The dynamics of a deformable body experiencing large displacements

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The Dynamics of a Deformable Body Experiencing Large Displacements

A general description of the dynamics of a deformable body experiencing large displacements is presented. These displacements are resolved into displacements due to deformation and displacements due to rigid body motion. The former are approximated with a linear combination of assumed displacement fields. D'Alembert's principle is used to derive the equations of motion. For this purpose, the rigid body displacements and the displacements due to deformation have to be independent. Commonly employed conditions for achieving this are reviewed. It is shown that some conditions lead to considerably simpler equations of motion and a sparser mass matrix, resulting in CPU time savings when used in a multibody program. This is illustrated with a uniform beam and a crank-slider mechanism.

Introduction

The dynamics of an individual deformable body constitute a building block for the dynamics of systems of deformable bodies. The dynamics of systems of deformable bodies have been studied by, among others, Sunada and Dubowsky (1981), Singh et al. (1984), Van der Werff and Jonker (1984), Agrawal and Shabana (1985), and Haug et al. (1986). In order to get tractable expressions for the elastic forces, they resolve the displacements into displacements due to deformation and displacements due to rigid body motion. They approximate the displacements due to deformation with a linear combination of assumed displacement fields.

In order to obtain a unique resolution of displacements, the displacements due to deformation are not allowed to represent a rigid body motion. This can be accomplished in various ways. Consequently, different options are available in the literature about the dynamics of systems of deformable bodies.

McDonough (1976) considered a resolution of displacements that has not found acceptance in literature, despite its potential merits such as the partial decoupling of rigid body motion from displacements due to deformation. This decoupling results in a simplification of the equations of motion and a more sparse mass matrix. Only Agrawal and Shabana (1985) have described this subdivision but they did not show its potential merits. It is the purpose of this paper to point out the merits of the resolution considered by McDonough (1976) and to apply it to analyzing the dynamics of systems of deformable bodies.

This paper considers the dynamics of an individual deformable continuous body. Using an optional resolution of displacements into displacements due to deformation and displacements due to rigid body motion, the equations of motion for the body are derived. Next, the two options that can be found in literature for bringing about a unique resolution of displacements are reviewed, namely, the option usually used in literature (Sunada and Dubowsky, 1981; Singh et al., 1984; Agrawal and Shabana, 1985; and Haug et al., 1986) and the option considered by McDonough (1976) and Agrawal and Shabana (1985). The displacements due to deformation are approximated with a linear combination of assumed displacement fields. It is shown that, using the resolution described by McDonough (1976), the equations of motion become more simple and the mass matrix becomes more sparse. This is illustrated with a description of the two-dimensional motion for a uniform beam. The method has been implemented in the multibody program DADS (CADSI, 1986) and is applied to a crank-slider mechanism.

The Kinematics of a Deformable Body

Consider a body $\mathcal{B}$ in its configuration G_t at time t (see Fig. 1). Let $\vec{x}$ be the position vector of an arbitrary material point P of the body, measured from an inertial point O.

Let G be a time-independent reference configuration of which all relevant quantities are known. Let $\vec{X}$ be the position vector of the point of G corresponding to P, measured from O. An invertible continuous transformation will exist mapping $\vec{X}$ on to $\vec{x}$, i.e., $\vec{x}=\vec{x}(\vec{X}, t)$.

The body in G_t can be considered as the result of a deformation of the body in G with displacement field $\vec{u}(\vec{X}, t)$, followed by a rigid body displacement defined by a translation vector $\vec{c}(t)$ and a rotation tensor $\mathbf{Q}(t)$

$$\vec{x}(\vec{X}, t)=\vec{c}(t)+\mathbf{Q}(t)\cdot\{\vec{X}+\vec{u}(\vec{X}, t)\}. \qquad (1)$$

