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Highest-Order Coupler Points of Watt-1 Linkages, Tracing Symmetrical 6-Bar Curves

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Abstract

Certain non-symmetrical Watt-1 linkage mechanisms may produce symmetrical curves. The dimensions of such mechanisms then have to meet *five* precise conditions. They have been derived from the two possibilities that symmetrize the Assembly Configuration containing the four 6-bar curve cognates. Each possibility leads to a different mechanism. The one that can be driven without additional conditions, contains a kite 4-bar, carrying rigid triangles that are similar and otherwise related. Then, a 'kite-cell' is involved, transforming the input-circle into a symmetrical Watt-1 curve of the 8th or of the 12th degree, depending on the choice of coupler-plane. The second possibility leads to a mechanism in which not only a point, but also a straight-line reaches the image positions. This type, however, must move through a 'stretched position' with a two-fold coupler motion. Then, gear-pairs or the like are necessary to overcome such a bifurcated position.

1. Introduction

SIX-BAR curves may be produced by coupler points of 6-bar linkage mechanisms of the *Watt*-type or of the *Stephenson*-type. Each type contains a 4-bar loop to which a linkage-*dyad* is adjoined in order to form a kinematic chain with six links. For Watt's type the adjoined dyad connects two points that are attached to adjacent sides of the 4-bar, whereas for Stephenson's type, points attached to the opposite sides are connected. A further distinction is made by appointing the frame in the linkage. For the *Watt*-1 *mechanism*, for instance, this is done by appointing a *binary* link as frame. We further have the Watt-2 mechanism, and also the Stephenson-1, -2 and -3 mechanisms.

Conditions for symmetry of the curves for the Stephenson-3 mechanism have been investigated by Levitskii[1] and Antuma[2], whereas the symmetry-conditions for the Stephenson-1 mechanism have been treated by the author[3] of this paper.

The author[4] has also investigated the symmetry-conditions for the Watt-1 mechanism in which the tracing point was attached to a floating link of '*lower order*' implying that the floating link contained at least *one* point tracing a circle‡. For particular mechanisms such as the *focal* linkages of the Watt-1 type, for which the tracing point is of '*higher order*'§, the author[5] similarly investigated the conditions of symmetry, in order to cause them to produce so-called '*half-symmetrical*' curves.

Here, in this paper, however, a more general case will be considered, for which the coupler points E are of higher order and are, therefore, attached to the dyad-link KD which is directly hinged to the coupler point K of the 4-bar A_0ABB_0. (See Fig. 1.)

Primrose *et al.*[6], who among others studied the Watt-1 mechanism, found that such coupler points generally produce 6-bar curves of degree 16 and *genus* 5. If the curves are symmetrical, they may be of lower degree. We intend to show that this is the case when *kite 4-bars* are involved. The main issue of this paper, however, is to find the conditions that have to be imposed on the mechanism for a higher order coupler point to produce symmetrical curves of the most general form.

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‡The coupler of a 4-bar mechanism is of lower order again, as then the coupler contains even *two* points tracing a circle.

§Of higher order in the sense that the floating link, containing such a point, does not include points tracing a circle.

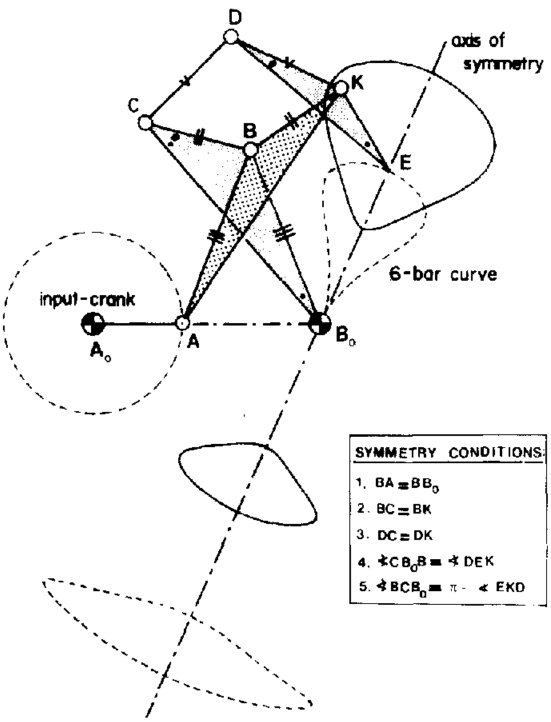

Figure 1. Source mechanism of type Watt-1, producing 4 symmetrical branches of a complete, 12th degree, Watt-1 curve.

We propose to use *cognate theory* [7] to derive the symmetry-conditions to be met by the Watt-1 mechanism. This may be carried out by combining the mechanism with one of its *curve*-cognates to form a symmetrical configuration. The symmetry-conditions that have to be imposed on the (initial) Watt-1 mechanism are then derived from this configuration.

2. The Assembly Configuration made Symmetrical

It is known from cognate theory[7], that a *four*-fold generation exists for the curve produced by a tracing-point E attached to the dyad-link KD. Thus, including the initial mechanism, four *curve* cognates exist, producing the same 6-bar curve. They are (a) the initial Watt-1 mechanism, (b) its double Roberts' *coupler* cognate, producing identical motion for the bar KD, (c) the curve cognate of the initial mechanism, obtained by an exchange of the bars of the adjoined dyad, and finally, (d) the curve cognate of the double Roberts' coupler cognate, obtained in a similar way.

The four curve cognates together, form the so-called '*Assembly Configuration*'. Part of this configuration is demonstrated in Fig. 2, showing the cognates (a) and (d). It is possible to make them *reflected identical* with respect to the common axis B_0E. As a consequence, not only (a) and (d) but also (b) and (c) then are images of each other with respect to this axis. To carry this out, it is necessary to arrange the triads A_0AKE and $A_0'''A'''K'E$ symmetrically about the axis B_0E. In addition, the dyads BCD and $B''C'D''$ will each have to be the image of the other with respect to this axis of symmetry. With due observance of the rules of cognation, this then leads to a result which is also obtainable directly through the introduction of an auxiliarly point B^v on the axis of symmetry. For the sake of brevity, however, only this direct way of derivation will be explained.

