\lambda}{\mu}\left((\bar{N}+1) - (N+1)\ p(N)\right)$$

Similarly, equations (4.12) and (4.13) yield

$$(4.15) \qquad \bar{N}_c = \frac{\alpha}{\beta}\bar{N}_a + \frac{\lambda}{\beta}\left(A_c - p(c,N)\right), \quad \text{and}$$

$$(4.16) \qquad \bar{N}_r = \frac{\gamma}{\phi}\bar{N}_a + \frac{\lambda}{\phi}\left(A_r - p(r,N)\right)$$

From equations (4.10), (4.14), (4.15) and (4.16) we obtain the following expression for $\bar{N}$

$$(4.17) \qquad \bar{N} = \frac{1}{(1-\frac{\lambda}{\mu A})}\left[\frac{\lambda}{\mu A}(1-(N+1)\ p(N)) + \frac{\lambda}{\beta}(A_c - p(c,N)) + \frac{\lambda}{\phi}(A_r - p(r,N))\right]$$

For a system with infinite waiting room $p(N) \to 0$ and $N\ p(N) \to 0$, thus equation (4.17) reduces to

$$(4.18) \qquad \bar{N} = \frac{1}{(1-\frac{\lambda}{\mu A})}\left[\frac{\lambda}{\mu A} + \lambda A\left(\frac{\alpha}{\beta^2} + \frac{\gamma}{\phi^2}\right)\right]$$

which is indentical to Gelenbé's result [5].

It can also be shown that $\bar{N}$ may be expressed as follows

$$(4.19) \qquad \bar{N} = A \frac{A-p(a,o)}{p(a,o)} + A_c \frac{A_c-p(c,o)}{p(c,o)} + A_r \frac{A_r-p(r,o)}{p(r,o)}$$

4.2 Analytic optimization

In this section we consider the analytic determination of the two optimum values of the checkpointing rate; $\hat{\alpha}_A$ which maximizes the system availability and $\hat{\alpha}_{\bar{N}}$ which minimizes the average number of transactions in the system. The two optimums are found to be different.

So far we have not considered the dependence of the mean recovery time (ϕ^{-1}) on the mean available time between checkpoints (α^{-1}). It can be proved [5] for Poisson failure occurrences and exponential available time between checkpoints (with mean α^{-1}), that the available time intervals between the failure occurrences and the most recent checkpoint are exponentially distributed (with mean α^{-1}). These time intervals are independent when the failure rate is much smaller than the checkpointing rate (i.e. $\gamma \ll \alpha$). Furthermore, we assume that the recovery time after a failure is equal to the available busy time between the failure occurrence and the most recent checkpoint. It follows, for a failure rate which is much smaller than the processing rate (i.e. $\gamma \ll \mu$) or for a heavily-loaded system, that the recovery time is proportional to the available time interval between the failure occurence and the most recent checkpoint. The above assumptions yield recovery periods which are independent and exponentially distributed with a mean (ϕ^{-1}) equal to the mean available busy time between checkpoints. The probability that the system is busy, given that it is available, is

$$\frac{A-p(a,o)}{A} = \frac{\lambda}{\mu A}$$

with A as given in equation (4.5), and thus

$$(4.20) \qquad \phi^{-1} = \frac{\lambda}{\alpha \mu A}$$

Now, we are able to use the analytic results of section 4.1 for the optimization of A or $\bar{N}$ with respect to α (the checkpointing rate). Substituting from (4.20) into (4.5), differentiating with respect to α and equating to zero yields $\hat{\alpha}_A$ for which A is maximum

$$(4.21) \qquad \hat{\alpha}_A = \left(\frac{\lambda\beta\gamma}{\mu\hat{A}}\right)^{\frac{1}{2}}$$

with

$$(4.22) \qquad \hat{A} = \left(1+ \frac{2\hat{\alpha}_A}{\beta}\right)^{-1}$$

With some manipulations we get the following expression for $\hat{\alpha}_A$,

$$(4.23) \qquad \hat{\alpha}_A = \beta\left((1+ \frac{\mu\beta}{\lambda\gamma})^{\frac{1}{2}} - 1 \right)^{-1}$$

For values of $\hat{A}$ close to 1, $\hat{\alpha}_A$ reduces to $(\frac{\lambda\beta\gamma}{\mu})^{\frac{1}{2}}$ which is analagous to the results obtained in earlier papers [3,5,9].