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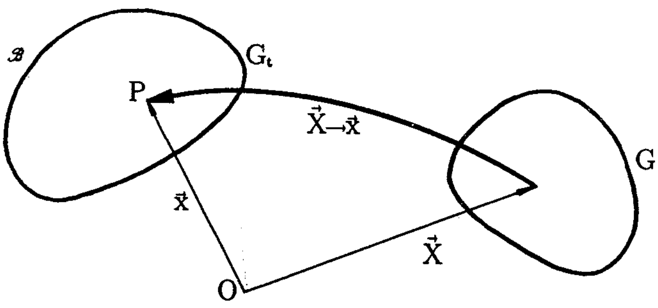

Fig. 1 Configurations G and G_t of body 𝓑

The vector $d\vec{x}$ between two neighboring points in G_t and the vector $d\vec{X}$ between the corresponding points in G are related by

$$d\vec{x}=\mathbf{Q}(t)\cdot\{d\vec{X}+\vec{u}(\vec{X}+d\vec{X},\ t)-\vec{u}(\vec{X},\ t)\}. \tag{2}$$

This equation can be rewritten as

$$d\vec{x}=\mathbf{F}\cdot d\vec{X}, \tag{3}$$

where the deformation tensor $\mathbf{F}$ is given by

$$\mathbf{F}=\mathbf{Q}\cdot\{\mathbf{I}+(\vec{\nabla}\vec{u})^c\}. \tag{4}$$

Here, $\vec{\nabla}$ is the gradient operator with reference to G, and $\mathbf{I}$ is the identity tensor. The superscript c denotes conjugation: the conjugate to a tensor $\mathbf{A}$ is the tensor $\mathbf{A}^c$ such that $\vec{v}\cdot\mathbf{A}\cdot\vec{w}=\vec{w}\cdot\mathbf{A}^c\cdot\vec{v},\ \forall\ \vec{v},\ \vec{w}$.

The acceleration and the virtual displacement of a material point are obtained by differentiating (1) twice with respect to time and by taking the variation of (1), respectively. This yields

$$\ddot{\vec{x}}=\ddot{\vec{c}}+\mathbf{Q}\cdot\{\vec{\omega}\times[\vec{\omega}\times(\vec{X}+\vec{u})]+\dot{\vec{\omega}}\times(\vec{X}+\vec{u})+2\vec{\omega}\times\dot{\vec{u}}+\ddot{\vec{u}}\}, \tag{5}$$

$$\delta\vec{x}=\delta\vec{c}+\mathbf{Q}\cdot\{\delta\vec{\pi}\times(\vec{X}+\vec{u})+\delta\vec{u}\}, \tag{6}$$

where $\vec{\omega}$ and $\delta\vec{\pi}$ are the axial vectors of the skew-symmetric tensors $\mathbf{Q}^c\cdot\dot{\mathbf{Q}}$ and $\mathbf{Q}^c\cdot\delta\mathbf{Q}$, respectively. The axial vector of a skew-symmetric tensor $\mathbf{S}$ is the vector $\vec{p}$, such that $\mathbf{S}\cdot\vec{v}=\vec{p}\times\vec{v}$, $\forall\vec{v}$.

The Equations of Motion

The equations of motion are given by (cf. Malvern, 1969)

$$\vec{\nabla}\cdot(\mathbf{T}\cdot\mathbf{F}^c)+\rho\vec{b}=\rho\ddot{\vec{x}}, \tag{7}$$

where $\mathbf{T}$ is the second Piola-Kirchhoff stress tensor, ρ is the mass density of G and $\vec{b}$ is a specific body force vector. Without losing the general validity of the conclusions of this paper, we may let $\vec{b}\equiv\vec{0}$.