Derivation of half the Symmetric Assembly Configuration

In order to make the curve cognates (a) and (d) images with respect to the axis of symmetry, we proceed as follows: (See Fig. 3) A 4-bar A_0ABB_0 with coupler point B^v, and in which $AB = B_0B = BB^v$, is drawn in its *design* position (also called the *symmetry* position). In this position the crank A_0A falls along the fixed link A_0B_0. The point B^v attached to the coupler AB

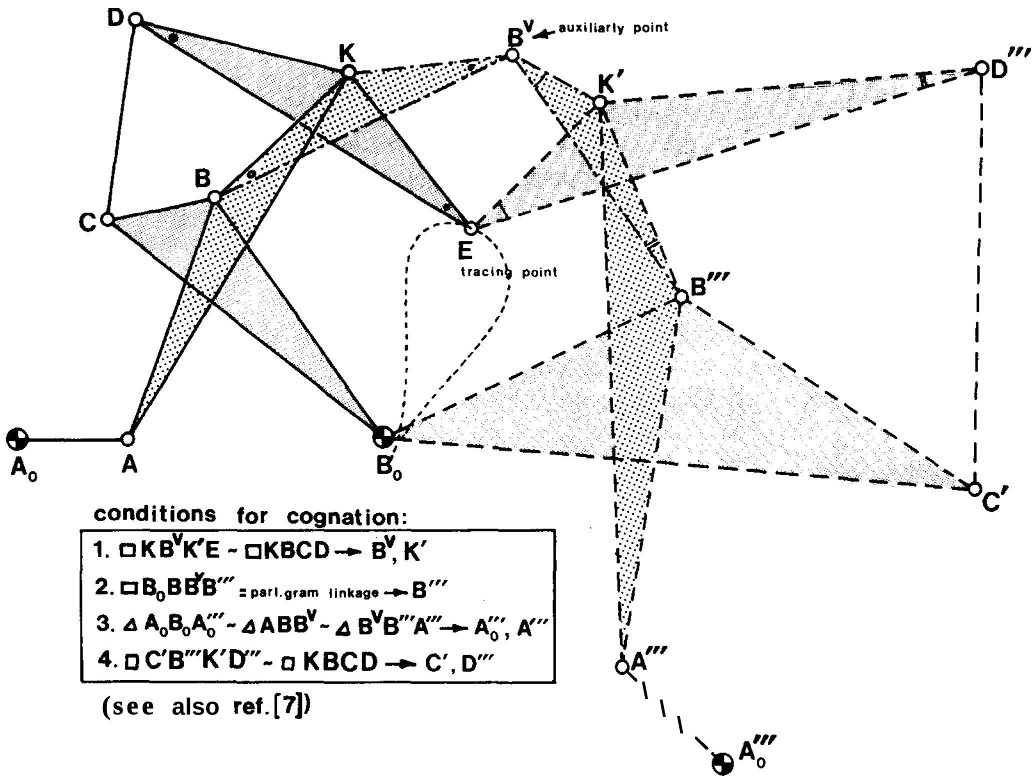

Figure 2. Half the Assembly Configuration,' containing the initial Watt-I mechanism as well as the curve cognate of the double Roberts coupler cognate.

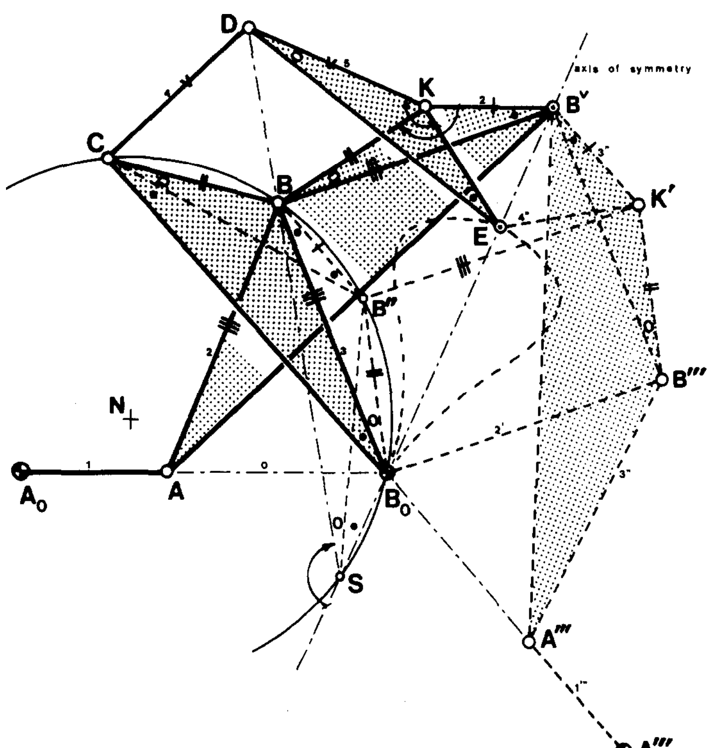

Figure 3. Symmetrical linkage and derivation of Watt-I 6-bar producing a symmetrical curve, traced by *E*.

traces a symmetrical 4-bar coupler curve. The line connecting B_0 to the design position of B^v is the axis of symmetry of the curve.

Next, we draw the *image 4-bar* $A_0'''A''' - B^v - B'''B_0$ of the original mechanism with respect to the axis of symmetry. Then, according to Roberts' Law, the common coupler point B^v of these two 4-bars produces the same coupler curve. The image and the initial 4-bars are, therefore, 4-bar *curve* cognates of one another. Their common coupler point B^v, when moved, causes the cranks A_0A and $A_0'''A'''$ to rotate at the same angular speed. It can also be seen that the configuration, being part of the Roberts' Configuration, contains a parallelogram linkage which in this case is the *rhombus* $B_0B'''B^vB$.

A linkage dyad KEK' is now added. This dyad has equal arms and connects two symmetrically located 4-bar coupler points (K and K') in the design position of the configuration. Thus, the dyad-joint E lies on the axis of symmetry in that position. As it is symmetrically attached to a symmetric configuration, joint E clearly moves along a symmetric curve.