Differentiating equation (4.18) for $\bar{N}$ with respect to α and making use of equation (4.20) yields

$$(4.24) \qquad \frac{\partial\bar{N}}{\partial\alpha} = (A- \frac{\lambda}{\mu})^{-2} \Big\{\lambda A^2 (A- \frac{\lambda}{\mu}) (\frac{1}{\beta^2} - \frac{2\gamma}{\alpha\phi^2})$$

$$+ \left(\lambda A^2(A- \frac{2\lambda}{\mu}) (\frac{\alpha}{\beta^2} + \frac{\gamma}{\phi^2}) - \frac{\lambda}{\mu} A\right)\left(\frac{\gamma}{\alpha\phi} - \frac{1}{\beta}\right) \left(\frac{1}{A} - \frac{\gamma}{\phi}\right)^{-1}\Big\}$$

Equating (4.24) to zero yields an equation for $\hat{\alpha}_{\bar{N}}$ for which $\bar{N}$ is minimum. The analytical expression for $\hat{\alpha}_{\bar{N}}$ is quite tedious and numerical techniques should be employed to determine $\hat{\alpha}_{\bar{N}}$.

It is interesting to evaluate $\frac{\partial\bar{N}}{\partial\alpha}$ at $\hat{\alpha}_A$. From equations (4.21) to (4.24) we obtain:

$$(4.25) \qquad \frac{\partial\bar{N}}{\partial\alpha}\Big|_{\alpha=\hat{\alpha}_A} = (\hat{A}- \frac{\lambda}{\mu})^{-1} \left(\frac{\lambda\hat{A}^2\gamma}{\hat{\alpha}_A\phi^2} \left(\frac{\gamma}{\hat{\alpha}_A} - 2\right)\right)$$

It is obvious from equation (4.25) that $\hat{\alpha}_A$ which maximizes the system availability, does not, in general, yield a minimum for the average number of transactions in the system $\bar{N}$. There is a minimum for $\bar{N}$ at $\hat{\alpha}_A$ if the following condition is satisfied,

$$\left(\frac{\gamma}{\hat{\alpha}_A} - 2\right) = 0 \quad \text{(or equivalently, } \frac{4\lambda\beta}{\mu\gamma\hat{A}} = 1).$$

Substituting for $\hat{\alpha}_A$ fom equation (4.23) yields the condition

(4.26) $\beta = \gamma(\frac{\mu}{4\lambda} - 1)$

for which $\hat{\alpha}_{\bar{N}} = \hat{\alpha}_A$

Considerable simplification arises in the determination of $\hat{\alpha}_{\bar{N}}$ if $\frac{\partial^2\bar{N}}{\partial\alpha^2} >> \frac{\partial^2 A}{\partial\alpha^2}$ in the neighbourhood of $\hat{\alpha}_{\bar{N}}$, since then we may put $\frac{\partial A}{\partial\alpha} \cong 0$ in the equation for $\frac{\partial\bar{N}}{\partial\alpha}$.

It follows that

(4.27) $\frac{\partial\bar{N}}{\partial\alpha} \cong (A - \frac{\lambda}{\mu})^{-1} \left(\lambda A^2 (\frac{1}{\beta^2} - \frac{2\gamma}{\alpha\phi^2})\right)$

which yields an approximate value for $\hat{\alpha}_{\bar{N}}$, given by

(4.28) $\hat{\alpha}_{\bar{N}} = \left(2\gamma (\frac{\lambda\beta}{\mu A})^2\right)^{\frac{1}{3}}$

It is easy to show that

(4.29) $\frac{\hat{\alpha}_{\bar{N}}}{\hat{\alpha}_A} \cong (\frac{4\lambda\beta}{\mu A\gamma})^{\frac{1}{6}}$

which is equal to one if $4\lambda\beta = \mu\gamma A$.

Note that maximizing A yields a maximum for p(a,0) (since $\frac{\lambda}{\mu}$ is invariant in eqution (4.2)) which is a measure for the maximum additional load which can be added to the system (recall the ergodicity condition $\lambda < \mu A$). The maximum limit on the arrival rate of transactions at maximum availability is determined from the equality

(4.30) $\lambda_{max} = \mu \hat{A} (\lambda_{max})$

since $\hat{A}$ is a function of λ.