Assume the body is stress-free in the reference configuration. Then, for isotropic linear elastic material behavior, $\mathbf{T}$ is related to the strain by (cf. Gurtin, 1981)

$$\mathbf{T}=2\mu\mathbf{E}+\lambda\ \mathrm{tr}(\mathbf{E})\mathbf{I}, \tag{8}$$

where μ and λ are the Lamé elastic constants, and $\mathbf{E}$ is the Green-Lagrange strain tensor, defined by

$$\mathbf{E}=\tfrac{1}{2}\{\mathbf{F}^c\cdot\mathbf{F}-\mathbf{I}\}. \tag{9}$$

An equivalent form of the equations of motion can be obtained with d'Alembert's principle. Multiplying (7) with a virtual displacement $\delta\vec{x}$ and integration over the reference volume Ω yields

$$\int_\Omega\{\vec{\nabla}\cdot(\mathbf{T}\cdot\mathbf{F}^c)-\rho\ddot{\vec{x}}\}\cdot\delta\vec{x}\ d\Omega=0. \tag{10}$$

Applying the divergence theorem, we arrive at

$$\int_\Omega\mathbf{T}:\delta\mathbf{E}\ d\Omega+\int_\Omega\rho\ddot{\vec{x}}\cdot\delta\vec{x}\ d\Omega=\int_\Gamma(\mathbf{F}\cdot\mathbf{T}\cdot\vec{n})\cdot\delta\vec{x}\ d\Gamma, \tag{11}$$

where Γ is the surface of G and $\vec{n}$ is the unit outward normal vector to Γ. The surface integral vanishes for that part of the surface where the displacements have been prescribed since there $\delta\vec{x}\equiv\vec{0}$. On the remaining part of the surface, a surface loading of $\vec{q}$ per unit of undeformed area is prescribed and $\mathbf{T}$ has to satisfy

$$\vec{q}=\mathbf{F}\cdot\mathbf{T}\cdot\vec{n}. \tag{12}$$

Without losing the general validity of the conclusions of this paper, we may let $\vec{q}\equiv\vec{0}$.

Substituting (8) into (11) yields

$$\int_\Omega\{2\mu\mathbf{E}+\lambda tr(\mathbf{E})\mathbf{I}\}:\delta\mathbf{E}\ d\Omega+\int_\Omega\rho\ddot{\vec{x}}\cdot\delta\vec{x}\ d\Omega=0. \tag{13}$$

The first term represents the variation in strain energy δU of the body due to a virtual displacement $\delta\vec{x}$. In general, this expression is too complicated to evaluate; consequently approximations are used instead.

Discretization of the Body

Approximating the displacement field $\vec{u}(\vec{X},\ t)$ with a linear combination of assumed displacement fields gives

$$\vec{u}(\vec{X},\ t)=\sum_{i=1}^{N}\alpha_i(t)\vec{\Phi}_i(\vec{X})=\underset{\sim}{\alpha}^T(t)\underset{\sim}{\vec{\Phi}}(\vec{X}), \tag{14}$$

where $\alpha(t)$ is a column matrix of generalized coordinates and $\underset{\sim}{\vec{\Phi}}(\vec{X})$ is a column matrix of the assumed displacement fields. The wavy underscore denotes a column matrix and the superscript T denotes transposition of a matrix. This approach leads to a finite element approximation when the displacement fields are nonzero in just a part of the body; otherwise, it leads to an assumed-modes approximation. A finite element approximation usually results in a model of the body with many degrees of freedom, which is undesirable as for CPU time. For this reason usually an assumed-modes approximation is preferred. Different types of displacement fields are dealt with by Yoo and Haug (1986).

Substituting (5), (6) and (14) into (13) yields

$$\begin{aligned}&\delta U\\&+\delta\vec{c}\cdot\{m\ddot{\vec{c}}+\mathbf{Q}\cdot[\vec{\omega}\times(\vec{\omega}\times\vec{d}_1)+\dot{\vec{\omega}}\times\vec{d}_1+2\vec{\omega}\times(\dot{\underset{\sim}{\alpha}}^T\underset{\sim}{\vec{c}_2})+\ddot{\underset{\sim}{\alpha}}^T\underset{\sim}{\vec{c}_2}]\}\\&+\delta\vec{\pi}\cdot\{\vec{d}_1\times(\mathbf{Q}^c\cdot\ddot{\vec{c}})+\vec{\omega}\times(\mathbf{J}\cdot\vec{\omega})+\mathbf{J}\cdot\dot{\vec{\omega}}+2(\dot{\underset{\sim}{\alpha}}^T\underset{\sim}{\mathbf{D}_2})\cdot\vec{\omega}+\ddot{\underset{\sim}{\alpha}}^T\underset{\sim}{\vec{d}_3}\}\\&+\delta\underset{\sim}{\alpha}^T\{\ddot{\vec{c}}\cdot(\mathbf{Q}\cdot\underset{\sim}{\vec{c}_2})-\vec{\omega}\cdot(\underset{\sim}{\mathbf{D}_2}\cdot\vec{\omega})+\dot{\vec{\omega}}\cdot\underset{\sim}{\vec{d}_3}-2\vec{\omega}\cdot(\underline{\vec{c}_7}\dot{\underset{\sim}{\alpha}})+\underline{c_8}\ddot{\underset{\sim}{\alpha}}\}=0,\end{aligned} \tag{15}$$