4. Derivation and properties of the Watt-1 mechanism

The mechanism assembled in section 3, contains part of a Roberts' Configuration. It is therefore an *overconstrained* mechanism which, because of its symmetry, guides points B^v and E along symmetrical paths. The non-symmetrical Watt-1 mechanism can now be derived from this symmetrical mechanism. To carry this out, we *stretch-rotate* the kite $KB^vK'E$ about K so that B^v moves to B. We so obtain the 4-bar kite $KBCD$ which is similar to the original kite $KB^vK'E$. The factor f_K of stretch-rotation, being equal to the vector ratio $\mathbf{KB}/\mathbf{KB}^v$, is independent of time. The two kites therefore may *remain* similar, causing corresponding links to rotate at the same angular speed. (In notation: $\square \underset{\uparrow}{K}BCD \cong f_K . \square \underset{\uparrow}{K}B^vK'E$, appointing K as its center of stretch-rotation.) Thus, KD may be attached to KE and, similarly, BC to BB_0, the latter having the same angular speed as B^vB''' or B^vK'. If we now obliterate the image mechanism and B^v, we are left with the Watt-1 mechanism $A_0ABB_0 - CDK$, in which the (higher) coupler point E clearly traces a symmetrical 6-bar curve. The properties to be met by this mechanism are derived as follows: From the stretch-rotation applied about the point K, we find that

$$\Delta \mathbf{K}BD = f_{\mathbf{K}} \cdot \square \mathbf{K}B^vE,$$

hence

$$\mathbf{KB}/\mathbf{KB}^v = \mathbf{KD}/\mathbf{KE},$$

so that

$$\triangle BKB^v \sim \triangle DKE.$$

Further, if we define the point B'' through the relationship

$$\triangle B_0B''B \cong \triangle B'''K'B^v,$$

we may write that

$$\mathbf{BC} = f_{\mathbf{K}} \cdot \mathbf{B}^v\mathbf{K}' = f_{\mathbf{K}} \cdot \mathbf{BB}'',$$

hence

$$\mathbf{BC}/\mathbf{BB}'' = \mathbf{KB}/\mathbf{KB}^v.$$

Thus, with $BC = KB$, it follows that

$$\triangle CBB'' \cong \triangle BKB^v.$$

Therefore, as the triangles $B'''K'B^v$ and BKB^v are reflected congruent so are the triangles CBB'' and $B_0B''B$. Thus, the rigid quadrilateral $CBB''B_0$ is an isosceles trapezium and so a cyclic quadrilateral.
From this, it follows that

$$\sphericalangle CB_0B = (\sphericalangle B''CB_0 = \sphericalangle CB''B = \sphericalangle BB^vK =)\sphericalangle DEK.$$

Also,

$$\sphericalangle BCB_0 = \sphericalangle DEK + \sphericalangle KDE.$$

As further the corresponding kite diagonals B^vE and BD are stretch-rotated about the angle $\sphericalangle BKB^v = \sphericalangle DKE = \pi - \sphericalangle B_0CB$, they meet in a point S of the trapezium. Therefore, the axis of symmetry, the diagonal DB and the circumcircle of $\triangle B_0BC$ are concurrent at S.

(For random positions, for which $\pi \neq \sphericalangle AA_0B_0 \neq 0$, the line B_0E, the diagonal DB and the circumcircle of $\triangle B_0BC$ equally have a common point S.) Thus,

$$\begin{aligned}\sphericalangle A_0B_0E &= \sphericalangle A_0B_0B^v = \pi - \sphericalangle SB_0A_0\\ &= \pi - \frac{1}{2}\sphericalangle A_0'''B_0A_0 = \pi - \frac{1}{2}\sphericalangle B^vBA\\ &= \pi - \frac{1}{2}(\sphericalangle KBA - \sphericalangle KBB^v)\\ &= \pi - \frac{1}{2}(\sphericalangle KBA + \sphericalangle CB_0B - \sphericalangle BCB_0)\end{aligned}$$

which is an expression for the direction of the axis of symmetry.

The properties derived will be used for a direct design of our Watt-1 mechanism. Clearly, the Watt-1 mechanism, drawn in full lines in the figure, will produce the same symmetric curve as did the point E of the symmetric mechanism, from which we started. In conclusion, we have found altogether 5 conditions for our Watt-1 mechanism to produce a symmetrical 6-bar curve. These conditions are respectively

$$\begin{cases} 1.\quad AB = B_0B \\ 2.\quad BC = BK \\ 3.\quad CD = DK \\ 4.\quad \sphericalangle DEK = \sphericalangle CB_0B \\ 5.\quad \sphericalangle EKD = \pi - \sphericalangle BCB_0. \end{cases}$$

Instead of the conditions (4) and (5), it is possible to use the set of conditions

$$\begin{cases} \sphericalangle DEK = \pi - \sphericalangle BB_0C & \text{(4A)} \\ \sphericalangle EKD = \sphericalangle B_0CB. & \text{(5A)} \end{cases}$$

In Fig. 4, for instance, the conditions (4) and (5) are applicable, whereas Fig. 5 demonstrates the application of the conditions (4A) and (5A).
In order to avoid this ambiguity, it is possible to relate the triangles EKD and B_0CB otherwise (See again Fig. 4):
Let then the circumcircle of $\triangle B_0BC$ intersect the circle about B, having the radius BC, at the points C and G. then,