5. Conclusions

An M/M/1/N system subject to Poisson breakdowns of exponential duration is considered. In the case of state-dependent parameters, efficient numerical algorithms were presented for the computation of the state probabilities and their sensitivities with respect to the system parameters (they are used in the numerical optimization of performance variables). In the case of state-independent parameters, a state-space analysis approach was presented in order to derive analytic expressions for the system availability and the average queue length. The analysed system can be used to represent the operation of a transactional database system, subject to random failures and supported with checkpointing and rollback recovery strategies. This representation is valid under various assumptions such as the Poisson occurrences of arrivals and breakdowns, and the exponential time distribution of transaction serivce and checkpoint duration. Furthermore, it was necessary to assume a heavily-loaded situation and a failure rate which is much smaller than the checkpointing rate in order to agree with the exponential assumption of recovery times. The recovery periods are independent when the failure rate is much smaller than the checkpointing rate. The optimum value of the checkpointing rate which maximizes the system availability is determined, depending on the system load and found to be different from the value which minimizes the average number of transactions in the system.

Although the underlying assumptions may not all be realistic, the obtained results may agreeably fit in practical situations. It remains interesting to develop and analyse more realistic models.

Acknowledgement

It is a pleasure to thank Prof. ir. F.J. Kylstra for his constant support and useful comments. I am grateful to Dr. ir. J. van der Wal for the interest he has shown during several fruitful discussions.

References:

[1] Bacelli, F.
Analysis of a service facility with periodic checkpointing.
Acta Informatica, Vol. 15 (1981), p. 67-81.

[2] Bacelli, F. and T. Znati
Queueing algorithms with breakdowns in database modelling.
In: Performance '81: Proc. 8th Int. Symp. on Computer Performance Modelling, Measurement and Evaluation, Amsterdam, 4-6 Nov. 1981.
Ed. by F.J. Kylstra. Amsterdam: North-Holland, 1981. P. 213-231.

[3] Chandy, K.M., J.C. Browne, C.W. Dissly and W.R. Uhrig
Analytic models for rollback and recovery strategies in database systems. IEEE Trans. Software Eng., Vol. SE-1 (1975) p. 100-110.

[4] Chandy, K.M.
A survey of analytic models of rollback and recovery strategies.
Computer, vol. 8, No. 5 (May 1975), p. 40-47.

[5] Gelenbe, E. and D. Derochette.
Performance of rollback recovery systems under intermittent failures.
Commun. ACM, Vol. 21 (1978), p. 493-499.

[6] Gelenbe, E.
On the optimum checkpoint interval.
J. Assoc. Comp. Mach., Vol. 26 (1979), p. 259-270.

[7] Gelenbe, E.,
Model of information recovery using the method of multiple checkpoints.
Autom. & Remote Control, Vol. 40 (1979), p. 598-605.
Translated from Avtom. & Telemekh., No. 4 (April 1979), p. 142-151.

[8] Kleinrock, L.
Queueing systems. Vol. I: Theory. New York: Wiley, 1975.

[9] Young, J.W.
A first order approximation to the optimum checkpoint interval.
Commun. ACM, Vol. 17 (1974), p. 530-531.

EINDHOVEN UNIVERSITY OF TECHNOLOGY
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Reports: EUT Reports are a continuation of TH-Reports.

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123) Ponomarenko, M.F.
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125) Saranummi, N.
DETECTION OF TRENDS IN LONG TERM RECORDINGS OF CARDIOVASCULAR SIGNALS.
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Eindhoven University of Technology Research Reports (ISSN 0167-9708)

(127) Damen, A.A.H., P.M.J. Van den Hof and A.K. Hajdasiński
THE PAGE MATRIX: An excellent tool for noise filtering of Markov parameters, order testing and realization.
EUT Report 82-E-127. 1982. ISBN 90-6144-127-7

(128) Nicola, V.F.
MARKOVIAN MODELS OF A TRANSACTIONAL SYSTEM SUPPORTED BY CHECKPOINTING AND RECOVERY STRATEGIES. Part 1: A model with state-dependent parameters.
EUT Report 82-E-128. 1982. ISBN 90-6144-128-5

(129) Nicola, V.F.
MARKOVIAN MODELS OF A TRANSACTIONAL SYSTEM SUPPORTED BY CHECKPOINTING AND RECOVERY STRATEGIES. Part 2: A model with a specified number of completed transactions between checkpoints.
EUT Report 82-E-129. 1982. ISBN 90-6144-129-3