where m is the mass of the body; the quantities $\underset{\sim}{\vec{c}_2}$, $\underline{\vec{c}_7}$, $\underline{c_8}$, $\vec{d}_1$, $\underset{\sim}{\mathbf{D}_2}$, $\underset{\sim}{\vec{d}_3}$ and $\mathbf{J}$ are defined in the appendix. An underscore denotes a matrix.

Preventing Rigid Body Motion in the Displacement Field $\vec{u}$

We want to describe the motion of the body in terms of displacements due to deformation defined by $\underset{\sim}{\alpha}$ and in terms of a rigid body motion defined by $\vec{c}$ and $\mathbf{Q}$. Then virtual variations of $\delta\vec{c}$, $\delta\vec{\pi}$, and $\delta\underset{\sim}{\alpha}$, must be independent. Consequently, the displacement fields $\vec{\Phi}_i$ must be mutually independent and such that a rigid body motion can not be represented by the linear combination (14).

Two kinds of conditions for preventing rigid body motion can be found in literature, namely, conditions for displacements of selected material points of the body and conditions for mean displacements of the body. The first kind of conditions is used by, among others, Sunada and Dubowsky (1981), Singh et al. (1984), Van der Werff and Jonker (1984), Agrawal and Shabana (1985), and Haug et al. (1986). The second kind of conditions is used by Agrawal and Shabana (1985), McDonough (1976), and others.

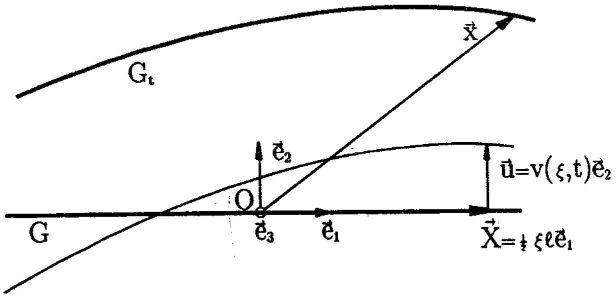

Fig. 2 Free beam

Conditions for Displacements of Selected Material Points. This kind of condition is also used in finite element analysis of structures. It comes to prescribing a minimum number of displacements of selected material points to prevent rigid body motion. For example, the displacements due to deformation of one material point P_c are required to be zero. This condition can be imposed on all displacement fields:

$$\vec{\Phi}_i(P_c) = \vec{0},\ i=1,2,\ \ldots,\ N. \tag{16}$$

Now, the body can still perform rigid body rotations around P_c. Hence, in addition, rigid body rotations have to be prevented. This may be achieved by constraining the rotation of a body-fixed frame (cf. Singh et al., 1984).

Conditions for Mean Displacement of the Body. A rigid body translation involves a displacement of the center of mass. Consequently, a linear combination of the assumed displacement fields $\vec{\Phi}_i$ can not represent a rigid body translation when the assumed displacement fields $\vec{\Phi}_i$ do not cause a displacement of the center of mass. This can be expressed mathematically as

$$\frac{1}{m}\int_\Omega \rho\vec{\Phi}_i d\Omega = \vec{0},\ i=1,2,3,\ \ldots,\ N. \tag{17}$$

The translation vector $\vec{c}$ will represent the translation of the center of mass of the body when this condition is used.