$$\sphericalangle EKD = \pi - \sphericalangle BDB_0 = \sphericalangle B_0GB.$$

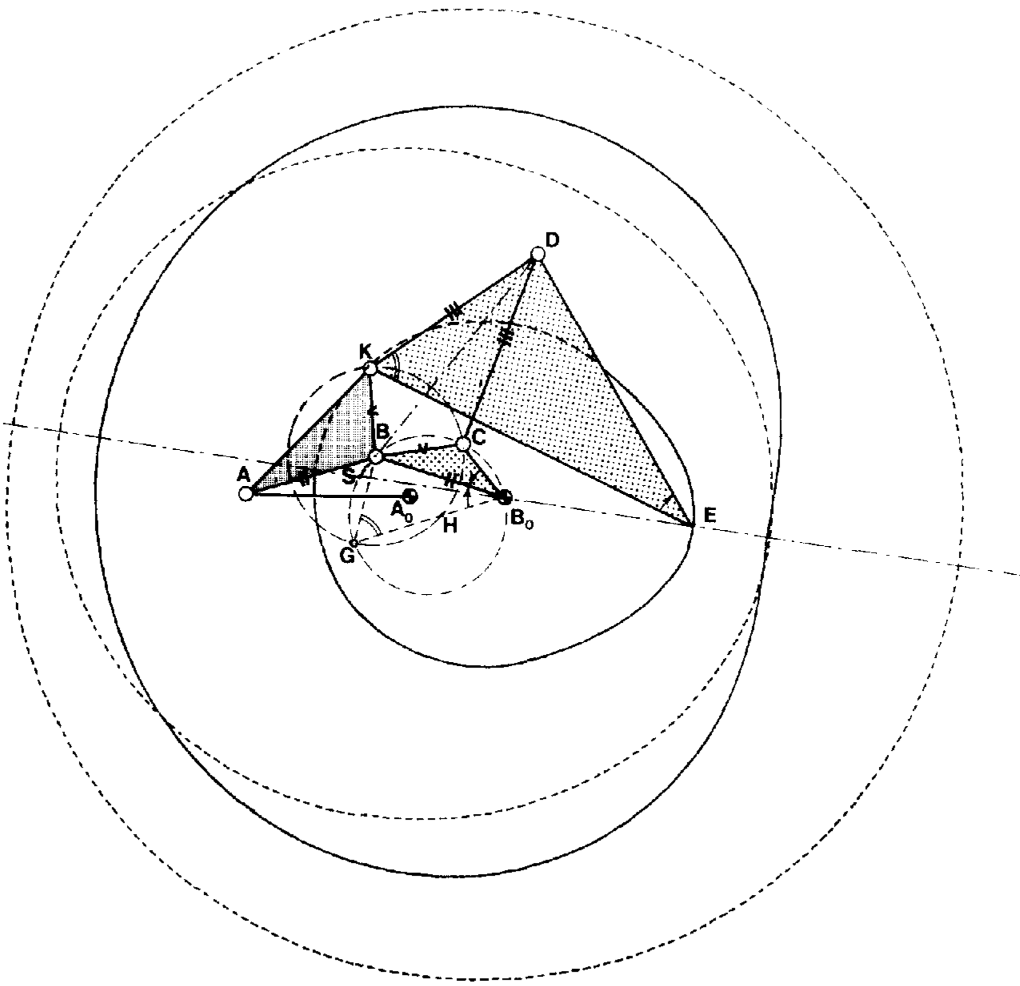

Figure 4. Simple design of a Watt-1 linkage, producing 4 symmetrical branches of a 12th degree 6-bar curve.

Also, because $GB = CB$,

$$\sphericalangle BB_0G = -\sphericalangle BB_0C = \sphericalangle DEK.$$

Hence,

$$\triangle BB_0G \sim \triangle DEK. \qquad (4B), (5B)$$

This similarity replaces the conditions (4) and (5) as well as the conditions (4A) and (5A), unambiguously.

Two unambiguous relations between the *sides* of the triangles EKD and B_0CB are derived as follows: For this, we intersect the ray B_0G and the circle about B, at the points G and H. Then, for the (geometric) *power* of point B_0 with respect to this circle, we may write

$$B_0H * B_0G = B_0B^2 - BC^2.$$

As further the triangles B_0CB and B_0HB are reflected identical, we have that $B_0H = B_0C$, so that with the similarity condition (4B), (5B), this power turns into

$$B_0C \cdot (EK/DK) \cdot BC = B_0B^2 - BC^2.$$

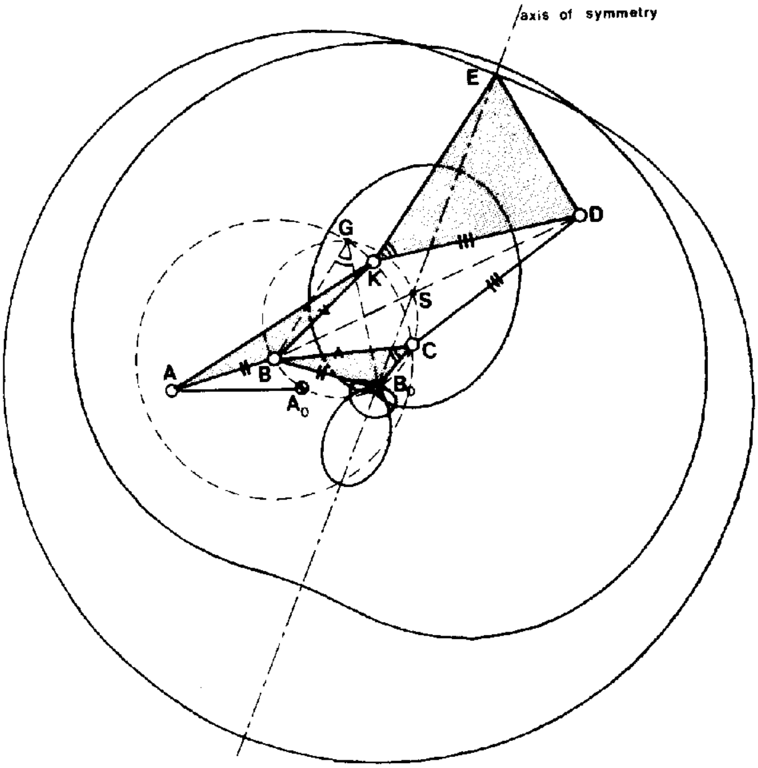

Figure 5. Design of the Watt-1 mechanism, producing 4 symmetrical branches of a 12th degree and complete Watt-1 curve.

