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# Index Profile Measurement of Asymmetrical Elliptical Preforms or Fibers

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*Abstract* An extension of the beam-deflection method to the case of elliptical preforms with eccentric core (asymmetrical elliptical preforms) is presented, which can be easily implemented on automatic measurement equipment.

## Introduction

The method of measuring the deflection function of a circular preform and determining the index profile using the Abel transformation was first proposed by Chu [1]. An extension to elliptical profiles, with one of the axes oriented orthogonally to the incoming beam, was given by Chu [2]. An extension to elliptical preforms with arbitrary orientation of the axes was proposed by Barrell and Pask [3]. An excellent review of index-profiling methods has been given by Stewart [4], a more extensive treatment by Marcuse [5].

In this paper the Abel-transformation method is extended to the case of an arbitrarily oriented elliptical preform in which the iso-

Received June 13, 1985
*Fiber and Integrated Optics*, Volume 6, Number 1
0146–8030/87/010055–00$02.00/0

index ellipses may have a unilateral center displacement. Although eccentric preforms are normally undesired, some eccentricity will always occur in practice. The present method offers an accurate means for the determination of residual eccentricity. The algorithm is suited for implementation on automatic measurement equipment.

## The Deflection Function for Arbitrarily Oriented Elliptical Profiles with Unilateral Center Displacement

The deflection function will be derived for preforms with elliptical isoindex contours with ellipticity $e^2 = 1 - b^2/a^2$, as sketched in Figure 1.

We assume that the isoindex contours have a center displacement $x = d(u)$ relative to the origin, where $d(u)$ is an arbitrary function of the ellipse parameter $u$:

$$u = \frac{1}{ab}\left[b^2(x')^2 + a^2(y')^2\right]^{1/2} \tag{1a}$$

in which

$$x' = [x - d(u)] \cos \gamma + y \sin \gamma \tag{1b}$$

and

$$y' = -[x - d(u)] \sin \gamma + y \cos \gamma \tag{1c}$$

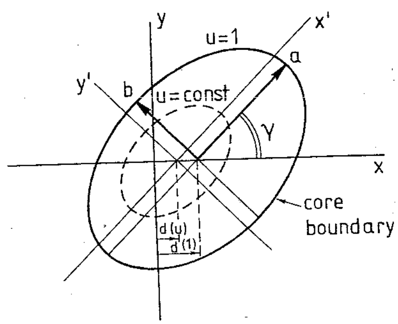


**FIGURE 1.** Preform or fiber geometry.

(For physical reasons $d(u)$ has to be such that its derivative $d/du$ is small enough to avoid intersection of the isoindex contours.)

The deflection of incident beams parallel to the $x$-axis (see Figure 2) can be calculated from

$$\tan[\phi(t)] = [dy/dx]_{x=x_2} = \int_{x_1}^{x_2} \frac{d^2y}{dx^2}\,dx \tag{2}$$

For small deflections $[\phi(t) \ll 1]\, d^2y/dx^2$ follows from the $y$-component of the ray equation:

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n \tag{3}$$

With the approximations $d/ds \approx d/dx$ and $n(d\mathbf{r}/ds) \approx n_{cl}(dy/dx)$, in which the refractive index of the cladding $n_{cl} = n(u \geqq 1)$, this $y$-component simplifies to

$$d^2y/dx^2 = \frac{1}{n_{cl}}\frac{\partial n}{\partial y} \tag{4}$$

which gives, upon substitution in equation 2,

$$\tan[\phi(t)] = \frac{1}{n_{cl}}\int_{x_1}^{x_2}\frac{\partial n}{\partial y}\,dx \tag{5}$$

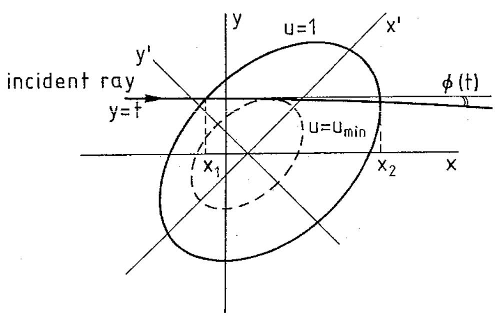


**FIGURE 2.** Ray deflection geometry.

Transformation of the integration over $x$ into an integration over $u$ yields $\left(\text{with } \frac{\partial n}{\partial y} = \frac{\partial n}{\partial u} \cdot \frac{\partial u}{\partial y} \text{ and } du = \frac{\partial u}{\partial x}\, dx\right)$:

$$\tan[\phi(t)] = \frac{1}{n_{cl}} \left[ \int_1^{u_{\min}} \frac{\partial n}{\partial u} \left( \frac{\partial u/\partial y}{\partial u/\partial x} \right)_- du + \int_{u_{\min}}^1 \frac{\partial n}{\partial u} \left( \frac{\partial u/\partial y}{\partial u/\partial x} \right)_+ du \right] \tag{6}$$

where $u_{\min}$ is the minimum value of $u$ which occurs on the path $y = t$:

$$u_{\min} = t/r_\gamma \tag{7}$$

and $r_\gamma$ is the projection of the core boundary on the $y$-axis:

$$r_\gamma = (b^2 \cos^2 \gamma + a^2 \sin^2 \gamma)^{1/2} \tag{8}$$

Using the identity $r_\gamma^2(a^2 \cos^2 \gamma + b^2 \sin^2 \gamma) = a^2b^2 + c^4$, the ellipse equation ($1a$) takes the following form in the $x$-$y$ coordinate system:

$$(abu)^2 = r_\gamma^2[x - d(u)]^2 + 2c^2[x - d(u)]y + (a^2b^2 + c^4)(y/r_\gamma)^2 \tag{9}$$

where

$$c^2 = (b^2 - a^2) \sin \gamma \cos \gamma$$

Partial differentiation of equation 9 with respect to $y$ and $x$ respectively yields:

$$\left( \frac{\partial u/\partial y}{\partial u/\partial x} \right)_\pm = \frac{c^2[x - d(u)] + (a^2b^2 + c^4)(y/r_\gamma^2)}{r_\gamma^2[x - d(u)] + c^2 y} = \frac{N_\pm}{D_\pm} \tag{10}$$

As a solution of equation 9 we find two roots $[x - d(u)]$. The largest is to the right of the point of contact with the ellipse $u = u_{\min}$ (Figure 2); the smallest is to the left. Substitution of these roots and $y = t = r_\gamma u_{\min}$ (equation 7) yields

$$D_\pm = \pm abr_\gamma(u^2 - u_{\min}^2)^{1/2} = \pm|D| \tag{11}$$

Substitution of equation 11 into equation 6 yields

$$\tan[\phi(t)] = \frac{1}{n_{cl}} \int_{u_{\min}}^{1} \frac{\partial n}{\partial u} \frac{N_- + N_+}{|D|} \tag{12}$$

By substituting

$$N_- + N_+ = 2(ab/r_\gamma)^2 t \tag{13}$$

into equation 12 we finally obtain

$$\begin{aligned} \tan[\phi(t)] &= \tan[\phi(r_\gamma u_{\min})] \\ &= 2\frac{u_{\min}}{n_{cl}} \cdot \frac{ab}{r_\gamma^2} \int_{u_{\min}}^{1} \frac{\partial n}{\partial u} \cdot \frac{1}{(u^2 - u_{\min}^2)^{1/2}}\, du \end{aligned} \tag{14}$$

which is independent of the center displacement $d(u)$. Equation 14 is the well-known Abel integral which has the following inversion:

$$n(u) - n_{cl} = -\frac{n_{cl}}{\pi} \cdot \frac{r_\gamma^2}{ab} \int_{u}^{1} \frac{\tan[\phi(r_\gamma u_{\min})]}{(u_{\min}^2 - u^2)^{1/2}}\, du_{\min} \tag{15}$$

Equations 14 and 15 hold only for center displacements parallel to the $x$-axis. This orientation can be found experimentally by rotating the preform such as to obtain an antisymmetrical deflection function.

## *Conclusion*

The index profile $n(u)$ of an elliptical preform with an arbitrary unilateral center displacement $d(u)$ is determined similarly to the index profile of a symmetrical preform, independent of the form of $d(u)$.

To compute the actual profile $n(x, y)$ from $n(u)$ we need to know the parameters $a$, $b$, and $\gamma$ and the function $d(u)$. The size of the core follows directly from the measured deflection function as the width of the non-zero-deflection region. The parameters $a$ and $b$ correspond to the half maximum and the half minimum of this width measured when the preform is rotated. The angle $\gamma$ is the difference between the angle at which the minimum width of the deflection

For small center displacements it will be reasonable to assume a linear dependence of $d$ on $u$:

$$d(u) = \delta f u \tag{19}$$

where $f$ is included for later convenience. Substitution of equation 19 and $t = 0$ into equation 16 yields

$$\tan\left[\phi_{\perp}(t = 0)\right] = 2\frac{ab}{f^2}\frac{n(0) - n_{cl}}{n_{cl}}\frac{\delta}{(1 - \delta^2)^{1/2}} \tag{20}$$