An infinitesimal small rigid body rotation around O can be represented by the vector field

$$\vec{u}_{\text{rotation}} = \chi\vec{e}\times\vec{X}, \tag{18}$$

where χ is the rotation angle and $\vec{e}$ is a unit vector parallel to the rotation axis. It can be seen that the displacements due to the rigid body rotation are perpendicular to $\vec{X}$. Consequently, a linear combination of the assumed displacement fields $\vec{\Phi}_i$ can not represent a rigid body rotation when the assumed displacement fields $\vec{\Phi}_i$ are on a mean parallel to $\vec{X}$. This can be expressed mathematically as

$$\int_\Omega \rho\vec{X}\times\vec{\Phi}_i\ d\Omega = \vec{0},\ i=1,2,3,\ \ldots,\ N. \tag{19}$$

(ρ has been used as a weighting factor in order to cancel the term $\underset{\sim}{\vec{c}_6}$ (see appendix) and $\vec{d}_3$ becomes proportional with $\underset{\sim}{\alpha}$.) Examples of displacement fields satisfying (17) and (19) are modes of free vibration (Ashley, 1967).

When in addition to conditions (17), the center of mass in the reference configuration is chosen to coincide with O, all terms involving $\vec{d}_1$ and $\underset{\sim}{\vec{c}_2}$ in the equations of motion (15) cancel. When this is taken into account, (15) reduces to

$$\begin{aligned}
&\delta U\\
&+\delta\vec{c}\cdot\{m\ddot{\vec{c}}\}\\
&+\delta\vec{\pi}\cdot\{\vec{\omega}\times(\mathbf{J}\cdot\vec{\omega})+\mathbf{J}\cdot\dot{\vec{\omega}}+2(\dot{\underset{\sim}{\alpha}}^T\underset{\sim}{\mathbf{D}_2})\cdot\vec{\omega}+\ddot{\underset{\sim}{\alpha}}^T\underset{\sim}{\vec{d}_3}\}\\
&+\delta\underset{\sim}{\alpha}^T\{-\vec{\omega}\cdot(\underset{\sim}{\mathbf{D}_2}\cdot\vec{\omega})-\dot{\vec{\omega}}\cdot(\underset{\sim}{\vec{c}_7}\underset{\sim}{\alpha})-2\vec{\omega}\cdot(\underset{\sim}{\vec{c}_7}\dot{\underset{\sim}{\alpha}})+\underset{\sim}{c_8}\ddot{\underset{\sim}{\alpha}}\}=0.
\end{aligned} \tag{20}$$

There is just a weak coupling between displacements due to deformation and displacements due to rigid body motion. Cancellation of all terms involving $\vec{d}_1$ and $\underset{\sim}{\vec{c}_2}$ makes the mass matrix more sparse, which may be advantageous when sparse matrix techniques are used for solving the equations of motion. This will be illustrated now for a uniform beam.

Example: Free Uniform Beam

Consider the uniform slender beam of length l and mass m shown in Fig. 2. We choose a reference configuration G with straight centroidal axis and with its center of mass coinciding with O. Introduce an orthonormal right-handed vector base $\underset{\sim}{\vec{e}}$, such that $\vec{e}_1$ is parallel to the centroidal axis of the beam.

The motion of the beam is restricted to the plane spanned by $\vec{e}_1$ and $\vec{e}_2$. Using the Bernoulli-Euler beam theory, only the displacements of the centroidal axis need to be considered. The centroidal axis is assumed to be inextensible. The lateral displacements due to deformation are approximated by

$$v(\xi,\ t) = a_0(t) + a_1(t)\xi + a_2(t)\xi^2 + a_3(t)\xi^3, \tag{21}$$

where ξ is the dimensionless distance in the reference configuration for a material point of the centroidal axis measured from the center of mass and made dimensionless with $l/2$. The rotary inertia of the cross-section of the beam will be neglected and the expression for the strain energy of the beam which will be used is