Thus,

$$EK/DK = (B_0B^2 - BC^2)/(B_0C * BC) \qquad (4C)†$$

and

$$DK/DE = BC/BB_0 \qquad (5C)$$

For obvious reasons we then say, that the triangles EKD and B_0CB are '*quasi-similar*' to one another.

If one studies the complete Assembly Configuration, one finds that the symmetry-conditions, here derived, remain the same for all 4 curve-cognates. Thus, through cognation, no other sets of conditions are to be obtained.

5. Design of the Watt-1 Mechanism

The design of the 6-bar mechanism to produce a symmetrical curve, may be carried out as follows: (See Fig. 5)

(a) Draw the 4-bar linkage A_0ABB_0, for which $AB = B_0B$, with point A lying on the frame-link A_0B_0.

(b) Attach a random point C to the rocker, or secondary crank, B_0B

(c) Attach a point K to the coupler AB such that $BK = BC$

(d) Adjoin an isosceles linkage dyad CDK

(e) Intersect the circle circumscribed about the triangle B_0BC and the circle about B with radius BC, at the points G and C.

(f) Make the floating $\triangle DKE \sim \triangle BGB_0$.

(g) Verify that the diagonal BD, the circumcircle of $\triangle B_0BC$ and the symmetry-axis B_0E all intersect at the one point S. (Note that the points S, K, E and D always lie on a circle.)

†The orientation of $\triangle EKD$ being decided by the sign of the expression for EK/DK.

Generally, a Watt-1 mechanism involves 15 degrees of freedom in design, not counting the motion variable of the mechanism. They consist of the 4 coordinates that determine the fixed link and a further 11 link-lengths. Thus, 10 parameters remain to be chosen by the designer in this case. They are the 4 coordinates for the frame-link, one parameter for the crank-length A_0A, one for the link-length AB, two for the sides BC and B_0C, and, finally, two parameters for the sides AK and DK.

A computer program, set up by Mr. A. T. J. M. Smals†, has been used on a Hewlett and Packard table-top calculator (type no. 9825A) in combination with a Hewlett and Packard plotter (type no. 9872A) in order to demonstrate the symmetry of the curves that are traced by the Watt-1 mechanisms of this design.

Figure 5, for instance, shows an example, in which the basic 4-bar linkage is a double crank. The complete curve is a *quadrucursal* one. Each branch belongs to a chosen orientation of the isosceles dyads B_0BA and CDK. (The orientations of the three rigid triangles of the 6-bar, however, remain the same for all branches that are traced by the (higher) coupler point.)

We further note that all branches have the same axis of symmetry through the fixed center B_0. Finally, a line perpendicular to axis of symmetry intersects the curve at up to 12 real points of intersection.

In Fig. 6, the basic 4-bar linkage is a non-Grashof one, with the result that twice two branches merge so that only two branches actually appear.

In Fig. 7, finally, a special case has been considered in which all the rigid triangles are stretched, so that each branch is traced twice.

6. The Degree of the Curve, Traced by Point *E*

Figure 8 shows a particular 8-bar mechanism that has one degree of freedom in motion. It consists of the 4-bar A_0ABB_0, the kite $KEK'B^v$, the rhombus B^vBB_0B''', and the rigid and reflected similar triangles $B'''K'B^v$ and BKB^v, the latter being rigidly attached to the isosceles tirangle ABB^v. This mechanism really is half the Assembly Configuration without the redundant bar $A_0'''A'''$. It therefore produces the same Watt-1 curve as before. The points B^v, E and B_0 of this mechanism remain collinear. So, we can regard the mechanism as a transformer, that transforms the symmetrical 4-bar coupler curve, traced by B^v, into a symmetrical 6-bar Watt-1 curve, traced by point E of the mechanism.

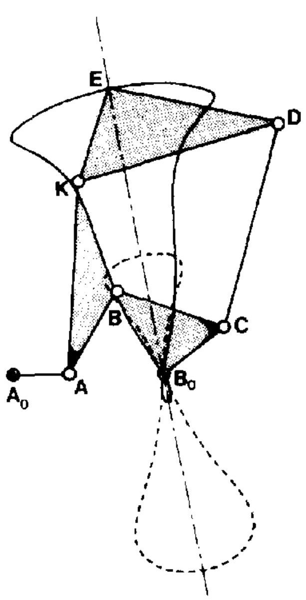

Figure 6. The complete Watt-1 curve that consists of two symmetrical branches.

†Ir. Smals is a senior research officer at the Department of Mechanical Engineering at the Eindhoven University of Technology in The Netherlands.

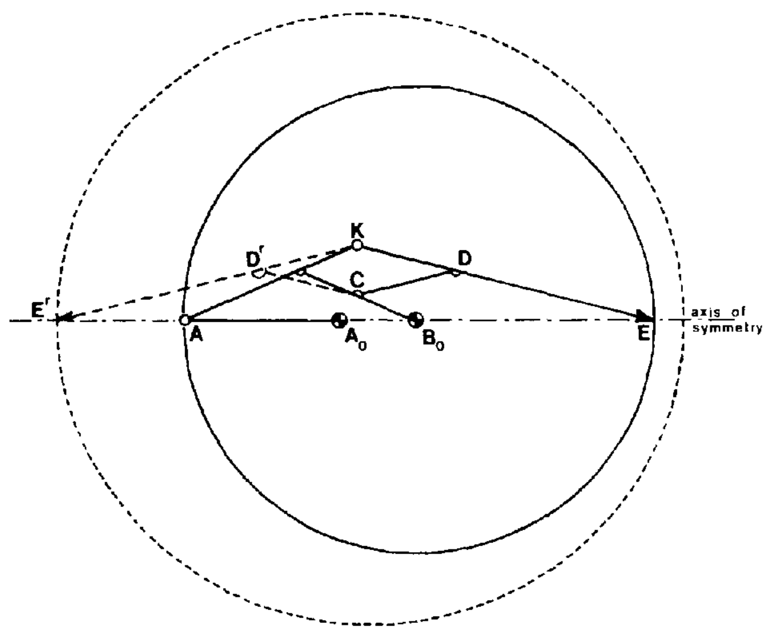

Figure 7. When two symmetric branches are traced twice.