The projections of the core boundary onto the $t$-axis are $t = f(1 + \delta)$ and $t = -f(1 - \delta)$. Introducing $t' = t - \delta f$, these projections become $t' = f$ and $t' = -f$, and equation 20 becomes

$$\tan\left[\phi_{\perp}(t' = -\delta f)\right] = 2\frac{ab}{f^2}\frac{n(0) - n_{cl}}{n_{cl}}\frac{-t'}{[f^2 - (t')^2]^{1/2}} = g(t') \tag{21}$$

By plotting $\tan[\phi_{\perp}(t')]$ and $g(t')$ in the same figure, $\delta f$ is directly found as the value of $t'$ at the intersection of both curves (Figure 4), and $\delta$ follows from

$$\delta = -t'_{\text{intersect}}/f \tag{22}$$

## Experiments

The asymmetrical model has been tested on an extremely asymmetrical graded index preform with $\delta = 0.05$. If the geometry of

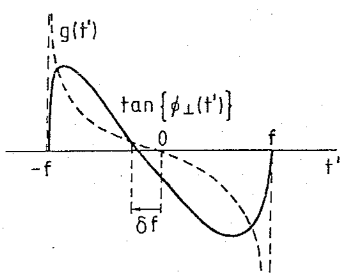


**FIGURE 4.** Determination of $\delta$ from the intersection of $\tan\left[\phi_{\perp}(t')\right]$ and $g(t')$.

function occurs, and the angle at which the deflection function becomes antisymmetrical. The last entity to be determined is the center displacement.

## The Determination of the Center Displacement

To determine the center displacement, the deflection function has to be measured with the preform rotated 90 degrees with respect to the orientation of Figure 2. The orientation and the deflection function, which will be called $\phi_\perp(t)$, are depicted in Figure 3.

Proceeding analogously to equations 5–14, we find

$$\tan[\phi_\perp(t)] = \frac{2}{n_{cl}} \frac{ab}{f^2} \int_{u_{\min\perp}}^{1} \frac{\partial n}{\partial u} \frac{t - d(u)}{[f^2u^2 - \{t - d(u)\}^2]^{1/2}} \, du \quad \text{for } -f + d(1) < t < f + d(1) \tag{16}$$

where

$$f = (b^2 \sin^2 \gamma + a^2 \cos^2 \gamma)^{1/2} \tag{17}$$

and $u_{\min_\perp}$ is related to $t$, $d(u)$, and $f$ according to

$$u_{\min_\perp} = \frac{t - d(u_{\min_\perp})}{f} \tag{18}$$

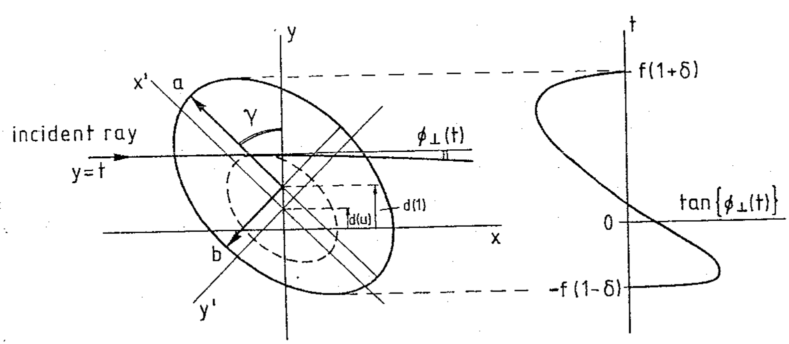


**FIGURE 3.** Deflection geometry and the corresponding deflection function $\tan[\phi_\perp(t)]$ for orthogonal preform orientation.

the isoindex contours is determined as described above, $n(u)$ can be computed from both halves of the deflection function $\phi_{\perp}(t)$ by computing the operator transforming a set of values $N(u)$ into a set of values $\Phi_{\perp}(t)$ and inverting it numerically [if $\Phi_{\perp}(t) = A \cdot N(u)$, then $N(u) = A^{-1} \cdot \Phi_{\perp}(t)$, where $A$ is the integral operator of equation 16].

Comparison of $N(u)$ computed in this way with $N(u)$ computed without correction for asymmetry showed that the differences between the profiles computed from both halves of the deflection function were reduced by 50%, the remaining differences probably being due to the imperfect ellipticity of the contours. The reduction demonstrates the potential of the model for reducing measurement inaccuracy.

## Acknowledgment

The authors wish to thank Professor G. A. Acket for his helpful suggestions.

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