$$U = (4EI/l^3)\int_{-1}^{1}(\partial^2 v/\partial\xi^2)^2 d\xi. \tag{22}$$

The vectors and tensor used to describe the kinematics of the beam are resolved into their components in the base $\underset{\sim}{\vec{e}}$. This yields

$$\vec{c} = c_1\vec{e}_1 + c_2\vec{e}_2, \tag{23}$$

$$\vec{X} = ½\xi\ l\vec{e}_1, \tag{24}$$

$$\vec{u} = v(\xi,\ t)\vec{e}_2, \tag{25}$$

$$\mathbf{Q} = (\vec{e}_1\vec{e}_1 + \vec{e}_2\vec{e}_2)\cos\varphi + (\vec{e}_2\vec{e}_1 - \vec{e}_1\vec{e}_2)\sin\varphi + \vec{e}_3\vec{e}_3, \tag{26}$$

where φ is the rigid body rotation of the body,

$$\vec{\omega} = \dot{\varphi}\vec{e}_3, \qquad \delta\vec{\pi} = \delta\varphi\vec{e}_3. \tag{27}$$

The displacement field (21) contains rigid body displacements. These are eliminated by using the two kinds of conditions discussed earlier.

1 Conditions on displacements of selected material points. We choose for the material point the middle of the beam ($\xi = 0$), and require that:

a the lateral displacement is zero,

b the rotation is zero.

The first condition yields $v(0,\ t) \equiv 0$. Hence

$$a_0 \equiv 0. \tag{28}$$

The second condition yields $dv/d\xi\,_{|0,t} \equiv 0$. Hence

$$a_1 \equiv 0. \tag{29}$$

Eliminating rigid body motions from the displacement field (21) using the conditions (28) and (29) yields

$$v(\xi,\ t) = \alpha_1(t)\phi_1(\xi) + \alpha_2(t)\phi_2(\xi), \tag{30}$$

where the assumed displacement fields $\phi_1(\xi)$ and $\phi_2(\xi)$ are

$$\phi_1(\xi) = \xi^2, \tag{31}$$

$$\phi_2(\xi) = \xi^3. \tag{32}$$

Elaboration of (15) for the strain energy expression (22) and the assumed displacement fields (31) and (32) yields for the coefficients of δc_1, δc_2, $\delta\varphi$, $\delta\alpha_1$ and $\delta\alpha_2$, respectively

$$\delta c_1\!:\ m\{\ddot{c}_1 - [\alpha_1(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi) + 2\dot{\varphi}\ \dot{\alpha}_1\cos\varphi + \ddot{\alpha}_1\sin\varphi]/3\},$$

$$\delta c_2\!:\ m\{\ddot{c}_2 - [\alpha_1(\ddot{\varphi}\sin\varphi + \dot{\varphi}^2\cos\varphi) + 2\dot{\varphi}\ \dot{\alpha}_1\sin\varphi - \ddot{\alpha}_1\cos\varphi]/3\},$$

$$\begin{aligned}\delta\varphi\!:\ m\{&-\alpha_1(\ddot{c}_1\cos\varphi + \ddot{c}_2\sin\varphi)/3 + \ddot{\varphi}(l^2/12 + \alpha_1^2/5 + \alpha_2^2/7)\\ &+ 2\dot{\varphi}(\alpha_1\dot{\alpha}_1/5 + \alpha_2\dot{\alpha}_2/7) + l\ddot{\alpha}_2/10\},\end{aligned}$$

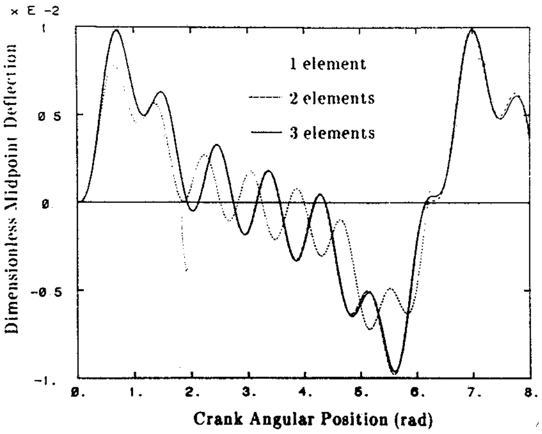

Fig. 3 Dimensionless deflection of the midpoint of the connecting rod

$$\delta\alpha_1: m\{(-\ddot{c}_1\sin\varphi + \ddot{c}_2\cos\varphi)/3 - \dot{\varphi}^2\alpha_1/5 + \ddot{\alpha}_1/5\} + 64\alpha_1 EI/l^3,$$