In order to derive the degree of this Watt-1 curve, we note that for each point B^v on the axis of symmetry *two* points E exist. These are obtained by reflection of the isosceles dyad KEK' in the diagonal KK' of the kite. Further, as B^v moves along a coupler curve of degree 6, a total of 6 points B^v lies on the axis of symmetry. Two of them, however, are located at B_0, which is a double point of the (4-bar) coupler curve. The corresponding points E (to this double point) lie somewhere on the tangents to the (4-bar) coupler curve at B_0, not on the axis of symmetry. (As point B^v approaches B_0 along a tangent of the curve, this tangent, also being the connecting line of B^v and E, joins E at a certain distance from B_0, but not on the axis of symmetry.) Thus, this axis intersects the symmetrical Watt-1 curve at $(6\text{-}2) * 2 = 8$ points outside B_0. This number, however, does not account for the possibility in which E lies on the axis, and B^v does not. Such a possibility, namely, may arise when point E coincides with B_0. If $E = B_0$, a triangle BKE

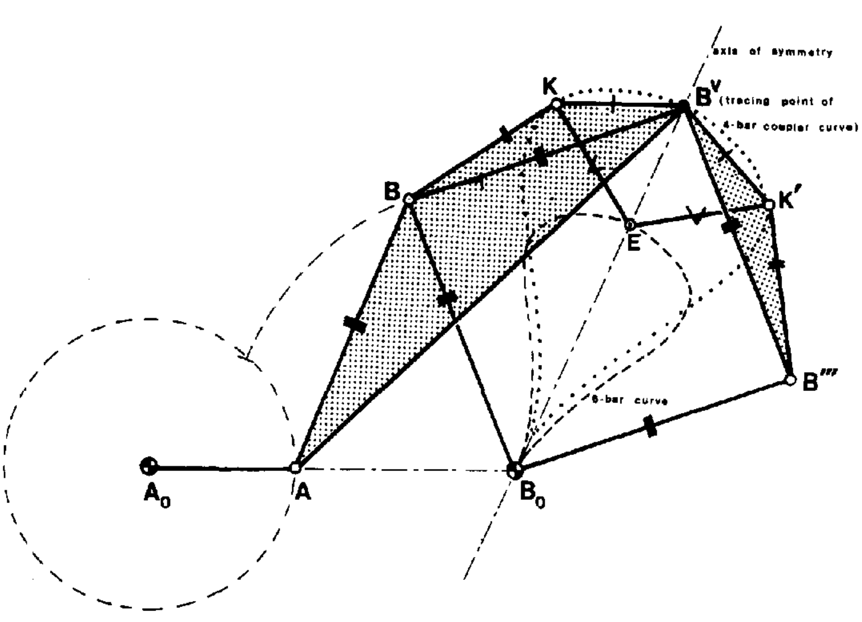

Figure 8. Symmetrical driven cell transforming a 4-bar coupler curve into a symmetrical Watt-1 curve of degree 12.

exists, that is rigidly attached to the coupler triangle ABK. Thus, two different distances EA exist, that correspond to the two possible orientations of ΔBKE. A circle about B_0 with radius EA intersects the input-circle at two real or at two imaginary points A. This occurs twice. So, a total of four positions A may exist, that correspond to four positions E at B_0. Hence, B_0 is a *quadruple* point E of the Watt-1 curve. The Watt-1 curve, therefore, intersects its symmetry-axis at $(6\text{-}2)*2+4=12$ points. Hence, the curve that is traced by the higher order coupler point E is of *degree* 12.

The lower order coupler point E^X, Fig. 9, which attached to the lower order coupler plane CD of the mechanism, also traces a symmetrical Watt-1 curve[4]. The latter, however, is only of degree 8.

7. The Kite-Cell, Transforming a Circle into a Symmetrical Watt-1 Curve of Degree 12

The Watt-1 mechanism, assembled in section 5, may be separated into two parts, namely the input-crank A_0A and a remaining two-degree-of-freedom mechanism, which is to be named the *kite-cell*. (See Fig. 9)

In doing so, we may regard the curve traced by point A as the input curve and the curve traced by E (or E^x) as the output curve. The particulars of the kite-cell are easily extracted from the symmetry conditions of the Watt-1 mechanism, in which the cell forms a part. The dimensions of the kite-cell, are therefore subjected to the following conditions

1. $AB = B_0B$
2. $BC = BK$
3. $CD = DK$
4. $EK/DK = (B_0B^2 - BC^2)/(B_0C * BC) = (E^xD^2 - DC^2)/(E^xC * DC)$
5. $DK/DE = BC/BB_0 = DC/DE^x$.

The tracing point E^x, apparently, is attached to the bar CD of the cell, such that $\Delta E^xDC \sim \Delta B_0BC$. In case the cell contains point E^x as well as point E, the cell is named a *complete* one.

From section 5 we deduce, that the kite-cell defined above transforms *any* circle traced by A into a symmetrical Watt-1 curve, traced by E. As further $\sphericalangle AB_0E = \pi - \frac{1}{2}\sphericalangle B^vBA = \text{constant}$, we find that E traces a symmetrical curve if A follows any curve that is symmetrical about a