$$\delta\alpha_2: m\{l\ddot{\varphi}/10 - \dot{\varphi}^2\alpha_2/7 + \ddot{\alpha}_2/7\} + 192\alpha_2 EI/l^3. \quad (33)$$

2 Conditions on mean displacements of the body. The conditions discussed above are:

a the displacement of the center of mass due to the displacements (21) equals zero,

b the displacements $\vec{u} = v(\xi, t)\vec{e}_2$ and the position vector $\vec{X}$ are on a mean parallel.

The first condition is equivalent to (cf. (17))

$$\int_{-1}^{1} v(\xi, t)d\xi \equiv 0. \quad (34)$$

Substituting (21) into this equation yields

$$3a_0 + a_2 \equiv 0. \quad (35)$$

The second condition is equivalent to (cf. (19))

$$\int_{-1}^{1} \xi\, v(\xi, t)d\xi \equiv 0. \quad (36)$$

substituting (21) into this equation yields

$$5a_1 + 3a_3 \equiv 0. \quad (37)$$

Eliminating rigid body motions from the displacement field (21) using conditions (35) and (37) yields

$$v(\xi, t) = \alpha_1(t)\phi_1(\xi) + \alpha_2(t)\phi_2(\xi), \quad (38)$$

where the assumed displacement fields $\phi_1(\xi)$ and $\phi_2(\xi)$ are

$$\phi_1(\xi) = \tfrac{1}{2}(-1 + 3\xi^2), \quad (39)$$

$$\phi_2(\xi) = \tfrac{1}{2}(-3\xi + 5\xi^3). \quad (40)$$

Elaborating (20) for the strain energy expression (22) and the assumed displacement fields (39) and (40) yields for the coefficients of δc_1, δc_2, $\delta\varphi$, $\delta\alpha_1$ and $\delta\alpha_2$, respectively

$$\delta c_1: m\ddot{c}_1,$$

$$\delta c_2: m\ddot{c}_2,$$

$$\delta\varphi: m\{\ddot{\varphi}(l^2/12 + \alpha_1^2/5 + \alpha_2^2/7) + 2\dot{\varphi}(\alpha_1\dot{\alpha}_1/5 + \alpha_2\dot{\alpha}_2/7)\},$$

$$\delta\alpha_1: (m/5)(\ddot{\alpha}_1 - \dot{\varphi}^2\alpha_1) + 144\alpha_1 EI/l^3,$$

$$\delta\alpha_2: (m/7)(\ddot{\alpha}_2 - \dot{\varphi}^2\alpha_2) + 1200\alpha_2 EI/l^3. \quad (41)$$

Comparison of (33) and (41) reveals that the equations of motion become more simple when the given conditions on mean displacements are used: there is no coupling between the rigid body translational motion and the displacements due to deformation; the coupling between the rigid body rotation and the displacements due to deformation is nonlinear in terms of the displacements due to deformation.

Numerical Results

In order to demonstrate the merits of using the description based on mean displacement conditions for describing the dynamics of multibody systems with deformable bodies, the consequences of this description have been implemented in the two-dimensional version of the multibody program DADS (CADSI, 1986), i.e., multiplications involving $\tilde{d}_1$, $\tilde{c}_2$ and $\tilde{c}_6$ are skipped and the increased sparseness of the mass matrix is taken into account. This version is used to analyze the crank-slider mechanism considerd by Song and Haug (1980).