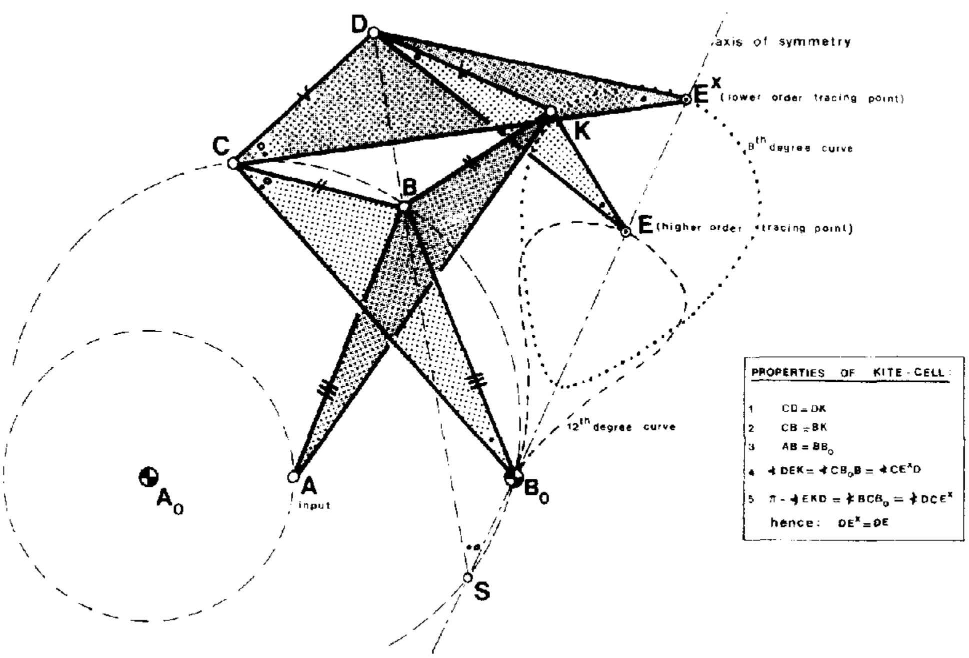

Figure 9. Complete kite-cell transforming a circle into symmetrical Watt-1 curves of degree 8 and 12.

line through B_0. If, for instance, point A traces a 4-bar coupler curve that is symmetrical about an axis through B_0, then point E will equally trace a symmetrical curve. This may be realized by forcing another point A of plane BK, that lies on the same distance from B, to move along a circle.

From Ref. [4] we derive, that if A traces a circle or any other curve that is symmetrical about an axis through the fixed center B_0, then also point E^x will trace a symmetrical curve. We further derive, (See again Ref. [4]), that the tracing points E and E^x of the cell, always remain aligned with point B^v and with the fixed center B_0. Therefore, the curves traced by the points E and E^x have a common axis of symmetry.
As further

$$DC/DE^x = BC/BB_0 = DK/DE = DC/DE,$$

it follows that

$$DE^x = DE.$$

We finally note that the circumcircles of the triangles B_0BC, DKE and CDE^x of the cell all have a common point S at any point of time.

8. Symmetrical Arrangement of a Source Mechanism in Conjunction with its Double Roberts' Coupler Cognate

Another way to symmetrize the Assembly Configuration is to make the first cognate (a) and the second cognate (b) reflected images of each other with respect to the axis of symmetry. This means, that the double Roberts' coupler cognate must be obtained from the source mechanism through reflection. As before, such a reflection has to comply with the normal rules for cognation; see Ref. [7] and under. Consequently the common joints B_0, K and E, the latter being the actual tracing point, all have to lie on the axis of reflection when the two cognates are in the symmetrical position. The remaining joints, such as A_0', A', B', C' and D' then are the reflected images of the initial joints A_0, A, B, C and D. (See Fig. 10.)

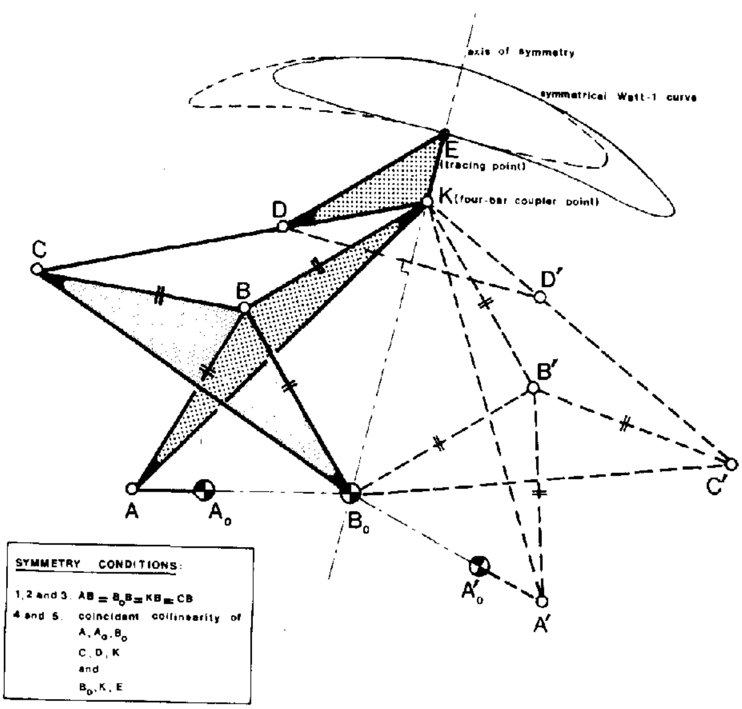

Figure 10. Source mechanism and its double Roberts coupler cognate, generating symmetrical positions for their common coupler plane *DKD'*.

Thus, all the cognate dimensions are the same as for the source mechanism. The *cognate conditions*, extracted from p. 185 of Ref.[7] are

$$\begin{cases} 1.\ \triangle BB_0C \sim \triangle B'C'B_0 \sim \triangle KD'D \\ 2.\ \triangle ABK \sim \triangle KB'A' \sim \triangle A_0B_0A_0' \\ 3.\ \triangle KDC \sim \triangle KD'C' \text{ (2 similar dyads)} \\ 4.\ \triangle B_0A_0A \sim \triangle B_0A_0'A' \text{ (2 similar dyads)} \\ 5.\ \square B_0BKB' \text{ is a parallelogram linkage.} \end{cases}$$

The conditions 1 and 5, in conjunction with the fact that $KD' = KD$ as well as $KB' = KB$, then lead to the equalities:

$$BC = BB_0 = KB.$$

As further $A_0B_0 = A_0'B_0$, condition 2 additionality leads to the fact that $AB = KB$. So, in total, we have

$$AB = KB = B_0B = CB,$$

which we will call the *star*-conditions of the mechanism. From the cognate-condition 3, we next derive that

$$\mathbf{KC'/KC = KD'/KD},$$

so that

$$\sphericalangle CKC' = \sphericalangle DKD'.$$

As, also, K lies on the axis of symmetry, which is the axis of reflection between the bars CD and $C'D'$, these angles are only equal if $K = CD \times C'D'$. Therefore, K, C and D are collinear. We may similarly prove that the joints A_0, A and B_0 are aligned in the symmetry position of the mechanism.