The model consists of three rigid bodies (the crank, the slider block, and a supporting body) and a deformable connecting rod. In order to demonstrate the influence of the number of deformable bodies in the system, the system was analyzed with the connecting rod divided into one, two, and three deformable beams. The deformation fields used for approximating the deflection of these beams are given by (38) and (39).

Figure 3 shows the deflection at the midpoint of the connecting rod, made dimensionless with its length. The ratios of the required CPU time for the revised version and the standard version for the system with one, two, and three deformable beams are .97, .93, and .92, respectively. Hence, the revised version of DADS requires less CPU time; the reduction in CPU time increases when the number of deformable bodies increases.

Concluding Remarks

The kinematics of a deformable body experiencing large displacements is described. The displacements are resolved into displacements due to deformation and displacements due to rigid body motion. This resolution is not unique. A review of conditions leading to a unique resolution is given. It is shown that the infrequently used conditions for mean displacements lead to simpler equations of motion and a more sparse mass matrix. This is implemented in the multibody program DADS, which uses sparse matrix techniques for solving the equations of motion. The results show that a reduction of the required CPU time is achieved when the conditions for mean displacements are used. The proposed description is in particular advantageous for multibody programs that use finite elements for approximating deformable bodies.

One of the reviewers, Professor Haug of the University of Iowa, is suspicious that there is an additional advantage. Absolute coordinate formulations of equations of motion, particularly including flexibility effects, can lead to challenges in stable symbolic factorization of sparse matrices in the equations of motion. He expects that the sparse matrix symbolic factorization will be substantially more stable when there is a larger number of precisely zero terms in the system mass matrix.

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APPENDIX

$$\vec{c}_1 = \int_\Omega \rho \vec{X}\, d\Omega$$

$$\underset{\sim}{\vec{c}_2} = \int_\Omega \rho \underset{\sim}{\vec{\Phi}}\, d\Omega$$

$$\mathbf{C}_3 = \int_\Omega \rho\{(\vec{X}\cdot\vec{X})\mathbf{I} - \vec{X}\vec{X}\}\, d\Omega$$

$$\underset{\sim}{\mathbf{C}}_4 = \int_\Omega \rho\{(\underset{\sim}{\vec{\Phi}}\cdot\vec{X})\mathbf{I} - \underset{\sim}{\vec{\Phi}}\vec{X}\}\, d\Omega$$

$$\underline{\mathbf{C}}_5 = \int_\Omega \rho\{(\underset{\sim}{\vec{\Phi}}\cdot\underset{\sim}{\vec{\Phi}}^T)\mathbf{I} - \underset{\sim}{\vec{\Phi}}\underset{\sim}{\vec{\Phi}}^T\}\, d\Omega$$

$$\underset{\sim}{\vec{c}_6} = \int_\Omega \rho\{\vec{X}\times\underset{\sim}{\vec{\Phi}}\}\, d\Omega$$

$$\underline{\vec{c}}_7 = \int_\Omega \rho\{\underset{\sim}{\vec{\Phi}}\times\underset{\sim}{\vec{\Phi}}^T\}\, d\Omega$$

$$\underline{c}_8 = \int_\Omega \rho\{\underset{\sim}{\vec{\Phi}}\cdot\underset{\sim}{\vec{\Phi}}^T\}\, d\Omega$$

$$\mathbf{J} = \mathbf{C}_3 + \underset{\sim}{\alpha}^T(\underset{\sim}{\mathbf{C}}_4 + \underset{\sim}{\mathbf{C}}_4^c) + \underset{\sim}{\alpha}^T\underline{\mathbf{C}}_5\underset{\sim}{\alpha}$$

$$\vec{d}_1 = \vec{c}_1 + \underset{\sim}{\alpha}^T\underset{\sim}{\vec{c}_2}$$
$$\underset{\sim}{\mathbf{D}}_2 = \underset{\sim}{\mathbf{C}}_4 + \underline{\mathbf{C}}_5\underset{\sim}{\alpha}$$
$$\underset{\sim}{\vec{d}_3} = \underset{\sim}{\vec{c}_6} - \underline{\vec{c}}_7\underset{\sim}{\alpha}$$