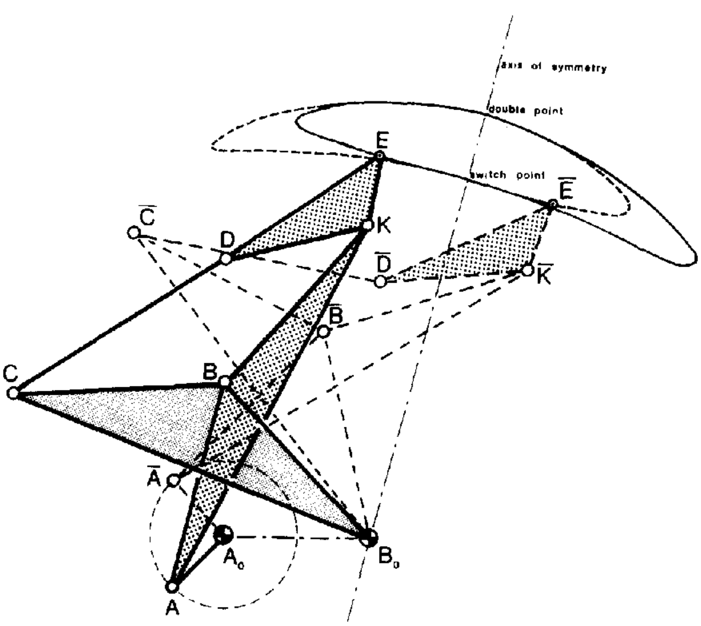

Figure 11. Two symmetrical positions of a 6-bar mechanism, producing two identically reflected branches of a 6-bar curve.

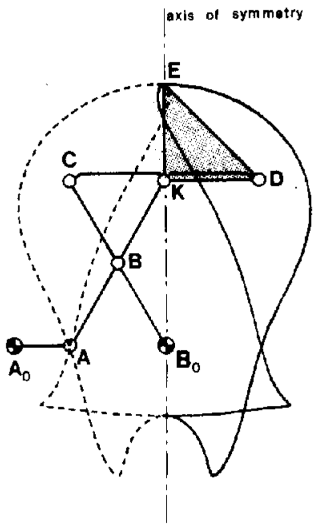

Figure 12. Two identical reflected branches of a Watt-1 curve.

So, in total, there are *five* conditions for symmetry of the mechanism. They are successively:

1. 2 and 3. $AB = B_0B = KB = CB$
4. Points C, D and K are collinear when A_0, A and B_0 are aligned.
5. In the symmetrical design-position for which A lies on A_0B_0, the tracing-point E must lie on the axis of symmetry.

The source mechanism, demonstrating these conditions, actually produces symmetrical positions of the (common) coupler-triangle DKD'. In particular, the altitude KE of this triangle obtains symmetrical positions.

In fact, any point of this altitude, produced or not, traces a symmetrical Watt-1 curve of highest order. Point K is an exception, for it traces merely a symmetrical 4-bar coupler curve.

We deduce from the *Assembly Configuration* that symmetrical positions of $\triangle DKD'$ are obtained from each other by successive reflection in the altitude KE and in the axis of symmetry. (See also Fig. 11.) They may also be obtained by a singular rotation about a point that is common to the altitude as well as to the axis of symmetry. Another way to arrive at the 'image position' is first to rotate $\triangle DKE$ about K until dyad CDK has reached its image position with respect to the diagonal CK. We then rotate the result about the fixed center B_0 until the 'image position' of the dyad C-DKE has been reached.

Two corresponding 'image positions' of this dyad blend only in the *stretched position* of the dyad, which is in fact the design position of the mechanism. Therefore, in its design position, point E lies at a blending point—or *switch point*—for two branches of the curve, each being the other's reflection with respect to the axis of symmetry. Thus, the two branches intersect at two points on the axis of symmetry of the curve, one of them being an ordinary double point, the other a switch point of the curve with coincident tangents. Figure 12 finally, demonstrates an example in which the basic 4-bar is a non-Grashof one. Also the two triangles ABK and B_0BC are stretched. Here too, the two branches are each other's image with respect to the axis of symmetry of the (complete) curve.

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Symmetrische Kurven erzeugende Koppelpunkte höchster Ordnung eines Watt-1-Gelenkgetriebes

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KURZFASSUNG Nicht-symmetrische Watt-1-Gelenkgetriebe erzeugen unter Umständen auch symmetrische Kurven. Dafür genügen offenbar fünf notwendige Bedingungen zwischen den Abmessungen dieser Getriebe. Diese Bedingungen wurden von den zwei Möglichkeiten zur Symmetrisierung der "Sammel-Konfiguration", die die vier sechsgliedrigen Verwandten enthält, abgeleitet. Dabei führt jede Möglichkeit zu einem anderen Lösungsgetriebe. Das Lösungsgetriebe, das ohne weiteres angetrieben werden kann, enthält ein gleichschenkliges Gelenkviereck mit zu bestimmten Dreiecken erweiterten Gliedern. Das erweiterte gleichschenklige Gelenkviereck ist im wesentlichen eine allgemeine "Drachenzelle", die einen Antriebskreis in eine symmetrische Watt-1-Kurve achten oder zwölften Grades umwandeln kann, je nach dem gewählten Koppeldreieck. Die zweite Möglichkeit führt zu einem Getriebe, wobei nicht nur ein Punkt, sondern auch eine Strecke spiegelsymmetrische Lagen einnehmen kann. Dann aber durchlaufen solche Getriebe eine Verzweigungslage, wobei die Koppelbewegung zweideutig ist. Zur Erzeugung der vollständigen Koppelbewegung sind demzufolge dann Hilfsverzahnungen oder andere Massnahmen erforderlich.